Generating continuous variable nonclassical states using superposition of number-conserving quantum operations

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MOTIVATION

We consider an experimentally realizable scheme for manipulating CV optical fields using a number-conserving, general superposition of products of field annihilation ($\hat{a}$) and creation ($\hat{a}^\dagger$) operators of the type $s\hat{a}\hat{a}^\dagger + t\hat{a}^\dagger\hat{a}$, with $s^2 + t^2 = 1$. Such an operation, when applied on states with classical features, is shown to introduce strong nonclassicality. We generate two-mode entangled and EPR correlated states that give interesting results when applied as quantum channels for CV teleportation.

OUTLINE

- Nonclassicality using photon subtraction and addition
- Generalized number conserving operator
- Characterization of nonclassicality
- Generation of two-mode entanglement
- Analyzing continuous variable teleportation
- Summary and outlook

OUTLINE

- **Nonclassicality using photon subtraction and addition**
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NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

PHOTON ADDITION

$$|\Psi\rangle' = N\left(\hat{a}^{\dagger}\right)^{m}|\Psi\rangle,$$

where N is the normalization.

For a coherent state input, $|\alpha\rangle$ and a single photon addition

$$|\alpha, 1\rangle = \frac{\hat{a}^{\dagger}|\alpha\rangle}{\sqrt{1+|\alpha|^2}}.$$

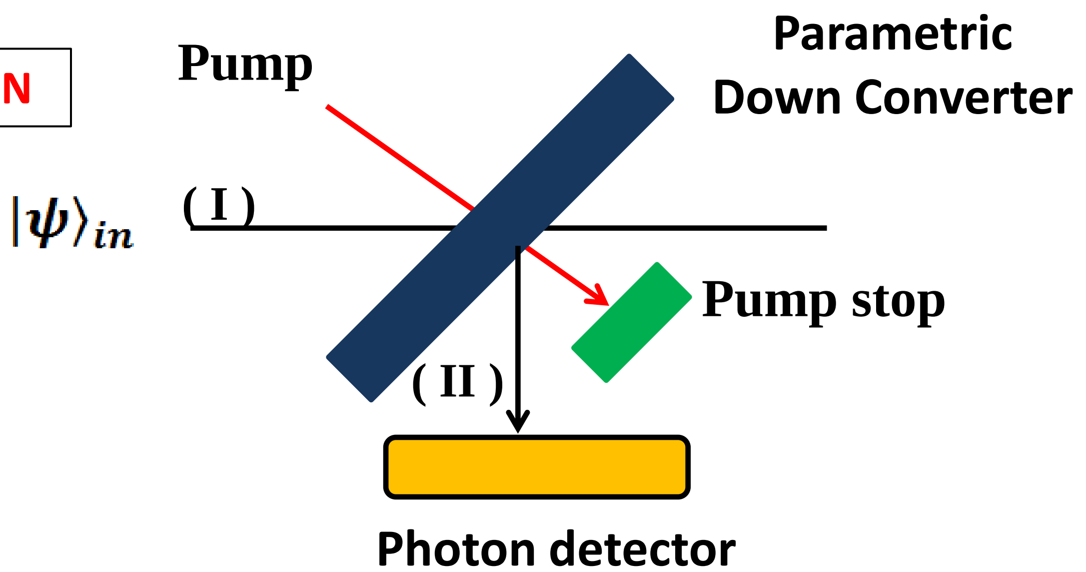
$|\alpha, 1\rangle$ is no more a classical coherent state.

Agarwal and Tara, Phys. Rev. A **43**, 492 (1991) – Study nonclassicality of coherent states upon multiple photon addition

NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

Experimentally Realizable:

PHOTON ADDITION

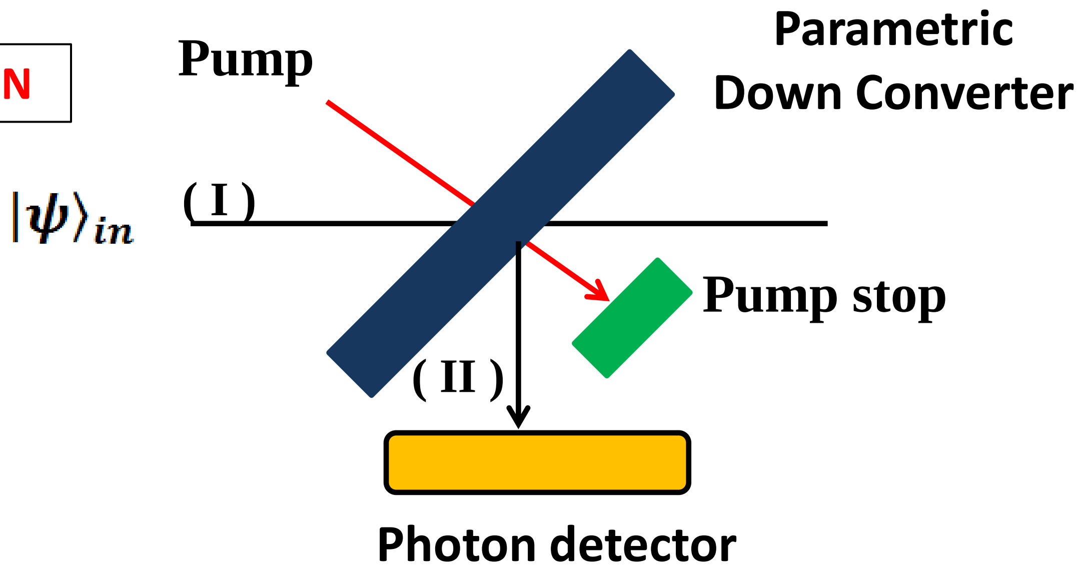


Zavatta et. al, Science, **306**, 660 (2004) – Experimentally study classical-nonclassical transition using photon addition

NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

Experimentally Realizable:

PHOTON ADDITION



$$P(g) = e^{\left(-g\hat{a}_I{}^{\dagger}\hat{a}_{II}{}^{\dagger} + g\hat{a}_I\hat{a}_{II}\right)}$$

$$P(g)|\Psi\rangle_I|0\rangle_{II} \approx \left(1 - g\hat{a}_I{}^{\dagger}\hat{a}_{II}{}^{\dagger}\right)|\Psi\rangle_I|0\rangle_{II}$$

For small coupling stregth , $g \ll 1$

NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

$$|\Psi\rangle' = N\,(\hat{a})^m|\Psi\rangle,$$

PHOTON SUBTRACTION

where N is the normalization.

For a coherent state input, $|\alpha\rangle$ and a single photon subtraction

$$\alpha|\alpha\rangle = \hat{a}|\alpha\rangle$$

does not introduce nonclassicality

NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

$$|\Psi\rangle' = N\,(\hat{a})^m|\Psi\rangle,$$

PHOTON SUBTRACTION

where N is the normalization.

For a two-mode squeezed vacuum state,

$$|\psi\rangle = \hat{S}(r)|0,0\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle$$

$\hat{a}|\psi\rangle$ does enhance nonclassicality

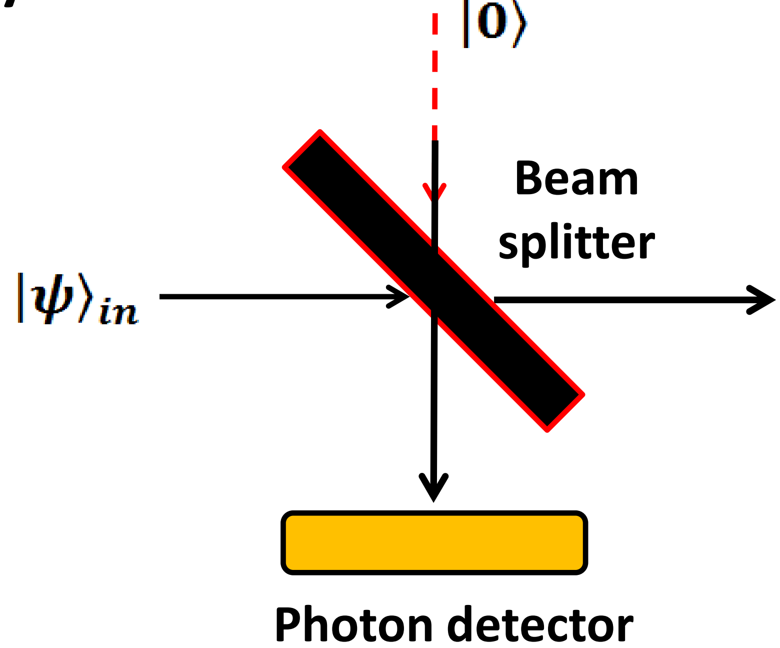
Kitagawa et. al, Phys. Rev. A **73**, 042310 (2006) - Entanglement enhancement generated by photon subtraction from squeezed states

Navarrete-Benlloch et. al, Phys. Rev. A **86**, 012328 (2012)- Enhancing quantum entanglement by photon addition and subtraction

NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

Experimentally Realizable:

PHOTON SUBTRACTION



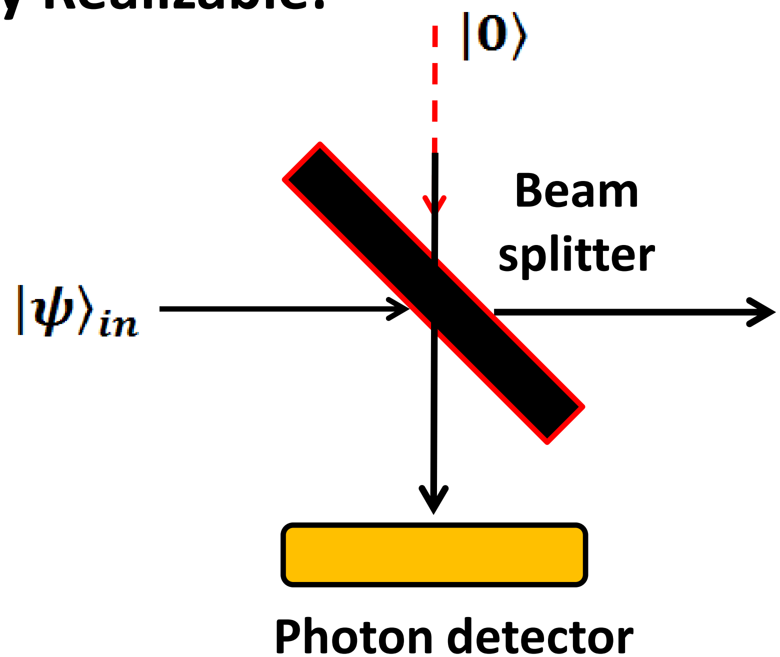
Parigi et. al, Science, **317**, 1890 (2007) – Subtraction of single photons

A. Ourjoumtsev et. al, Science **312**, 83 (2006) – Use of high-transmittive beams plitters.

NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

Experimentally Realizable:

PHOTON SUBTRACTION



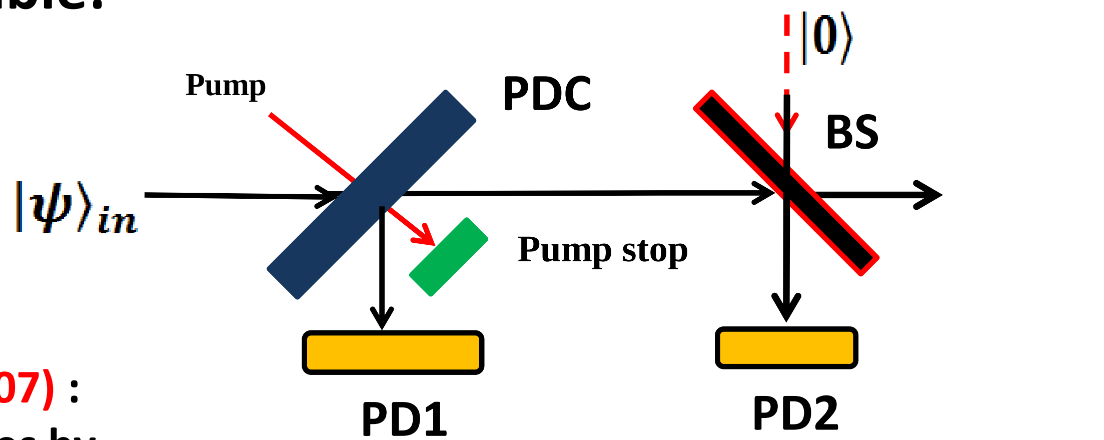
$$\hat{B}|\Psi\rangle_I|0\rangle_{II} = e^{-\frac{r}{t}\hat{a}_I\hat{a}_{II}^\dagger} \approx \left(1 - \frac{r}{t}\hat{a}_I\hat{a}_{II}^\dagger\right)|\Psi\rangle_I|0\rangle_{II},$$

where $\hat{B}$ is a lossless, high-transmissivity beam-splitters, with transmissivity t and reflectivity r

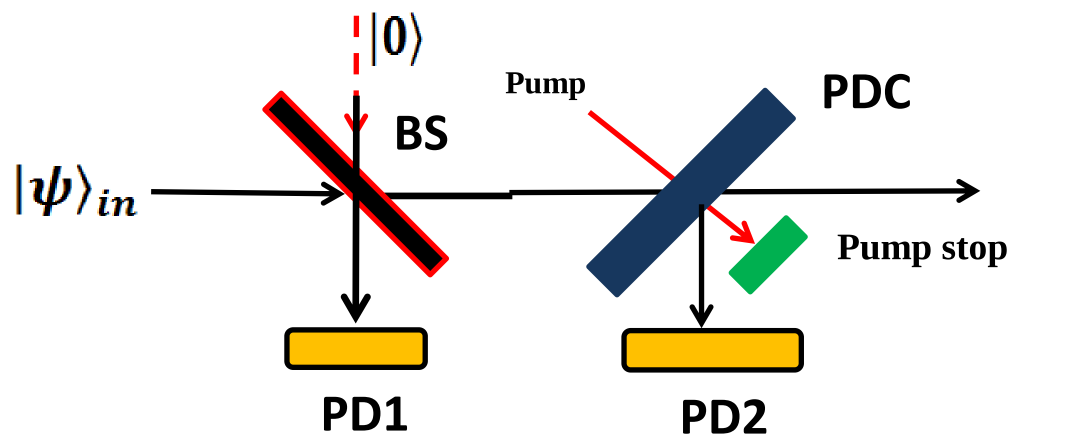
NONCLASSICALITY USING PHOTON SUBTRACTION AND ADDITION

Experimentally Realizable:

PHOTON ADDITION FOLLOWED BY SUBTRACTION



Parigi et. al, Science, 317, 1890 (2007) : Probing Quantum Commutation Rules by Addition and Subtraction of Single Photons to/from a Light Field



PHOTON SUBTRACTION FOLLOWED BY ADDITON

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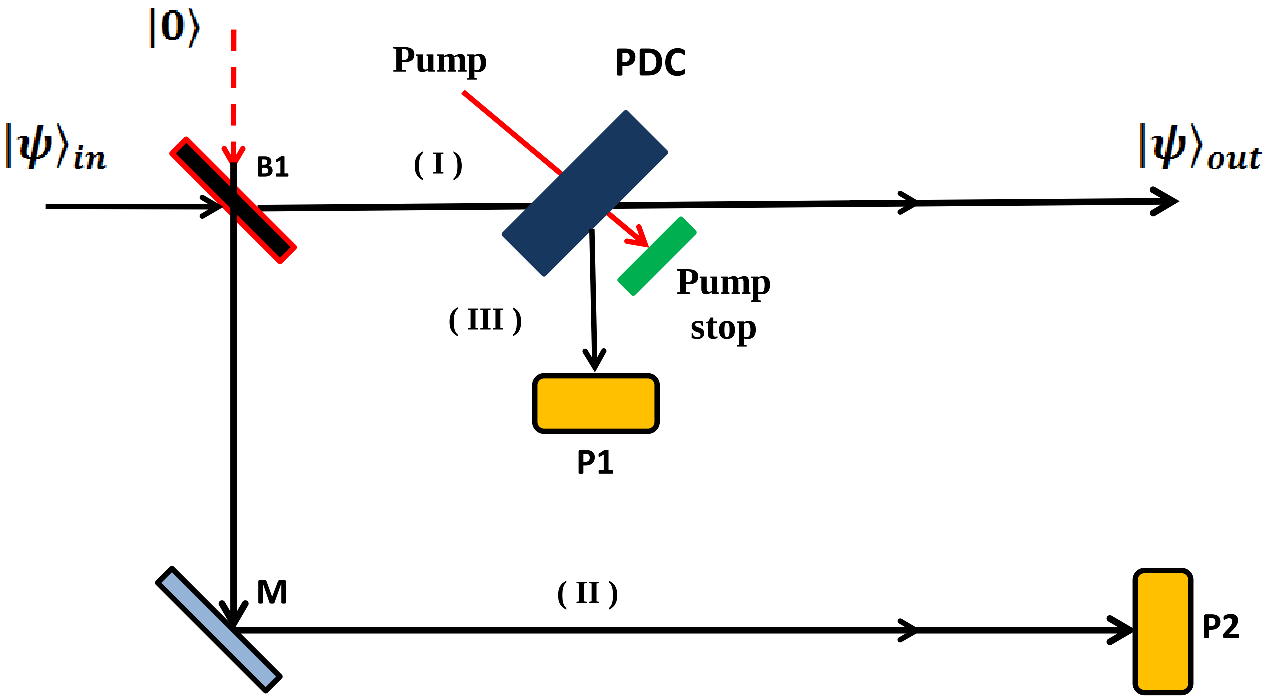
GENERALIZED NUMBER CONSERVING OPERATOR

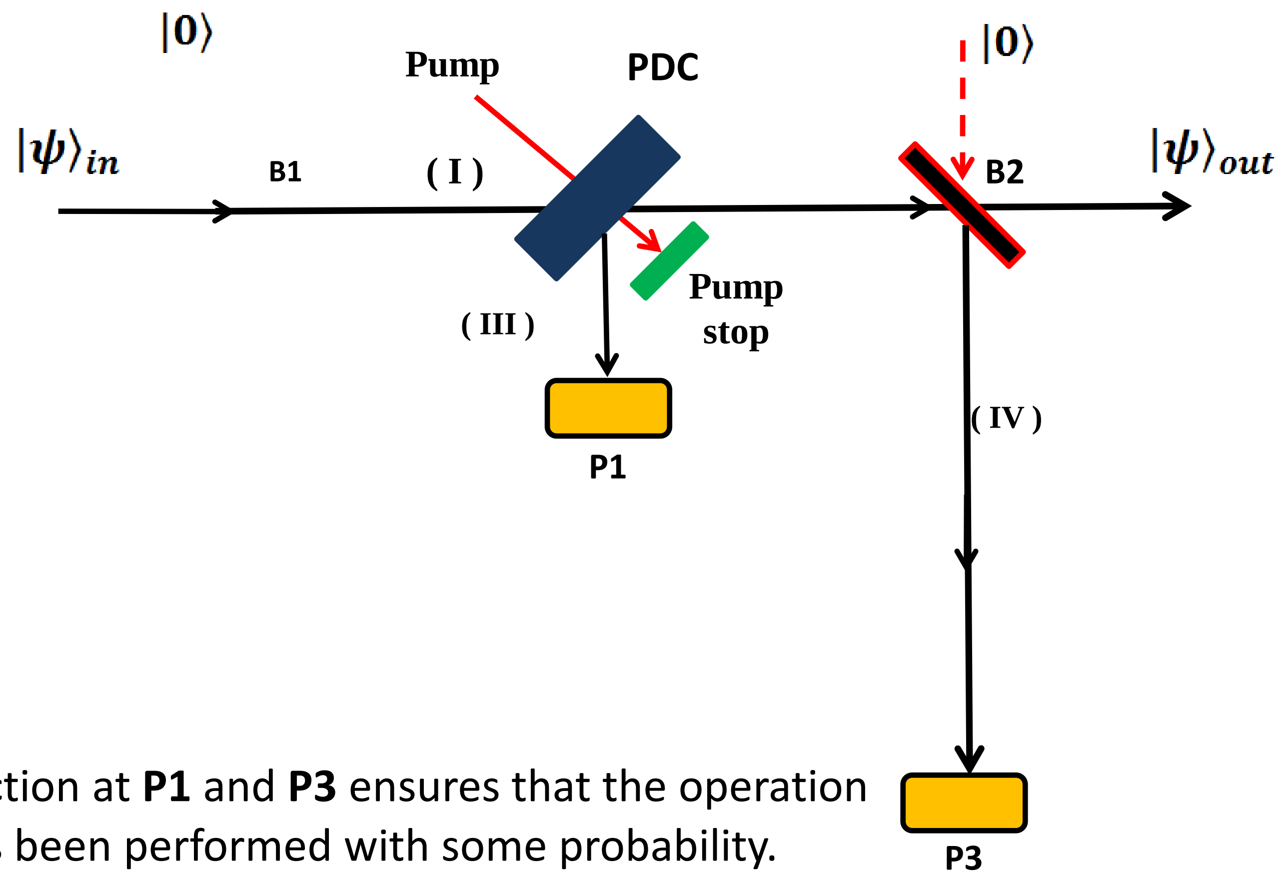
Experimental Scheme:



Photon detection at **P1** and **P2** ensures that the operation $\hat{a}\,\hat{a}^{\dagger}$ has been performed with some probability. **M** is a highly reflective, lossless mirror.

GENERALIZED NUMBER CONSERVING OPERATOR

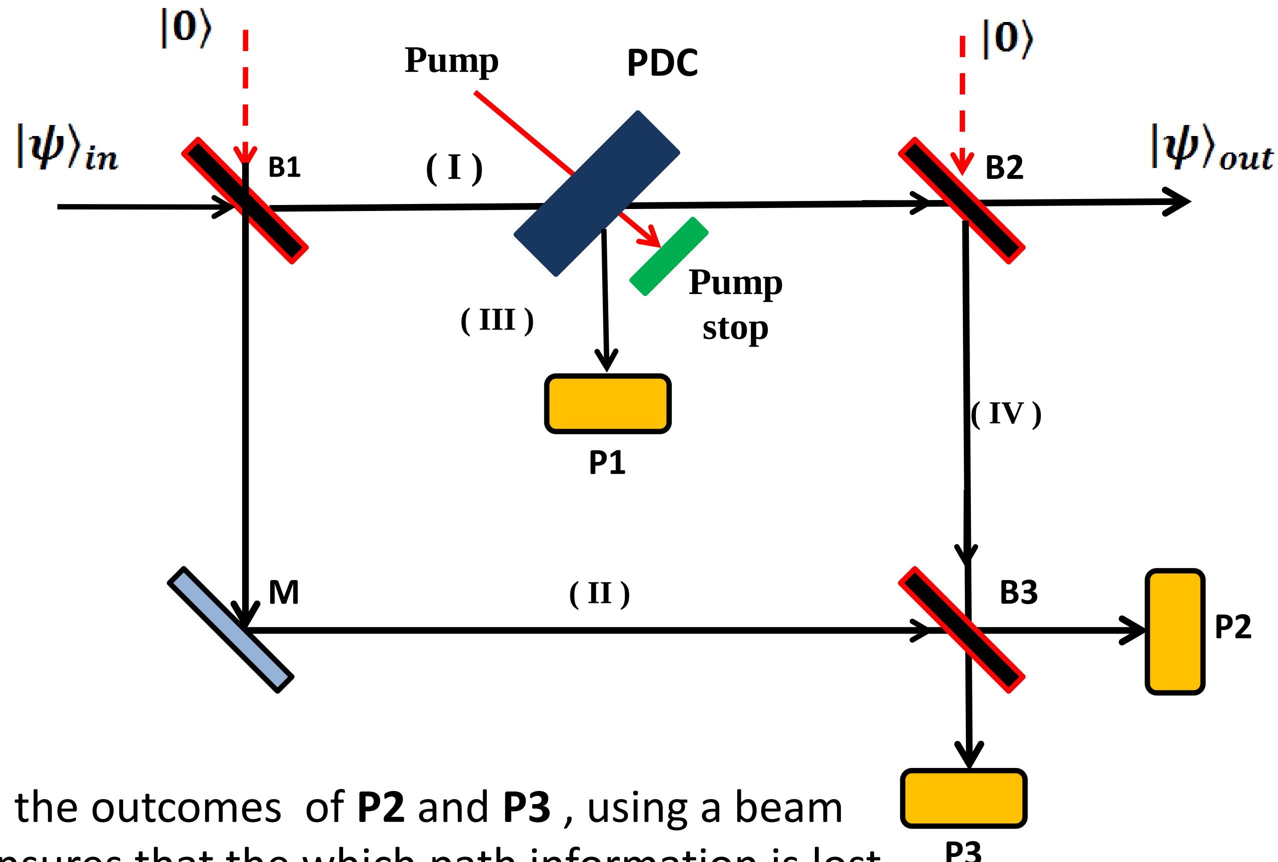
Experimental Scheme:



Photon detection at **P1** and **P3** ensures that the operation $\hat{a}^{\dagger}\hat{a}$ has been performed with some probability.

GENERALIZED NUMBER CONSERVING OPERATOR

Experimental Scheme:



Interfering the outcomes of **P2** and **P3** , using a beam splitter **B3**, ensures that the which path information is lost and we have the operation $s\hat{a}\hat{a}^{\dagger} + t\hat{a}^{\dagger}\hat{a}$.

GENERALIZED NUMBER CONSERVING OPERATOR

Theory of the experimental scheme:

Beamsplitter1 (B1, mode II) operating on **input (mode I):**

$$\hat{B}_{B1}|\psi\rangle_{\text{in,I}}|0\rangle_{\text{II}} \simeq \left(1 - \frac{r_1^*}{t_1}\hat{a}_{\text{I}}\hat{a}_{\text{II}}^\dagger\right)|\psi_{\text{in}},0\rangle_{\text{I,II}}.$$

PDC (mode III) operating on input after **B1**:

$$(1 - g\hat{a}_{\text{I}}\hat{a}_{\text{III}}^\dagger)\hat{B}_{B1}|\psi_{\text{in}},0,0\rangle_{\text{I,II,III}}$$

$$\simeq \left(1 - g\hat{a}_{\text{I}}\hat{a}_{\text{III}}^\dagger - \frac{r_1^*}{t_1}\hat{a}_{\text{I}}\hat{a}_{\text{II}}^\dagger + g\frac{r_1^*}{t_1}\hat{a}_{\text{I}}\hat{a}_{\text{I}}^\dagger\hat{a}_{\text{II}}^\dagger\hat{a}_{\text{III}}^\dagger\right)|\psi_{\text{in}},0,0\rangle_{\text{I,II,III}}.$$

State after detection of **1 photon** at **PDC**:

$$\left(-g\hat{a}_{\text{I}}^\dagger + g\frac{r_1^*}{t_1}\hat{a}_{\text{I}}^\dagger\hat{a}_{\text{I}}\hat{a}_{\text{II}}^\dagger\right)|\psi_{\text{in}},0\rangle_{\text{I,II}}.$$

GENERALIZED NUMBER CONSERVING OPERATOR

Theory of the experimental scheme:

Beamsplitter2 (B2, mode IV) operating after **B1** and **PDC** to obtain final state

$$\left(1 - \frac{r_2^*}{t_2}\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{IV}}^\dagger\right)\left(-g\hat{a}_{\mathrm{I}}^\dagger + g\frac{r_1^*}{t_1}\hat{a}_{\mathrm{I}}^\dagger\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{II}}^\dagger\right)|\psi_{\mathrm{in}}, 0, 0\rangle_{\mathrm{I,II,IV}}$$

$$\simeq \left(-g\hat{a}_{\mathrm{I}}^\dagger + g\frac{r_1^*}{t_1}\hat{a}_{\mathrm{I}}^\dagger\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{II}}^\dagger - g\frac{r_2^*}{t_2}\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{I}}^\dagger\hat{a}_{\mathrm{IV}}^\dagger - g\frac{r_1^*}{t_1}\frac{r_2^*}{t_2}\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{I}}^\dagger\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{II}}^\dagger\hat{a}_{\mathrm{IV}}^\dagger\right)$$

$$\times\,|\psi_{\mathrm{in}}, 0, 0\rangle_{\mathrm{I,II,IV}}.$$

The superposed operations on the input state after **beamsplitter (B3)** interference

$$\left(gt_3\frac{r_1^*}{t_1}\hat{a}_{\mathrm{I}}^\dagger\hat{a}_{\mathrm{I}} - r_3 g\frac{r_2^*}{t_2}\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{I}}^\dagger\right)|\psi\rangle_{\mathrm{in}},$$

$$\left(-gr_3^*\frac{r_1^*}{t_1}\hat{a}_{\mathrm{I}}^\dagger\hat{a}_{\mathrm{I}} - t_3^* g\frac{r_2^*}{t_2}\hat{a}_{\mathrm{I}}\hat{a}_{\mathrm{I}}^\dagger\right)|\psi\rangle_{\mathrm{in}},$$

Suitably adjusting the optical paramteres we get the number-conserving superposed operation $s\hat{a}\hat{a}^\dagger + t\hat{a}^\dagger\hat{a}$.

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CHARACTERIZATION OF NONCLASSICALITY

Nonclassicality indicators: Wigner distribution

$$W(\beta, \beta^*) = \frac{2}{\pi^2} e^{2|\beta|^2} \int d^2\gamma \langle -\gamma | \hat{\rho} | \gamma \rangle e^{-2(\beta^*\gamma - \beta\gamma^*)}$$

is the **Wigner distribution** in terms of the coherent state

$$|\gamma\rangle = \exp(-|\gamma|^2/2 + \gamma \hat{a}^\dagger)|0\rangle$$

The expression can be written in a series form,

$$W(\beta, \beta^*) = \frac{2}{\pi} \sum_{k=0}^{\infty} (-1)^k \langle \beta, k | \hat{\rho} | \beta, k \rangle,$$

where, $|\beta, k\rangle$ is the displaced number state.

The **positivity** of the Wigner function is a **necessary but not sufficient** condition of classicality

CHARACTERIZATION OF NONCLASSICALITY

Nonclassicality indicators: Mandel's Q-parameter

Mandel's Q-parameter is a measure of the photon statistics and its deviation from the Poissonian nature.

$$Q \equiv \frac{\langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2}{\langle \hat{a}^\dagger \hat{a} \rangle}.$$

$Q < 0$ corresponds to states with sub-Poissonian statistics that have no classical analogue.

CHARACTERIZATION OF NONCLASSICALITY

Nonclassicality indicators: Quadrature squeezing

A state is said to be **squeezed** if any of its quadrature deviation is less than its coherent state value.

$$S_{\rm opt} = \langle : (\Delta \hat{X}_\theta)^2 : \rangle_{\min}$$
$$= -2|\langle \hat{a}^{\dagger 2}\rangle - \langle \hat{a}^\dagger\rangle^2| + 2\langle \hat{a}^\dagger \hat{a}\rangle - 2|\langle \hat{a}^\dagger\rangle|^2$$

The nonclassical states correspond to negative values of $S_{\rm opt}$ where $-1 \leqslant S_{\rm opt} < 0.$

CHARACTERIZATION OF NONCLASSICALITY

Coherent state input

$$|\alpha\rangle = \exp\left(\frac{-|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

$$\hat{\rho}_{\rm coh} = N_1^{-1}[s(\hat{a}\hat{a}^\dagger) + t(\hat{a}^\dagger\hat{a})]|\alpha\rangle\langle\alpha|[s(\hat{a}\hat{a}^\dagger) + t(\hat{a}^\dagger\hat{a})].$$

$$N_1 = s^2 + (s+t)(3s+t)|\alpha|^2 + (s+t)^2|\alpha|^4$$

$$\begin{aligned} W_{\rm SOCS}(\beta,\beta^*) &= W_{\rm coh}(\beta,\beta^*) \\ &\quad \times N_1^{-1}[M_1{}^2 + 2(s+t)M_1(\alpha^*\beta + \alpha\beta^*) \\ &\quad + (s+t)^2|\alpha|^2(4|\beta|^2 - 1)], \end{aligned}$$

$$W_{\rm coh}(\beta,\beta^*) = \frac{2}{\pi}\,{\rm e}^{-2|\beta-\alpha|^2}$$

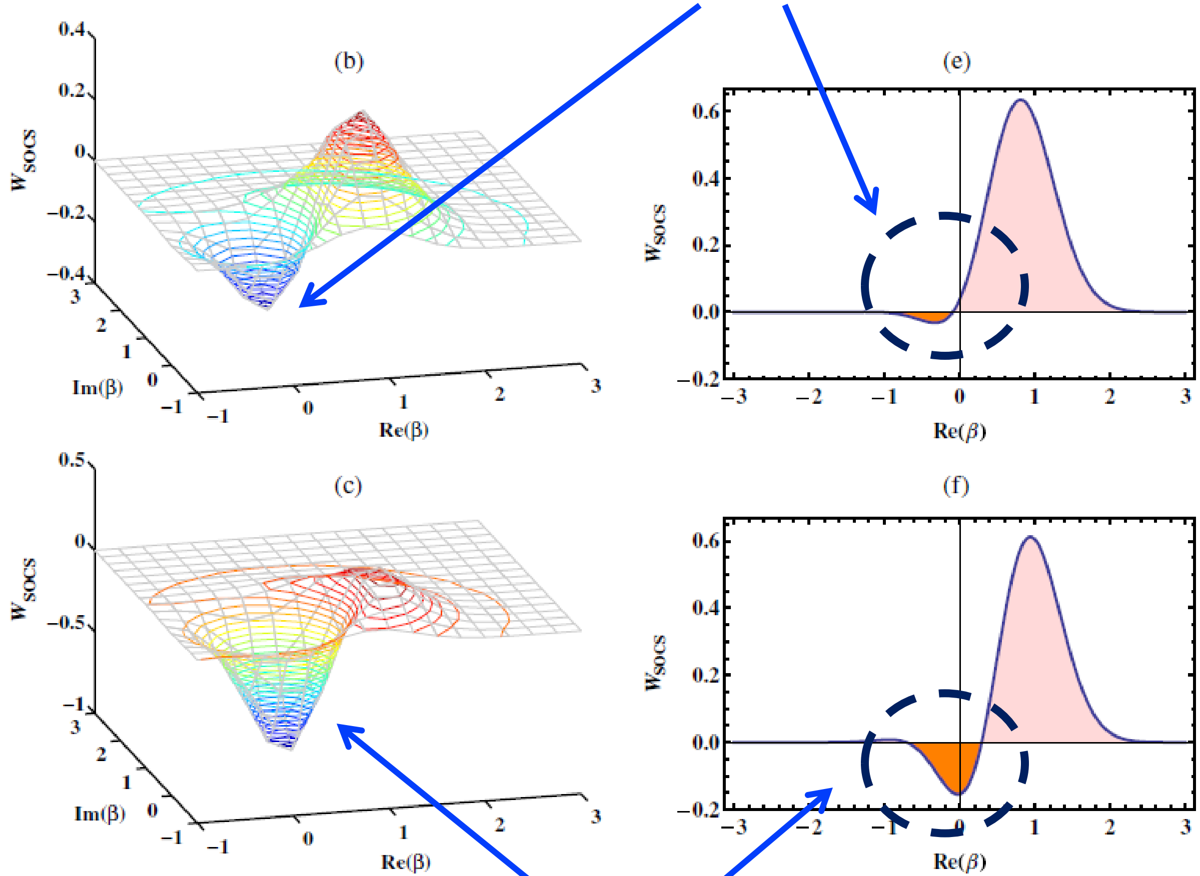
CHARACTERIZATION OF NONCLASSICALITY

Coherent state input

Negative region of Wigner dist.

$|\alpha| = 0.4,$
$t = 0.5$

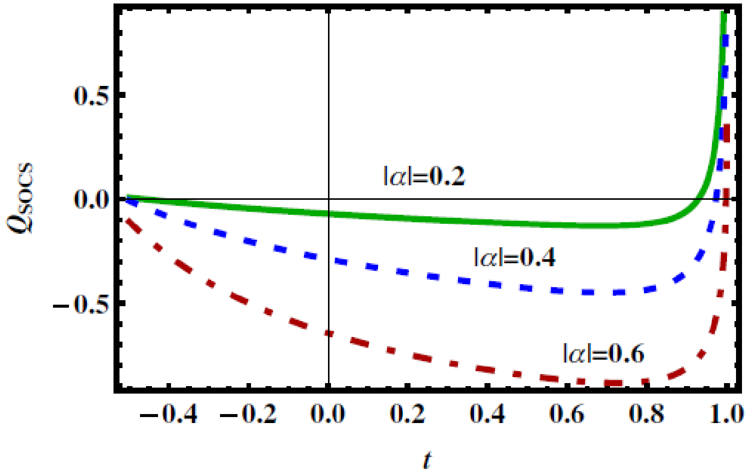
$|\alpha| = 0.4,$
$t = 0.9$



Negative region of Wigner dist.

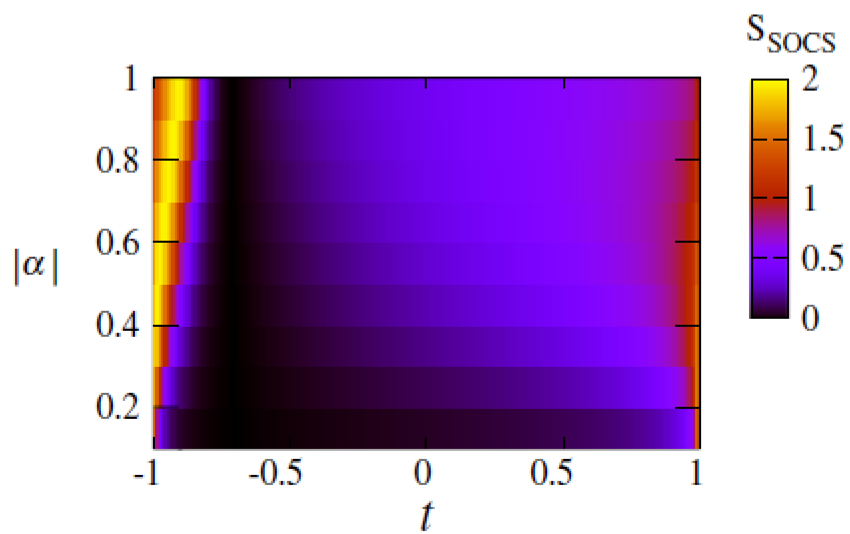
CHARACTERIZATION OF NONCLASSICALITY

Coherent state input



Negative region of Q parameter.

No negative values of the S parameter.



CHARACTERIZATION OF NONCLASSICALITY

Thermal state input $$\hat{\rho}_{\text{in}} = \frac{1}{(1+\bar{n})} \sum_n \left(\frac{\bar{n}}{1+\bar{n}} \right)^n |n\rangle\langle n|$$

$$\hat{\rho}_{\text{th}} = N_2^{-1}[s(\hat{a}\hat{a}^\dagger) + t(\hat{a}^\dagger\hat{a})]\hat{\rho}_{\text{in}}[s(\hat{a}\hat{a}^\dagger) + t(\hat{a}^\dagger\hat{a})],$$

$$N_2 = s^2(1+\bar{n})(1+2\bar{n}) + 4st\bar{n}(1+\bar{n}) + t^2\bar{n}(1+2\bar{n})$$

$$W_{\text{SOTS}}(\beta, \beta^*) = W_{\text{th}}(\beta, \beta^*)N_2^{-1}[(M_2+s)^2 + (s+t)M_2],$$

$$W_{\text{th}}(\beta, \beta^*) = \frac{2}{\pi}\frac{1}{(1+2\bar{n})}\,\mathrm{e}^{-\frac{2|\beta|^2}{1+2\bar{n}}}$$

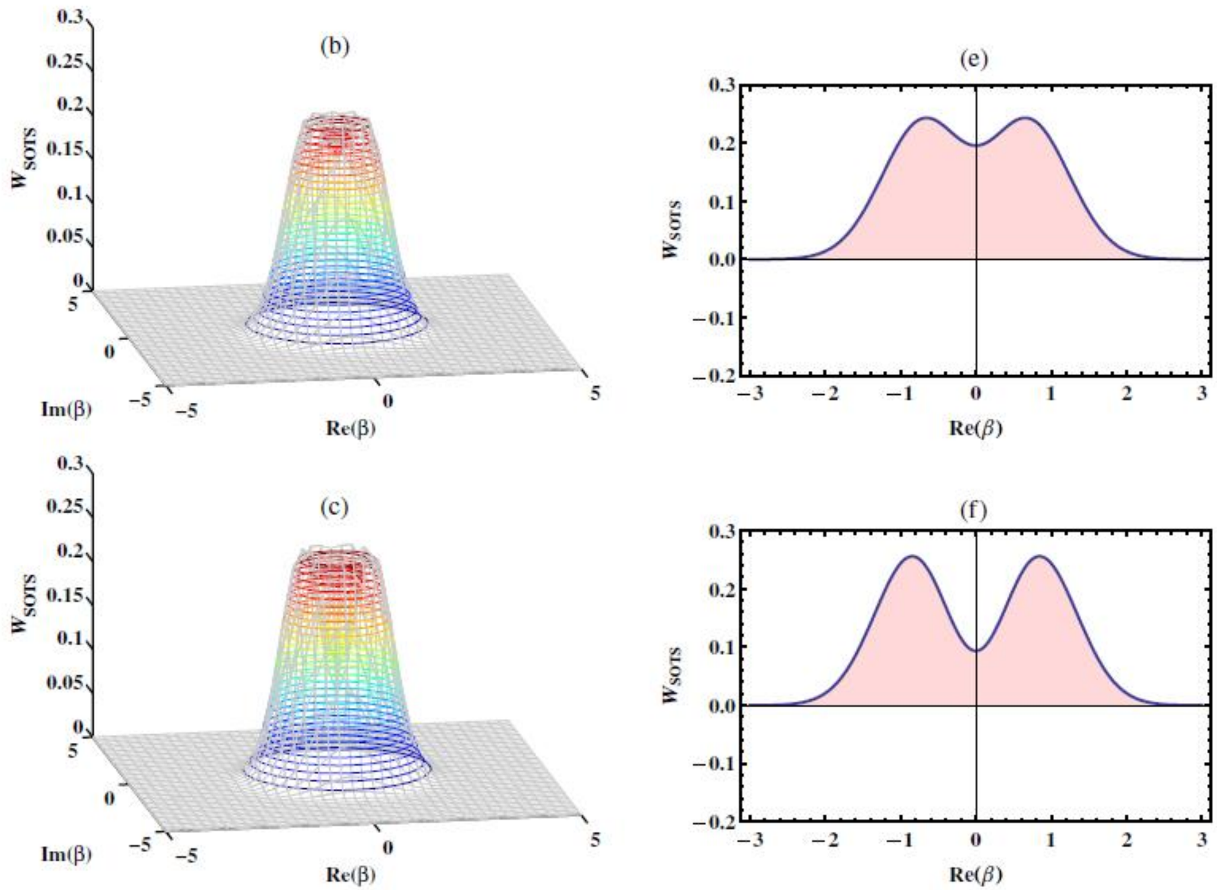
$$M_2 = \frac{4\bar{n}(1+\bar{n})}{(1+2\bar{n})^2}(s+t)|\beta|^2$$

CHARACTERIZATION OF NONCLASSICALITY

Thermal state input

$|\alpha| = 0.4,$
$t = 0.5$

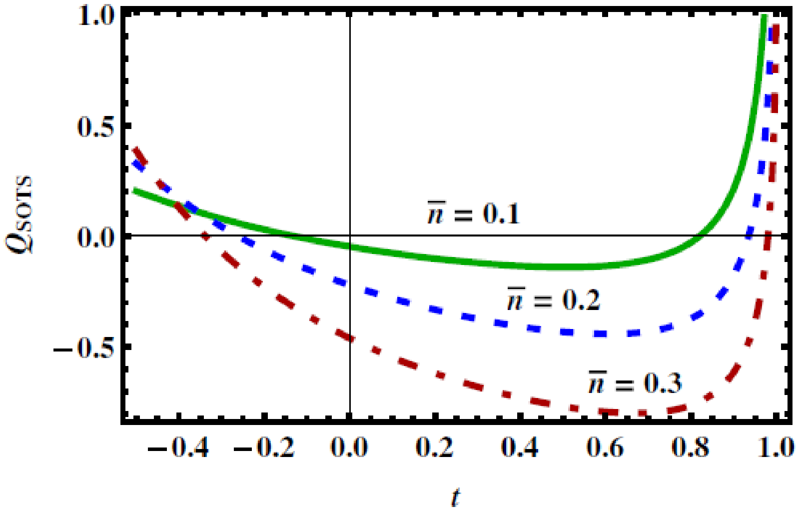
$|\alpha| = 0.4,$
$t = 0.9$



No negative region of Wigner dist.

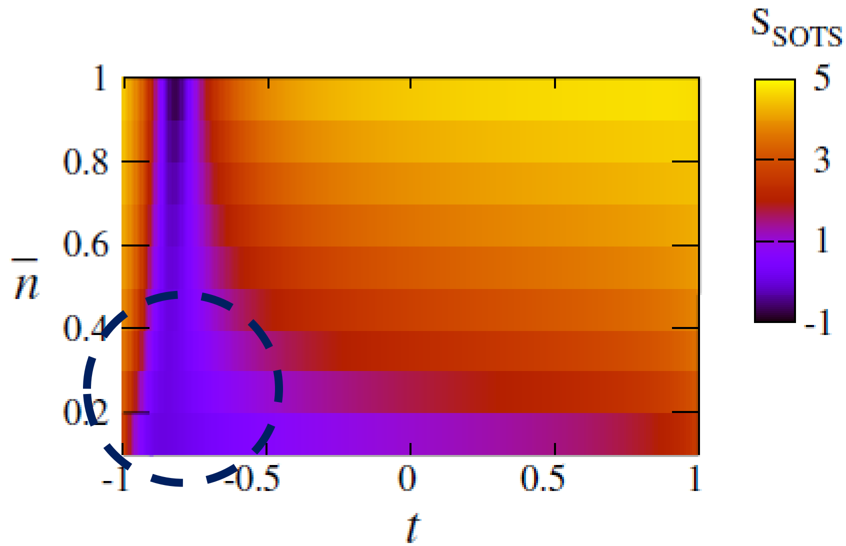
CHARACTERIZATION OF NONCLASSICALITY

Thermal state input



Negative region of Q parameter.

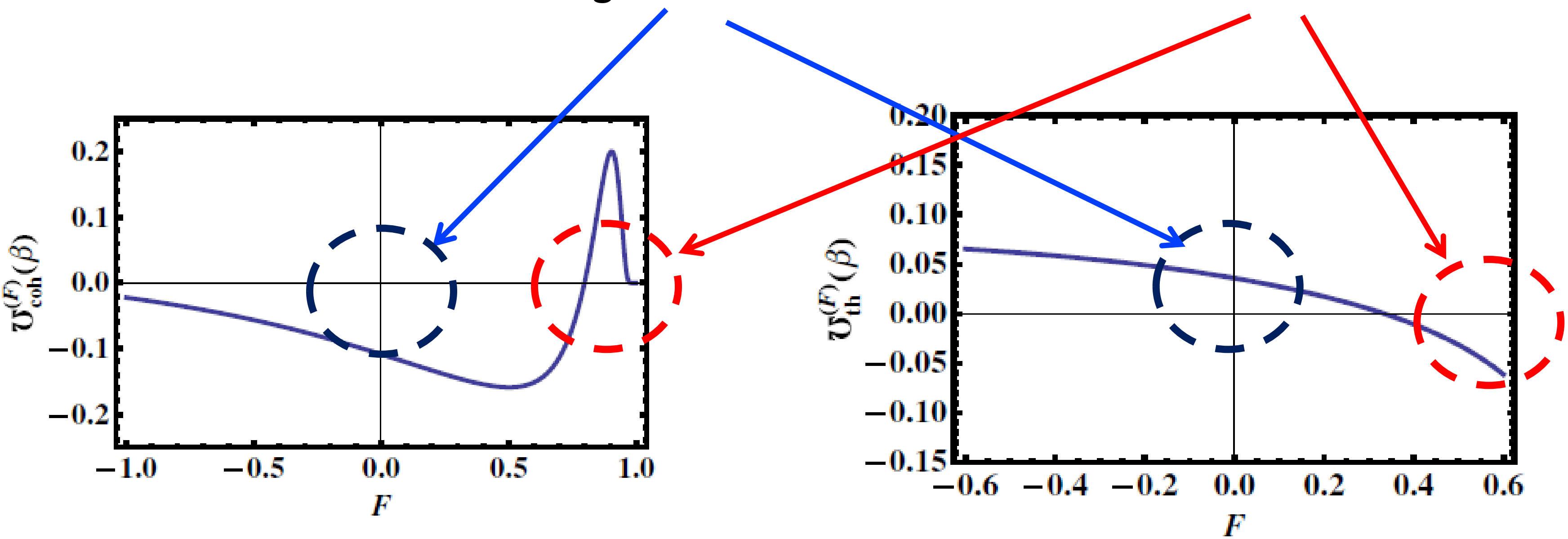
Negative values of the S parameter.



CHARACTERIZATION OF NONCLASSICALITY

Using parameterized quasiprobability

Wigner function dist. **Sudarshan *P* function dist.**



Coherent state **Thermal state**

The thermal state has a **negative** quasiprobability distribution corresponding to the *P*- function.

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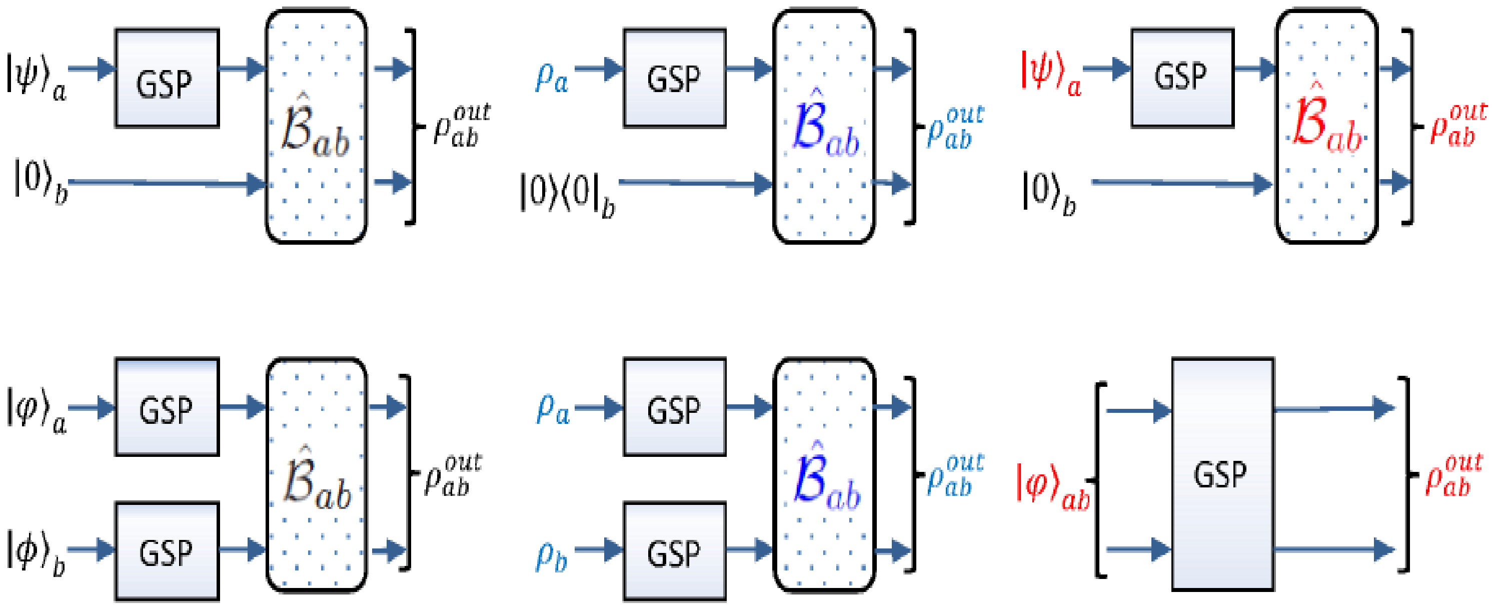
GENERATION OF TWO-MODE ENTANGLEMENT



Single mode input

Two mode input

Coherent state **Thermal state** **Squeezed state**

GSP correspond to the generalized number-conserving superposed operations.
$\widehat{\boldsymbol{B}}_{\boldsymbol{ab}}$ correspond to the mode interacting beamsplitter operation

In **single mode** operations the second mode is a vacuum mode. For squeezed state two-mode input the $\widehat{\boldsymbol{B}}_{\boldsymbol{ab}}$ operation is pre-considered in generating the state.

GENERATION OF TWO-MODE ENTANGLEMENT

Single mode input: Coherent state

GSP operation.
$D_a(\alpha)$ is the displacement operator

$$|\alpha'\rangle_a = \frac{1}{\sqrt{N}}(s\hat{a}\hat{a}^\dagger + t\hat{a}^\dagger\hat{a})|\alpha\rangle_a$$
$$= \frac{D_a(\alpha)}{\sqrt{N}}(s + \alpha(s+t)(\hat{a}^\dagger + \alpha^*))|0\rangle_a$$

Beamsplitter ($\hat{B}_{ab}$) operation with a vacuum input in mode **b**.
A_{ab} are operators acting locally.

$$|\phi\rangle_{ab}^{out} = \hat{\mathcal{B}}_{ab}\,|\alpha'\rangle_a \otimes |0\rangle_b$$
$$= \mathcal{A}_{ab}\left[p_1|00\rangle_{ab} + \frac{p_2}{\sqrt{2}}(|01\rangle_{ab} + |10\rangle_{ab})\right]$$

Since, the operators A_{ab} act locally on ρ_{ab} hence, the Entanglement properties can be calculated using this.

$$\rho_{ab}^{out} = \mathcal{A}_{ab}\,\rho_{ab}^0\,\mathcal{A}_{ab}^\dagger$$

$$\rho_{ab}^0 = \begin{pmatrix} p_1^2 & p_1p_2/\sqrt{2} & p_1p_2/\sqrt{2} & 0 \\ p_1p_2/\sqrt{2} & p_2^2/2 & p_2^2/2 & 0 \\ p_1p_2/\sqrt{2} & p_2^2/2 & p_2^2/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

GENERATION OF TWO-MODE ENTANGLEMENT

Single mode input: Thermal states

GSP operation

$$\rho_a^{'\,th} = \frac{M^{-1}}{1+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n (s + n(s+t))^2 |n\rangle\langle n|_a.$$

Beamsplitter ($\hat{B}_{ab}$) operation with a vacuum input in mode **b**.

$$\begin{aligned} \rho_{ab} &= \hat{\mathcal{B}}_{ab} \left(\rho_a^{'\,th} \otimes |0\rangle\langle 0|_b \right) \hat{\mathcal{B}}^{\dagger}_{ab} \\ &= \hat{\mathcal{B}}_{ab} \left(\sum_{n=0}^{\infty} \frac{q_n}{n!} (\hat{a}^{\dagger})^n |00\rangle\langle 00|_{ab} (\hat{a})^n \right) \hat{\mathcal{B}}^{\dagger}_{ab} \end{aligned}$$

Using low-field intensity ($\bar{n} \leq 0.1$) to allow optical truncation, we obtain a low dimensional $\boldsymbol{\rho_{ab}}$.

$$\rho_{ab}^{out} = \begin{pmatrix} q_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{q_1}{2} & 0 & \frac{q_1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{q_2}{4} & 0 & -\frac{q_2}{2\sqrt{2}} & 0 & \frac{q_2}{4} & 0 & 0 \\ 0 & \frac{q_1}{2} & 0 & \frac{q_1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{q_2}{2\sqrt{2}} & 0 & \frac{q_2}{2} & 0 & -\frac{q_2}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{q_2}{4} & 0 & -\frac{q_2}{2\sqrt{2}} & 0 & \frac{q_2}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

GENERATION OF TWO-MODE ENTANGLEMENT

Single mode input: Squeezed states

GSP operation

$$|\psi\rangle = \frac{1}{\sqrt{N}}(s\hat{a}\hat{a}^\dagger + t\hat{a}^\dagger\hat{a})(1-\lambda^2)^{1/4} \times \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!}\left(-\frac{\lambda}{2}\right)^n |2n\rangle_a$$

$$= \sqrt{\frac{(1-\lambda^2)^{1/2}}{N}} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!}\left(-\frac{\lambda}{2}\right)^n \times (s + 2n(s+t))|2n\rangle_a,$$

Beamsplitter (**$\hat{B}_{ab}$**) operation with a vacuum input in mode **b**.

$$|\psi'\rangle = \hat{\mathcal{B}}_{ab} \sum_{n=0}^{\infty} c_n \frac{(\hat{a}^\dagger)^{2n}}{\sqrt{(2n)!}} |0\rangle_a |0\rangle_b$$

Using low squeezing $(\lambda \ll 1)$ to allow optical truncation, we obtain a low dimensional output.

$$|\psi'\rangle = c_0|00\rangle_{ab} + c_1 \hat{\mathcal{B}}_{ab} (\hat{a}^\dagger)^2 \hat{\mathcal{B}}_{ab}^\dagger |00\rangle_{ab}$$

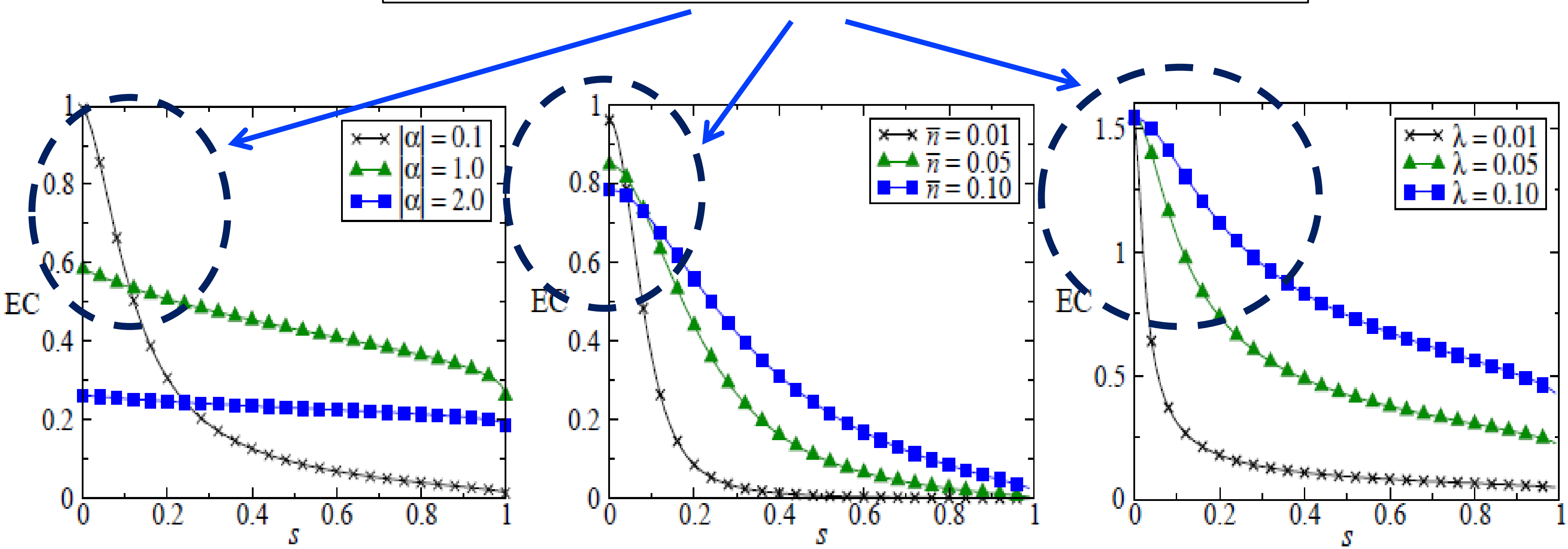
$$= c_0|00\rangle_{ab} + c_1 \left(\frac{\hat{a}^\dagger - \hat{b}^\dagger}{\sqrt{2}}\right)\left(\frac{\hat{a}^\dagger - \hat{b}^\dagger}{\sqrt{2}}\right)|00\rangle_{ab}$$

$$= c_0|00\rangle_{ab} + \frac{c_1}{\sqrt{2}}|20\rangle_{ab} + \frac{c_1}{\sqrt{2}}|02\rangle_{ab} - c_1|11\rangle_{ab}$$

GENERATION OF TWO-MODE ENTANGLEMENT

BIPARTITE ENTANGLEMENT BETWEEN THE MODES:
SINGLE MODE INPUT

REGIONS OF HIGH ENTANGLEMENT

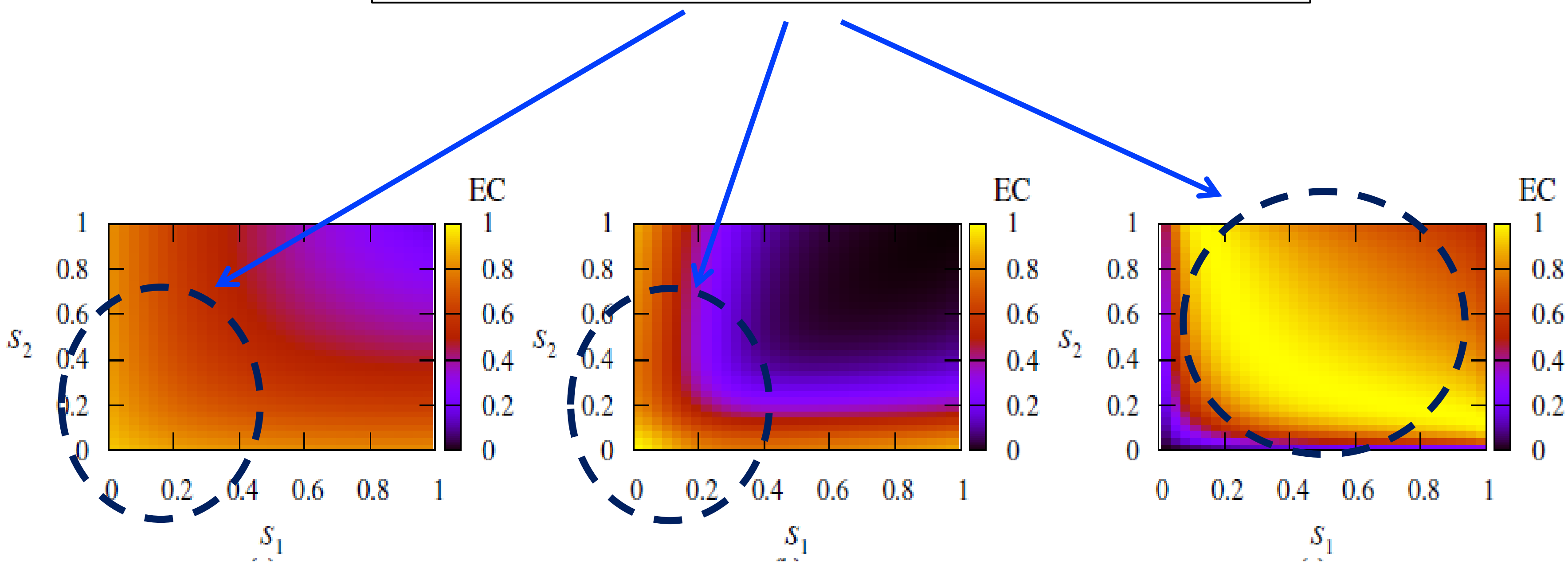


Coherent state **Thermal state** **Squeezed state**

GENERATION OF TWO-MODE ENTANGLEMENT

BIPARTITE ENTANGLEMENT BETWEEN THE MODES: TWO MODE INPUT

REGIONS OF HIGH ENTANGLEMENT



Coherent state | **Thermal state** | **Squeezed state**

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ANALYZING CONTINUOUS VARIABLE TELEPORTATION

BRAUNSTEIN KIMBLE PROTOCOL
FOR CONTINUOUS VARIABLE TELEPORTATION

$$W(\alpha,\beta,\gamma) = W_{in}(\gamma)\otimes W_{ch}(\alpha,\beta),$$

Where, $W_{in}(\gamma)$ and $W_{ch}(\alpha,\beta)$ are the Wigner distribution of the state to be teleported and the quantum channel respectively. After performing homodyne measurement and classical communication, the output state is given by

$W_{out}(\beta) = \int d^2z\, P(z)\, W(\beta - gz, z)$, $\chi_{out} = \chi_{in}(\alpha)\chi_{ch}(\alpha,\alpha^*)$, where χ_{in} and χ_{out} are characteristic functions of the input and output state.

post homodyne measurement

$$W(\beta,z) = \frac{2}{P(z)}\int d^2x_\nu d^2p_\mu W_{in}\left(\frac{\mu-\nu}{\sqrt{2}}\right)\otimes W_{ch}\left(\frac{\mu+\nu}{\sqrt{2}}\right)$$

AVERAGE FIDELITY

$$F = \pi\int d^2\beta W_{in}(\beta)W_{out}(\beta) = \frac{1}{\pi}\int d^2\alpha\chi_{in}(\alpha)\chi_{out}(-\alpha)$$

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

Single mode input: Coherent state

Characteristic function of channel:

$$\chi_{ch}(\xi,\eta) = N^{-1}(s^2 + (s+t)^2|\alpha|^2\{1+(\alpha+X)(\alpha^*-X^*)\} \\ +s(s+t)\{\alpha(\alpha^*-X^*)+\alpha^*(\alpha+X)\}) \\ \times \exp\left[\frac{1}{2}(|X|^2+|Y|^2)\right]\exp(\alpha^*X-\alpha X)$$

$\chi_{ch}(\alpha,\alpha^*)$

Characteristic function of single mode coherent input:

$$\chi^{coh}(\gamma) = \exp\left[-\frac{1}{2}|\gamma|^2\right]\exp(\alpha^*\gamma-\alpha\gamma^*)$$

$\chi_{in}(\alpha)$

Characteristic function of single mode squeezed input:

$$\chi_{sqz}(\gamma) = \exp\left[-\frac{\cosh 2r'}{2}|\gamma|^2 - \frac{\sinh 2r'}{4}(\gamma^2+\gamma^{*2})\right]$$

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

Single mode input: Coherent state

Characteristic function of channel:

$$\chi_{ch}(\xi,\eta) = N^{-1}(s^2 + (\ldots \ldots (\alpha + X)(\alpha^* - X^*)\}$$

$$+s(\ldots)$$

$\chi_{ch}(\alpha,\alpha^*)$

$$\chi_{out} = \chi_{in}(\alpha)\chi_{ch}(\alpha,\alpha^*)$$

$$F = \frac{1}{\pi}\int d^2\alpha \;\; \chi_{in}(\alpha)\chi_{out}(-\alpha)$$

Characteristic func…

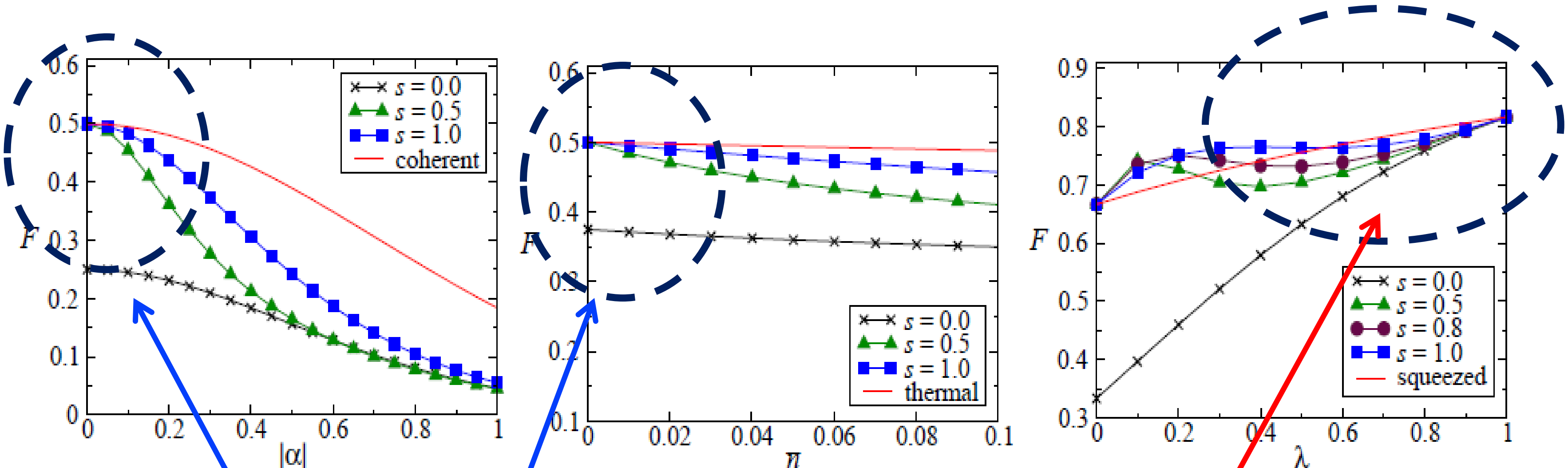
Characteristic function of …

$\chi_{in}(\alpha)$

$$\chi_{sqz}(\gamma) = \exp\left[- \frac{\ldots}{2}|\gamma|^2 - \frac{\sinh 2r'}{4}(\gamma^2 + \gamma^{*2}) \right]$$

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

Single mode GSP channel and single mode coherent input



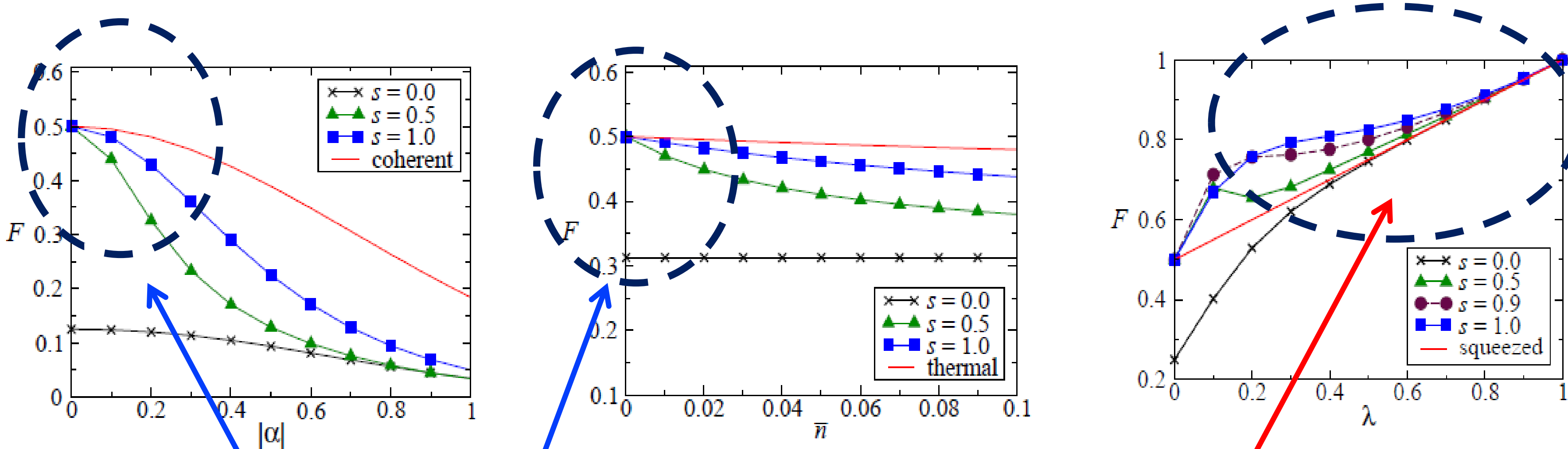
Coherent state **Thermal state** **Squeezed state**

Fidelity below classical limit

Enhanced Fidelity

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

Two mode GSP channel and single mode coherent input



Coherent state

Thermal state

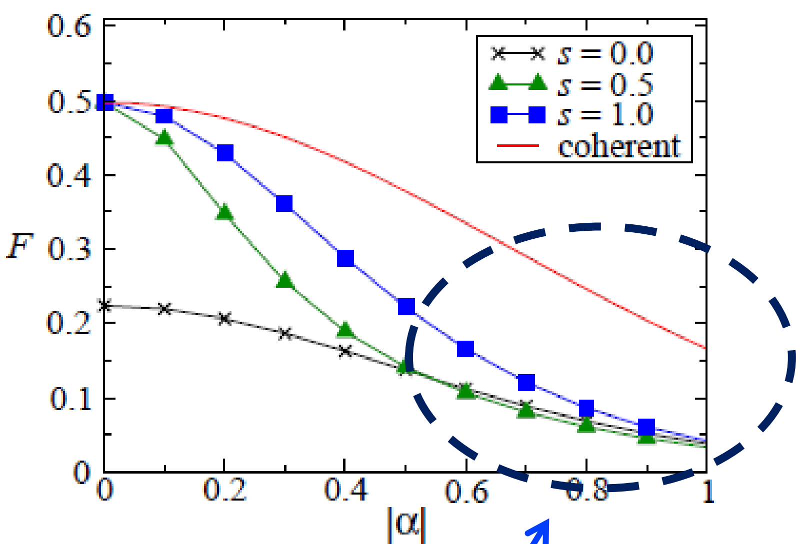
Squeezed state

Fidelity below classical limit

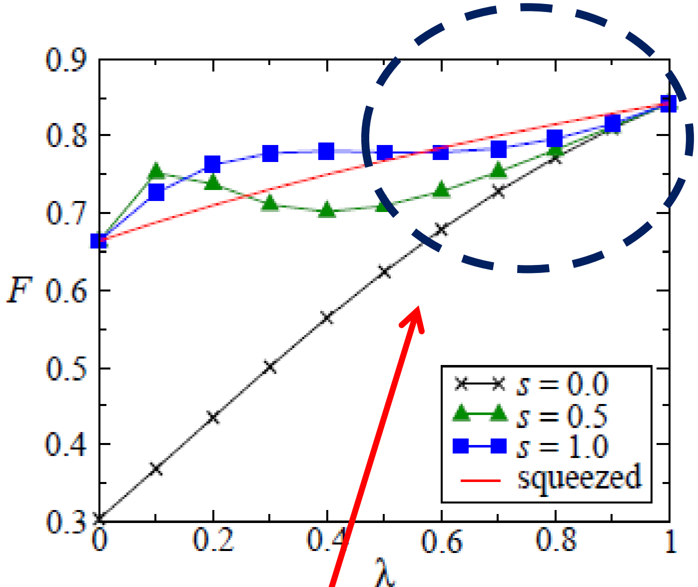
Enhanced Fidelity as high as F=0.8 for low squeezing

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

Single mode GSP channel and single mode squeezed input



Coherent state



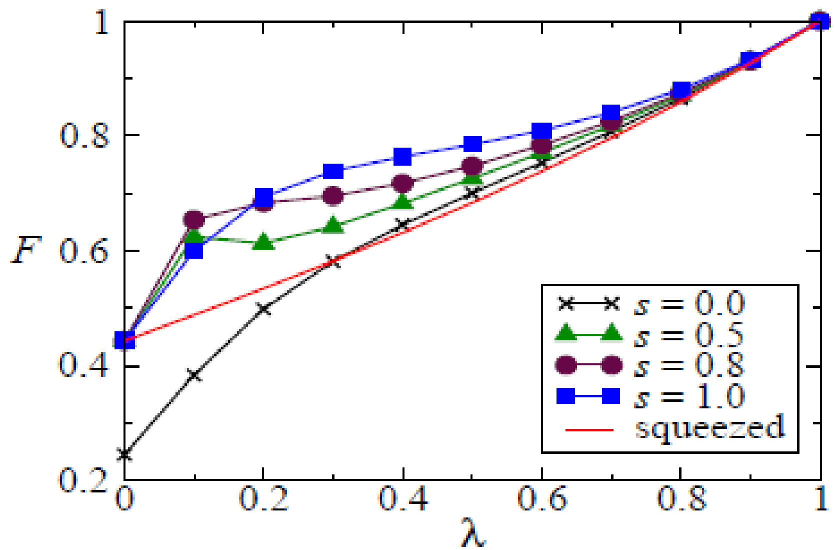
Squeezed state

Steady decrease of classical fidelity

Enhancement of quantum fidelity

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

Two mode GSP channel and single mode squeezed input



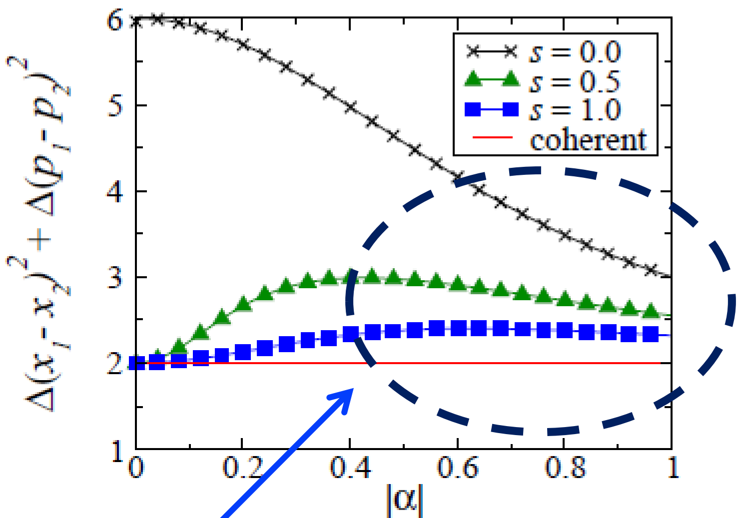
Squeezed state

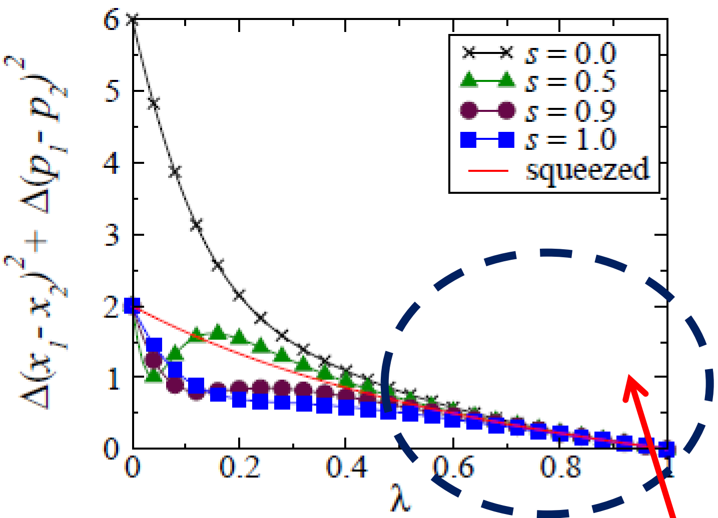
Unlike other results, the fidelity does not resound the effect of bipartite mode-entanglement

ANALYZING CONTINUOUS VARIABLE TELEPORTATION

EPR CORRELATIONS

Answer lies in the EPR correlations





Coherent state

Squeezed state

Classically correlated

EPR correlated

$$\Delta(\hat{x}_1-\hat{x}_2)^2+\Delta(\hat{p}_1+\hat{p}_2)^2=2$$ is the classical limit

OUTLINE

- Nonclassicality using photon subtraction and addition
- Generalized number conserving operator
- Characterization of nonclassicality
- Generation of two-mode entanglement
- Analyzing continuous variable teleportation
- **Summary and outlook**

SUMMARY AND OUTLOOK

- Generation of nonclassical and nonGaussian quantum states
- Useful in heralded generation of highly entangled low dimensional discrete quantum states
- Enhancement of the average fidelity CV quantum teleportation
- We observe that classical fidelity is suppressed while quantum fidelity is enhanced.
- Can be useful in investigation of noisy quantum protocols

SUMMARY AND OUTLOOK

- Experimentally realizable and can be harnessed for various quantum protocols

- Heralded generation of nonclassical states can be used for studying important aspects of quantum physics such as micro-macroscopic correlations

- Interesting theoretical interface of quantum optics and quantum information theory

RELATED PUBLICATIONS

Nonclassical properties of states engineered by superpositions of quantum operations on classical states.

Arpita Chatterjee, **Himadri Shekhar Dhar**, and Rupamanjari Ghosh,
J. Phys. B: At. Mol. Opt. Phys. **45**, 205501 (2012).

Generating continuous-variable entangled states for quantum teleportation using a superposition of number-conserving quantum operations.

Himadri Shekhar Dhar, Arpita Chatterjee, and Rupamanjari Ghosh,
arXiv:1312.6226v1 [quant-ph] (2013).

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