Multiparty entanglement
Vs
classical information transmission

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IPQI-2014, IoP, Bhubaneswar

OUTLINE

- Multiparty entanglement
- Dense coding - single to many
- Connecting both of them
  - Analytically
  - Numerically
- Dense coding - many to single
  - Noiseless
  - Noisy

OUTLINE

Multiparty entanglement

Dense coding - single to many

Connecting both of them

- Analytically
- Numerically

Dense coding - many to single

- Noiseless
- Noisy

Multiparty entanglement

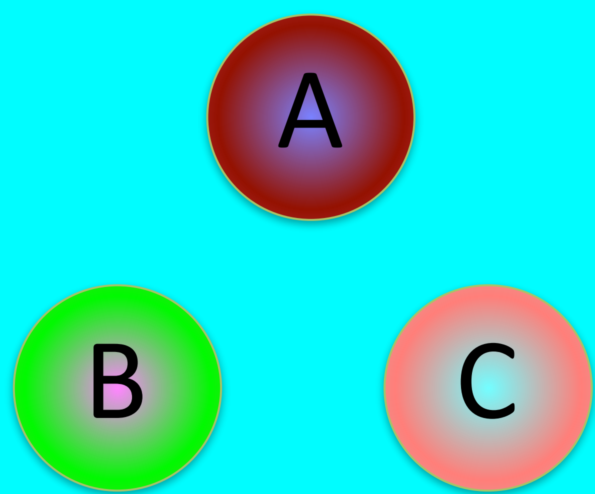
- Relative entropy of entanglement (REE)

Genuine multiparty entangled

- Generalized geometric measure (GGM)

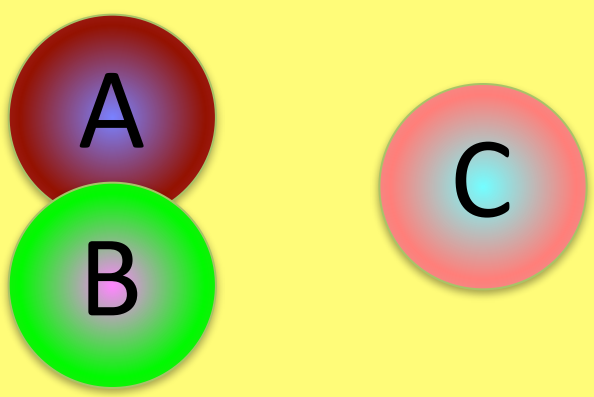
- Monogamy based measure

Types of entangled states



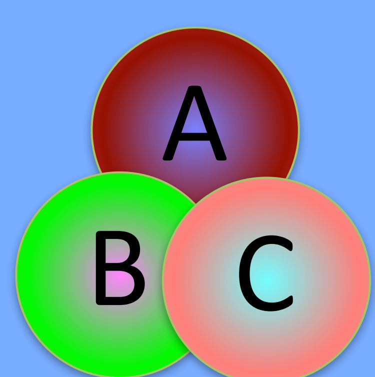
Fully separable:

$$|\phi\rangle \otimes |\xi\rangle \otimes |\chi\rangle$$



Biseparable:

$$|\psi\rangle_{AB} \otimes |\chi\rangle$$



Genuine multiparty entangled:

$$\neq |\phi\rangle \otimes |\xi\rangle \otimes |\chi\rangle$$

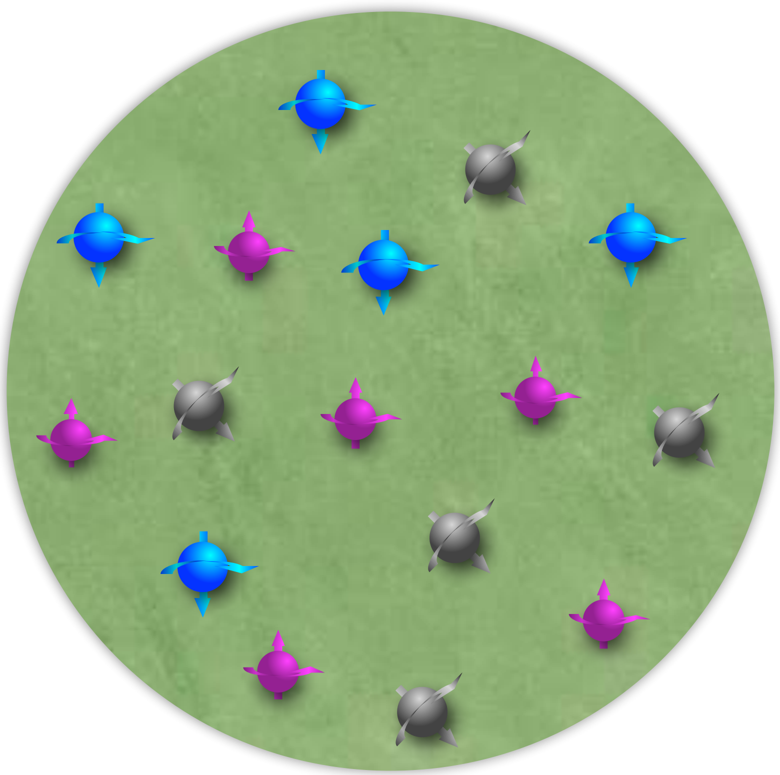
$$\neq |\psi\rangle_{AB} \otimes |\chi\rangle$$

Multiparty entanglement

- Relative entropy of entanglement (REE)
- Generalized geometric measure (GGM)
- Monogamy based measure

Multiparty entanglement

Relative entropy of entanglement



$$\varrho_{A_1 A_2 \ldots A_n}$$

Pure or mixed state

V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. 78, 2275 (1997)
V. Vedral, M. B. Plenio, K. Jacobs, and P. L. Knight, Phys. Rev. A 56, 4452 (1997)
V. Vedral, Rev. Mod. Phys. 74, 197 (2002)

Multiparty entanglement

Relative entropy of entanglement

$$E_R(\varrho_{A_1 A_2 \ldots A_n}) = \min_{\sigma \in n\text{-gen}} S(\varrho_{A_1 A_2 \ldots A_n} || \sigma)$$

{not genuinely multiparty entangled}

$$S(\varrho || \sigma) = \mathrm{tr}(\varrho \log_2 \varrho - \varrho \log_2 \sigma)$$

Relative entropy

V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. 78, 2275 (1997)

V. Vedral, M. B. Plenio, K. Jacobs, and P. L. Knight, Phys. Rev. A 56, 4452 (1997)

V. Vedral, Rev. Mod. Phys. 74, 197 (2002)

Multiparty entanglement

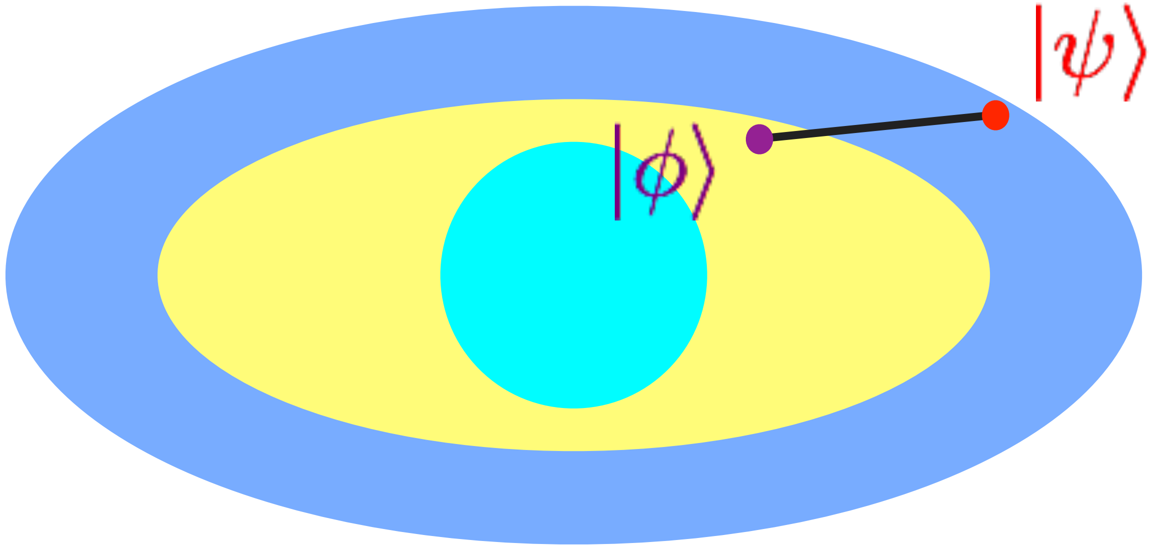
- Relative entropy of entanglement (REE)
- Generalized geometric measure (GGM)
- Monogamy based measure

Multiparty entanglement

Generalized geometric measure (GGM)

$$\mathcal{E}(|\psi_{A_1 A_2 \ldots A_n}\rangle) = 1 - \max_{|\phi\rangle} |\langle\phi|\psi_{A_1 A_2 \ldots A_n}\rangle|^2$$

$|\phi\rangle$ is not genuinely entangled state



A. Sen(De) and U. Sen, Phys. Rev. A 81, 012308 (2010); arXiv:1002.1253

Multiparty entanglement

Generalized geometric measure (GGM)

$$\mathcal{E}(|\psi_{A_1 A_2 \ldots A_n}\rangle) = 1 - \max\{\lambda^2_{\mathcal{A}:\mathcal{B}} | \mathcal{A}\cup\mathcal{B} = \{1, 2, \ldots, N\}, \mathcal{A}\cap\mathcal{B} = \emptyset\}$$

maximal Schmidt coefficients in the

$\mathcal{A} : \mathcal{B}$ bipartite split of $|\phi\rangle$

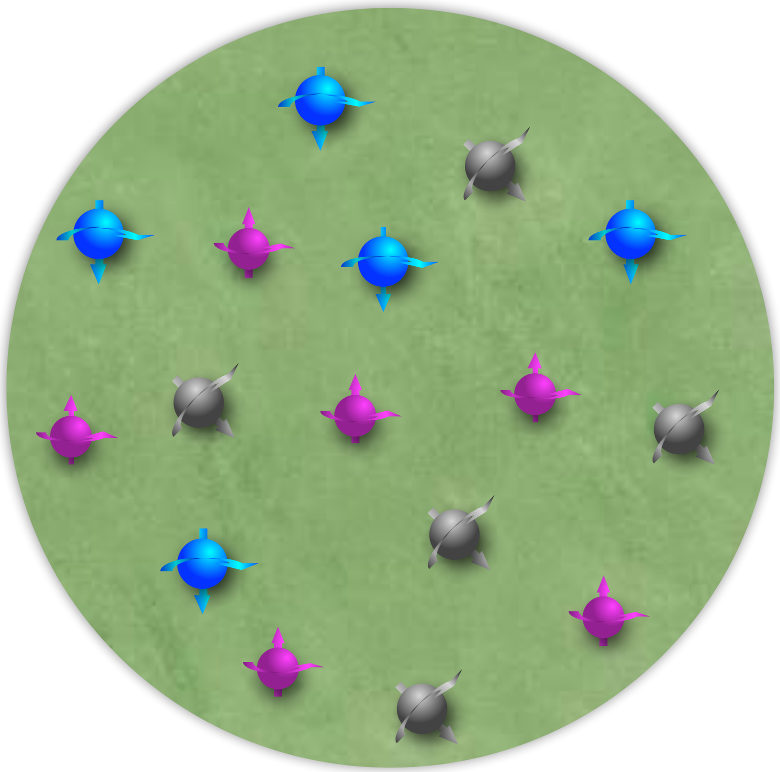
A. Sen(De) and U. Sen, Phys. Rev. A 81, 012308 (2010); arXiv:1002.1253

Multiparty entanglement

- Relative entropy of entanglement (REE)
- Generalized geometric measure (GGM)
- Monogamy based measure

Multiparty entanglement

Monogamy based measures



$$\sum_{i=1}^{N} \mathcal{Q}_{A_iB} \leq \mathcal{Q}_{A_1A_2\ldots A_N:B}$$

Quantum correlation measure

$$\delta_{\mathcal{Q}} = \mathcal{Q}_{A_1A_2\ldots A_N:B} - \sum_{i=1}^{N} \mathcal{Q}_{A_iB}$$

Positive → Monogamous

Negative → Non-Monogamous

V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A 61, 052306 (2000)
T. J. Osborne and F. Verstraete, Phys. Rev. Lett. 96, 220503 (2006)
R. Prabhu, A.K. Pati, A. Sen(De), and U. Sen, Phys. Rev. A 86, 052337 (2012)

Multiparty entanglement

Monogamy based measures

$$\delta_{\mathcal{Q}} = \mathcal{Q}_{A_1 A_2 \ldots A_N : B} - \sum_{i=1}^{N} \mathcal{Q}_{A_i B}$$

$$\mathcal{Q} = C^2 = \text{Concurrence squared}$$

V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A 61, 052306 (2000)
T. J. Osborne and F. Verstraete, Phys. Rev. Lett. 96, 220503 (2006)

$$\mathcal{Q} = D = \text{Quantum discord}$$

R. Prabhu, A.K. Pati, A. Sen(De), and U. Sen, Phys. Rev. A 85, 040102(R) (2012); 86, 052337 (2012)
G. L. Giorgi, Phys. Rev. A 84, 054301 (2011)

OUTLINE

Multiparty entanglement

Dense coding - single to many

Connecting both of them

- Analytically
- Numerically

Dense coding - many to single

- Noiseless
- Noisy

Classical information transfer

Quantum Channel

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$$

2 bits

requires

2 dimension state

C. H. Bennett and S. J. Wiesner, PRL 69 2881 (1992)

Dense coding capacity

$$\mathcal{C}(\varrho_{AB}) = \log_2 d_A + S(\varrho_B) - S(\varrho_{AB})$$

dimension of subsystem A

$$S(\rho) = -\mathrm{tr}(\rho \log \rho)$$

von Neumann entropy

T. Hiroshima, J. Phys. A 34, 6907 (2001)

Dense coding capacity

$$\mathcal{C}(\varrho_{AB}) = \log_2 d_A + S(\varrho_B) - S(\varrho_{AB})$$

Classical dense coding capacity

T. Hiroshima, J. Phys. A 34, 6907 (2001)

Dense coding capacity

$$\mathcal{C}(\varrho_{AB}) = \log_2 d_A + S(\varrho_B) - S(\varrho_{AB})$$

Quantum DC advantage

T. Hiroshima, J. Phys. A 34, 6907 (2001)

Quantum advantage

Quantum advantage in dense coding of AB channel:

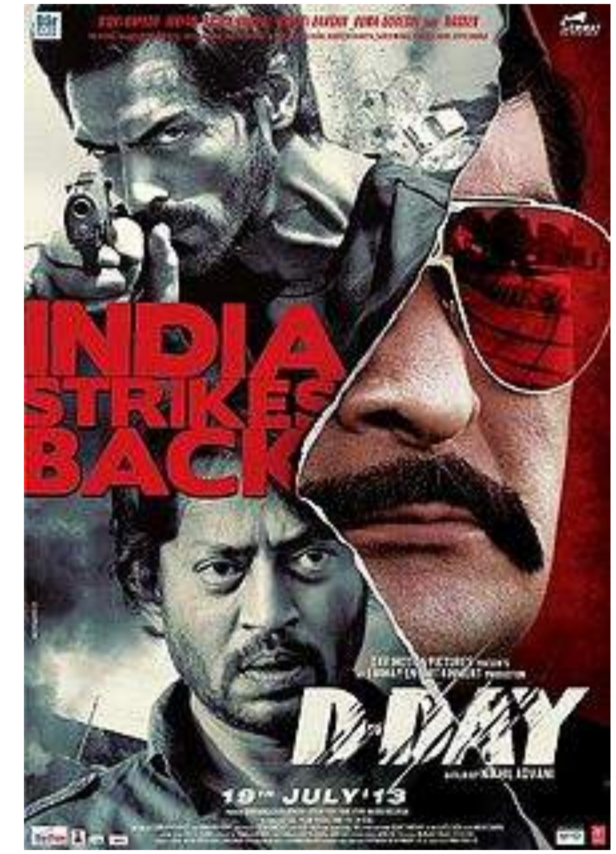
$$\mathcal{C}_{adv}(\varrho_{AB}) = \max\{S(\varrho_B) - S(\varrho_{AB}), 0\}$$

Therefore

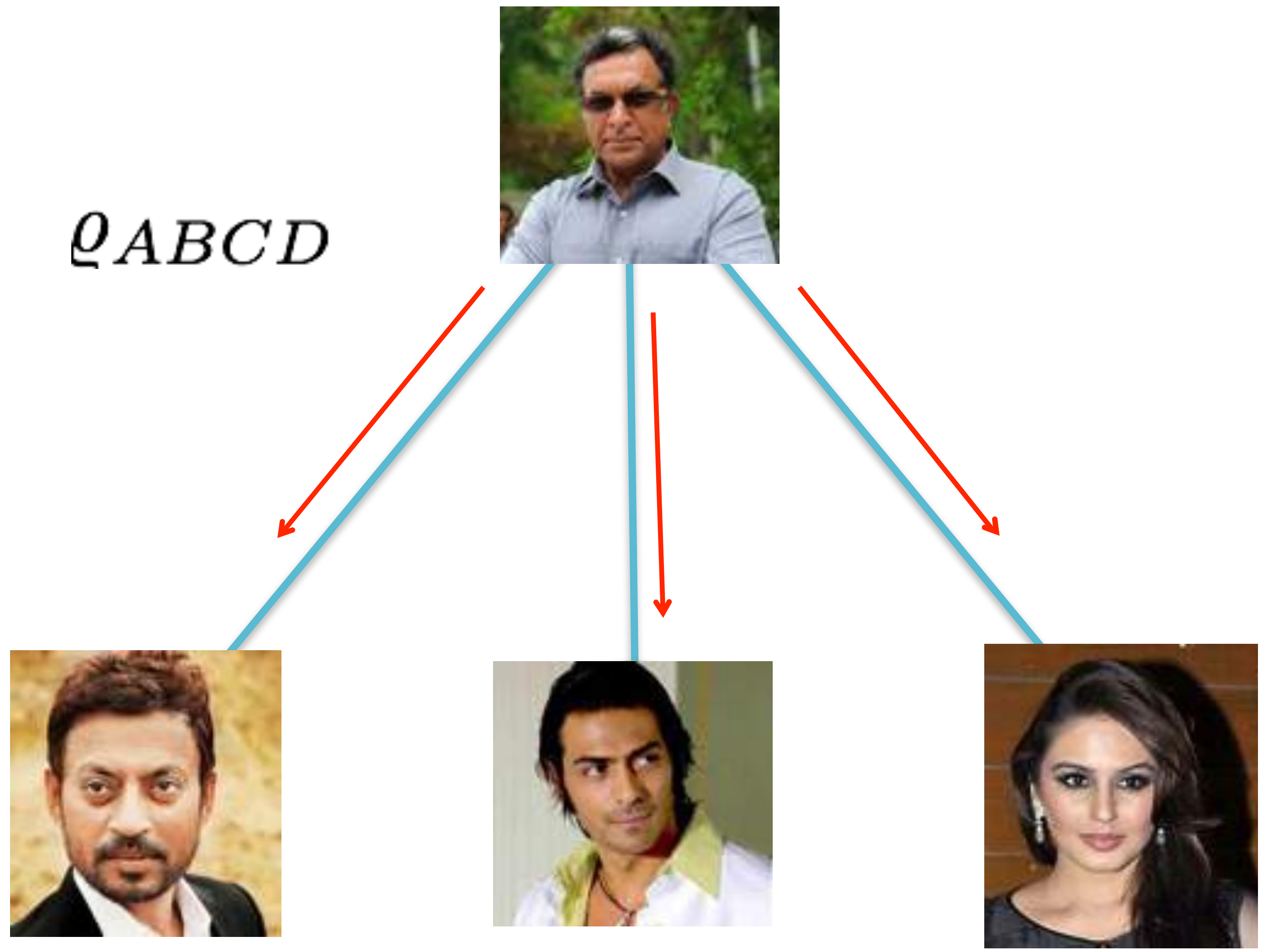
$$\mathcal{C}(\varrho_{AB}) = \log_2 d_A + \mathcal{C}_{\text{adv}}(\varrho_{AB})$$

R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

Quantum advantage in networks



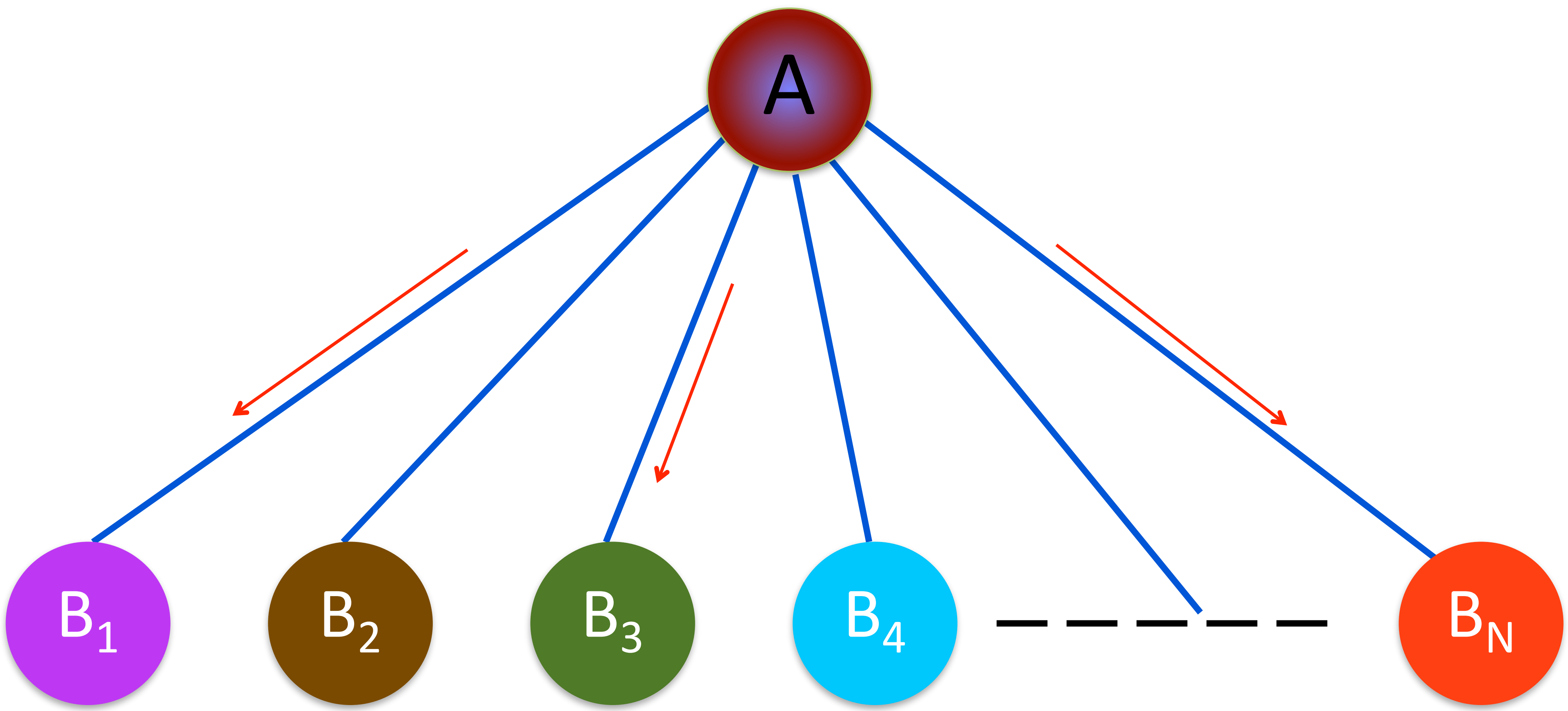
ϱ_{ABCD}



$$\mathcal{C}_{adv}^{max}(\varrho_{ABCD}) = \max\{\mathcal{C}_{adv}(\varrho_{AB}), \mathcal{C}_{adv}(\varrho_{AC}), \mathcal{C}_{adv}(\varrho_{AD})\}$$

R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

Quantum advantage in networks



$$\mathcal{C}_{\text{adv}}^{\max}(\varrho_{AB_1B_2\ldots B_N}) = \max\{\mathcal{C}_{\text{adv}}(\varrho_{AB_i})|i = 1,2,\ldots,N\}$$

R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

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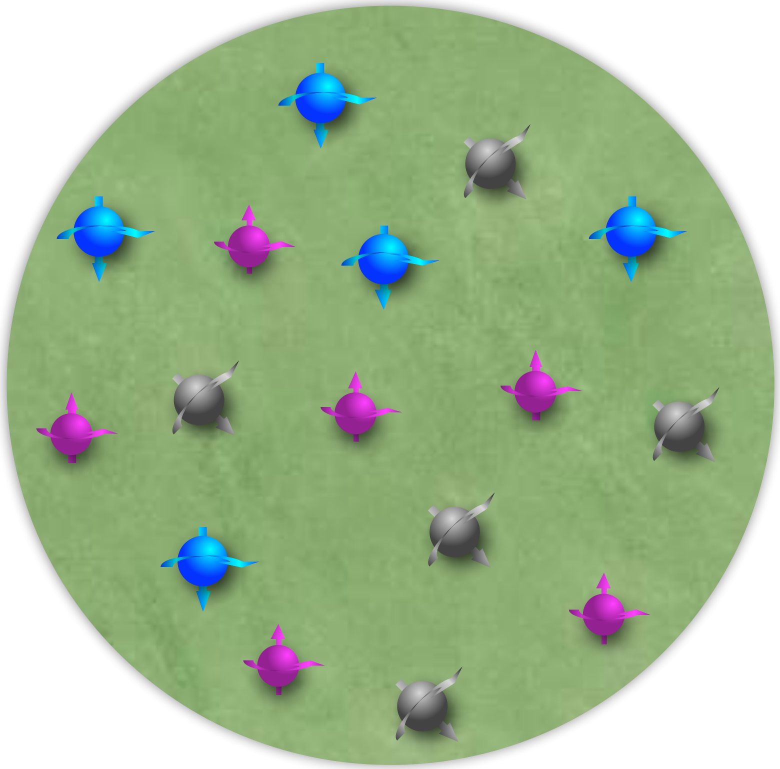
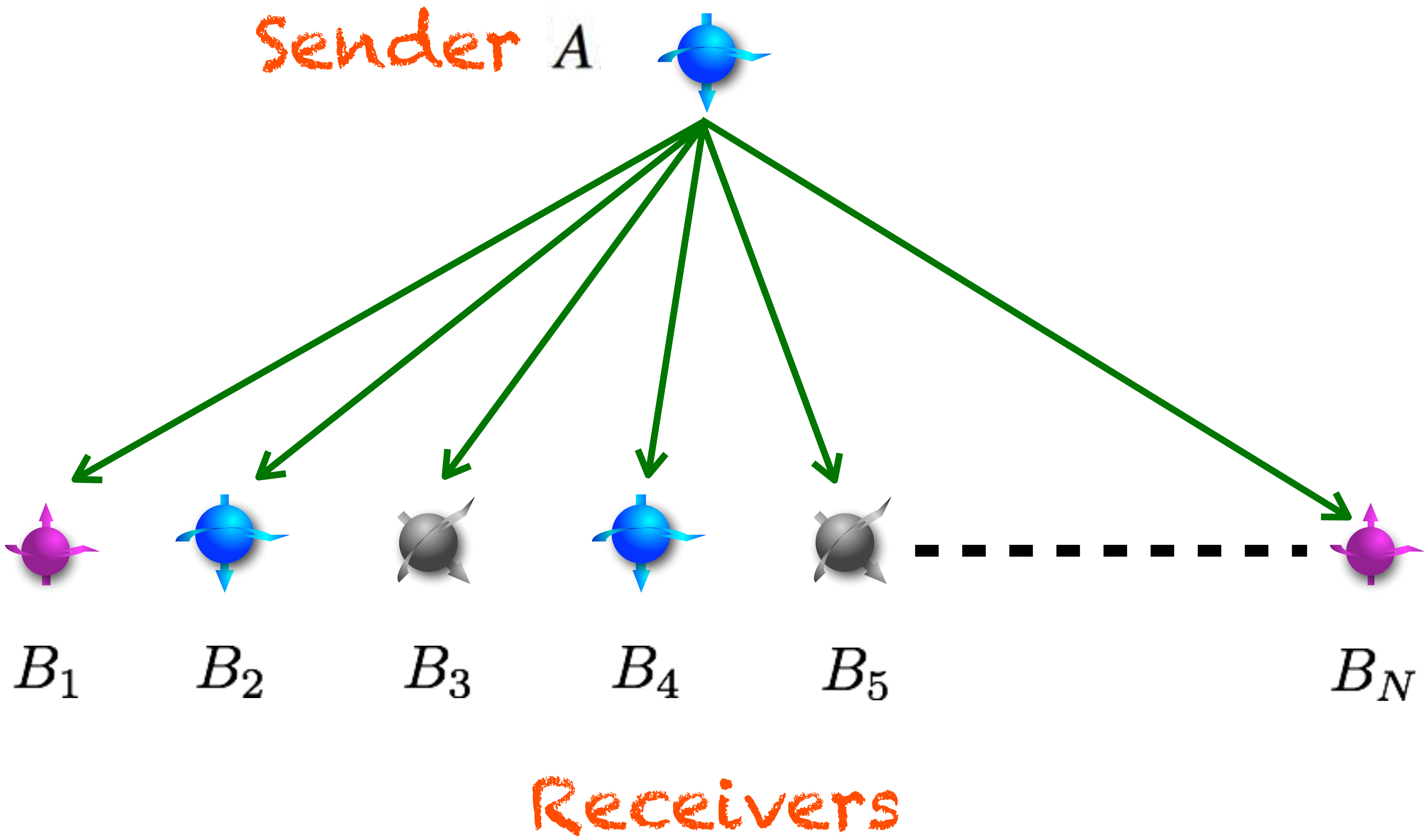
Multiparty DC & Entanglement



$$\varrho_{AB_1B_2\ldots B_N}$$

Multiparty DC & Entanglement



Multiparty DC & Entanglement

$$\mathcal{C}^{\max}_{\mathrm{adv}}(\varrho_{AB_1B_2\ldots B_N})$$

DC advantage

REE $$E_R(\varrho_{A_1A_2\ldots A_n}) = \min_{\sigma \in n\text{-gen}} S(\varrho_{A_1A_2\ldots A_n}||\sigma)$$

GGM $$\mathcal{E}(|\psi_{A_1A_2\ldots A_n}\rangle) = 1 - \max_{|\phi\rangle} |\langle\phi|\psi_{A_1A_2\ldots A_n}\rangle|^2$$

Monogamy score $$\delta_{\mathcal{Q}} = \mathcal{Q}_{A_1A_2\ldots A_N:B} - \sum_{i=1}^{N} \mathcal{Q}_{A_iB}$$

Multiparty DC & Entanglement

DC advantage & REE

$$E_R(\varrho_{AB_1B_2\ldots B_N}) = \min_{\sigma\in(N+1)\text{-gen}} S(\varrho||\sigma)$$
$$\leqslant \min_{\sigma'\in \text{sep}_1} S(\varrho||\sigma') \equiv E_R^{AB_1:\text{rest}}(\varrho_{AB_1:B_2\ldots B_N})$$
$$\leqslant E_f^{AB_1:\text{rest}}(\varrho_{AB_1:B_2\ldots B_N})$$
$$\leqslant S(\varrho_{AB_1})$$

separable states in $AB_1 : B_2 \ldots B_N$

Entanglement of formation

C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A 54, 3824 (1996)

$$E_R(\varrho_{AB_1B_2\ldots B_N}) \leq S(\varrho_{AB_1}) \equiv S_{AB_1}$$

V. Vedral and M. B. Plenio, Phys. Rev. A 57, 1619 (1998)

Multiparty DC & Entanglement

DC advantage & REE

$$C_{\text{adv}}^{\max}(\varrho_{AB_1B_2\ldots B_N}) = S_{B_1} - S_{AB_1}$$

$$E_R(\varrho_{AB_1B_2\ldots B_N}) \leq S_{AB_1}$$

$$S_{B_1} \leq \log_2 d_{B_1}$$

$$C_{\text{adv}}^{\max} + E_R \leqslant \log_2 d$$

R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

Multiparty DC & Entanglement

$\mathcal{C}_{\text{adv}}^{\max}(\varrho_{AB_1B_2\ldots B_N})$

DC advantage

REE $E_R(\varrho_{A_1A_2\ldots A_n}) = \min_{\sigma \in n\text{-gen}} S(\varrho_{A_1A_2\ldots A_n}||\sigma)$

GGM $\mathcal{E}(|\psi_{A_1A_2\ldots A_n}\rangle) = 1 - \max_{|\phi\rangle} |\langle\phi|\psi_{A_1A_2\ldots A_n}\rangle|^2$

Monogamy score $\delta_{\mathcal{Q}} = \mathcal{Q}_{A_1A_2\ldots A_N:B} - \sum_{i=1}^{N} \mathcal{Q}_{A_iB}$

Multiparty DC & Entanglement

DC advantage & GGM

$$\frac{1}{\log_2 d}\mathcal{C}_{\text{adv}}^{\text{max}} + \frac{d}{d-1}\mathcal{E} \leqslant 1$$

Normalization terms which makes individual quantities maximum

snippet

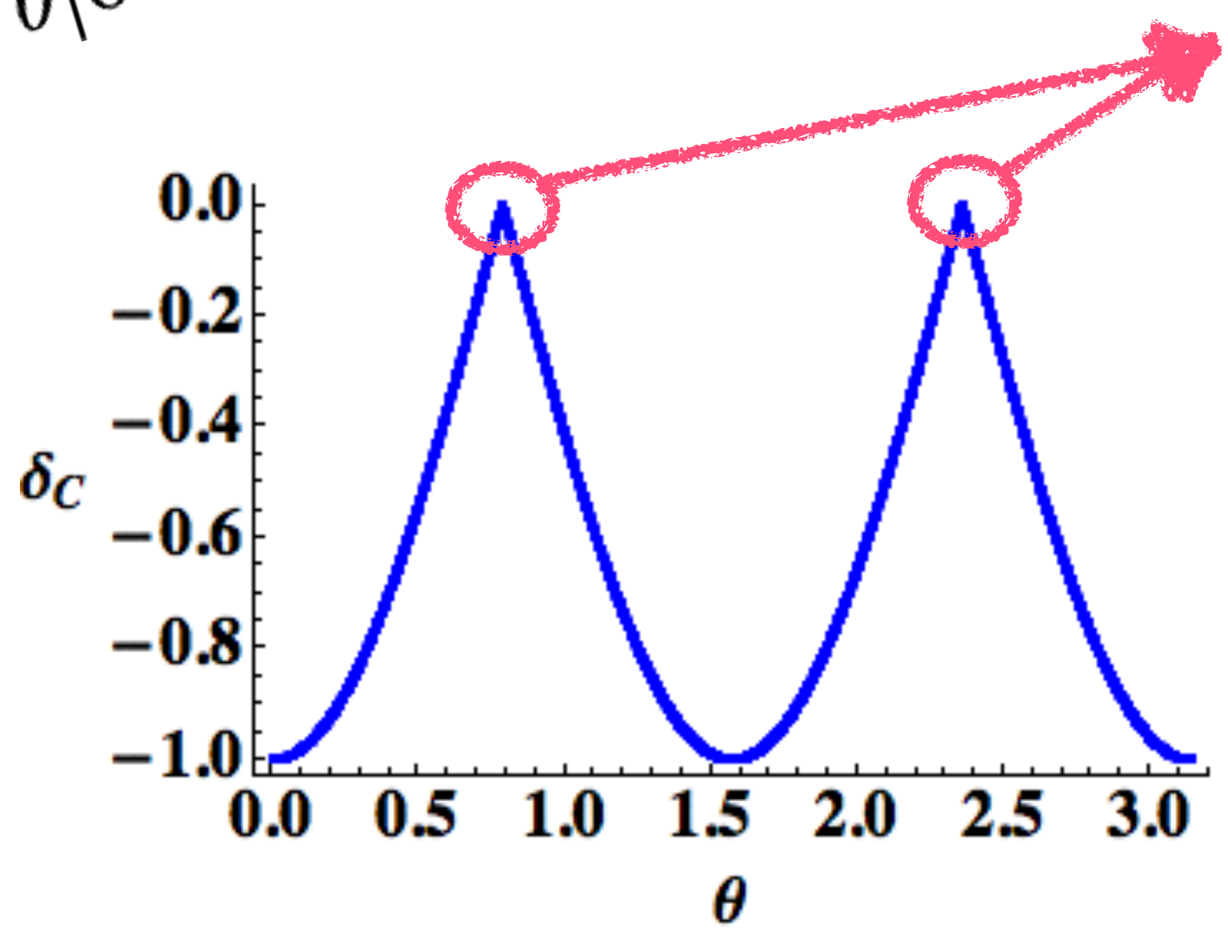
True for arbitrary states

R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

Multiparty DC & Entanglement

$$\delta_C = \frac{1}{\log_2 d}\mathcal{C}_{\text{adv}}^{\max} + \frac{d}{d-1}\mathcal{E} - 1$$

$$|\psi_{GHZ}\rangle = \cos\theta|000\rangle + \sin\theta|000\rangle$$

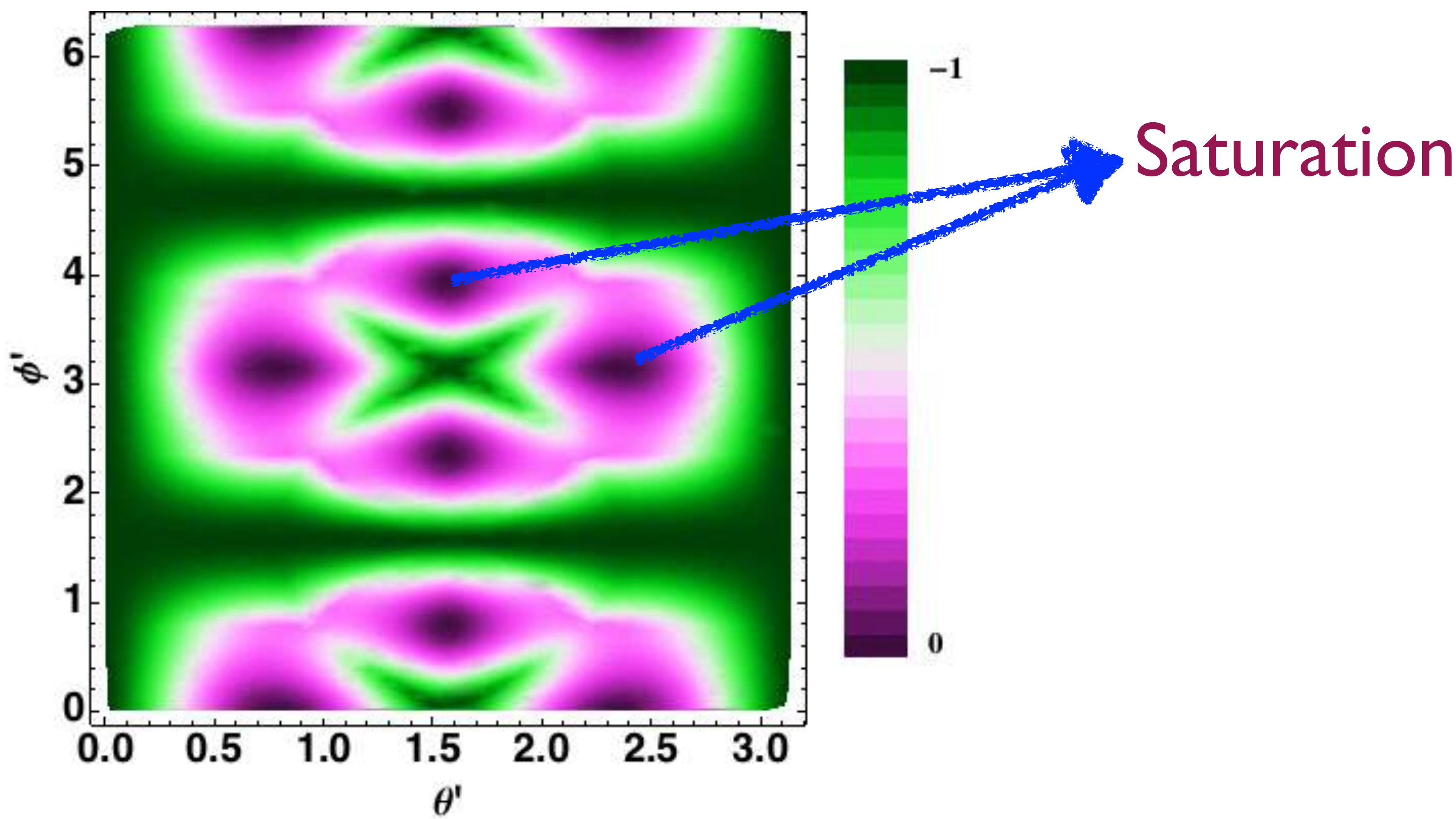


Saturation

R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

Multiparty DC & Entanglement

$$|\psi_W\rangle = \sin\theta' \cos\phi' |011\rangle + \sin\theta' \sin\phi' |101\rangle + \cos\theta' |110\rangle$$



R.P, A. Sen(De), and U. Sen, Phys. Rev. A 88, 042329 (2013)

Multiparty DC & Entanglement

$$C^{\max}_{\text{adv}}(\varrho_{AB_1B_2\ldots B_N})$$

DC advantage

REE $$E_R(\varrho_{A_1A_2\ldots A_n}) = \min_{\sigma\in n\text{-gen}} S(\varrho_{A_1A_2\ldots A_n}||\sigma)$$

GGM $$\mathcal{E}(|\psi_{A_1A_2\ldots A_n}\rangle) = 1 - \max_{|\phi\rangle} |\langle\phi|\psi_{A_1A_2\ldots A_n}\rangle|^2$$

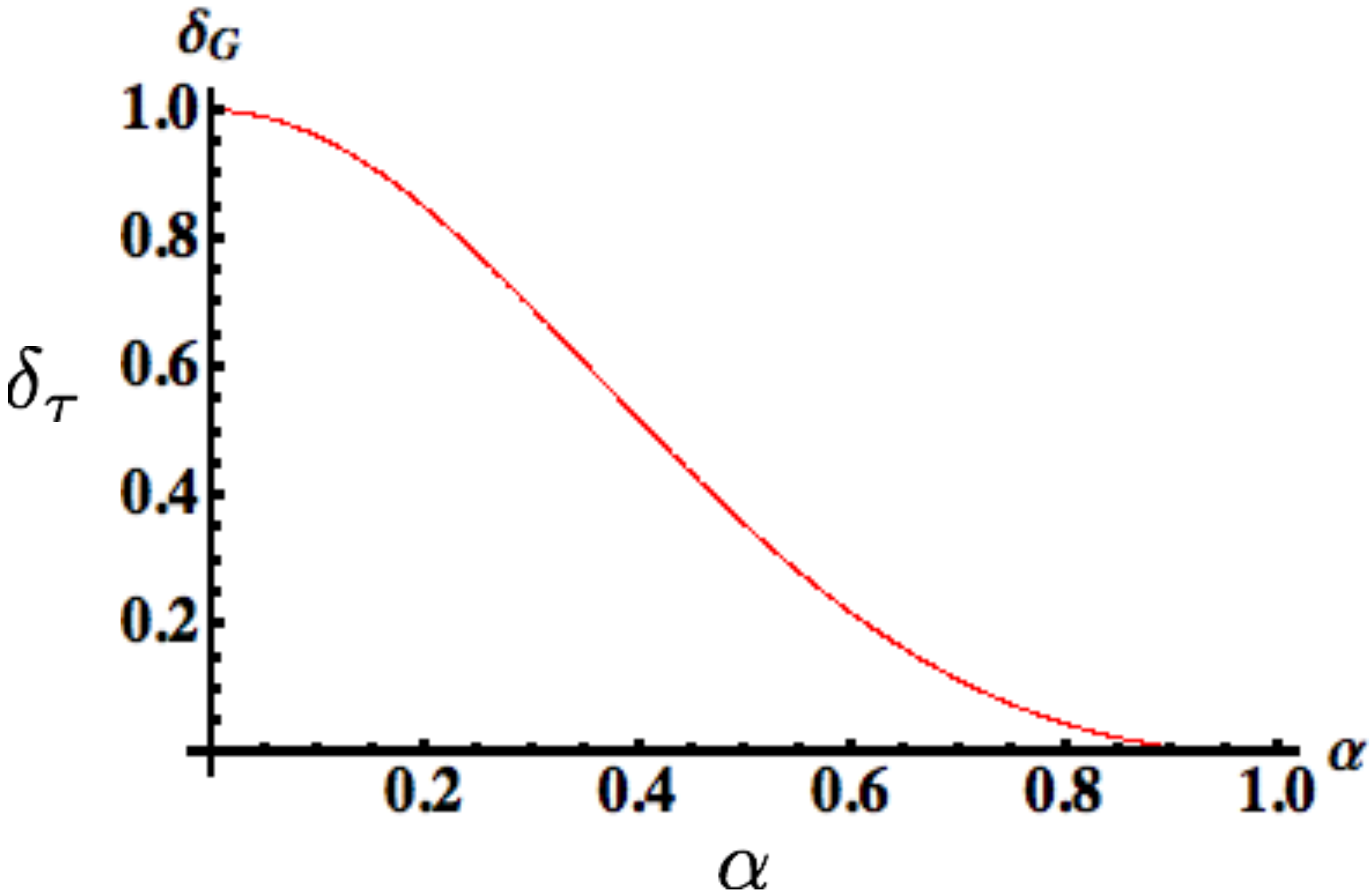
Monogamy score $$\delta_{\mathcal{Q}} = \mathcal{Q}_{A_1A_2\ldots A_N:B} - \sum_{i=1}^{N} \mathcal{Q}_{A_iB}$$

MDCC states

3-qubit

Maximally-dense-coding-capable states (MDCC)

$$|\psi_\alpha\rangle = |111\rangle + |000\rangle + \alpha(|101\rangle + |010\rangle)$$



MDCC states

3-qubit

Maximally-dense-coding-capable states (MDCC)

$$|\psi_\alpha\rangle = |111\rangle + |000\rangle + \alpha(|101\rangle + |010\rangle)$$

Belongs to GHZ class, except for $\alpha = 1$

Arbitrary state $|\psi\rangle$ and set $\mathcal{E}(|\psi_\alpha\rangle) = \mathcal{E}(|\psi\rangle)$

$$C_{\mathrm{adv}}(|\psi_\alpha\rangle) \geqslant C_{\mathrm{adv}}(|\psi\rangle)$$

MDCC states

3-qubit

Maximally-dense-coding-capable states (MDCC)

$$|\psi_\alpha\rangle = |111\rangle + |000\rangle + \alpha(|101\rangle + |010\rangle)$$

Eigenvalues of marginal density matrices

$$\lambda_A = \frac{1}{2}(\text{twice}) = \lambda_C \qquad \lambda_B = \frac{(1\pm\alpha)^2}{2(1+\alpha^2)}$$

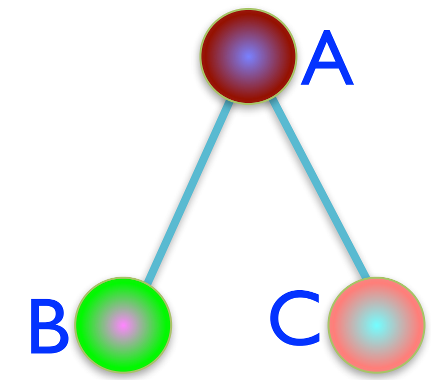
$$\mathcal{E}(|\psi_\alpha\rangle) = 1 - \frac{(1+\alpha)^2}{2(1+\alpha^2)} = \frac{1}{2} - \frac{\alpha}{1+\alpha^2}$$

$$\mathcal{E}(|\psi\rangle) = 1 - \max\{\lambda_A,\, \lambda_B,\, \lambda_C\} = 1 - \lambda$$

$$\frac{\alpha}{1+\alpha^2} = \lambda - \frac{1}{2}$$

MDCC states

3-qubit



$$\mathcal{C}_{adv}(|\psi_\alpha\rangle) = S_C^\alpha - S_B^\alpha$$

$$S_{AC}^\alpha = S_B^\alpha$$

$$\lambda_C = \frac{1}{2}(\text{twice})$$

$$\lambda_B = \frac{(1 \pm \alpha)^2}{2(1+\alpha^2)}$$

$$\frac{\alpha}{1+\alpha^2} = \lambda - \frac{1}{2}$$

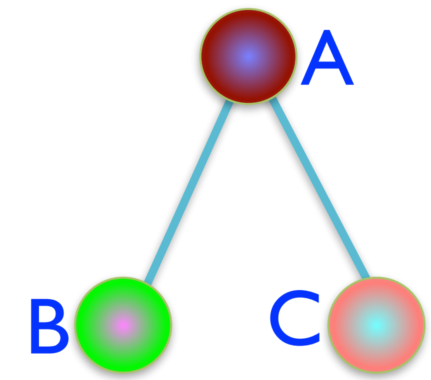
$$\mathcal{C}_{adv}(|\psi_\alpha\rangle) = 1 + \lambda \log \lambda + (1-\lambda)\log(1-\lambda)$$

$$= 1 - H(\lambda)$$

Shannon entropy

MDCC states

3-qubit



Arbitrary state $|\psi\rangle$

$$\max\{\lambda_A,\, \lambda_B,\, \lambda_C\} = \lambda_B$$

Since $\lambda_B,\ \lambda_C \geq \frac{1}{2}$ → $S_C - S_B \geq 0$

≤ 1

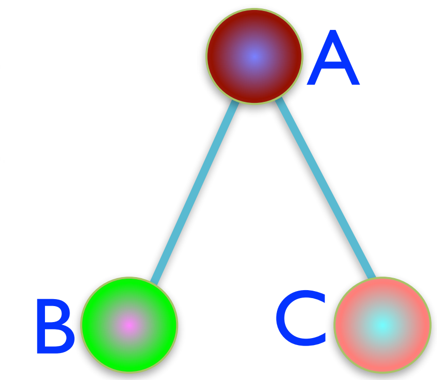
$$\mathcal{C}_{adv}(|\psi\rangle) = \max\{S_B - S_C,\, S_C - S_B,\, 0\}$$
$$= S_C - S_B$$

$$\mathcal{C}_{adv}(|\psi_\alpha\rangle) = 1 - H(\lambda_B)$$

$$\mathcal{C}_{\text{adv}}(|\psi_\alpha\rangle) \geqslant \mathcal{C}_{\text{adv}}(|\psi\rangle)$$

3-qubit

MDCC states



Arbitrary state $|\psi\rangle$

$$\max\{\lambda_A, \lambda_B, \lambda_C\} = \lambda_A$$

$$\lambda_A \geq \lambda_B \geq \lambda_C \qquad \lambda_A, \lambda_B, \text{and } \lambda_C \geq \frac{1}{2}$$

$$S_C - S_B \geq 0$$

$$S_A \leq S_B$$

$$\mathcal{C}_{adv}(|\psi_\alpha\rangle) = 1 - H(\lambda_A) = 1 - S_A \geq S_C - S_B = \mathcal{C}_{adv}(|\psi\rangle)$$

MDCC states

3-qubit

Maximally-dense-coding-capable states (MDCC)

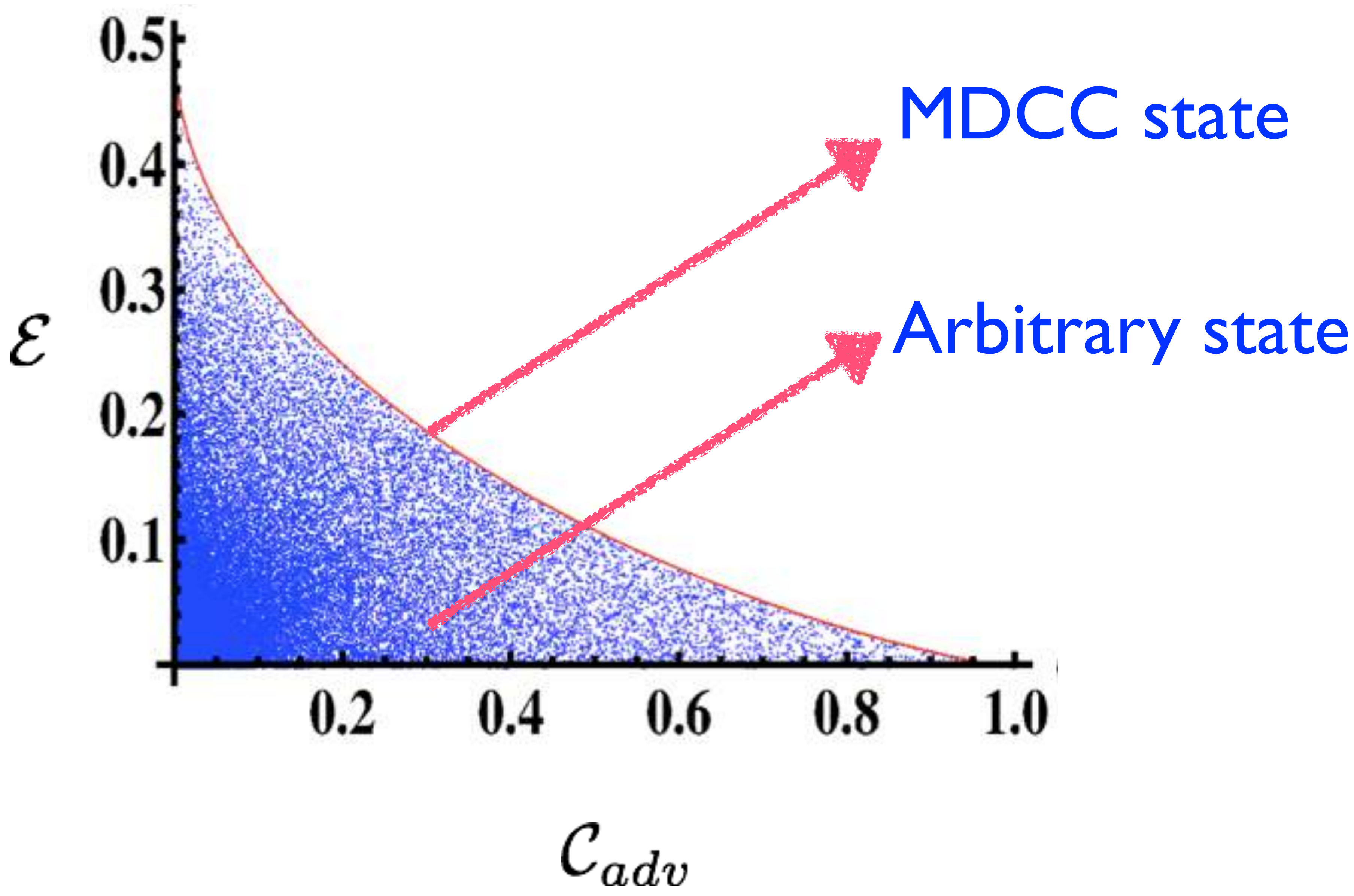
$$|\psi_\alpha\rangle = |111\rangle + |000\rangle + \alpha(|101\rangle + |010\rangle)$$

Arbitrary state $|\psi\rangle$ and set $\mathcal{E}(|\psi_\alpha\rangle) = \mathcal{E}(|\psi\rangle)$

$$\mathcal{C}_{\mathrm{adv}}(|\psi_\alpha\rangle) \geqslant \mathcal{C}_{\mathrm{adv}}(|\psi\rangle)$$

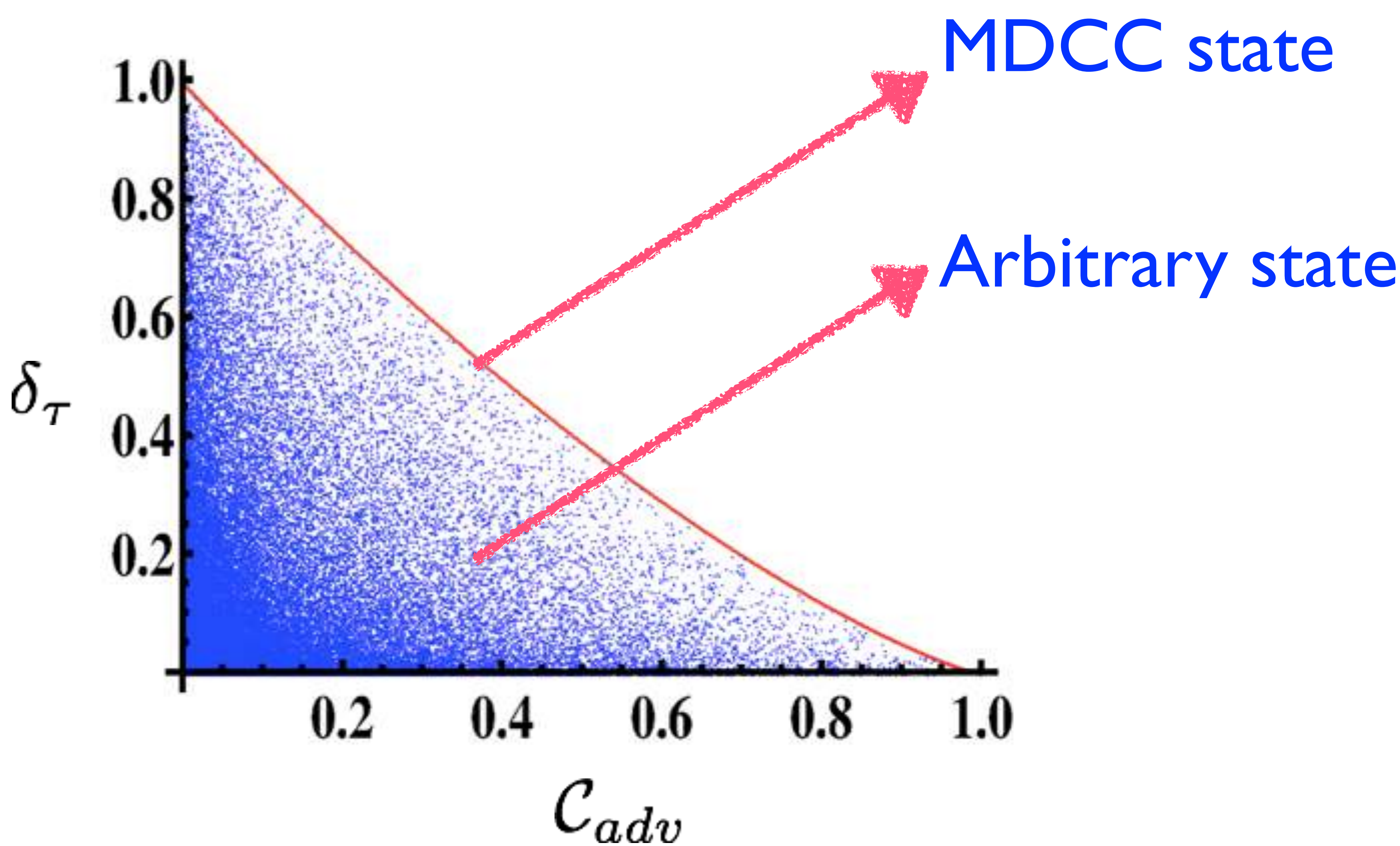
DC advantage & GGM

3-qubit



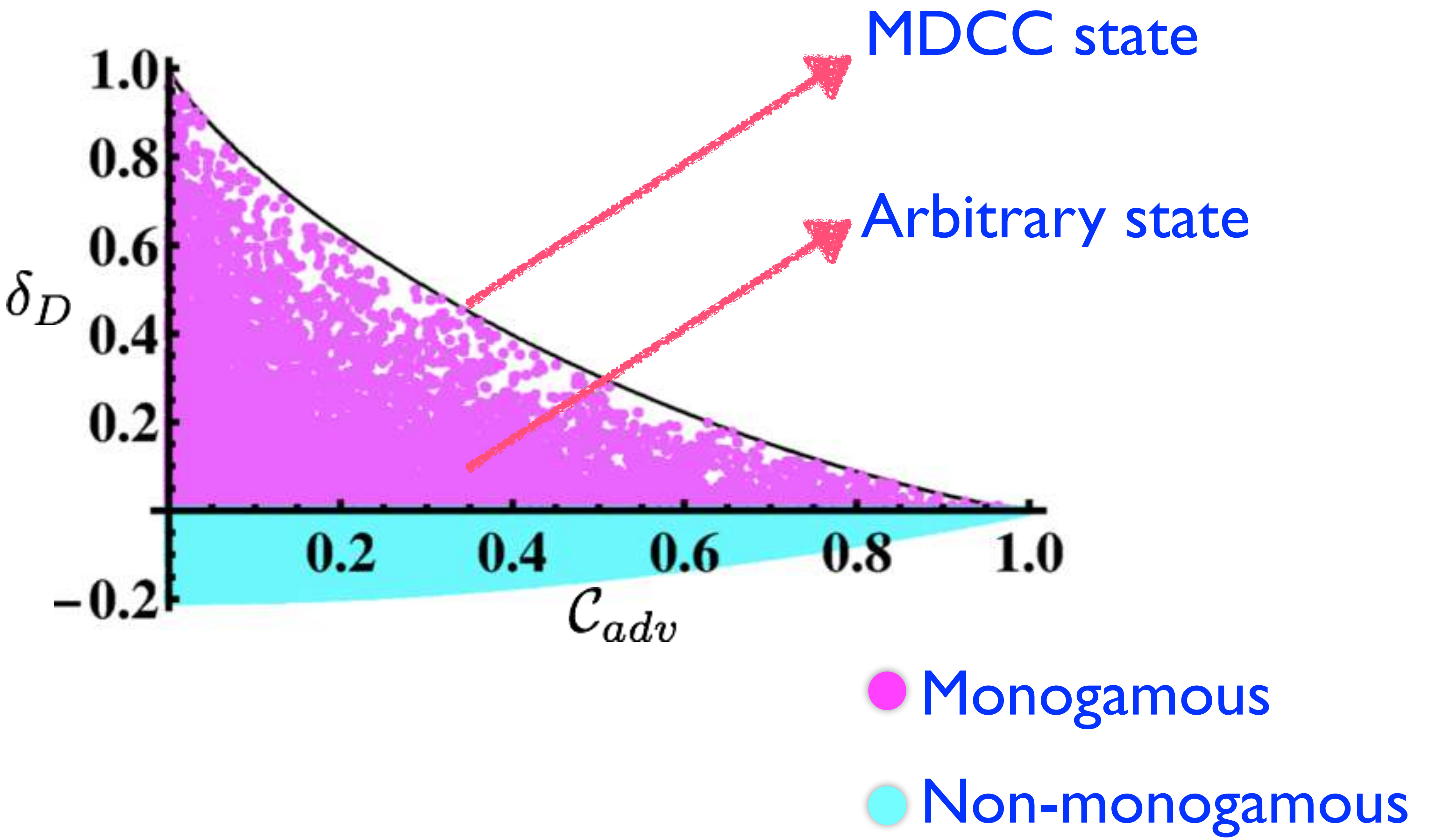
DC advantage & QC Score

3-qubit



DC advantage & QC Score

3-qubit



DC advantage & QC Score

Arbitrary state $|\psi\rangle$ and set $\delta_{\mathcal{Q}}(|\psi_\alpha\rangle) = \delta_{\mathcal{Q}}(|\psi\rangle)$

$$\mathcal{C}_{\text{adv}}(|\psi_\alpha\rangle) \geqslant \mathcal{C}_{\text{adv}}(|\psi\rangle)$$

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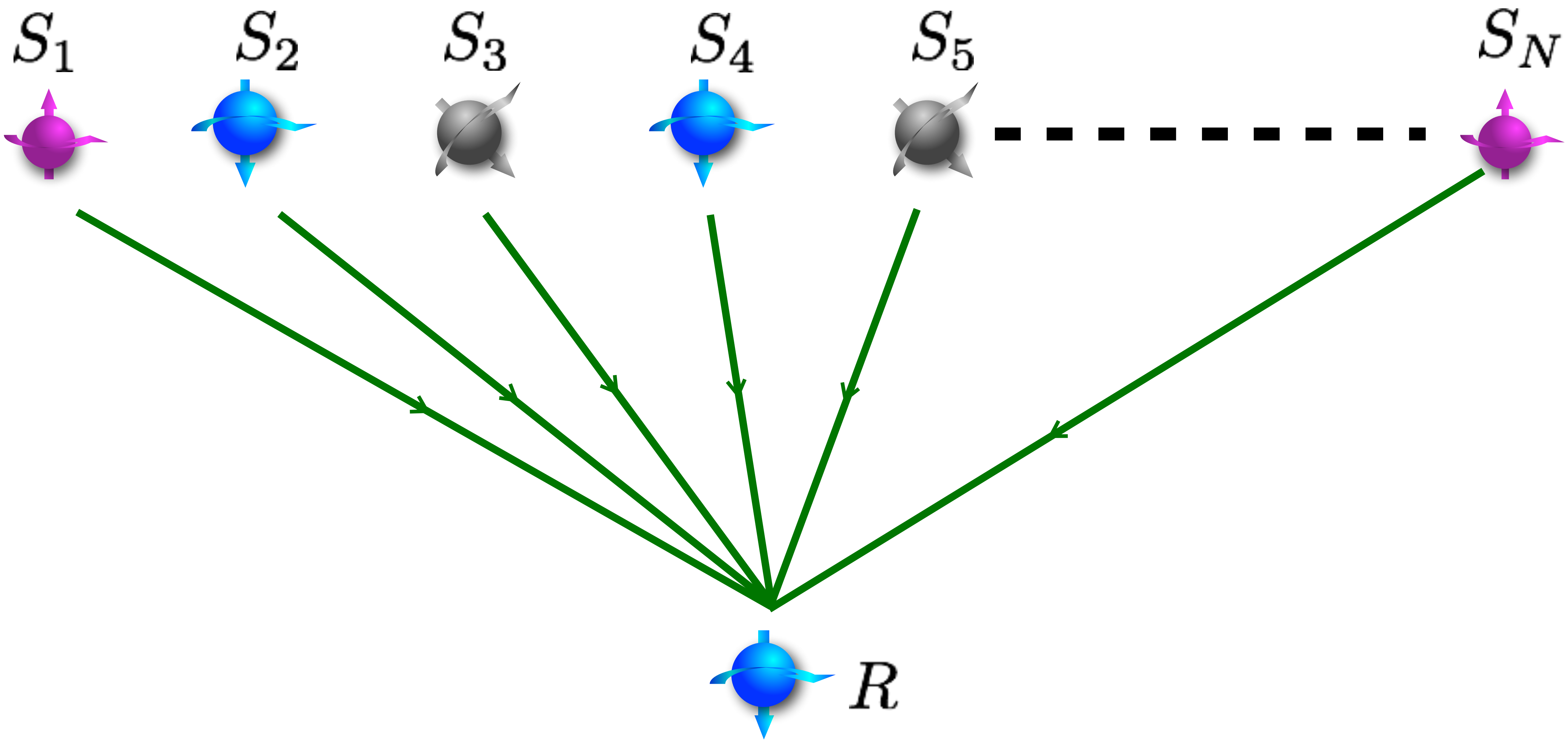
Tamoghna's poster

Dense coding - many to single



$$C(\rho_{S_1 \ldots S_N R}) = \frac{\log_2 d_{S_1 \ldots S_N} + S(\rho_R) - S(\rho_{S_1 \ldots S_N R})}{\log_2 d_{S_1 \ldots S_N R}}$$

$$d_{S_1 \ldots S_N} = d_{S_1} \ldots d_{S_N}$$

Dense coding - many to single

Generalized GHZ state

$$|GGHZ\rangle_{S_1S_2\ldots S_NR} = \sqrt{\alpha}|0_{S_1}\ldots 0_{S_N}\rangle|0_R\rangle + \sqrt{1-\alpha}e^{i\phi}|1_{S_1}\ldots 1_{S_N}\rangle|1_R\rangle$$

Arbitrary state $|\psi\rangle$ and set $C(|GGHZ\rangle) = C(|\psi\rangle)$

$$\mathcal{E}(|\psi\rangle) \leq \mathcal{E}(|GGHZ\rangle)$$

Dense coding - many to single

3-qubit

$$C(|GGHZ\rangle) = \frac{2}{3} + \frac{S(\rho_R)}{3}$$
$$= \frac{2}{3} - \frac{\alpha \log_2 \alpha + (1-\alpha)\log_2(1-\alpha)}{3}$$

$$C(|\psi\rangle) = \frac{2}{3} - \frac{\lambda_R \log_2 \lambda_R + (1-\lambda_R)\log_2(1-\lambda_R)}{3}$$

$$C(|GGHZ\rangle) = C(|\psi\rangle)$$

$$\alpha = \lambda_R$$

Dense coding - many to single

3-qubit

$$\mathcal{E}(|GGHZ\rangle) = 1 - \alpha$$

$\alpha \geq 1/2$

$$\mathcal{E}(|\psi\rangle) = 1 - \max[\{l_A\}]$$

Set of all max eigenvalues all bipartitions

If max eigenvalue is from receiver state (rank 2)

$$\mathcal{E}(|\psi\rangle) = 1 - \lambda_R = 1 - \alpha = \mathcal{E}(|GGHZ\rangle)$$

Dense coding - many to single

3-qubit

If $\lambda_R \neq \max[\{l_A\}]$

$$\lambda_R \leq \lambda_0 = \max[\{l_A\}]$$

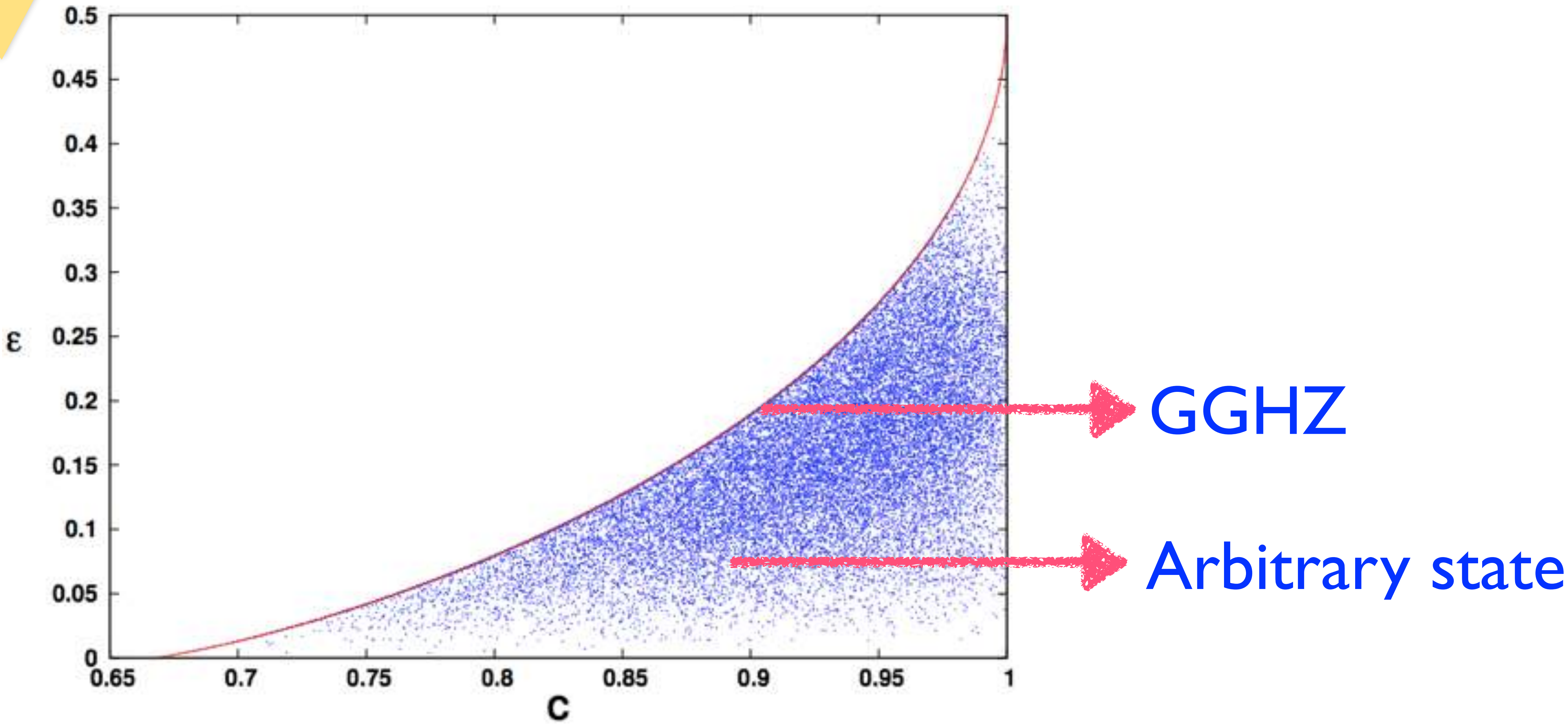
$$\mathcal{E}(|\psi\rangle) = 1 - \lambda_0 \leq 1 - \lambda_R = 1 - \alpha = \mathcal{E}(|GGHZ\rangle)$$

Arbitrary state $|\psi\rangle$ and set $C(|GGHZ\rangle) = C(|\psi\rangle)$

$$\mathcal{E}(|\psi\rangle) \leq \mathcal{E}(|GGHZ\rangle)$$

Dense coding - many to single

3-qubit



Dense coding - many to single

Generic ?

If receiver is set as nodal observer

$$\delta_D(|\psi\rangle) \leq \delta_D(|GGHZ\rangle)$$

$$\delta_\tau(|\psi\rangle) \leq \delta_\tau(|GGHZ\rangle)$$

Conclusions

DC advantage in network

Complimentarity b/w QC and DC adv

$$\mathcal{C}_{\text{adv}}^{\max} + E_R \leqslant \log_2 d$$

$$\frac{1}{\log_2 d}\mathcal{C}_{\text{adv}}^{\max} + \frac{d}{d-1}\mathcal{E} \leqslant 1$$

Found MDCC states

Related multi DC and multi QC

$$\mathcal{E}(|\psi\rangle) \leq \mathcal{E}(|GGHZ\rangle)$$

http://www.hri.res.in/~qic/

Thank you