Understanding Ghost Interference

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ENTANGLEMENT

Possible states of particle 1: $|Z_{1+}\rangle$, $|Z_{1-}\rangle$ also $|X_{1+}\rangle$, $|X_{1-}\rangle$
Possible states of particle 2: $|Z_{2+}\rangle$, $|Z_{2-}\rangle$ also $|X_{2+}\rangle$, $|X_{2-}\rangle$

Entangled state:

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}(|Z_{1+}\rangle|Z_{2+}\rangle + |Z_{1-}\rangle|Z_{2-}\rangle) \\
&= \frac{1}{\sqrt{2}}(|X_{1+}\rangle|X_{2+}\rangle + |X_{1-}\rangle|X_{2-}\rangle)
\end{aligned}
$$

Measure Z component of particle 1

$$|Z_{1-}\rangle \xrightarrow{\qquad \textit{implies} \qquad} |Z_{2-}\rangle$$

Alternatively, measure X component of particle 1

$$|X_{1+}\rangle \xrightarrow{\qquad \textit{implies} \qquad} |X_{2+}\rangle$$

Implication: Choice of measurement on particle 1 decides the final state of (the remotely located) particle 2.

Nonlocality!

Most amazing demonstration of nonlocality

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Observation of Two-Photon "Ghost" Interference and Diffraction

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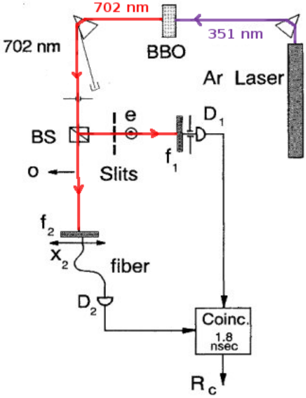
Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21228
(Received 11 August 1994)

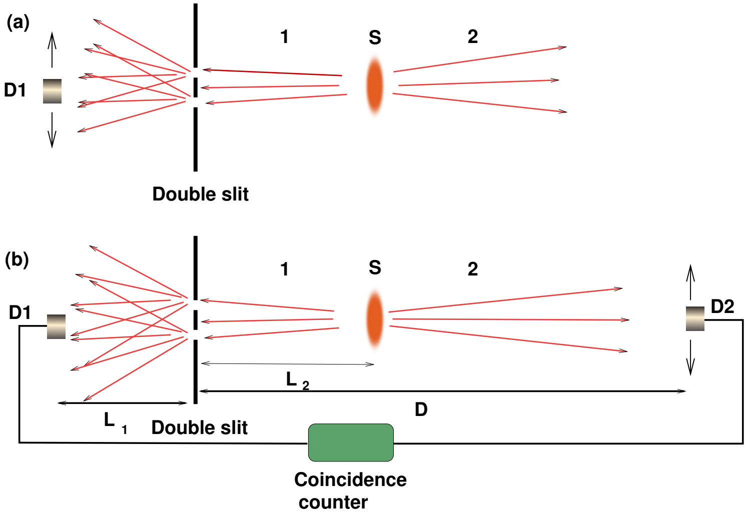
Experimental setup

Entangled photon pairs generated using Spontaneous Parametric Down Conversion (SPDC).

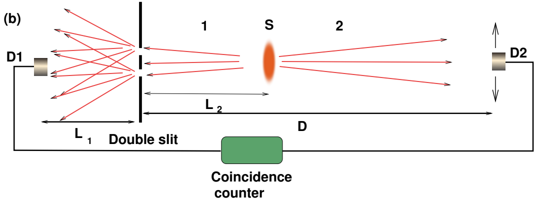
Photons move in different directions.



Effective experiment



Experimental results



- No first order interference is observed for photons 1.
- For photons 2, first order interference is neither expected, nor seen.
- Photons 2 detected *in coincidence with a* **fixed** *D1* show an interference pattern!
- But photons 2 do not pass through any double slit!
  Ghost interference!

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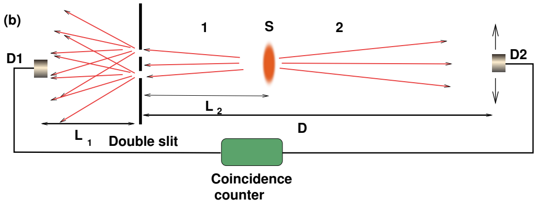

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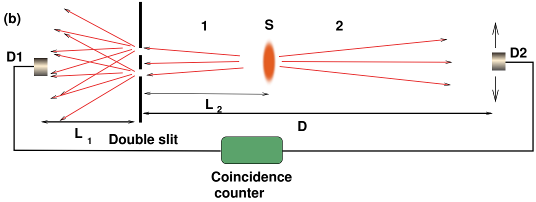

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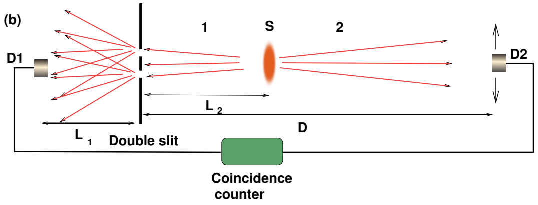

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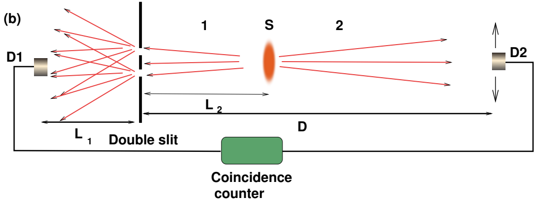
Fringe-width of ghost interference

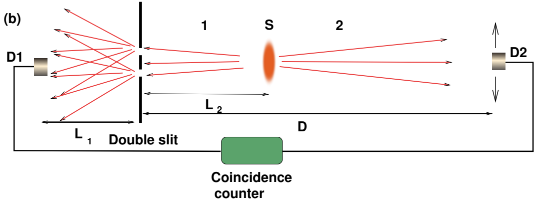


Fringe width follows Young's double-slit formula

$$w = \frac{\lambda D}{d}$$

with a twist:

Particle 2 doesn't travel the distance *D*!



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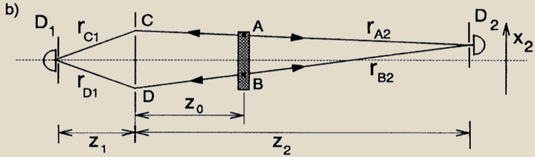

If the optical paths from the fixed detector D_1 to the two slits are equal, i.e., $r_{C1} = r_{D1}$, and if $z_2 \gg d^2/\lambda$ (which is true for this experiment), then $r_A - r_B = r_{C2} - r_{D2} \cong x_2 d/z_2$, and Eq. (7) can be written as

$$R_c(x_2) \propto \cos^2(x_2 \pi d/\lambda z_2)\,. \qquad (8)$$

Equation (8) has the form of standard Young's double-slit interference pattern. Here again $z_2 = 1.8$ m is the unusual distance described above.

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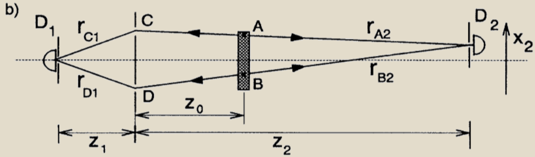

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mainly due to the divergence of the SPDC beam ($\gg \lambda/d$). In other words, the “blurring out” of the first order interference fringes is due to the considerably large angular propagation uncertainty of a single SPDC photon.

Theoretical analysis

Einstein-Podolsky-Rosen (EPR) state

Momentum entangled state discussed by Einstein, Podolsky and Rosen[1]

$$\psi(y_1, y_2) = \int_{-\infty}^{\infty} e^{ipy_1/\hbar} e^{-ipy_2/\hbar} dp$$

Problems with the EPR state:

1. Difficult to normalize
2. Wave-function is unbounded in the space variable $(y_1 + y_2)$.
3. Most likely, not realizable in practice

[1] A. Einstein, B. Podolsky N. Rosen, *Phys. Rev.* 47, 777 (1935).

Theoretical analysis

Generalized EPR state

$$\Psi(y_1, y_2) = C\int_{-\infty}^{\infty} dp e^{-p^2/4\sigma^2} e^{-ipy_2/\hbar} e^{ipy_1/\hbar} e^{-\frac{(y_1+y_2)^2}{4\Omega^2}},$$

Integration over p can be performed to obtain:

$$\Psi(y_1, y_2) = \sqrt{\frac{\sigma}{\pi\hbar\Omega}} e^{-(y_1-y_2)^2\sigma^2/\hbar^2} e^{-(y_1+y_2)^2/4\Omega^2}.$$

Uncertainty in momenta of the two particles:

$$\Delta p_{1y} = \Delta p_{2y} = \frac{1}{2}\sqrt{\sigma^2 + \frac{\hbar^2}{4\Omega^2}}.$$

Position uncertainty:

$$\Delta y_1 = \Delta y_2 = \sqrt{\Omega^2 + \hbar^2/4\sigma^2}.$$

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Time evolution

For massive particles

$$\Psi(y_1, y_2, t) = e^{-i\hat{H}t/\hbar}\Psi(y_1, y_2, 0), \qquad \hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y_2^2}$$

Alternately, for photons

$$\Psi(y_1, y_2, t) = \int dk_{y1} \int dk_{y2}\, \Phi(k_{y1}, k_{y2}, 0) e^{ik_{y1}y_1 - i\omega(k_{y1})t} e^{ik_{y2}y_2 - i\omega(k_{y2})t}$$

where $\qquad \omega(k_y) = c\sqrt{k_x^2 + k_y^2} \approx ck_0 + \frac{ck_y^2}{2k_0}$

In time t, particle travels distance L $\xrightarrow{\text{implies}}$ $t = L/v_0$ or $t = L/c$

Finally, $\qquad \hbar t/m = \hbar L/(mv_0) = \lambda L/2\pi \quad$ or $\quad ct/k_0 = \lambda L/2\pi$

After a time t_0 particle 1 reaches the double-slit.

$$\Psi(y_1, y_2, t_0) = \frac{1}{\sqrt{\pi(\Omega + \frac{i\hbar t_0}{m\Omega})(\hbar/\sigma + \frac{4i\hbar t_0}{m\hbar/\sigma})}} \exp\left[\frac{-(y_1 - y_2)^2}{\hbar^2/\sigma^2 + \frac{4i\hbar t_0}{m}}\right] \exp\left[\frac{-(y_1 + y_2)^2}{\left(4\Omega^2 + \frac{i\hbar t_0}{m}\right)}\right],$$

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Passing through the double-slit

Possibilities for particle 1:

Passes through slit A $\rightarrow$ $|\phi_A(y_1)\rangle$
Passes through slit B $\rightarrow$ $|\phi_B(y_1)\rangle$
Gets blocked by the slit $\rightarrow$ $\chi(y_1)$

These three states are orthogonal: any state of particle 1 can be written in terms of these:

$$|\Psi(y_1, y_2, t_0)\rangle = |\phi_A\rangle\langle\phi_A|\Psi\rangle + |\phi_B\rangle\langle\phi_B|\Psi\rangle + |\chi\rangle\langle\chi|\Psi\rangle. \tag{1}$$

The state after the double-slit:

$$|\Psi(y_1, y_2)\rangle = |\phi_A\rangle|\psi_A\rangle + |\phi_B\rangle|\psi_B\rangle + |\chi\rangle|\psi_\chi\rangle, \tag{2}$$

Gaussian states

Throwing away the blocked part of the wave-function,

$$|\Psi(y_1, y_2)\rangle = \frac{1}{C}(|\phi_A\rangle|\psi_A\rangle + |\phi_B\rangle|\psi_B\rangle), \quad (3)$$

where $C = \sqrt{\langle\psi_A|\psi_A\rangle + \langle\psi_B|\psi_B\rangle}$

$|\phi_A\rangle, |\phi_B\rangle$ are already normalized states.

Assume: $|\phi_A\rangle$, $|\phi_B\rangle$ are Gaussian states:

$$\phi_A(y_1) = \frac{1}{\sqrt{\epsilon\sqrt{\pi/2}}} e^{-(y_1-y_0)^2/\epsilon^2}, \qquad \phi_B(y_1) = \frac{1}{\sqrt{\epsilon\sqrt{\pi/2}}} e^{-(y_1+y_0)^2/\epsilon^2}, \quad (4)$$

where $d \equiv 2y_0$ = slit-separation, ϵ is slit-width.

Entangled Gaussian wave-packets

The state which emerges from the double slit, now assumes the form

$$\Psi(y_1, y_2) = c e^{-\frac{(y_1-y_0)^2}{\epsilon^2}} e^{-\frac{(y_2-y_0')^2}{\Gamma}} + c e^{-\frac{(y_1+y_0)^2}{\epsilon^2}} e^{-\frac{(y_2+y_0')^2}{\Gamma}}, \quad (5)$$

where $c = (1/\sqrt{\pi\epsilon})(\sqrt{\Gamma_r} + \frac{i\Gamma_i}{\sqrt{\Gamma_r}})^{-1/2}$ and

$$\Gamma = \frac{\frac{\hbar^2}{\sigma^2}(1 + \frac{\epsilon^2 + i\hbar t_0/m}{4\Omega^2}) + \epsilon^2 + 4i\hbar t_0/m + \frac{5i\hbar t_0}{4\Omega^2 m}}{1 + \frac{\epsilon^2}{\Omega^2} + \frac{5i\hbar t_0}{4\Omega^2 m} + \frac{\hbar^2}{4\Omega^2\sigma^2}} \quad (6)$$

After a further time t,
particle 1 reaches detector D1, particle 2 reaches D2.

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Coincident counts and ghost interference

The probability of coincident click of D1 and D2 is given by $P(y_1, y_2) = |\Psi_r(y_1, y_2, t)|^2$, which has the following form

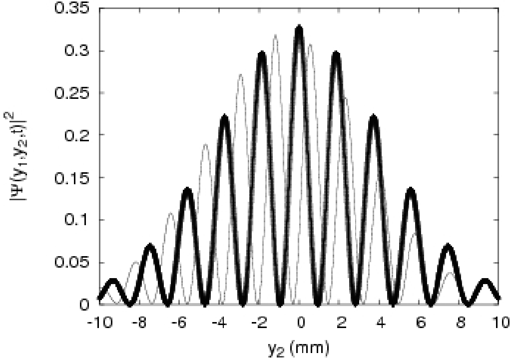
$$
\begin{aligned}
P(y_1, y_2) \quad = \quad & |C_1(t)C_2(t)|^2 \left(\exp\left[-\frac{2(y_1 - y_0)^2}{\epsilon^2 + (\lambda L_1/\pi\epsilon)^2} - \frac{2(y_2 - y_0)^2}{\gamma^2 + (\lambda D/\pi\gamma)^2} \right] \right. \\
& + \exp\left[-\frac{2(y_1 + y_0)^2}{\epsilon^2 + (\lambda L_1/m\epsilon)^2} - \frac{2(y_2 + y_0)^2}{\gamma^2 + (\lambda D/\pi\gamma)^2} \right] \\
& + \exp\left[-\frac{2(y_1^2 + y_0^2)}{\epsilon^2 + (\lambda L_1/\pi\epsilon)^2} - \frac{2(y_2^2 + y_0^2)}{\gamma^2 + (\lambda D/\pi\gamma)^2} \right] \\
& \left. \times 2\cos\left[\theta_1 y_1 + \theta_2 y_2\right] \right),
\end{aligned}
$$

where $\theta_1 = \frac{4y_0\lambda L_1/\pi}{\epsilon^4+\lambda^2 L_1^2/\pi^2}$, $\theta_2 = \frac{4y_0\lambda D/\pi}{\gamma^4+\lambda^2 D^2/\pi^2}$.
fringe width of the pattern for particle 2 is given by

$$
w_2 = \frac{\lambda_d D}{d} + \frac{\gamma^4\pi}{2d\lambda D} \approx \frac{\lambda D}{d}
$$

Ghost interference

Probability of coincident counting of D1 and D2



P. Chingangbam, T. Qureshi, *Prog. Theor. Phys.* **127**, 383-392 (2012).

Physics of Ghost interference

- Entanglement leads to formation of a *virtual double-slit* for particle 2 (in coincident counting).
- Because of entanglement each particle carried which-path information about the other.
- By detecting particle 2, one can tell which slit particle 1 passed through.
- By Bohr's complementarity principle, no interference is possible in such a situation.
- By detecting particle 1 (sufficiently far) behind the double-slit, one essentially erases the information about which slit the particle passed through.
- Consequently, one loses information on which virtual slit particle 2 passed through.
- Interference is possible in this situation - ghost interference.

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Two-color ghost interference

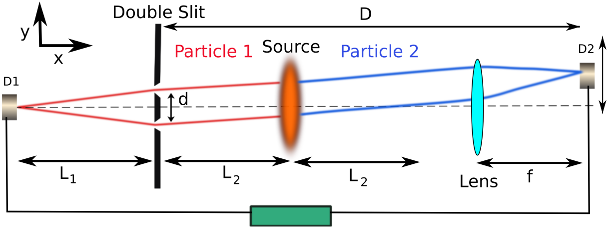
AIP ADVANCES **2**, 032177 (2012)

Two-color ghost interference with photon pairs generated in hot atoms

Dong-Sheng Ding, Zhi-Yuan Zhou, Bao-Sen Shi,[a] Xu-Bo Zou, and Guang-Can Guo
Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China

(Received 7 June 2012; accepted 11 September 2012; published online 19 September 2012)

Entangled photons generated via *spontaneous four-wave mixing* (SFWM) $\lambda_1 = 1530$ nm, $\lambda_2 = 780$ nm.



Two-color ghost interference: Theoretical analysis

$$\Psi(y_1, y_2) = \sqrt{\frac{\sigma}{\pi\hbar\Omega}} e^{-(y_1-y_2)^2\sigma^2/\hbar^2} e^{-(y_1+y_2)^2/4\Omega^2}.$$

Hamiltonian governing the time evolution,

$$\hat{H} = -\frac{\hbar^2}{2m_1}\frac{\partial^2}{\partial y_1^2} - \frac{\hbar^2}{2m_2}\frac{\partial^2}{\partial y_2^2}$$

$\hbar t/m_1 = \lambda_1 L/2\pi$ and $\hbar t/m_2 = \lambda_2 L/2\pi$.

After a time t_0,

$$\Psi(y_1, y_2, t_0) = \frac{1}{\sqrt{\pi(\Omega + \frac{i\hbar t_0}{2M\Omega})(\hbar/\sigma + \frac{2i\hbar t_0}{\mu\hbar/\sigma})}} \exp\left[\frac{-(y_1-y_2)^2}{\hbar^2/\sigma^2 + \frac{2i\hbar t_0}{\mu}}\right] \exp\left[\frac{-(y_1+y_2)^2}{\left(4\Omega^2 + \frac{2i\hbar t_0}{M}\right)}\right],$$

where $M = m_1 + m_2$ and $\mu = \frac{m_1 m_2}{m_1+m_2}$.

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Two-color ghost interference: Theoretical analysis

Without the converging lens

The fringe width of the pattern for particle 2 is given by

$$w_2 = \frac{2\pi}{\theta_2} = \frac{\lambda_2 D}{d} + \frac{(\lambda_1 - \lambda_2)L_2}{d} + \frac{\gamma^4\pi^2}{d\lambda_2 D + d(\lambda_1 - \lambda_2)L_2} \tag{7}$$

For $\pi\gamma^2 \ll \lambda_2 L_2, \lambda_2 L_1, \lambda_1 L_2$, we get a simplified double-slit interference formula,

$$w_2 \approx \frac{\lambda_2(L_1 + L_2)}{d} + \frac{\lambda_1 L_2}{d}. \tag{8}$$

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Effect of the converging lens

Effect of a lens → a unitary transformation [2]

$$\mathbf{U}_f \frac{(\pi/2)^{-1/4}}{\sqrt{\sigma + \frac{i\Lambda L}{\sigma}}} \exp\left(\frac{-y_1^2}{\sigma^2 + i\Lambda L}\right) = \frac{(\pi/2)^{-1/4}}{\sqrt{\tilde{\sigma} + \frac{i\Lambda(L-4f)}{\tilde{\sigma}}}} \exp\left(\frac{-y_1^2}{\tilde{\sigma}^2 + i\Lambda(L-4f)}\right),$$

σ → initial width of the wave-packet
L → distance travelled by the wave-packet before the lens.

$$\tilde{\sigma}^2 + \frac{\Lambda^2(L-4f)^2}{\tilde{\sigma}^2} = \sigma^2 + \frac{\Lambda^2 L^2}{\sigma^2}. \tag{9}$$

Satisfies the thin lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

[1] T. Qureshi, *Prog. Theor. Phys.* **127**, 645 (2012).

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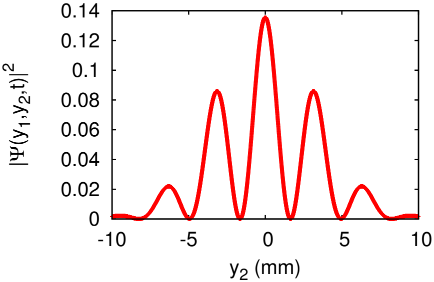
Two-color ghost interference

In the presence of a converging lens

A simplified double-slit interference formula, [3]

$$w_2 \approx \frac{\lambda_2(L_1 + L_2 - 4f)}{d} + \frac{\lambda_1 L_2}{d}.$$

Probability of coincident counting of D1 and D2



[3] S. Shafaq, T. Qureshi, *Eur. Phys. J.* (2014) (in press); arXiv:1308.4680 [quant-ph].

Conclusions

- Ghost interference is the combined effect of
  - virtual double-slit formation due to entanglement
  - quantum erasure of which-path information
- No first-order interference behind double-slit $\rightarrow$ because which-path information for particle 1 is carried by particle 2.
- Interference for particle 1 can also be obtained by coincident-counting it with a fixed detector for particle 2.
- Prediction: In the two-color ghost interference, the fringe width of photon 2 pattern depends on the wavelength of photon 1 too! Can be verified in a modified experiment

Pravabati Chingangbam, Tabish Qureshi
Ghost interference and quantum erasure
Prog. Theor. Phys. **127**, 383-392 (2012).

Sheeba Shafaq, Tabish Qureshi
Theoretical analysis of two-color ghost interference
Eur. Phys. J. D (2014) (in press) arXiv:1308.4680 [quant-ph]