Aspects of Uncertainty and Complementarity

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Outline

Bohr's Complementarity Principle

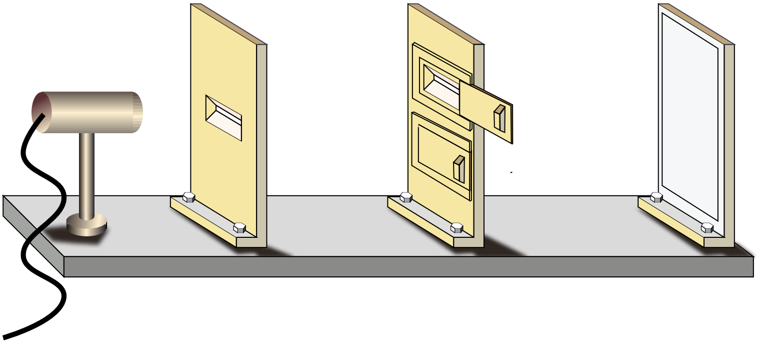
Niels Bohr in 1928

In describing the results of quantum mechanical experiments, certain physical concepts are complementary. If two concepts are complementary, an experiment that clearly illustrates one concept will obscure the other complementary one. . .

–*"The Quantum Postulate and the Recent Development of Atomic Theory," Supplement to Nature, April 14, 1928, p.580*

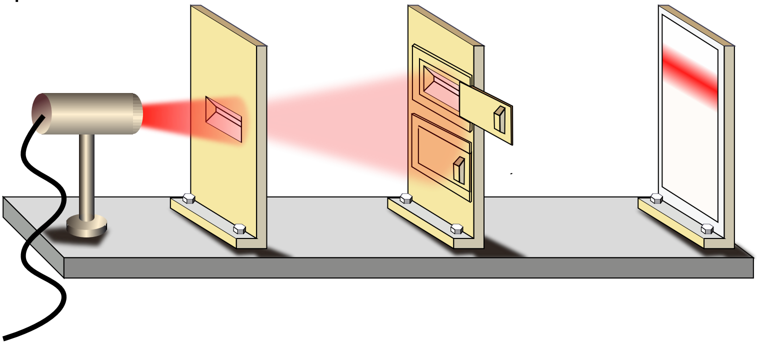
The Two-Slit Experiment with Quantum particles

Setup



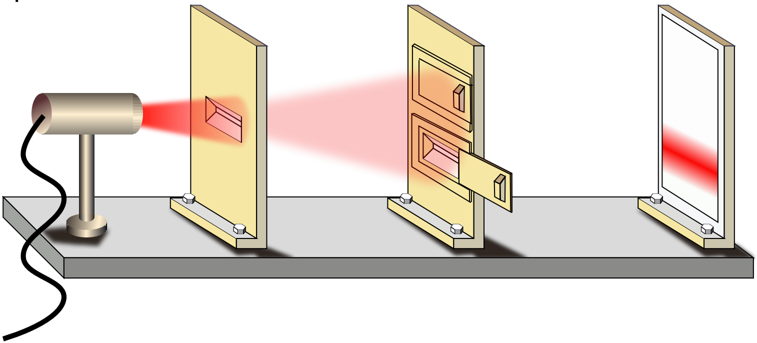
The Two-Slit Experiment with Quantum particles

Slit 1 open



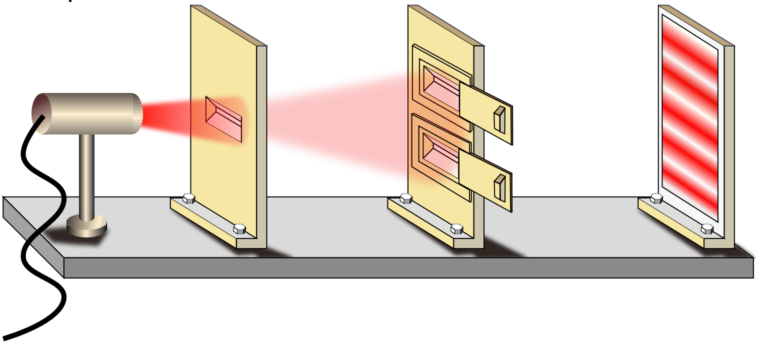
The Two-Slit Experiment with Quantum particles

Slit 2 open



The Two-Slit Experiment with Quantum particles

Both slits open



Two-slit experiment with electrons

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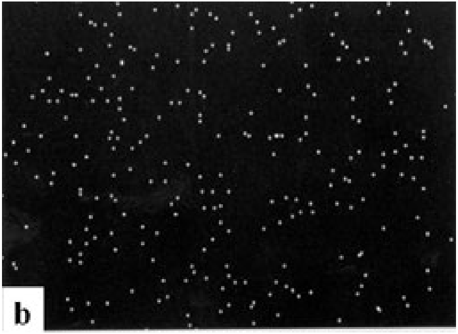

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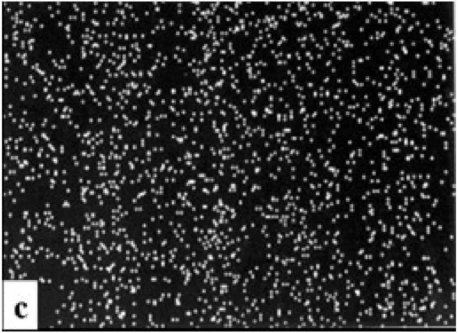

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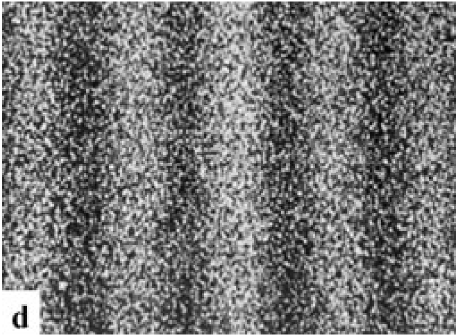
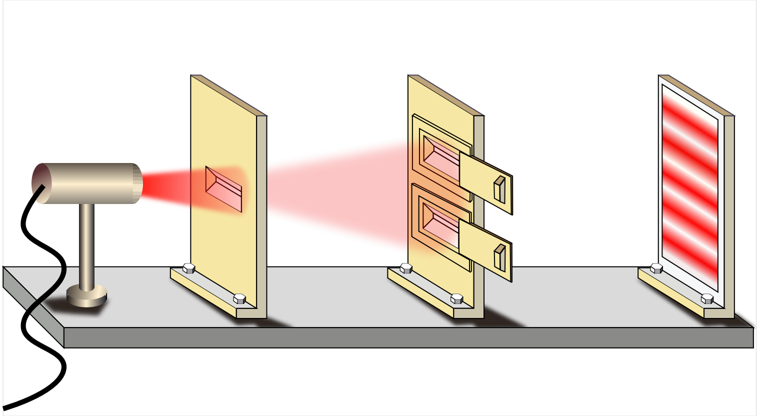


Which slit did the electron pass through?

Getting the “*Welcher-Weg*" (which-way) information



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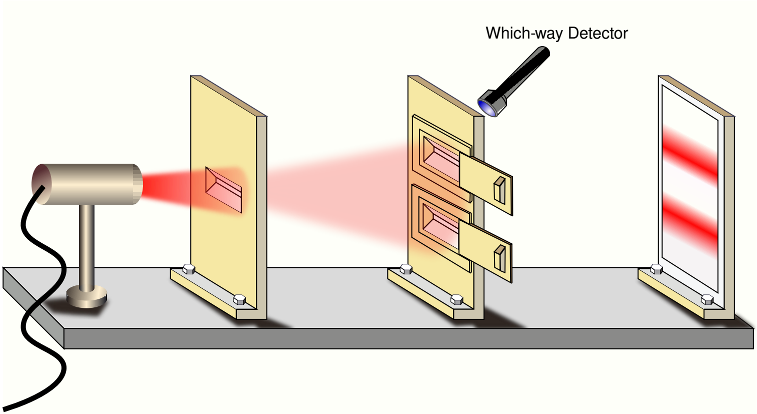

No Interference!

Complementarity in the 2-slit Experiment

Complementarity = Wave-particle duality?

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Complementarity in the 2-slit Experiment

Complementarity = Wave-particle duality?

- In the two-slit experiment: the **“which-way"** information vs existence of **interference** pattern.
- They can NEVER be observed at the same time, in the same experiment.

Uncertainty and Complementarity

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Measurement of particle position results in uncontrollable disturbance in its momentum, washing out the interference pattern.

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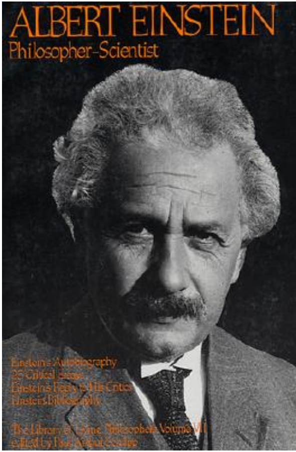
Determine which-way without disturbing the particle?

Einstein's Recoiling-Slit *Gedanken* Experiment



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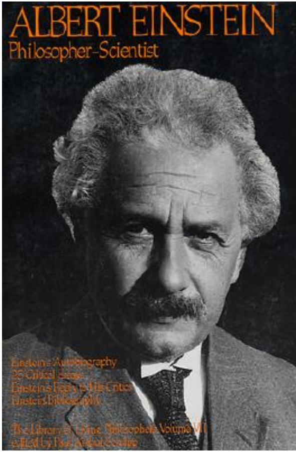
... Einstein thought he had found a counterexample to the uncertainty principle. *"It was quite a shock for Bohr .... he did not see the solution at once. During the whole evening he was extremely unhappy, going from one to the other and trying to persuade them that it couldn't be true, that it would be the end of physics if Einstein were right; but he couldn't produce any refutation.*



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ROSENFELD (1968)
Fundamental Problems in Elementary Particle Physics

Proceedings of the Fourteenth Solvay Conference, Interscience, New York, p. 232.

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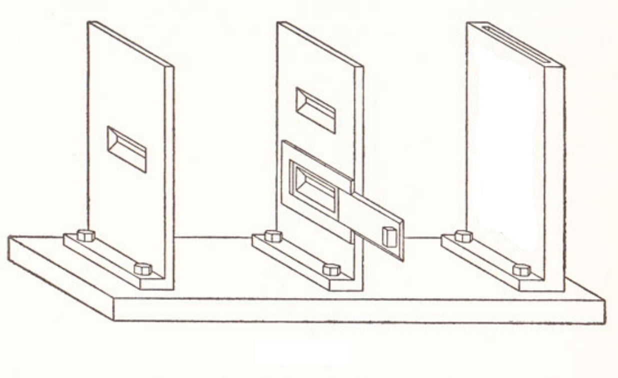

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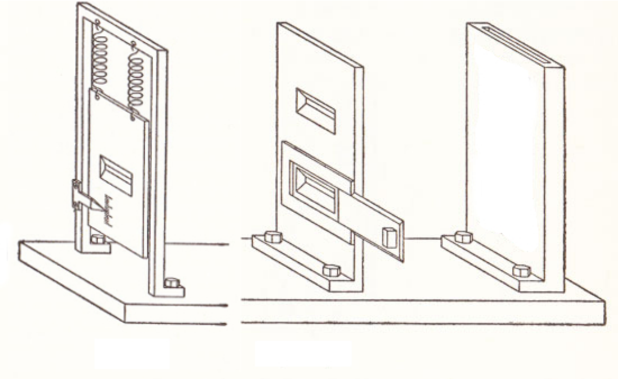

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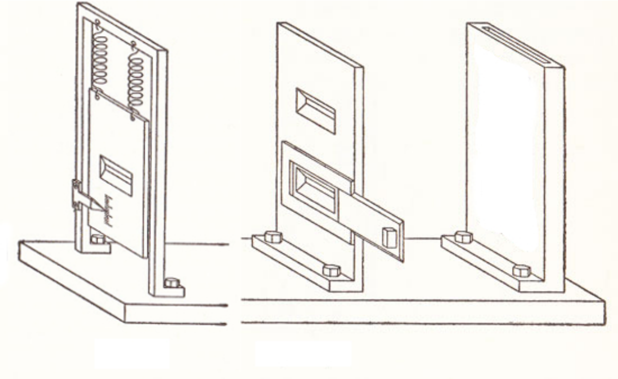

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Recoil of slit $\Longrightarrow$ which-way information without disturbing the particle

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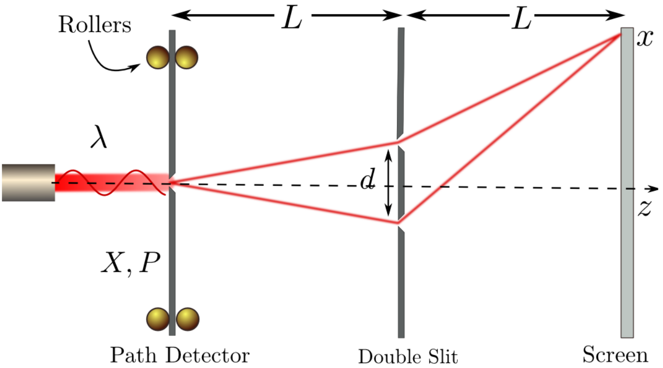

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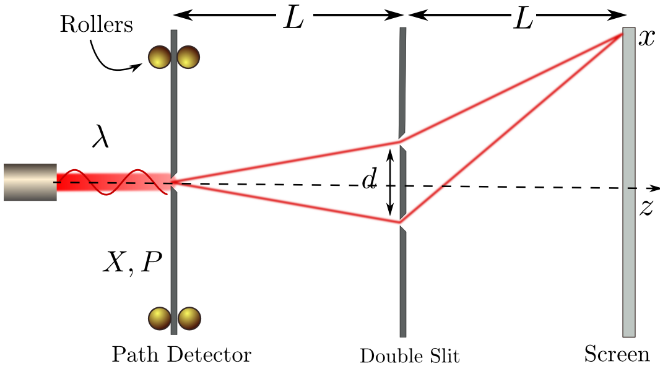

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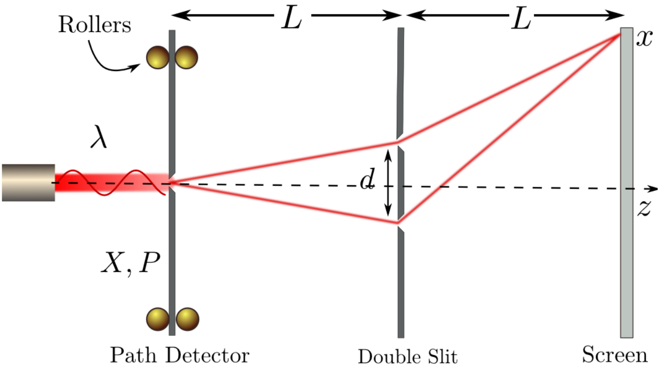

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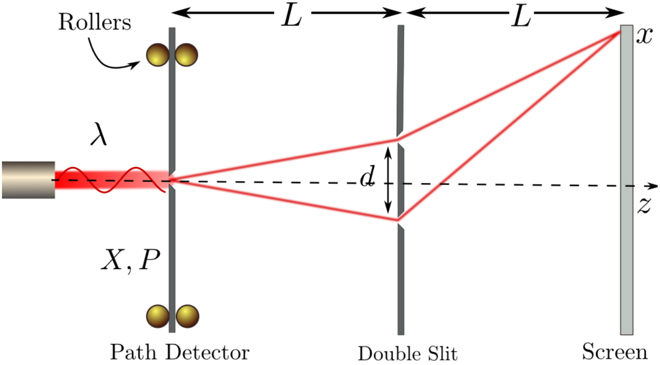

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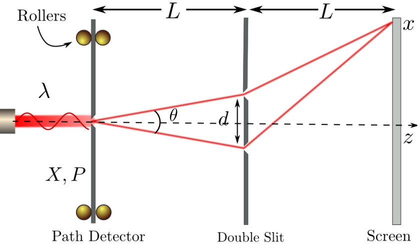
Bohr's reply



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$$\Delta p_x = 2p \sin(\theta/2)$$

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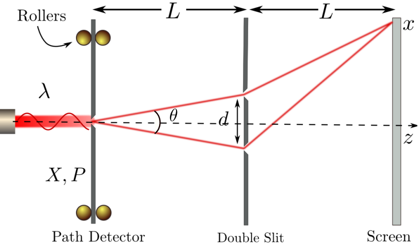

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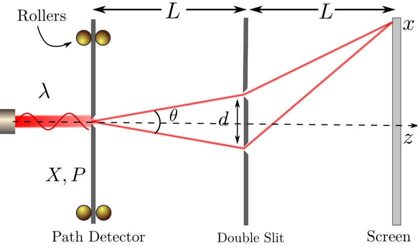

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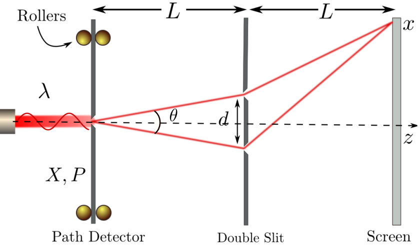

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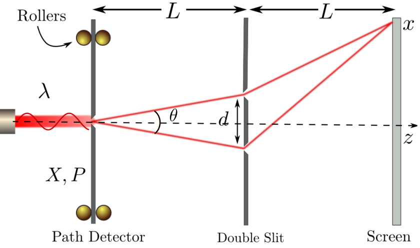

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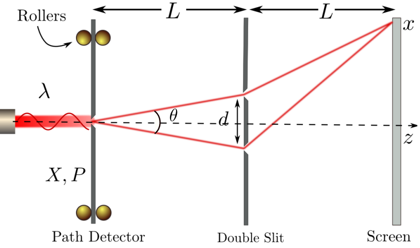

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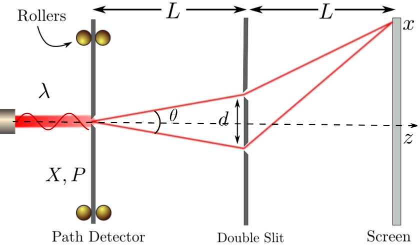

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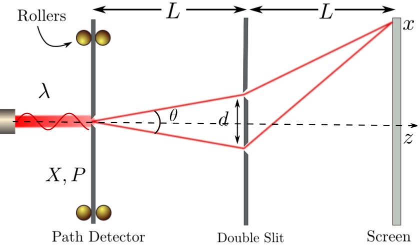

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- Interference pattern is lost!

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- This viewed as a restatement of Uncertainty Principle

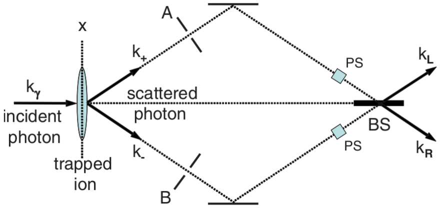
Realization of Recoiling-Slit Experiment

PHYSICAL REVIEW A **75**, 062105 (2007)

Trapped-ion realization of Einstein's recoiling-slit experiment

Robert S. Utter and James M. Feagin*
Department of Physics, California State University-Fullerton, Fullerton, California 92834, USA
(Received 10 July 2006; revised manuscript received 9 October 2006; published 13 June 2007)

We analyze photon scattering by a harmonically trapped ion using two-port interferometry of the scattered photon and coherent-state measurement of the ion's external recoil motion. We examine how the coherent-state measurement could be used to mimick both momentum and position ion measurements and thus a modern realization of Wootters and Zurek's pioneering analysis of Einstein's historic recoiling-slit gedanken experi-



Realization of Recoiling-Slit Experiment

Letters to Nature > Abstract

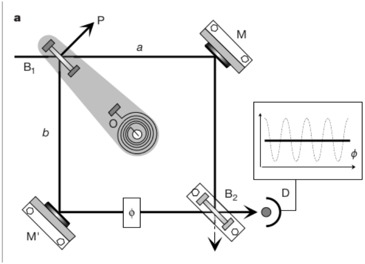
Letters to Nature

Nature **411**, 166-170 (10 May 2001) | doi:10.1038/35075517; Received 22 December 2000; Accepted 7 March 2001

A complementarity experiment with an interferometer at the quantum–classical boundary

P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. M. Raimond & S. Haroche ← Physics Nobel 2012

1. Laboratoire Kastler Brossel, Département de Physique, Ecole Normale Supérieure, 24 rue Lhomond, F-75231, Paris Cedex 05, France



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"The Measurement Problem".

Which-way Detection in Einstein's experiment

Using von Neumann's process 1

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and is enough to rule out interference!

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Which-way Information and Interference

- **Without which-way information**
  Amplitude for finding the particle at point x on the screen is

$$\Psi(x) = \psi_1(x) + \psi_2(x).$$

  Probability (intensity):

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Can this argument be made more quantitative?

Suppose our detector distinguishes the two paths inaccurately.

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Is there a relationship between them to capture complementarity?

Meaning of D

Consider an observable $\hat{P}$ of detector with eigenstates

$$\hat{P}|p_1\rangle = +|p_1\rangle, \quad \hat{P}|p_1\rangle = -|p_2\rangle$$

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i.e, if we measure $\hat{P}$ and get -1, probability of distinguishing the paths is

$$\begin{aligned} |\alpha_r|^2 &= |\langle p_2|d_2\rangle|^2 \\ &= 1 - |\langle p_1|d_2\rangle|^2 \\ &= 1 - |\langle d_1|d_2\rangle|^2 \\ &= D^2 \end{aligned}$$

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So D^2 is the probability of correctly distinguishing the two paths.

$t = 0$: particle emerges from the double-slit with amplitude

$$\Psi(x, 0) =$$

Path-distinguishability and Interference

Gaussian Wave-packet Model

$t = 0$: particle emerges from the double-slit with amplitude

$$\Psi(x,0) = A\left(|d_1\rangle e^{-\frac{(x-d/2)^2}{4\epsilon^2}} + |d_2\rangle e^{-\frac{(x+d/2)^2}{4\epsilon^2}}\right),$$

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After time t, traveling a distance L, amplitude for particle to arrive at x on screen:

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&\times \left(1 + |\langle d_1|d_2\rangle| \frac{\cos\left(\frac{xd\lambda L/2\pi}{4\epsilon^4+(\lambda L/2\pi)^2} + \theta\right)}{\cosh(xd/2\sigma_t^2)}\right)
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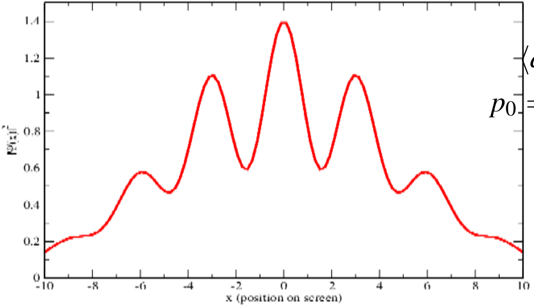

$$
\begin{aligned}
\langle d_1|d_2\rangle &= |\langle d_1|d_2\rangle| e^{i\theta} \\
p_0 = h/\lambda &\implies \hbar t/m = \lambda L/2\pi, \\
\sigma_t^2 &= \epsilon^2 + \left(\frac{\hbar t}{2m\epsilon}\right)^2
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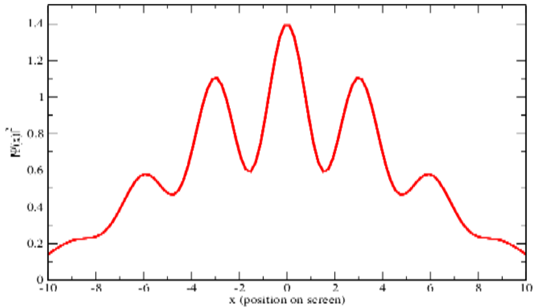

Fringe width =

$$
\frac{\lambda L}{d} + \frac{16\pi^2\epsilon^4}{\lambda d L}.
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$$\boxed{\mathcal{V}^2 + \mathcal{D}^2 \leq 1.}$$

Englert-Greenberger-Yasin duality relation

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A *quantitative* statement of complementarity

Origin of Complementarity?

- Quantum correlations?
  - D.M. Greenberger, A. Yasin, Phys. Lett. A 128, 391 (1988), "Simultaneous wave and particle knowledge in a neutron interferometer",
  - B-G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996), "Fringe visibility and which-way information: an inequality"
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  - H. Wiseman, F. Harrison, *Nature* **377**, 584 (1995), “Uncertainty over complementarity?"
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  - H. Wiseman, *Phys. Lett. A* **311**, 285 (2003), "Directly observing momentum transfer in twin-slit which-way experiments"
- Does the particle really receive a "momentum kick"?
  - S. Durr, T. Nonn, G. Rempe, *Nature* **395**, 33 (1998), "Origin of quantum-mechanical complementarity probed by a which-way experiment in an atom interferometer."
  - C.S. Unnikrishnan, *Phys. Rev. A* **62**, 015601 (2000), "Origin of quantum-mechanical complementarity without momentum back action in atom-interferometry experiments".

Uncertainty principle and complementarity

Other work

- G. Bjork, J. Soderholm, A. Trifonov, T. Tsegaye, A. Karlsson, *Phys. Rev. A* **60**, 1874 (1999), "Complementarity and the uncertainty relations".
- K-P Marzlin, B.C. Sanders, P.L. Knight, *Phys. Rev. A* **78**, 062107 (2008), "Complementarity and uncertainty relations for matter-wave interferometry",
- J-H Huang, S-Y Zhu, *arXiv:1011.5273 [physics.optics]*, "Complementarity and uncertainty in a two-way interferometer".
- G.M. Bosyk, M. Portesi, F. Holik, A. Plastino, *arXiv:1206.2992 [quant-ph]* "On the connection between complementarity and uncertainty principles in the Mach-Zehnder interferometric setting".
- Paul Busch, Christopher R. Shilladay. *arXiv:quant-ph/0609048, Phys Rep* **435**, 1-31 (2006)

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Uncertainty and Duality

Correlation of detector states with particle states:

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- The particle states can be correlated with these states:

$$\Psi(x) = \frac{c_1}{\sqrt{2}}[\psi_1(x) + \psi_2(x)]|q_1\rangle + \frac{c_2}{\sqrt{2}}[\psi_1(x) - \psi_2(x)]|q_2\rangle$$

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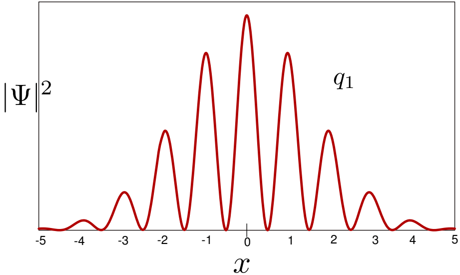

Two complementary interference patterns corresponding to $|q_1\rangle$ and $|q_2\rangle$.

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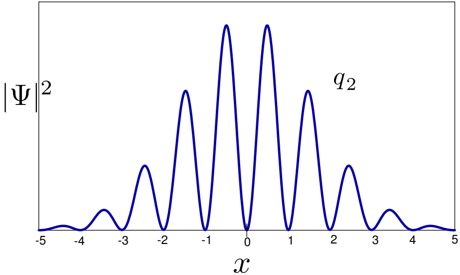

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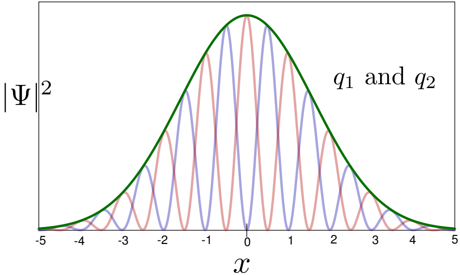

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Combining with the earlier result $\mathcal{D}^2 = 1 - \Delta P^2$, we get

$$\boxed{\mathcal{D}^2 + \mathcal{V}^2 \leq 2 - [\Delta P^2 + \Delta Q^2].}$$

Sum uncertainty relation for angular momenta [1]

$$\Delta L_x^2 + \Delta L_y^2 + \Delta L_z^2 \geq \ell$$

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The duality relation also emerges from the *sum* uncertainty relation.

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Conclusions

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- Momentum back-action of the recoiling slit on the particle plays no role in complementarity.

Tabish Qureshi, Radhika Vathsan
Einstein's Recoiling Slit Experiment, Complementarity and Uncertainty
Arxiv: 1210.4248 [quant-ph]
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THANK YOU!