Parton Shower Basics

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Note: These slides don't contain a number of schematic diagrams and analytical examples which were worked out on chalkboard.

Master Formula

$$\sigma_X = \sum_{a,b} \int dx_1 dx_2 \; f_a(x_1, \mu_F^2) \; f_b(x_2, \mu_F^2) \; \hat{\sigma}_{ab\to X} \left(x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2} \right)$$

- PDF: extracted from experiment, using evolution from theory
- $\hat{\sigma}_{ab\to X}$: short distance partonic cross section, perturbative behaviour
- Expansion over α_s: gives LO, NLO, NNLO and so on

Cross-section Calculation

$$
\begin{aligned}
[d\sigma]_{2\to n} &= \frac{|\mathcal{M}|^2}{4\sqrt{(p_1.p_2)^2 - m_1^2 m_2^2}} d\Phi_n \\
&= \frac{|\mathcal{M}|^2}{2\sqrt{\lambda(E_{cm}^2, m_1^2, m_2^2)}} d\Phi_n
\end{aligned}
$$

Integrated it gives collision rate:

$$N = \sigma \int \mathcal{L}(t)\ dt$$

Källén Function:

$$\lambda(a^2, b^2, c^2) = (a + b + c)(a + b - c)(a - b + c)(a - b - c)$$

n-body Phase Space

$$d\Phi_n = \left[\prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}\right] (2\pi)^4 \; \delta^{(4)} \left(p_0 - \sum_{i=1}^{n} p_i\right)$$

- a general and efficient way of phase space parametrisation is required due to large number of dimensions
- analytical methods become too complicated when different CUTS are applied on the final states
- numerical evaluation of the integrations are necessary
- however, δ-functions cannot be integrated numerically
- δ-function integrations are to be done analytically by choosing a set $\{p_i\}$, such that δ-function relation is already satisfied
- no other alternative than to calculate at least $d\Phi_2$ and use it recursively to calculate $d\Phi_n$

2-body Phase Space

$$d\Phi_2 = \frac{1}{16\pi^2}\frac{|\vec{p}|d\Omega}{E_1 E_2}$$

- with this relation in hand, we'll factorise 3-body phase space
- we'll use that relation recursively to factorise n-body phase space

Factorisation of 3-body Phase Space

$$d\Phi_3(P; p_1, p_2, p_3) = dm_{23}^2 \; [d\Phi_2(P; p_1, p_{23})] \; [d\Phi_2(p_{23}; p_2, p_3)]$$

Factorisation of n-body Phase Space

$$\begin{aligned} d\Phi_n(P; p_1, p_2, \ldots, p_n) = dm_{23\ldots n}^2 \; & [d\Phi_2(P; p_1, p_{23\ldots n})] \\ & \times [d\Phi_{n-1}(P; p_2, p_3, \ldots, p_n)] \end{aligned}$$

- adaptation of numerical techniques is necessary

Monte Carlo Integration

$$I = \int_{x_1}^{x_2} f(x)dx$$

- Mean Value theorem: basis of Monte Carlo integrations
- Draw N sample points uniformly

$$I_N = (x_2 - x_1)\,\frac{1}{N}\sum_{n=1}^{N} f(x_n)$$

$$V_N = \left\{(x_2 - x_1)^2\,\frac{1}{N}\sum_{n=1}^{N}[f(x_n)]^2\right\} - I_N^2$$

Central Limit Theorem

$$\boxed{I = I_N \pm \sqrt{V_N/N}}$$

- convergence is slow: $1/\sqrt{N}$
- error estimation is easy
- errors do not depend on the number of dimensions
- improvement in the result can be controlled by minimizing V_N
- optimal case: $f(x) = \text{constant} \quad \Longrightarrow \quad V_N = 0$

Importance Sampling

$$I = \int_{x_1}^{x_2} \frac{f(x)}{p(x)}\; p(x)\; dx$$

- method of minimizing V_N
- it corresponds to change of variables
- choose $p(x)$ in such a way that $\frac{f(x)}{p(x)} \sim$ constant
- error is now determined by $Var(f/p)$
- $p(x)$ is restricted to become a $+$ve valued function and can be normalised to unity
- $p(x)$ might be interpreted as probability density function $\Longrightarrow$ NON-uniform distribution of sample points

Drawback

Need to know a lot about $f(x)$ before starting the integration !

Adaptive Importance Sampling

An algorithm which learns about the integrand as it proceeds.

- If $p(x) = \frac{|f(x)|}{\int |f(x)| \, dx}$, the $Var(f/p)$ vanishes.

Example: VEGAS

- Learns about the integrand during the integration
- Uses numerical step functions which comes closer and closer to the true integrand
- Bins are of equal area
- Starts by sub-dividing the integration space into rectangular grid
- Performs integration in each sub-spaces
- These results are then used to adjust the grid for next iteration

Multi Channel Integration

- MC leads to poor results when $f(x)$ has sharp peaks
- Remapping of variables can make the integrand flat
- Variable transformation is difficult when $f(x)$ contains different peaks in different regions

Solution: use different transformation for different peaks

$$p(x) = \sum_{i=1}^{n} \alpha_i \; p_i(x) \quad \text{with} \quad \sum_{i=1}^{n} \alpha_i = 1$$

Drawback

- all $p_i(x)$ functions are to be calculated to determine $p(x)$
- time consuming
- relative weight (α_i) of each channel changes to minimize variance

Solution: Write the integrand in terms of a basis of n functions f_i such that,

$$f = \sum_{i=1}^{n} f_i$$

$$\Longrightarrow I = \sum_{i=1}^{n} I_i$$

Towards Event Generation

Example: $u\bar{u} \to gg$

- Three very different pole structures contributing to the same matrix element
- Basis: $f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{total}|^2$
- Choice of such basis divides integrations into pieces, based on diagrams
- No need to calculate weight functions from other channels
- Errors add in quadrature $\Longrightarrow$ no extra CPU cost
- Parallel in nature
- Interference terms never create new peaks

Event Selection

1. pick x at random
2. calculate $f(x)$
3. pick y at random, where $0 < y < f_{max}$
4. If $f(x) > y \quad \Longrightarrow$ Accept
5. Otherwise Reject

Weighted Events: Same number of events in areas of phase space with very different probabilities

Unweighted Events: No. of events $\propto$ probability of phase space area

Parton Shower

- Particles are by definition HARD, while calculating ME
- Accelerated particles radiate
- PS evolve the hard process down to the hadronisation scale
- They generate high multiplicity final states, which can readily be converted into hadrons
- In practice, $PP \to X \implies PP \to X$+n jets
- Logarithmically dominant contributions are universal

Collinear Factorisation

$$d\sigma_{n+1} = d\sigma_n \, \frac{dt}{t} \, dz \, \frac{\alpha_s}{2\pi} \, \hat{P}_{ba}(z)$$

- This relation appears after integration over azimuthal angle ϕ
- t: evolution parameter
- $\hat{P}$: unregulated splitting functions
- z: energy fraction E_b/E_a

Iteration of Parton Branching

$$d\sigma_{n+2} = d\sigma_n \, \frac{dt}{t} \, dz \, \frac{dt'}{t'} \, dz' \, \left(\frac{\alpha_s}{2\pi}\right)^2 \, \hat{P}_{ba}(z)\hat{P}_{db}(z')$$

- $a(t) \to b(z) + c$
- $b(t') \to d(z') + e$
- Markov chain process: probability of the next branching depends only on the present values of random variables
- Branching tree: $Q^2 \gg t_1 \gg t_2 \gg \ldots\ldots \gg Q_0^2$

Sudakov Factor

$$\Delta(Q_1, Q_2) = \exp\left[-\frac{\alpha_s}{2\pi}\int_{Q_2^2}^{Q_1^2}\frac{dt}{t}\sum_b\int_{z_{min}}^{z_{max}} dz\ \hat{P}_{ba}(z)\right]$$

- Probability of not finding a parton b from a, when evolution parameter varies from Q_1 to Q_2
- Basis of PS Monte Carlo

Parton Shower Monte Carlo

1. Start the evolution at the (virtual) mass scale t_0 and momentum fraction $x_0 = 1$
2. Given a virtual mass scale (t_1) and momentum fraction (x_1), generate the scale (t_2) of the next emission by solving: $\Delta(t_1, t_2) = R$
3. If $t_2 < t_{cut}$, shower has been finished
4. Otherwise, generate $z = x_2/x_1$ with a distribution proportional to $(\frac{\alpha_s}{2\pi})\hat{P}(z)$
5. For each emitted particle, iterate steps $2 - 4$ until branching stops

Angular Ordering

- Different MCs' uses different evolution parameters
- $p_a^2 = z(1-z)\ \theta^2\ E_a^2$
- $p_{T,a}^2 = z^2(1-z)^2\ \theta^2\ E_a^2$
- $\tilde{t}_a = \theta^2\ E_a^2$

- All of them have same angular behaviour
- Studing SOFT emission may give extra information on the proper choice of evolution parameter

Effects of Angular Ordering

- Radiation happens only for angles smaller that the colour connected opening angle
- $|M|^2$ gets factorised as if the is no interference
- Angles will become smaller and smaller while this construction is iterated
- Once the gluon is far enough from the two quark legs, it will not resolve their individual colour charges, but only feel the combined charges
- This screening leads to an additional suppression factor

Angular ordering is automatically satisfied in P_T and θ ordered showers

Limitations of Parton Shower

- It is based on soft/collinear approximation
- It cannot describe the hard radiation correctly
- Neither of the available codes give warning while they are used outside their range of validity

Solution:

- Use ME to describe the hard radiation together with PS
- ME+PS: calculate higher multiplicity ME to describe the hard part and merge them to PS
  (CKKW, MLM)
- NLO+PS: start from NLO corrected results for describing the hard part and match them with PS
  (MC@NLO, POWHEG)

ME+PS: Limitations of naive approach

- Partons far away can re-enter into the cone due the more radiation
- Relative weights of MEs' with different multiplicities are unspecified
- No secific way to determine the size of the cone
- Final event sample should be independent of cone size

K_T Algorithm

1. Define parton-beam distance: $d_i = p^2_{T,i}$
2. Define parton-parton distance: $d_{ij} = min(p^2_{T,i}, p^2_{T,j})\ R^2_{ij}$
3. Define a stopping scale d_{stop} below which clustering is not required
4. If $d_{ij} < d_{stop} \implies$ two partons are close, combine them
5. If $d_i < d_{stop} \implies$ partons are close to the beam, reject them
6. Iterate the whole process until partons are left far apart

CKKW Algorithm

1. Compute the probabilities: $P_i^{(0)} = \dfrac{\sigma_i^{(0)}}{\sum\limits_{i=1}^{n} \sigma_i^{(0)}}$
2. Choose a multiplicity $0 \leq i \leq n$ with probability $P_i^{(0)}$
3. Use the matrix element M_i to generate $X + i$-jet kinematic configuration for unweighted events
4. Use K_T algorithm to cluster the partons to reach to $X + i$-jet configuration
5. Apply coupling re-weighting factor
6. Apply Sudakov re-weighting factor
7. Unweight again the hard configuration: accept it if the product of coupling & Sudakov reweighting factors is larger than a random number, otherwise start from 2.

NLO+PS: Necessicity

- K-factors: The only way to include k-factor consistently and use the information in detector simulation
- Shapes: Observable shape has NLO correction and that has an impact on acceptance studies in general
- Theoretical Systematics: Scale dependency can be computed in a meaningful way
- Predictive Power: These MC tools can be used as a tool for "precision" physics

MC@NLO Formalism

- Calculate FO NLO first, removing all divergences
- Invoke PS after that
- Incorporate improved substraction scheme so that the $\mathcal{O}(\alpha_s)$ hard part remains unaffected while using PS

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Thank You !