Quantum Simulations: an overview

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Tel-Aviv University

IPQI, February 17-28, 2014

OUTLINE

1. QUANTUM SIMULATIONS.

    “DIGITIZED” SIMULATION

    “ANALOG” SIMULATIONS

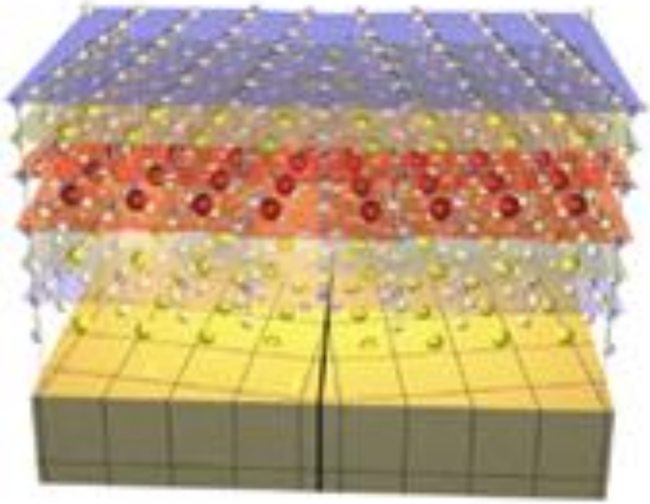
2. SIMULATED PHYSICS

3. THE SIMULATING SYSTEM:

    COLD TRAPPED ATOMS AND IONS

4. SIMULATIONS -AND NATURE’S “ANALOGUES”

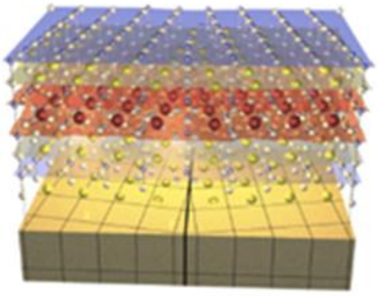
PHYSICAL SYSTEM



(Phenomenological) Hamiltonian

$$H = ...$$

Simulation

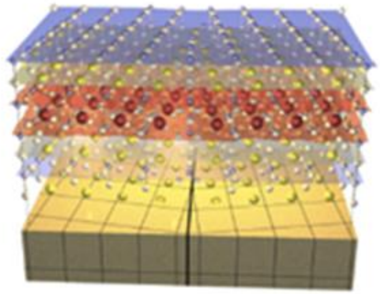


We want to compute :

- Ground state (correlation, entanglement propertis, many-body structure…)
- Excited states.
- Phase structure, phase transitions.
- Dynamics.
- Decoherence.

Simulations

$$|\psi> = \alpha_1 |000...0> + \alpha_2 |000...1> + ... + \alpha_{2^N} |111...1>$$



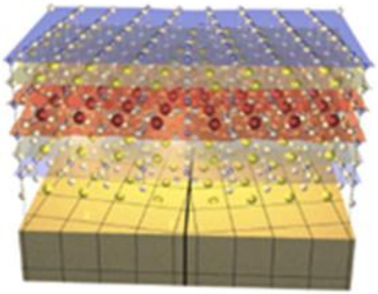
.

-*Exact simulations* on a classical computer are possible for 30-40 spins.

Each time we increase the size by one Qbit we need to double the memory and computational power.

Simulations

$$|\psi>=\alpha_1|000...0>+\alpha_2|000...1>+...+\alpha_{2^N}|111...1>$$



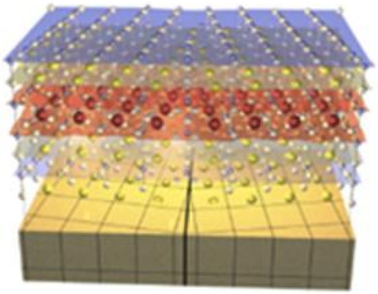
to circumvent the problem…:

-Various *approximations:* Mean field, variational, Hartree-Fock, etc.

-*Approximate simulation* methods: DMRG or MPS, Monte-Carlo, etc.

Simulations

$$|\psi>=\alpha_1|000...0>+\alpha_2|000...1>+...+\alpha_{2^N}|111...1>$$

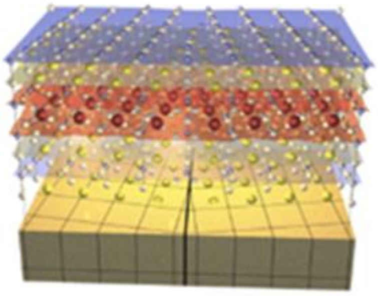


Insufficient when entanglement is too large:

- Close to phase transitions: all scales contribute.
- For Non-perturbative problem: all scales contribute

Simulations

$$|\psi\rangle = \alpha_1 |000...0\rangle + \alpha_2 |000...1\rangle + ... + \alpha_{2^N} |111...1\rangle$$



Lattice field theory: invented by K. Wilson in the 70ts.
to deal with non-perturbative effects (uses Monte Carlo methods)

But this fails for systems with too many Fermion.
(Known as the "sign problem") .
Same problem appears in Cond. Matter Frustrated systems.

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

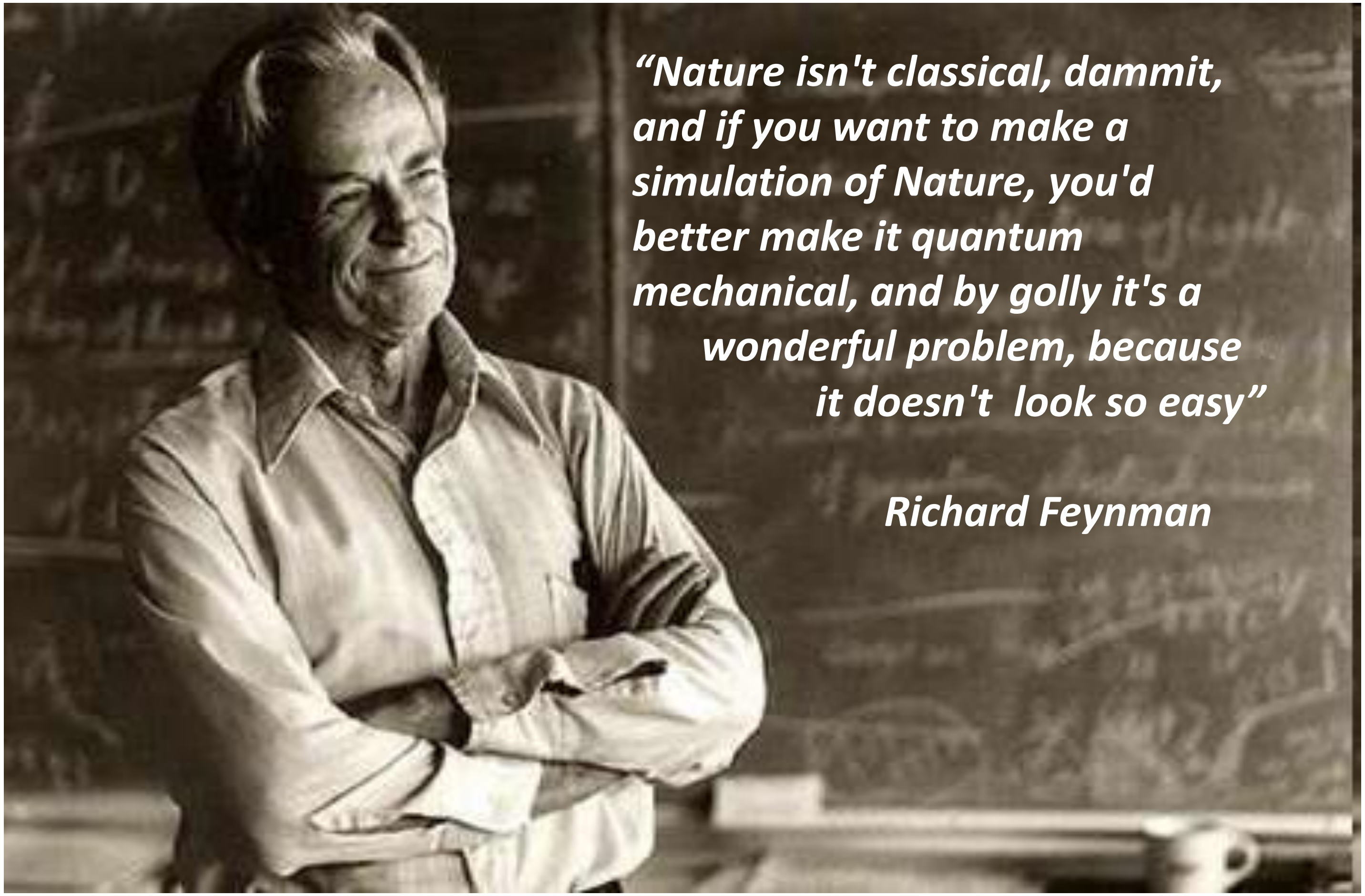
Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have

be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982



Quantum simulations !

”Simulating Physics with Computers”,
International Journal of Theoretical *Physics*, Vol 21, Nos. 6/7, *1982*

Quantum Simulators
Digital

Seth Lloyd (96): *Quantum computers are universal quantum simulators*.

Idea: use small time steps (Trotter's formula)

$$H = \sum_i h_i$$

$$U = e^{iHt} = (e^{ih_1 t/n} e^{ih_2 t/n} \ldots e^{ih_N t/n})^n$$

Where each time steps involves only few locally interacting qubits.
For example, in the spin chain.

$$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S_n^x S_m^x + J_y S_n^y S_m^y + J_z S_n^z S_m^z \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S_n^z$$

We need to perform long enough sequence of C-not interactions

Quantum Simulators

Digital

$$H = \sum_i h_i$$

$$U = e^{iHt} = (e^{ih_1 t/n} e^{ih_2 t/n} \ldots e^{ih_N t/n})^n$$

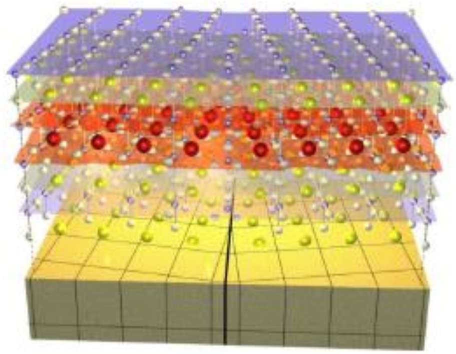
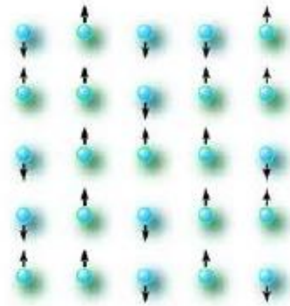
If the little h's involve only several body interactions as e.g for spin chains

$$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S_n^x S_m^x + J_y S_n^y S_m^y + J_z S_n^z S_m^z \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S_n^z$$

The number of gates scales only polynomials with the number of spins.

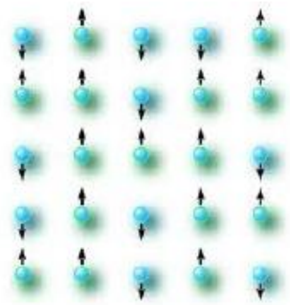
- ❑ Indeed quantum simulations could be the first main application of a quantum computer!!
- ❑ *Drawback*: at the moment we have "QC's" with around O(10) qubits.

QUANTUM SIMULATION
ANALOG

PHYSICAL SYSTEM	QUANTUM SIMULATOR
	
(Phenomenological) Hamiltonian	Physical Hamiltonian
$H = \dots$	$H = \dots$

Example: Hubbard model in 2D: $H = -t\sum_{k,\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} + V\sum_{k} n_{k\uparrow} n_{k\downarrow}$

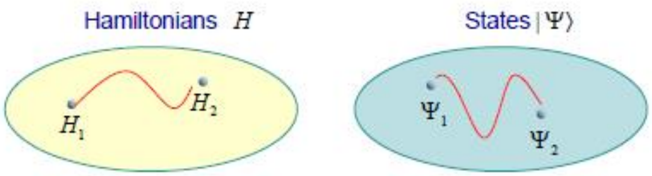
QUANTUM SIMULATION
ANALOG



$$H = \ldots$$

- **Questions:**
  - Dynamics: $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$
  - Ground state: $H|\Psi_0\rangle = E_0|\Psi_0\rangle$

Adiabatic algorithms



  - Physical properties: $\langle\sigma_n\rangle, \langle\sigma_n\sigma_m\rangle, \ldots$

Next we Focus mostly on Analog simulation examples.

Quantum simulations

Why shouldn't we just use the system itself?

- We would like to know if a toy model captures the expected physics.
  e.g. Does fermionic 2-D Hubbard model manifest's high-Tc superconductivity?

- Design new materials.

- Fundamental effects that are extremely difficult to test, are potentially observable in analog systems.

Example: Spins-chains, Fermions and Bosons

- Bosons/Fermions: $$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$$
- Spins: $$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$$

$\Rightarrow$ CONDENSED MATTER PHYSICS

Origin of High-TC SUPERCONDUCTIVITY ??

Example: Abelian U(1): QED

$$\alpha_{QED} \ll 1, \quad V_{QED}(r) \propto \frac{1}{r}$$

At low energies we don't need second quantization to understand the structure of atoms.

$$m_e c^2 \gg E_{Rydberg} \simeq \alpha_{QED}^2 \, m_e c^2$$

In HEP scattering, perturbation theory (Feynman diagrams) Works well. ✓

(g-2)/2= 1 159 652 180.73 *(0.28)* × 10^{-12}
anomalous electron magnetic moment
Harvard ion trapping Group

Non-abelian: QCD

$$\alpha_{QCD} > 1\,, V_{QCD}(r) \propto r$$

confinement !

=> <u>structure of Hadrons</u>: quark pairs form Mesons ,
triplets form Baryons.

Color Electric flux-tubes: "a non-abelian Meissner effect".

Lattice gauge theory

Main tool to extract non-perturbative physics!

... but:

($Z_{lattice}$ is obtained by Monte Carlo "sampling")

- ❑ Correlations but not time-dependence:

  e.g. no test of confinement with <u>dynamic</u> matter

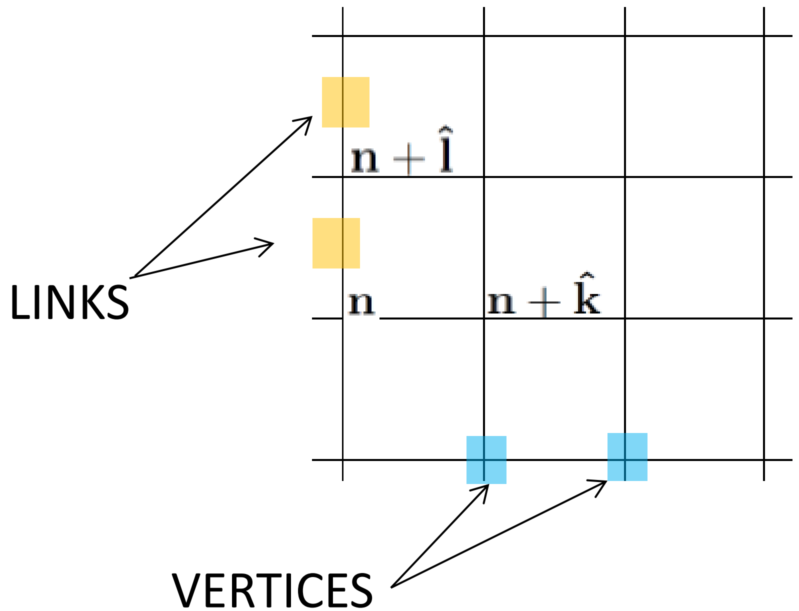
  pair creation (vacuum instability) in a strong field.

- ❑ Problems with too many (fermions) quarks

  Computationally hard: the "sign problem".

  (e.g. in exotic phases: color superconductivity, Quark-Gluon plasma.)

LATTICE GAUGE THEORIES
DEGREES OF FREEDOM



Gauge field degrees of freedom:
U(1), SU(N), etc, unitary matrices

Matter degrees of freedom :
Spinors

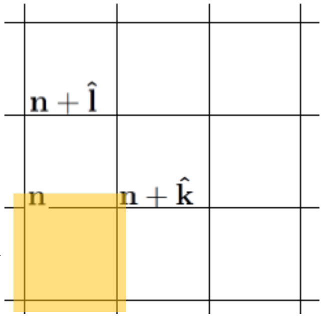
$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$

Example: Gauge fields and matter on a Lattice

Gauge field dynamics (Kogut-Susskind Hamiltonian):

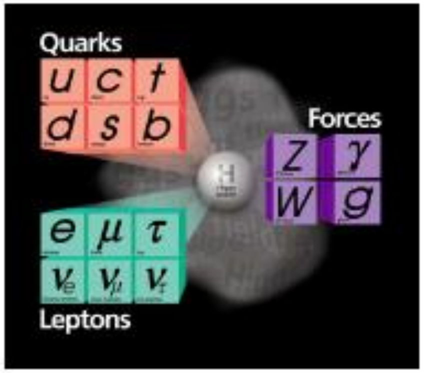
$$H_E = \frac{g^2}{2} \sum_{\mathbf{n},k,a} \left(E_{\mathbf{n},k}\right)_a \left(E_{\mathbf{n},k}\right)_a$$

$$H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger\right) + h.c.\right)$$



Strong coupling limit: $g >> 1$
Weak coupling limit: $g << 1$

HIGH ENERGY PHYSICS?



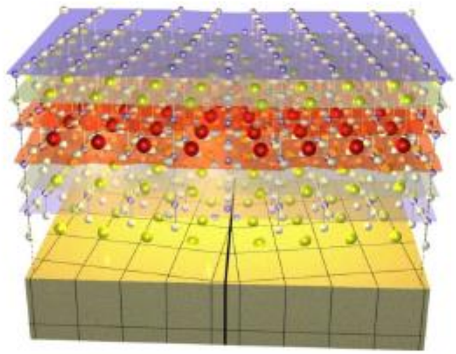
Structure of Matter?

Confinement of Quarks to mesons and baryons?

What Possible Simulating Systems?

Simulating systems?

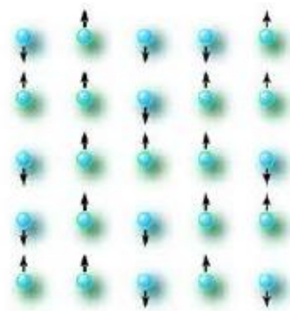
PHYSICAL SYSTEM



(Phenomenological) Hamiltonian

$$H = \ldots$$

QUANTUM SIMULATOR



Physical Hamiltonian

$$H = \ldots$$

Simulating systems

Should be a highly controllable,

There are many proposals and ideas :

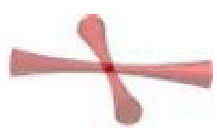
- Condensates BECs
- Atoms in optical lattices
- Rydberg Atoms
- Trapped Ions
- Superconducting Quantum Circuits
- NV centers
- …

COLD ATOMS

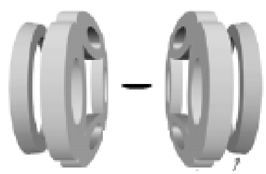
- Control: External fields

Trapping:

lasers

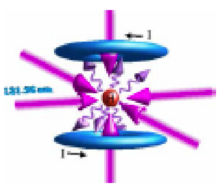


Magnetic fields



Cooling:

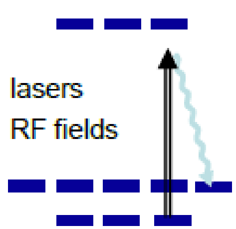
lasers



evaporation

Internal manipulation

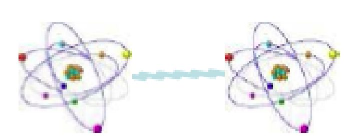
lasers
RF fields



purification
coherence
detection

Interactions

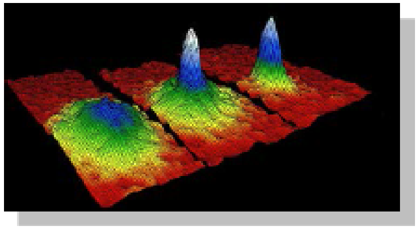
tune scattering length

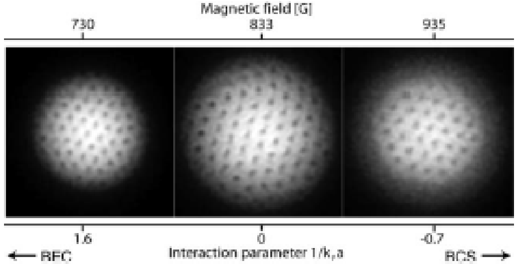


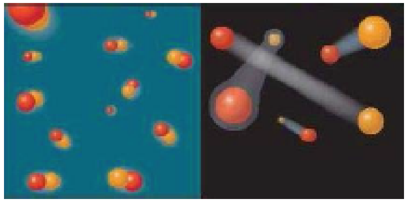
COLD ATOMS

- Many-body phenomena
  - Degeneracy: bosons and fermions (BE/FD statistics)
  - Coherence: interference, atom lasers, four-wave mixing, …
  - Superfluidity: vortices
  - Disorder: Anderson localization
  - Fermions: BCS-BEC

\+ many other phenomena







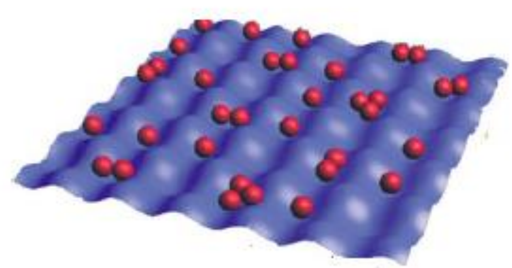
COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

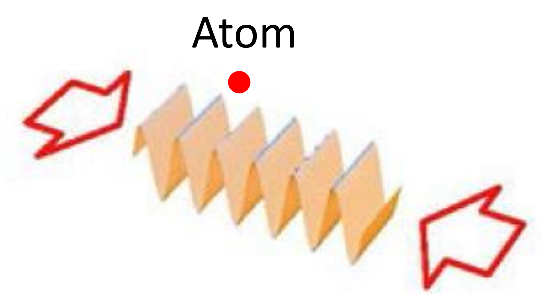
Cold Bosonic Atoms in Optical Lattices

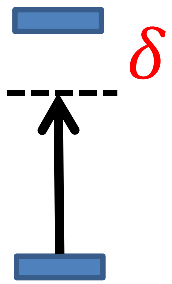
D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]



COLD ATOMS

OPTICAL LATTICES

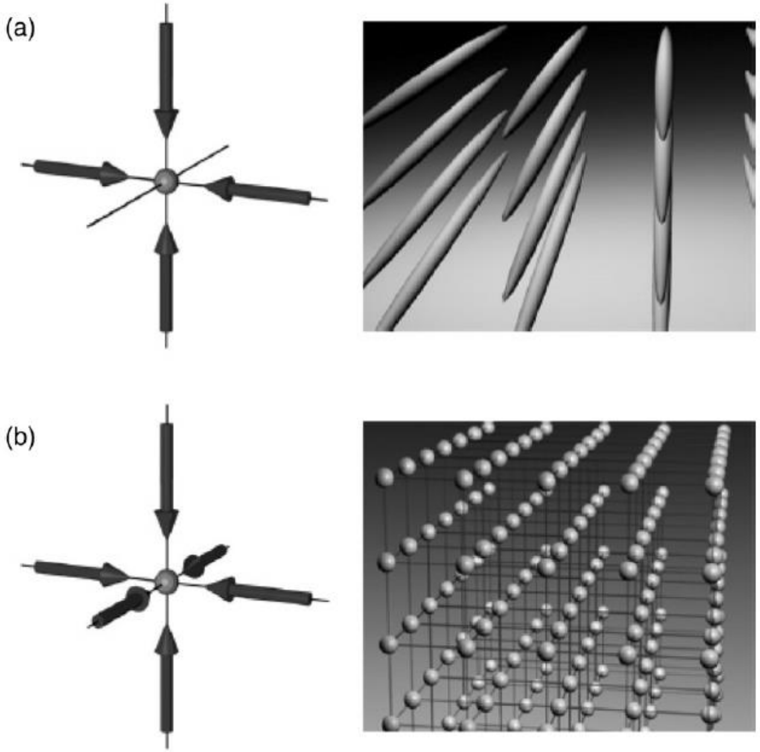




In the presence $\boldsymbol{E}(r,t)$ the atoms has a time dependent dipole moment $d(t) = \alpha(\omega)\,\boldsymbol{E}(r,t)$ of some non resonant excited states.
Stark effect:

$$\mathrm{V}(\mathrm{r}) \equiv \Delta\mathrm{E}(\mathrm{r}) = \alpha(\omega)\langle\, \boldsymbol{E}(r,t)\, \boldsymbol{E}(r,t)\rangle/\delta$$

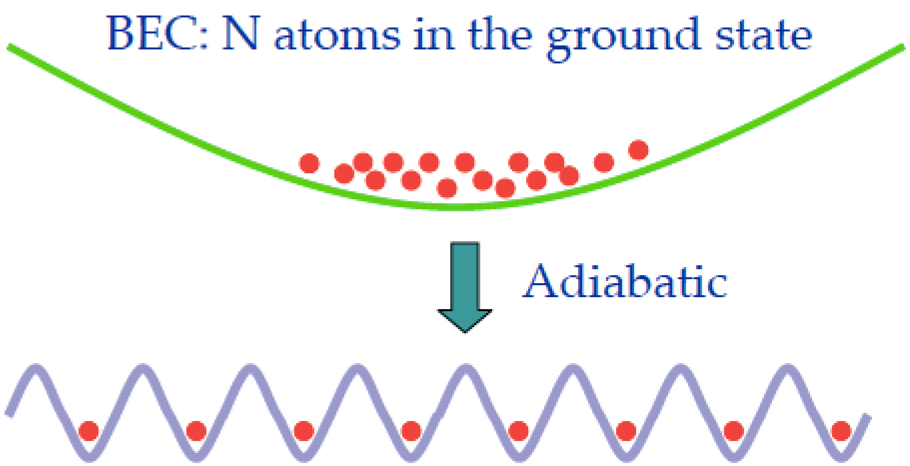
COLD ATOMS
OPTICAL LATTICES



(a) 2d array of effective 1d traps
(b) 3d square lattice

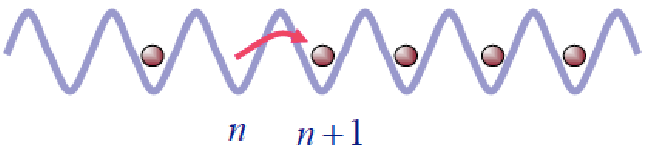
M. Lewenstein et. al, Advances in Physics, 2010. Book (2013).

Loading an optical lattice



Bose Hubbard Interactions

- Atoms may tunnel to neighboring sites:



$$-t\ a_{n+1}^{\dagger}a_n$$

- Atoms in the same site interact:

$$U\ a_n^{\dagger 2}a_n^2$$

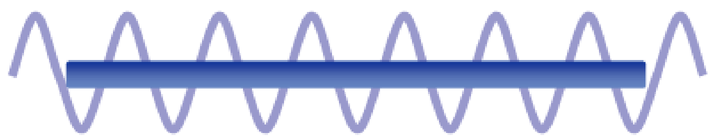
Atoms in optical lattices realize the Bose-Hubbard model:

$$H=-t\sum_n\left(a_n^{\dagger}a_{n+1}+h.c\right)+U\sum_n a_n^{\dagger 2}a_n^2$$

Quantum Phase transition: superfluid to insulator

- **Weak interactions:** t ? U

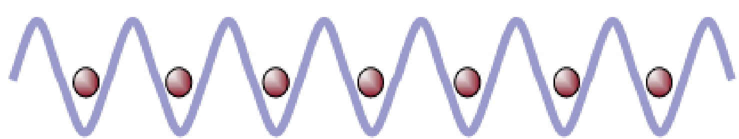
All atoms tend to delocalize, occupying the same state ➡ BEC (superfluid)



$$|\Psi\rangle : |\varphi_{k=0}\rangle^{\otimes N} \equiv \left(\sum_n a_n^\dagger\right)^N |\mathrm{vac}\rangle$$

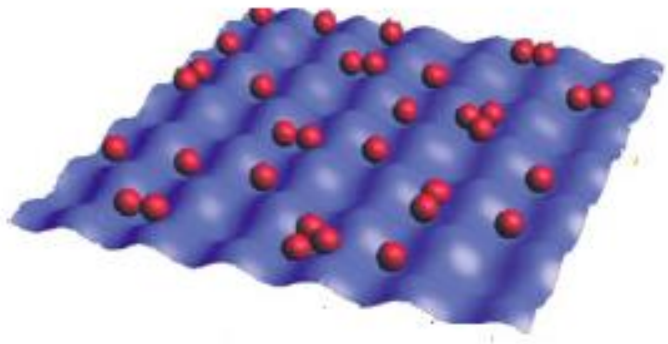
- **Strong interactions:** U ? t

Atoms tend to occupy different sites ➡ MOTT or Tonks



$$|\Psi\rangle : \; a_1^\dagger a_2^\dagger \ldots a_N^\dagger |\mathrm{vac}\rangle$$

COLD ATOMS
QUANTUM SIMULATIONS



- Bosons/Fermions: $H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$

- Spins: $H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$

→ CONDENSED MATTER PHYSICS

The field is rapidly advancing!

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

PRL 103, 080404 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice

Peter Würtz,[1] Tim Langen,[1] Tatjana Gericke,[1] Andreas Koglbauer,[1] and Herwig Ott[1,2,*]
[1]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[2]*Research Center OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany*
(Received 18 March 2009; published 21 August 2009)

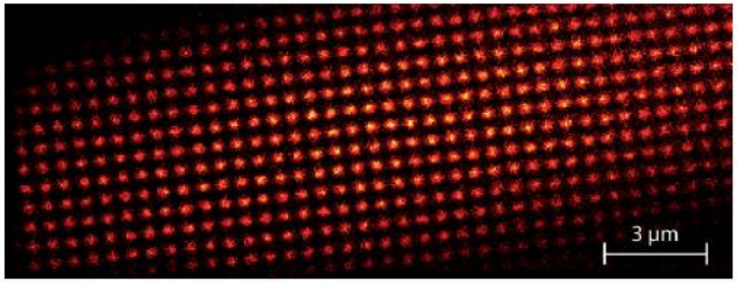


FIG. 1 (color online). Electron microscope image of a Bose-Einstein condensate in a 2D optical lattice with 600 nm lattice spacing (sum obtained from 260 individual experimental realizations). Each site has a tubelike shape with an extension of 6 μm perpendicular to the plane of projection. The central lattice sites contain about 80 atoms.

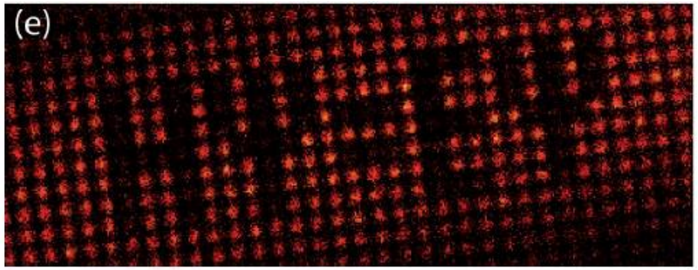


FIG. 2 (color online). Patterning a Bose-Einstein condensate in a 2D optical lattice with a spacing of 600 nm. Every emptied site was illuminated with the electron beam (7 nA beam current, 100 nm FWHM beam diameter) for (a),(b) 3, (c),(d) 2, and (e) 1.5 ms, respectively. The imaging time was 45 ms. Between 150 and 250 images from individual experimental realizations have been summed for each pattern.

QUANTUM SIMULATIONS
COLD ATOMS – EXPERIMENTS

nature Vol 462 | 5 November 2009 | doi:10.1038/nature08482

A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice

Waseem S. Bakr[1], Jonathon I. Gillen[1], Amy Peng[1], Simon Fölling[1] & Markus Greiner[1]

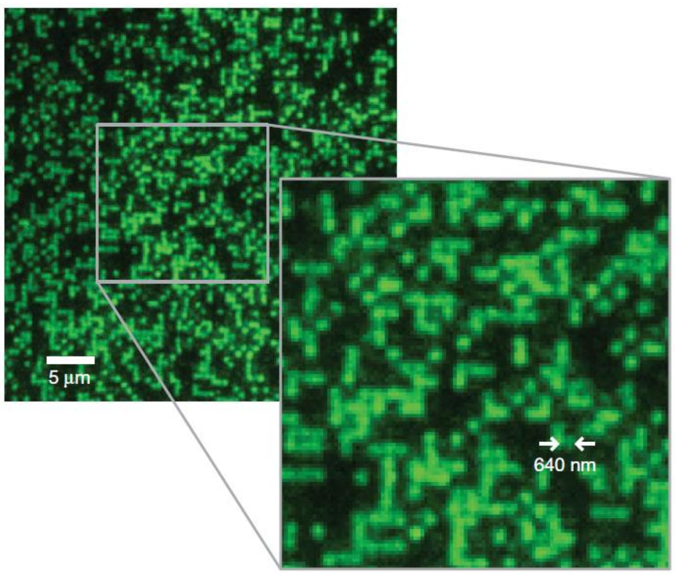


Figure 3 | Site-resolved imaging of single atoms on a 640-nm-period optical lattice, loaded with a high density Bose–Einstein condensate. Inset, magnified view of the central section of the picture. The lattice structure and the discrete atoms are clearly visible. Owing to light-assisted collisions and molecule formation on multiply occupied sites during imaging, only empty and singly occupied sites can be seen in the image.

mined through preparation and measurement. By implementing a high-resolution optical imaging system, single atoms are detected with near-unity fidelity on individual sites of a Hubbard-regime optical lattice. The lattice itself is generated by projecting a holographic mask through the imaging system. It has an arbitrary geometry, chosen to support both strong tunnel coupling between lattice sites and strong on-site confinement. Our approach can be

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

doi:10.1038/nature09827

Single-spin addressing in an atomic Mott insulator

Christof Weitenberg[1], Manuel Endres[1], Jacob F. Sherson[1]†, Marc Cheneau[1], Peter Schauß[1], Takeshi Fukuhara[1], Immanuel Bloch[1,2] & Stefan Kuhr[1]

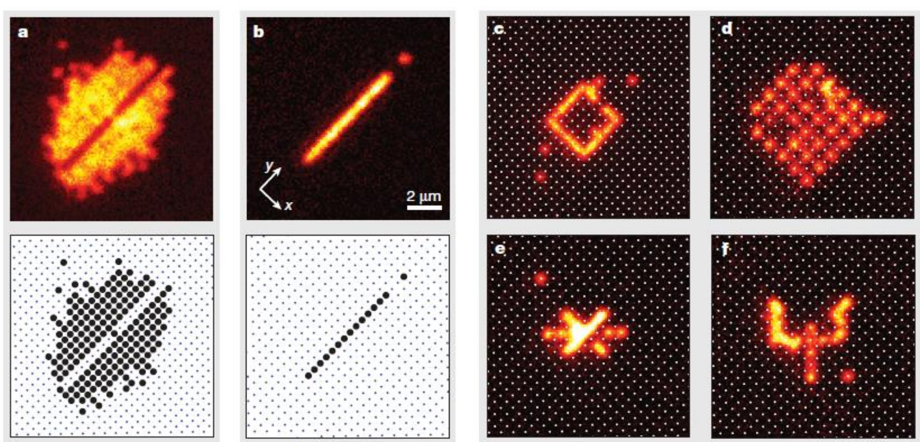
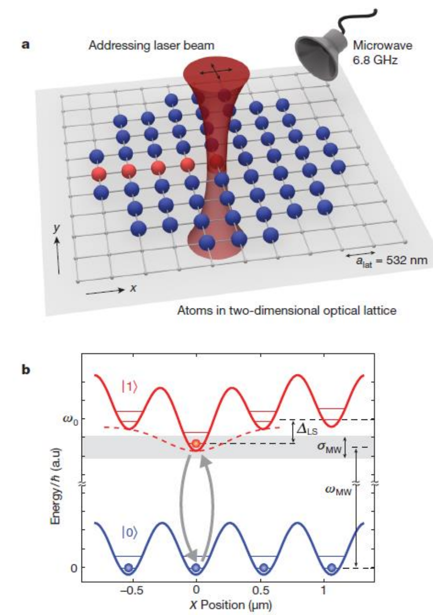
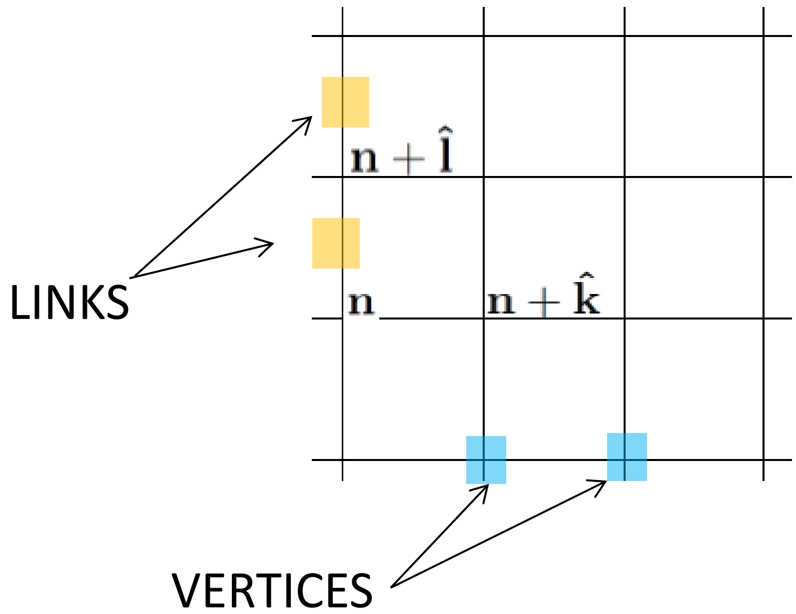


Figure 2 | Single-site addressing. **a**, Top, experimentally obtained fluorescence image of a Mott insulator with unity filling in which the spin of selected atoms was flipped from $|0\rangle$ to $|1\rangle$ using our single-site addressing scheme. Atoms in state $|1\rangle$ were removed by a resonant laser pulse before detection. Bottom, the reconstructed atom number distribution on the lattice. Each filled circle indicates a single atom; the points mark the lattice sites. **b**, Top, as for **a** except that a global microwave sweep exchanged the population in $|0\rangle$ and $|1\rangle$, such that only the addressed atoms were observed. Bottom, the reconstructed atom number distribution shows 14 atoms on neighbouring sites. **c–f**, As for **b**, but omitting the atom number distribution. The images contain 29 (**c**), 35 (**d**), 18 (**e**) and 23 (**f**) atoms. The single isolated atoms in **b**, **e** and **f** were placed intentionally to allow for the correct determination of the lattice phase for the feedback on the addressing beam position.



LATTICE GAUGE THEORIES

Gauge field degrees of freedom:
U(1), SU(N), etc, unitary matrices



Matter degrees of freedom :
Spinors

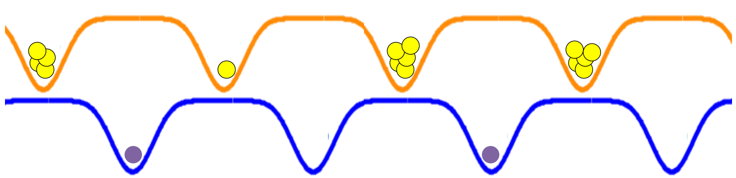
$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$

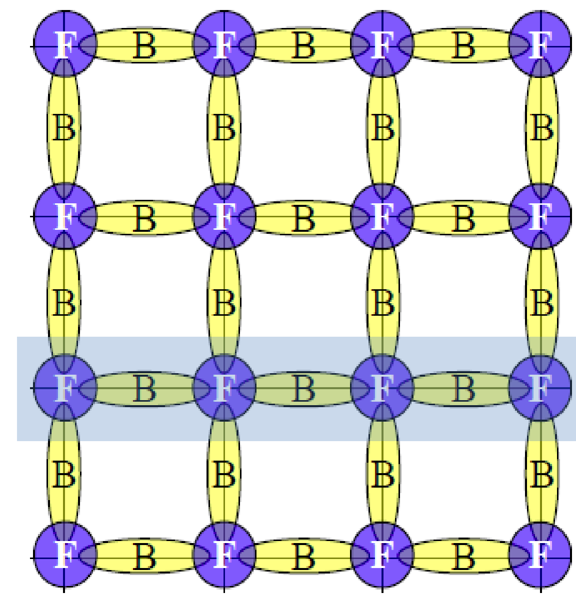
QUANTUM SIMULATION OF LATTICE GAUGE THEORY

COLD ATOMS

- Fermion matter fields
- Bosonic gauge fields

Super-lattice:

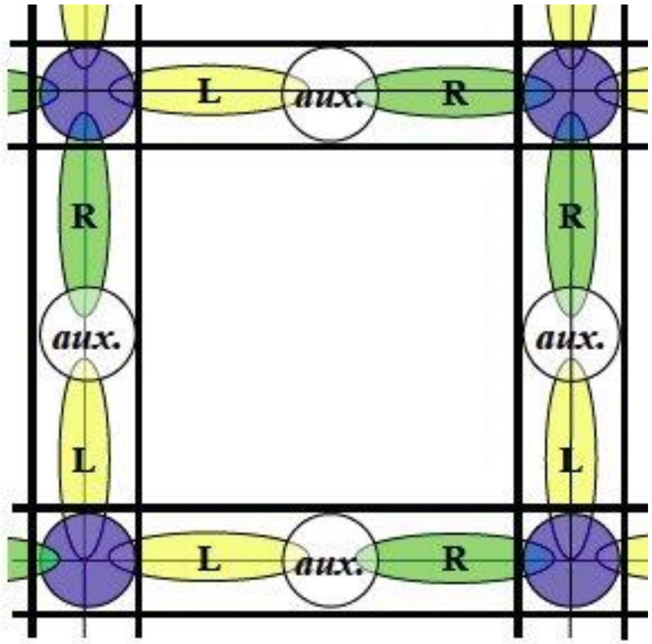




$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \dots \end{pmatrix} \longrightarrow$$ Atom internal levels

QUANTUM SIMULATIONS OF NONABELIAN MODELS

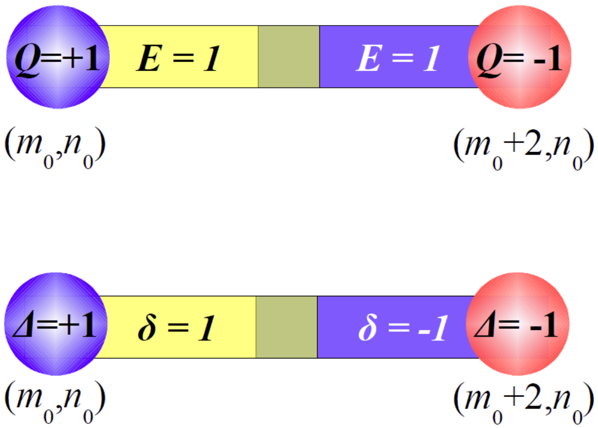
GENERAL STRUCTURE



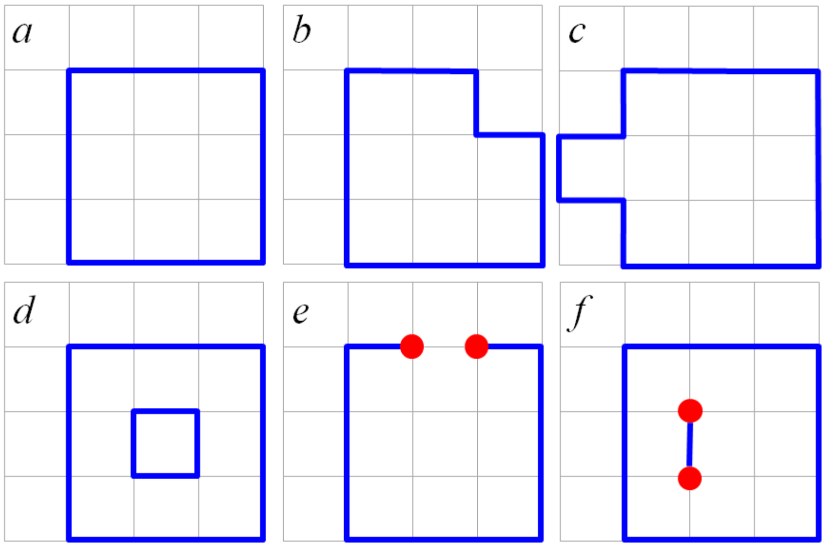
Each link has *left* and *right* degrees of freedom – forming together SU(N) elements. The "relative rotation" corresponds to the non-abelian charge on the link.

Quark Confinement, flux breaking & glueballs

Electric flux tubes



Flux loops deforming and breaking effects



E. Zohar, BR, Phys. Rev. Lett. **107**, 275301 (2011)

E. Zohar, I. Cirac, BR, PRL **109**, 125302 (2012)

E. Zohar, J. I. Cirac, BR, PRL **110,** 055302 (2013)

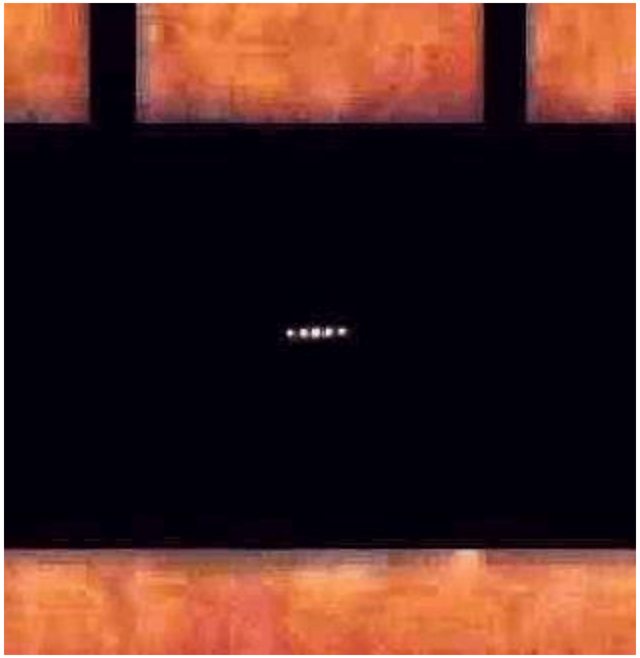
E. Zohar, I. Cirac, BR, PRL **110** 125304 (2013)

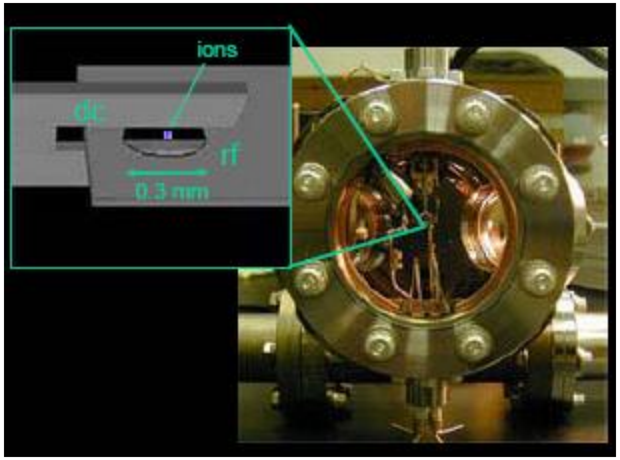
(*) E. Zohar, I. Cirac, BR, PRA (2013) arxiv 1303.5040

(*) – self contained detailed account.

Q. Simulations: Trapped Ions

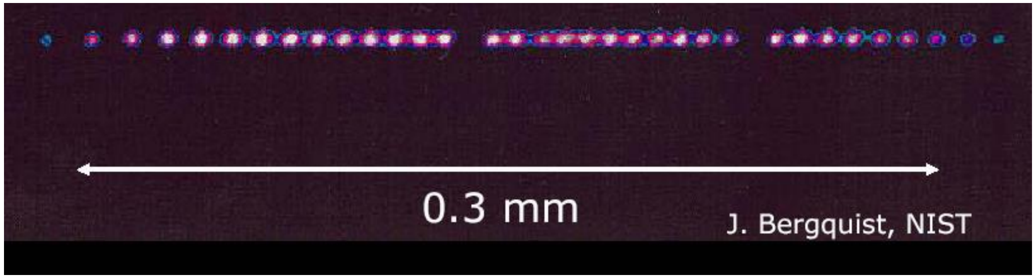
Trapped ions



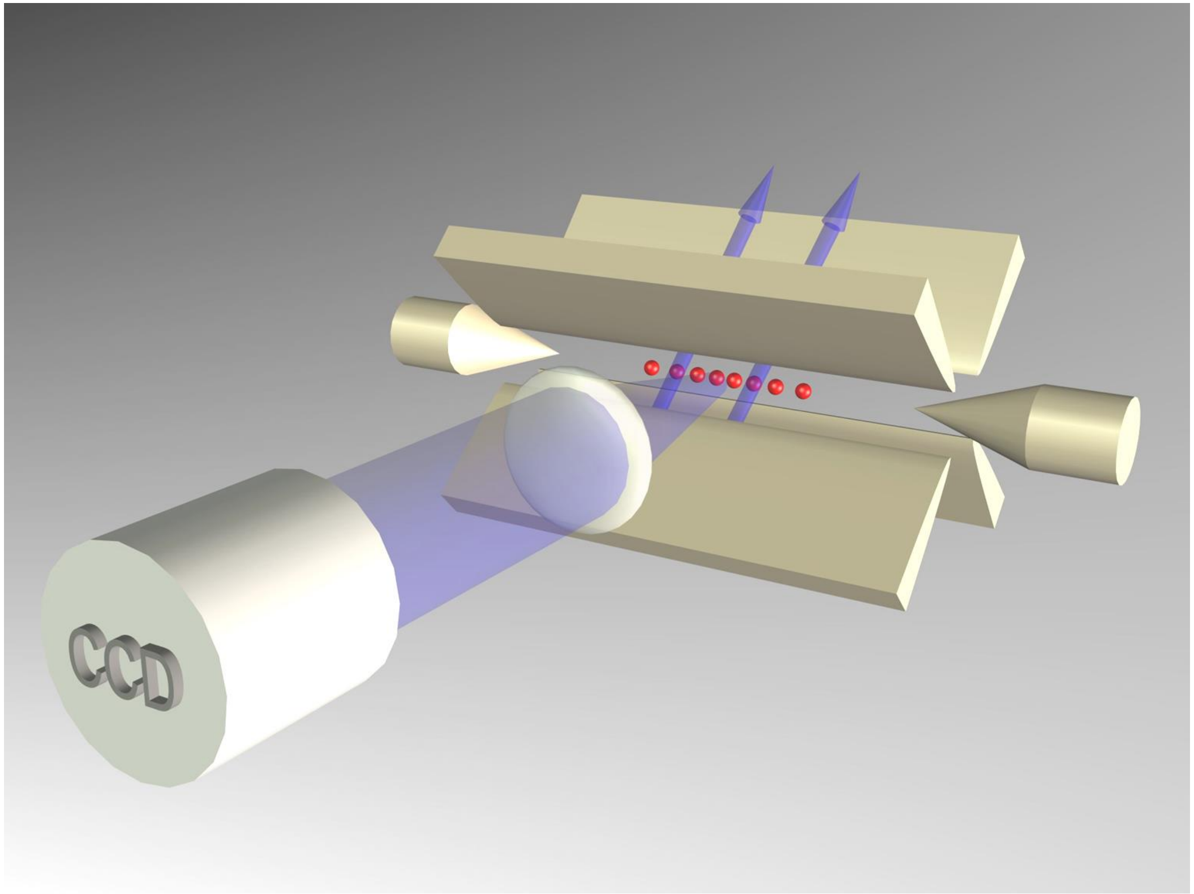


The Michigan experiment.

Five beryllium ions in a lithographycally fabricated RF trap.
The separation between ions is about 10 microns.

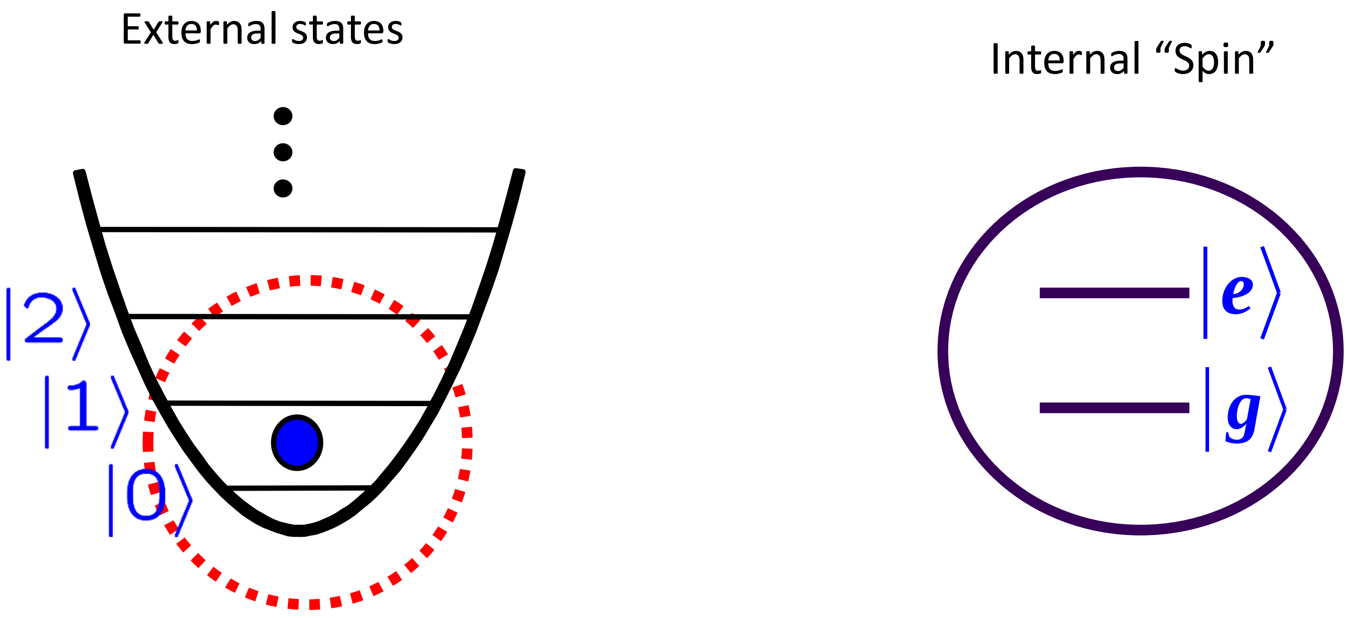


Linear Paul trap

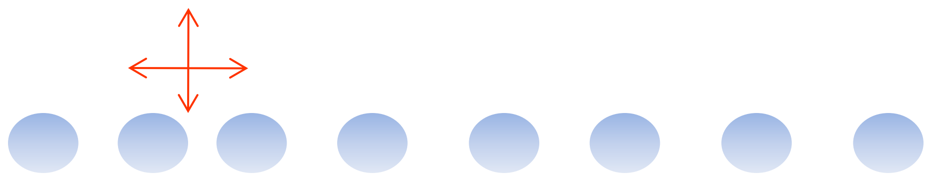


R. Blatt

“Spin-like” states



Motional states



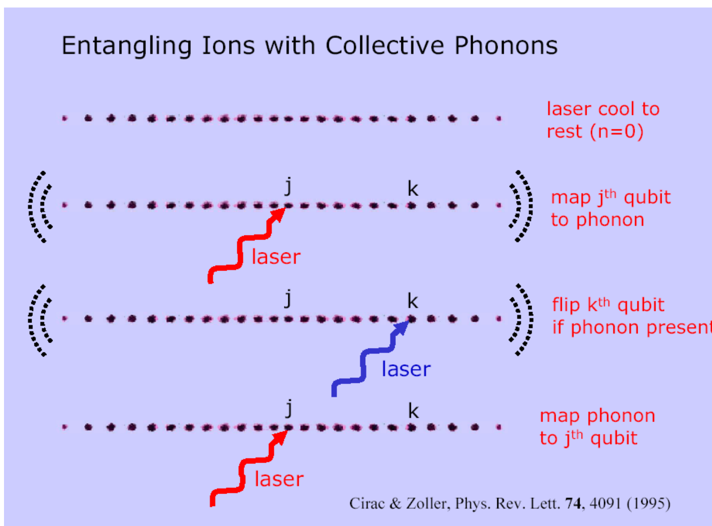
Axial excitations $\leftrightarrow$ $\beta >> 1$ ➡ Scalar phonons

Radial excitations $\updownarrow$ $\beta < 1$ ➡ Localized phonons

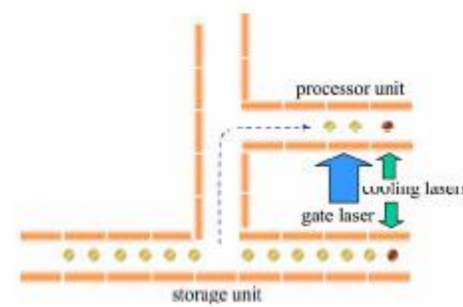
$$\beta \equiv \frac{\text{Coulomb interaction}}{\text{Trapping potential}}$$

Cirac-Zoller scheme


$$H = \sum_{i=1}^{N} \frac{1}{2}\omega_i(t)\hat{\vec{\sigma}}^{(i)} + \sum_{i,j=1}^{N} g_{ij}(t)\hat{\vec{\sigma}}^{(i)} \cdot \hat{\vec{\sigma}}^{(j)}$$

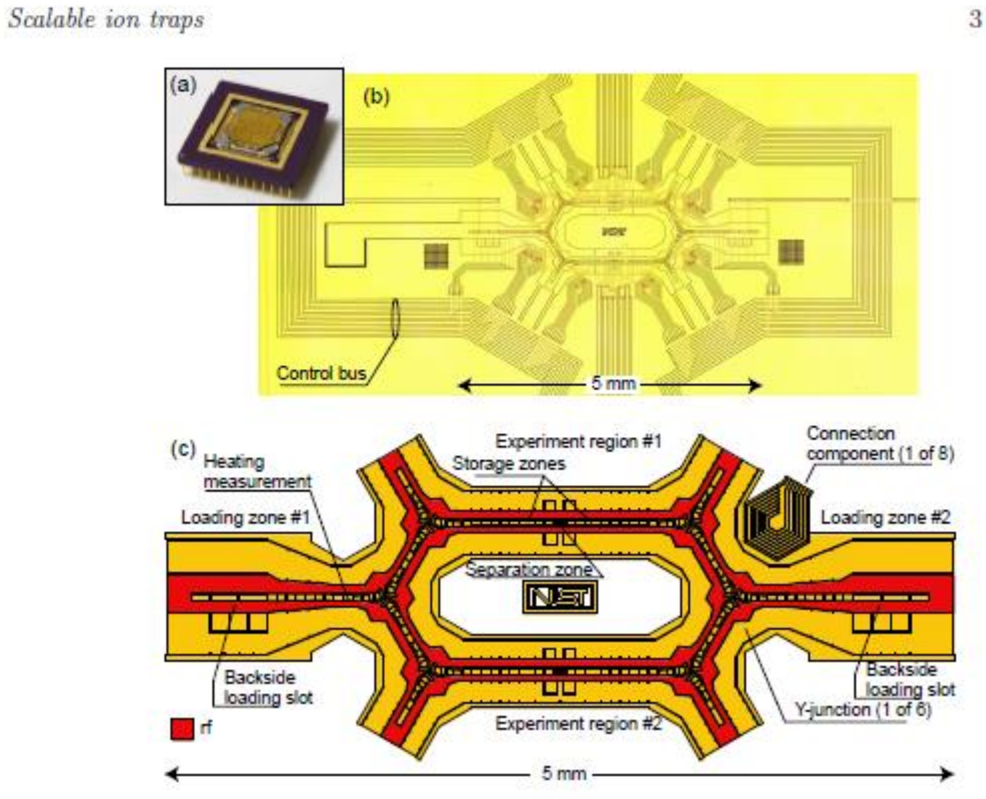


-Controlled sequence of gates
-Fidelity higher then99% fidelity
-requires ground state cooling.
-Up to 10 ions.
-hard to scale-up.



Cirac, Zoller, PRL (1995)

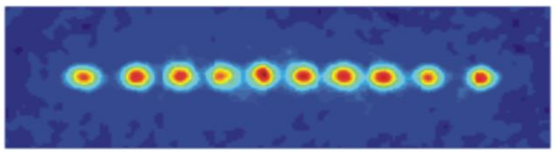
Scalable ion traps for quantum information processing



D. Wineland's group NIST, 2009.

… =>Road map for quantum computing

Quantum simulations?



trapped ion "crystal"

?

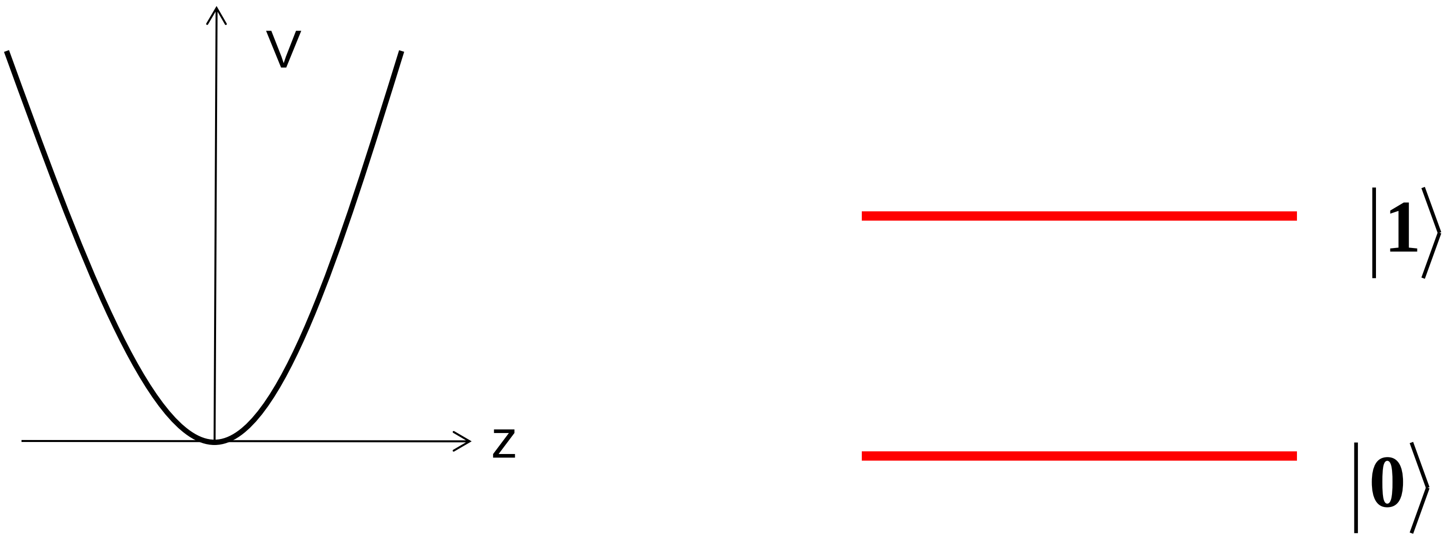
-Using Nature's given interactions
-Engineer the interaction, e.g.:

Bose-Hubbard , and xy models:

$$H_{BH} = -t\sum_n (a_n^\dagger a_{n+1} + h.c) + U\sum_n a_n^{\dagger 2} a_n^2$$

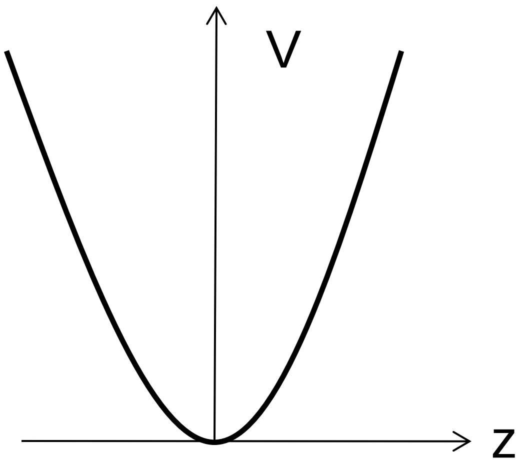
$$H_{XY} = \lambda \Sigma_i \sigma_z^i \; + \; \Sigma_{ij} J_{ij}\left(\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j \right)$$

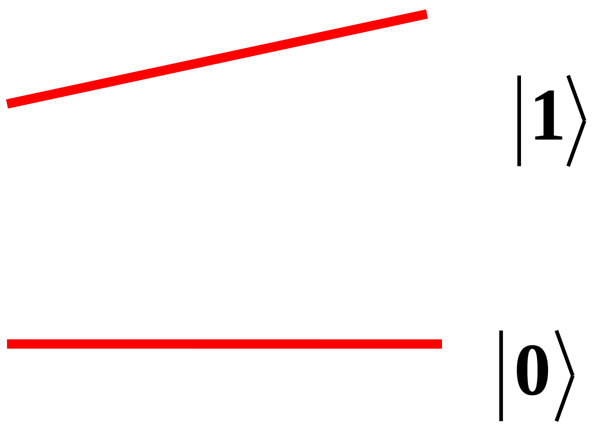
Spin-position coupling



F. Mintert, C. Wunderlich, PRL (2001);

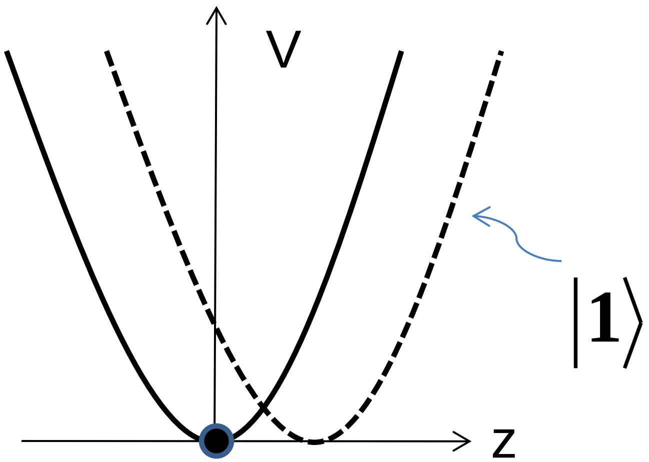
Spin-position coupling

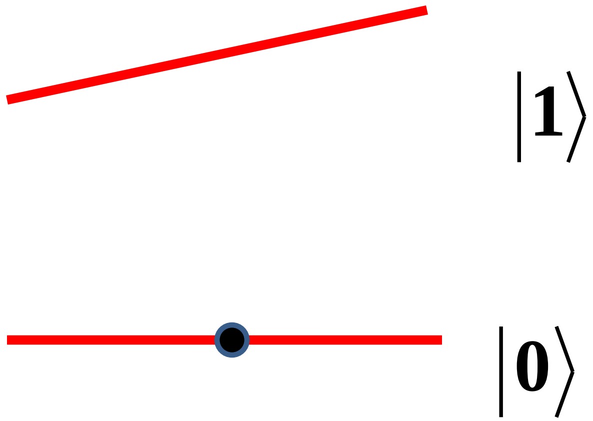




$$B_z \propto z$$

Spin-position coupling





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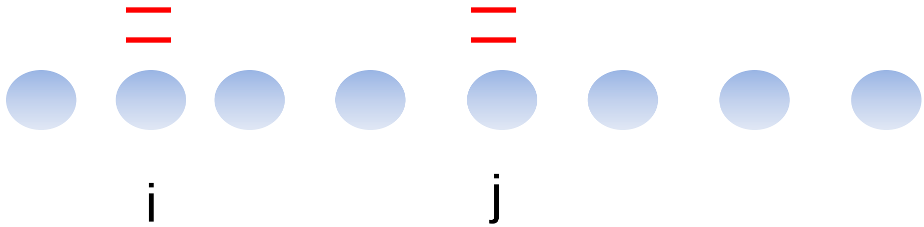
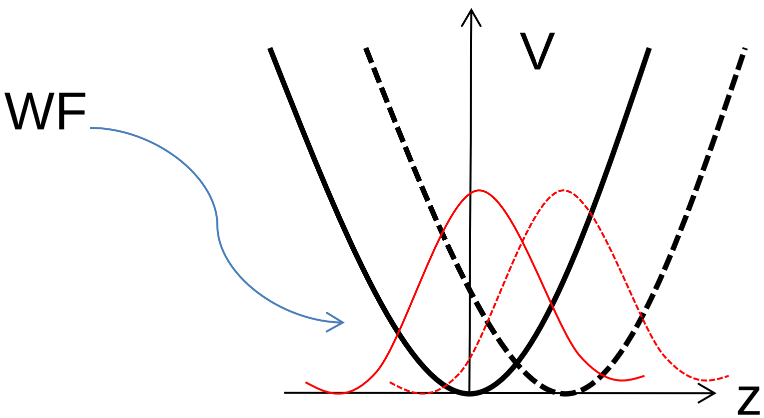


$$H_{eff} = \sum_{i,j} J_{ij} \sigma_z^i \sigma_z^j$$

F. Mintert, C. Wunderlich, PRL (2001);

Spin-Spin coupling

Phonon not excited!



The shift in position is small
Compared with the WF width.

$$\approx | s > \otimes | \gamma >$$

- ❑ An example of the general phenomena of adiabatic elimination of fast degree of freedom (phonons)

Spin-Chain models

- Less sensitive to the motional states, ground state cooling not required.

$$\boldsymbol{H_{eff} = \lambda \sum_i \sigma_x^i + \sum_{i,j} J_{ij} \sigma_z^i \sigma_z^j}$$

Transverse Ising

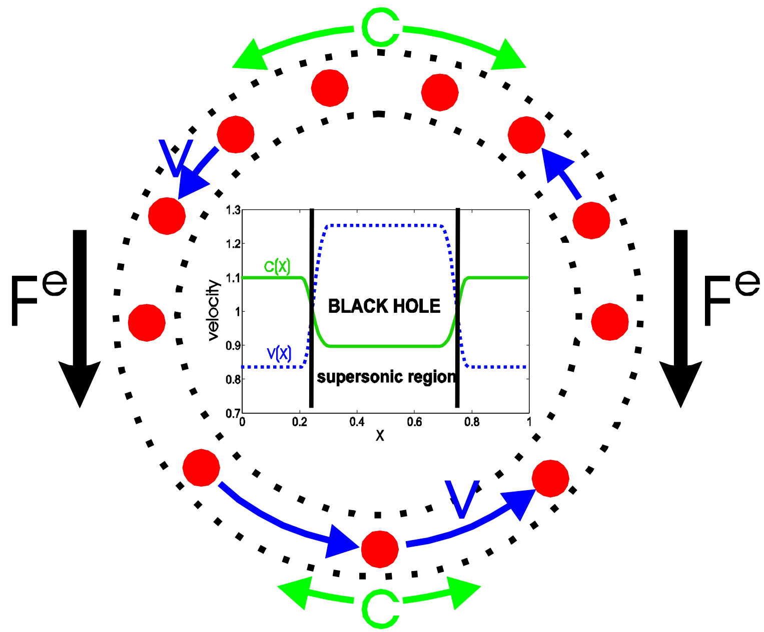
$$\boldsymbol{H_{eff} = \lambda \sum_i \sigma_z^i + \sum_{i,j} J_{ij} (\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j)}$$

XY -Model

- For the radial mode mediators $J_{ij} \approx \frac{1}{(i-j)^3}$
- Can be more localized in microtraps

Deng, Porras, Cirac 2005.

Gravity: discrete BH analogue with Ions in a ring trap



Inhomogeneous, but stationary velocity profile v (θ).
The necessary (de-)acceleration of the ions is guaranteed by a force F^e on the ions.

Harmonic oscillations around the equilibrium motion are phonons with velocities $c(\theta) \propto (v(\theta))^{-1/2}$.
When v increases the sound velocity decreases and a Black and White horizons can form.

Can Hawking radiation be detected is such simulators?

Summary

- Several Hard problems in High-energy physics,
  condensed matter physics, and gravity,
  can be studied with analog quantum simulators,
  building on current experimental methods
  with of cold atoms/ion.

- They are already non-trivial with O(100) atoms.

- Analog Quantum simulations:
  -- do not require a full-fledged quantum-computing.
  -- seem less sensitive to errors.
  -- are feasible in near future experimental.

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Thank YOU!