Statistical QCD

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IOP: 1994-1999.

The Institute of Mathematical Sciences

Hadronic world

Hadrons: Mesons (pions, kaons,....etc) Baryons (Proton, nutrons, hyperons,.....etc) in thousands

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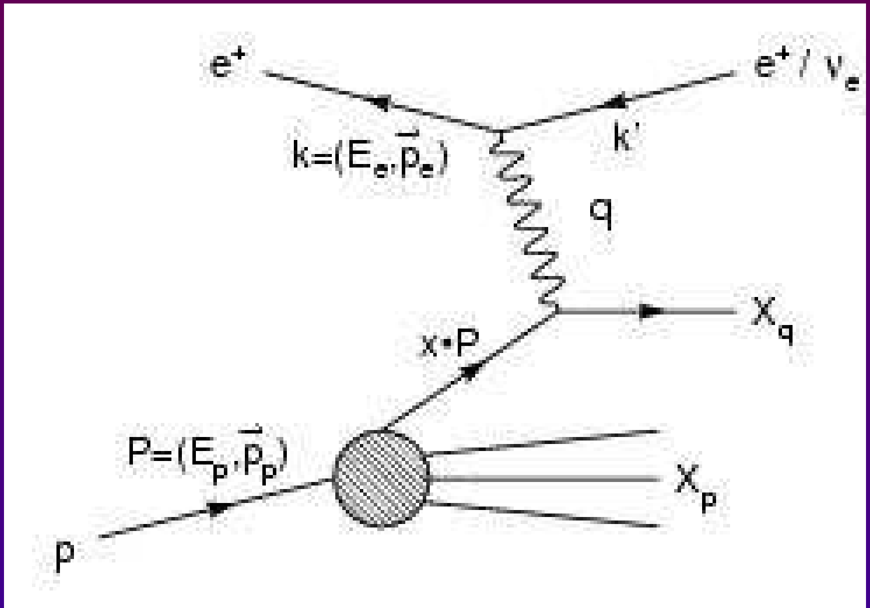
If hadrons are fundamental/elementary? What is the theory of hadron world?

Structure of Hadrons

High energy electron proton scattering showed hadrons are composites of quarks and gluons



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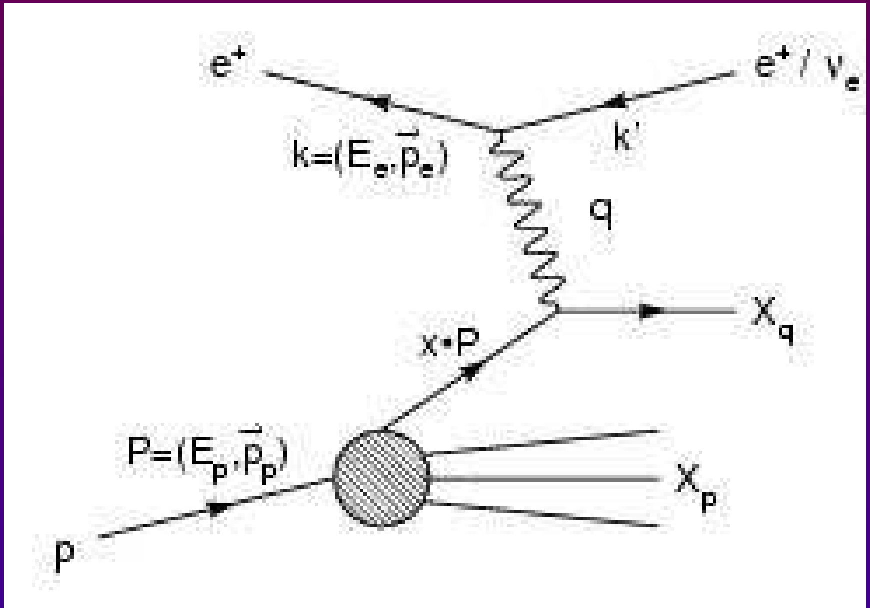

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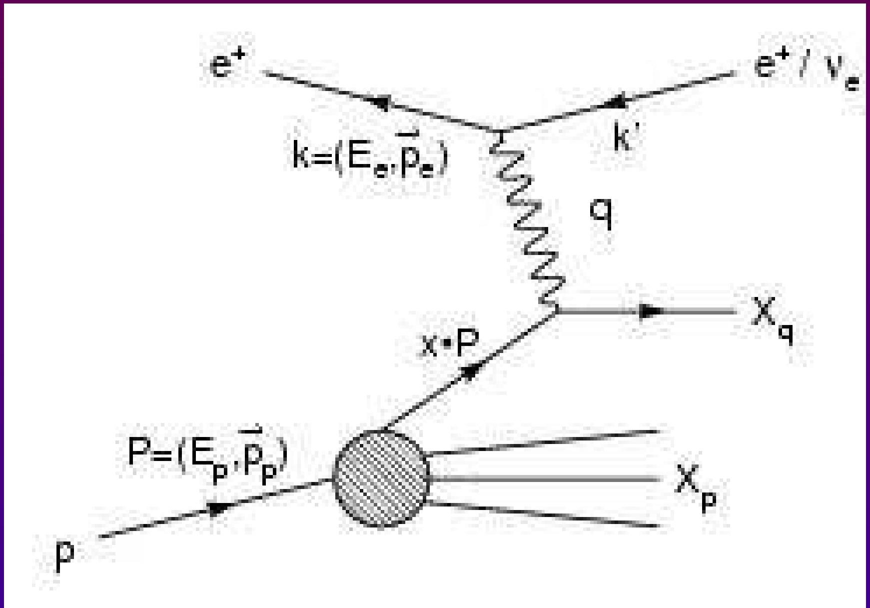

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Theory of quarks and gluons must answer questions of hadronic world

Theory of Quarks and Gluons (QCD)

Quantum electrodynamics (QED)	Quantum chromodynamics(QCD)

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Exchange boson − photon	Exchange boson − gluon
Photon − charge neutral	Gluons − colored (8)

QCD

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{f=u,d,s} \bar{\psi}^f_\alpha \left(i\gamma^\mu D_\mu - m_f\right)^{\alpha\beta} \psi^f_\beta \\
F^a_{\mu\nu} &= \partial_\mu G^c_\nu - \partial_\nu G^c + g f^{cba} G^b_\mu G^a_\nu \\
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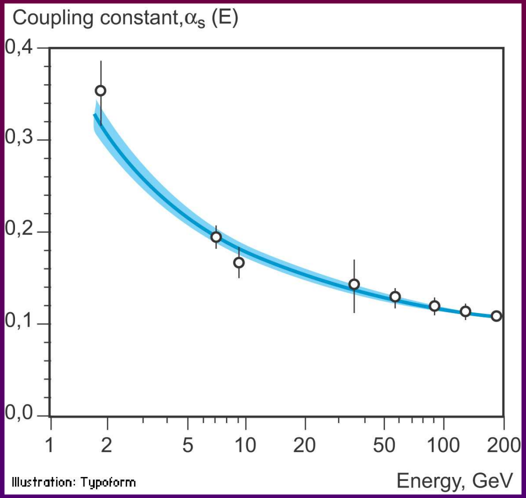
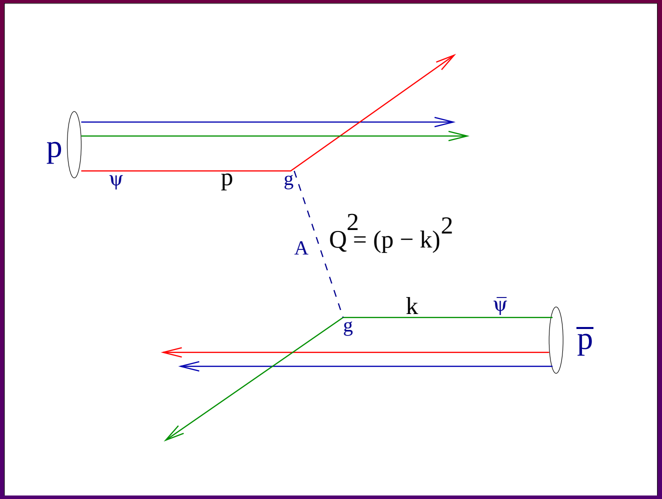
Analytical proof of color confinement is still a open problem.

Asymptotic Freedom





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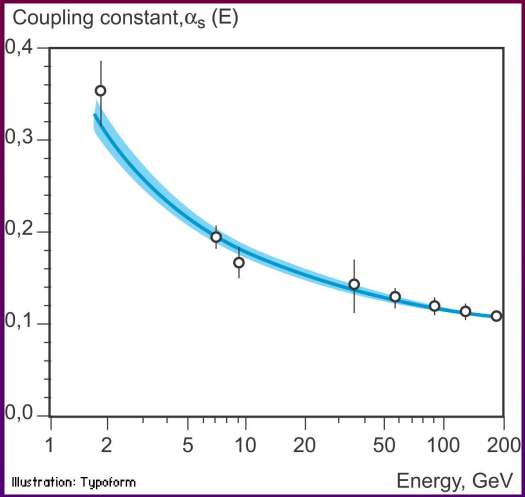

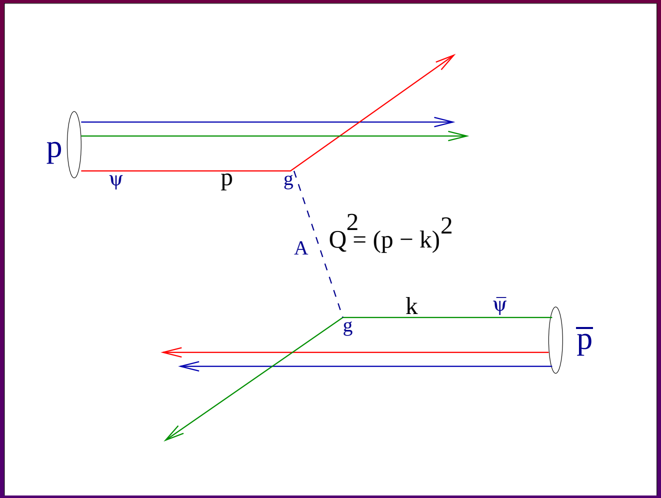

$$g \sim \frac{1}{log(Q^2/\lambda^2)}$$

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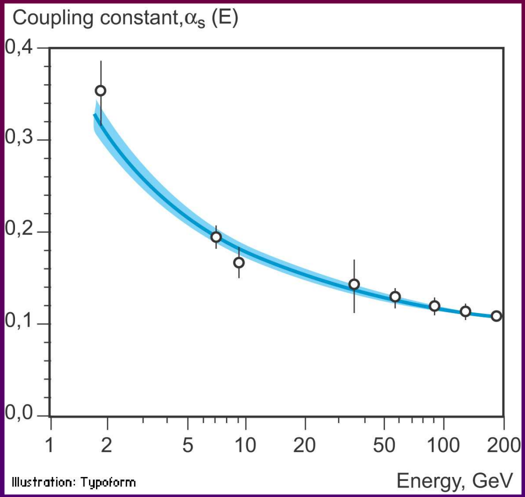

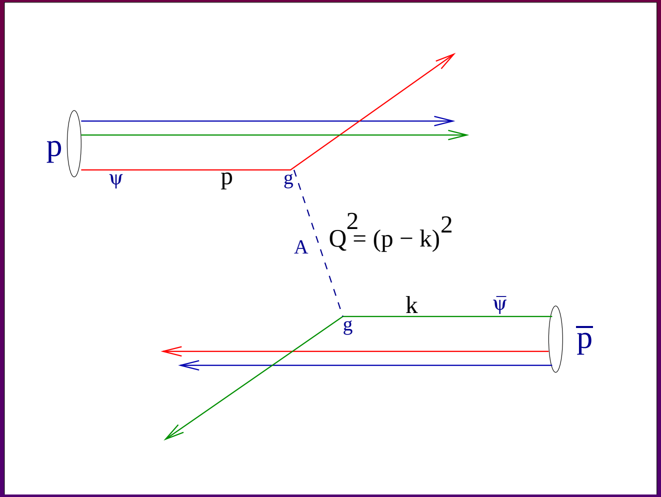

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Gross, Pulitzer, Wilczek....2004 Nobel Prize

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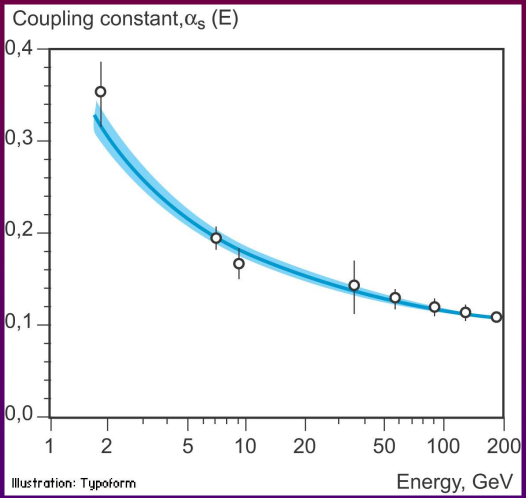

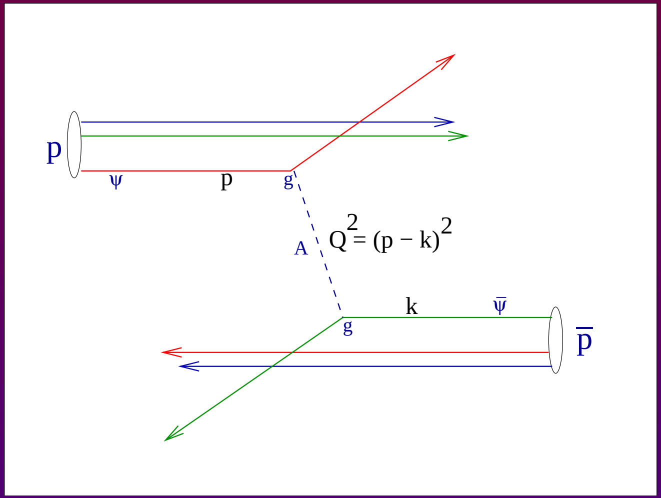

$$g \sim \frac{1}{log(Q^2/\lambda^2)}$$

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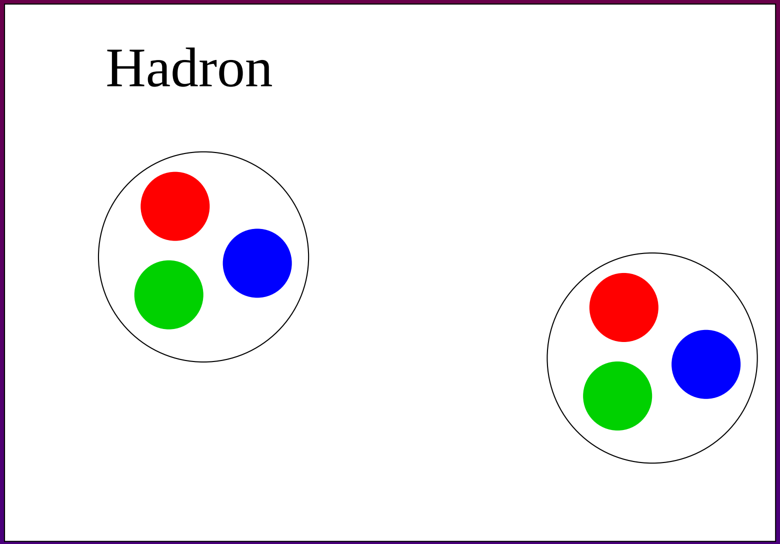
g is small for $Q^2 >> \lambda^2$, analytic (perturbative)

Few higher order corrections

Confinement



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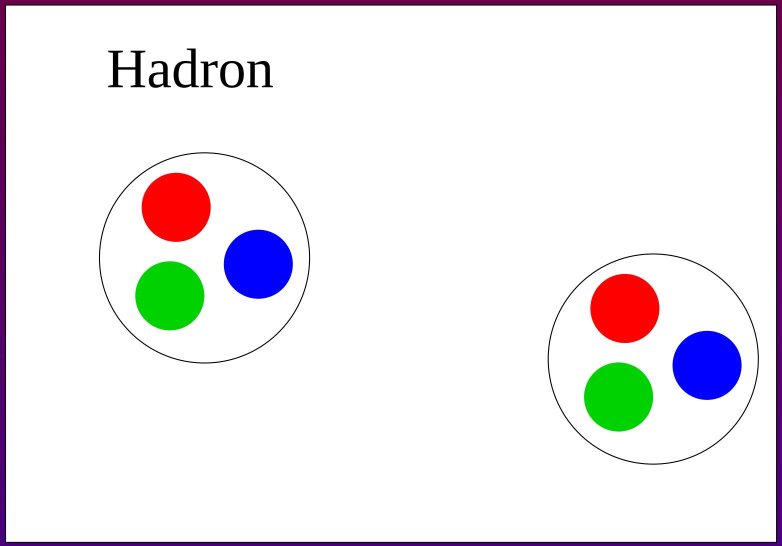
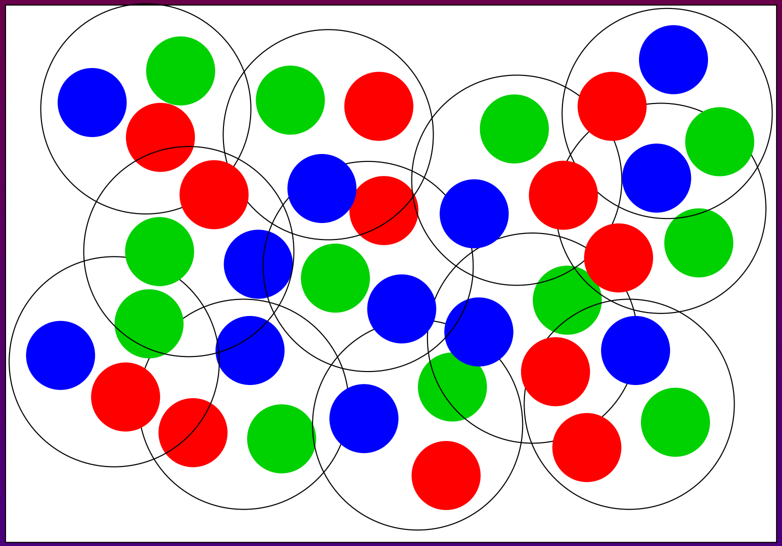


Hadrons are of size $\sim\ 1.0$ fm, so need $\sim\ 1.0$ fm^3 space to exist

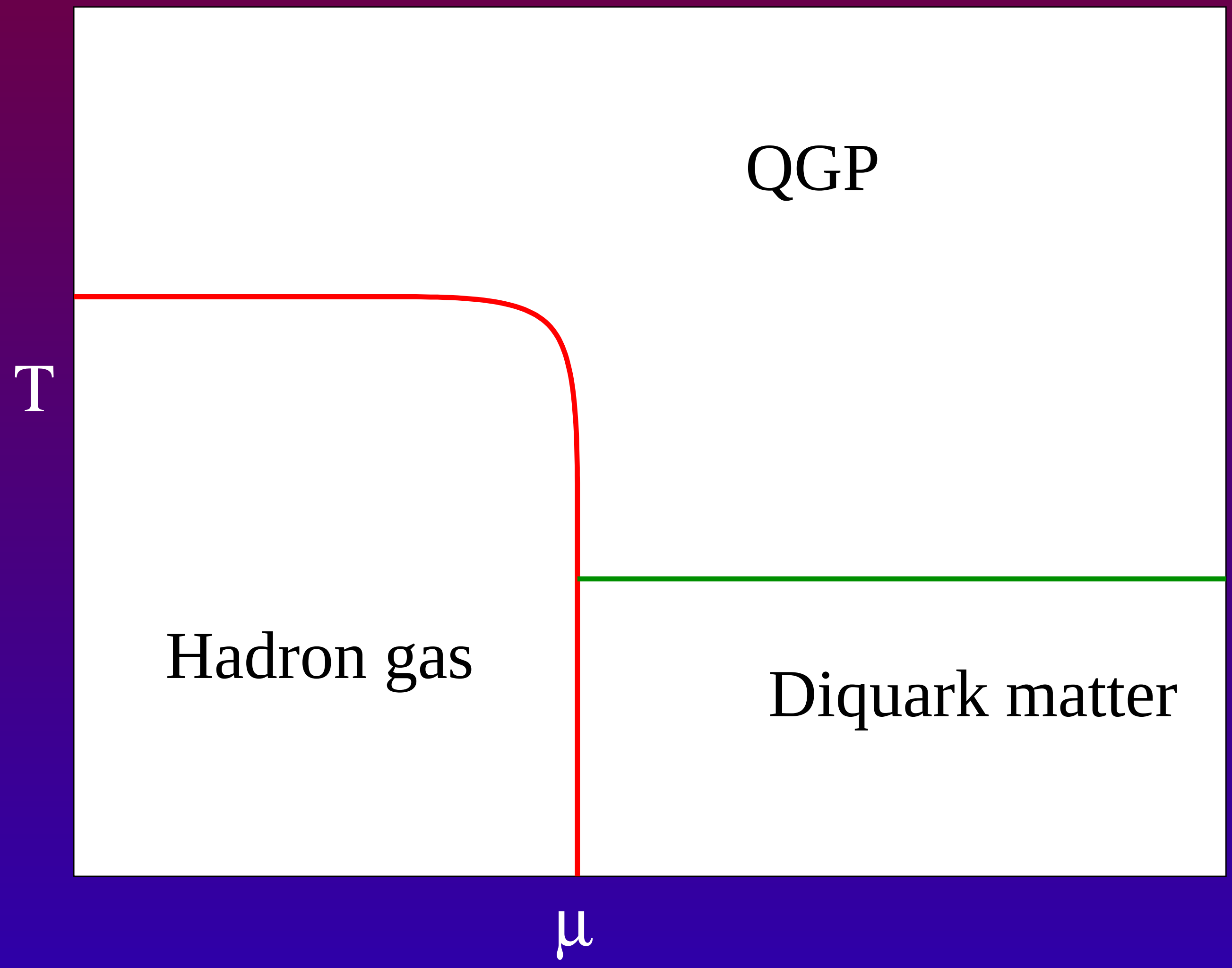
What happens when many hadrons are put in a small volume?

Deconfinement



Now quarks can propagate distances $>> 1$ fm
Hadrons have melted away........and the system is “Quark gluon plasma”

Hadronic matter at finite T and μ



QCD thermodynamics at finite T, μ

$$\begin{aligned}
\mathcal{Z} &= Tr[e^{-H/T}] \\
\textbf{free energy} &= -\frac{T}{V} ln\mathcal{Z} \\
\textbf{energy density} &= \frac{T^2}{V}\frac{\partial ln\mathcal{Z}}{\partial T} \\
\textbf{pressure} &= \frac{T}{V}\frac{\partial ln\mathcal{Z}}{\partial V}
\end{aligned}$$

QCD at finite T

$$
\begin{aligned}
\mathcal{Z} &\propto Tr[e^{-H/T}] \longrightarrow \mathcal{Z}_{QCD} \propto \int D[G,\bar{\psi},\psi]e^{-S} \\
S &= \int_0^{\frac{1}{T}} dt \int d^3x \left[\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_f \bar{\psi}^f_\alpha \left(\gamma^\mu_E D_\mu + m_f\right)^{\alpha\beta} \psi^f_\beta\right] \\
\mathcal{Z} &\propto \int DG det(M) e^{-S_{gluon}} = \int DG e^{-S_{QCD}}
\end{aligned}
$$

$S_{QCD} = S_{gluon} - log(det M)$. Changing temperature amounts to changing the extent of imaginary time direction.

QCD at finite T

$$\langle O(G) \rangle = \frac{\int DG\, O(G)\, e^{-S_{QCD}}}{\int DG e^{-S_{QCD}}}$$

Non-perturbative method, lattice QCD is the only alternative

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Non-perturbative method, lattice QCD is the only alternative
Suppose the system makes a random walk in $A(x)-$space........

$$G_1(x) \rightarrow G_2(x) \rightarrow G_3(x) \rightarrow G_4(x) \ldots\ldots\ldots.$$

$$\langle O(G)_{ap} \rangle = \frac{\sum_{i=1}^{N} O(G_i)}{N}$$

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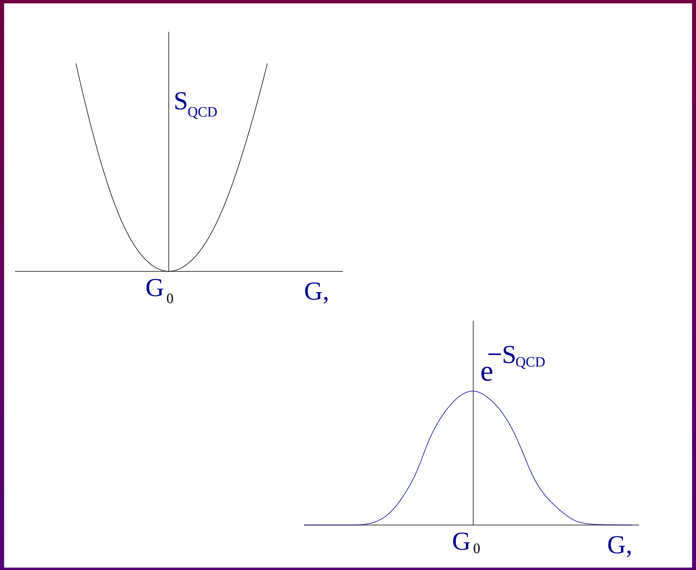
$\langle O(A)_{ap}\rangle \neq \langle O(A)\rangle$

Lattice QCD



If in the random walk the system spends more “time” near G_0..... if histogram $H(G)$.....follows $e^{-S_{QCD}}$....then

$$\langle O(G)\rangle == \frac{\int DG\ O(G)\ e^{-S_{QCD}}}{\int DG e^{-S_{QCD}}} = \frac{\sum_{i=1}^{N} O(G_i)}{N}$$

Lattice QCD

How to generate N number of G's with probability $e^{-S_{QCD}}$?

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$$G \to G', \qquad \triangle S = S'_{QCD} - S_{QCD}$$

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$$G \rightarrow G', \qquad \triangle S = S'_{QCD} - S_{QCD}$$

Accept G' with probability $exp(-\triangle S)$.......
$G \rightarrow G' \rightarrow G'' \rightarrow G'''......$

$$G(x,y,z,t) \rightarrow G'(x,y,z,t)$$
$$G(x_i,y_i,z_i,t_i) \rightarrow G'(x_i,y_i,z_i,t_i)$$

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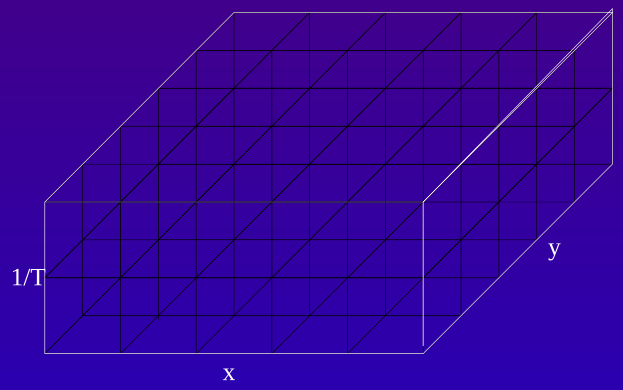
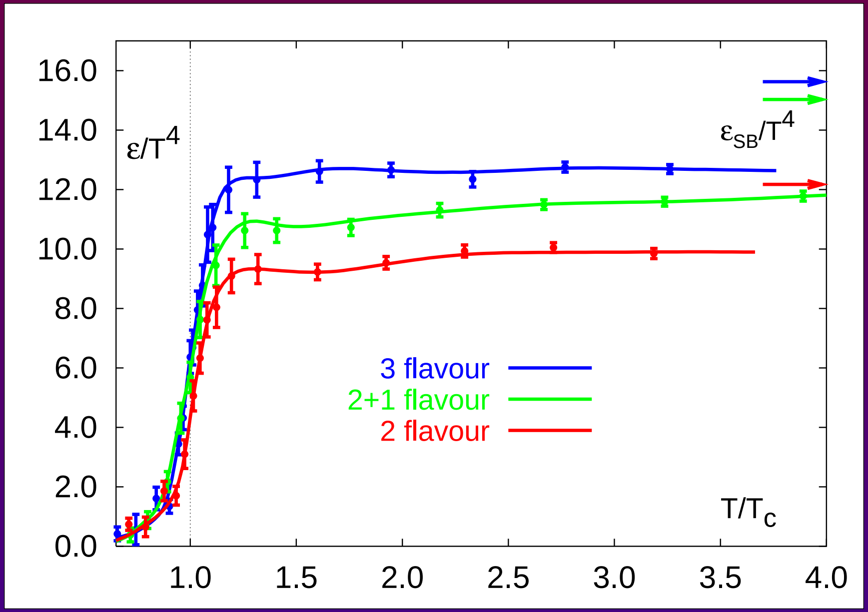


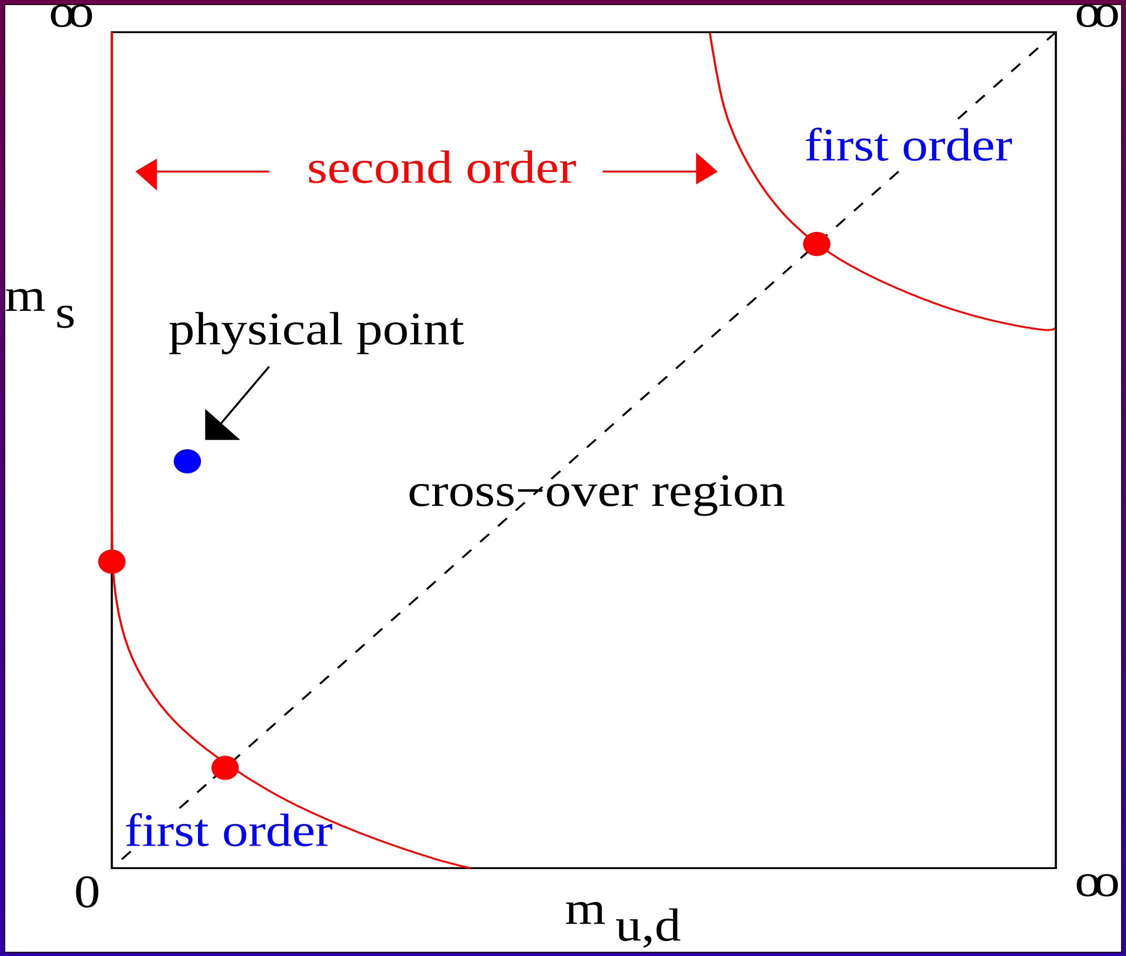
QCD at finite T



A sharp increase in the number of degrees of freedom
$=>$ Melting of hadrons $=>$ Color deconfinement

This is the only theoretical evidence of deconfinement, success of non-perturbative methods

QCD Phase Diagram ($\mu = 0$)



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