Bell's theorem with and without inequalities for graph states

Adán Cabello[1], Otfried Gühne[2], Pilar Moreno[1], David Rodríguez[1], and Ignacio Villanueva[3].

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International Conference on Quantum Information (Bhubaneswar, India).
March 9th, 2008.

Problems

- Which is the maximum degree of nonlocality *D* for a six-qubit graph state allowing bipartite elements of reality?
- Which is the maximum *D* for the perfect correlations of a *n*-qubit graph state?
- Which is the relation between *D* and η?
- Can these results help us to make a loophole-free experiment?

(*D* is the ratio between the QM value and the bound of the Bell inequality. η is the minimum overall detection efficiency required for a loophole-free experiment.)

Plan

- Previously on the School…
- Problem #1… Solved
- Problem #2… Solved for $n < 7$
- Problem #3… Solved for GHZ states
- Problem #4… Work in progress

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The Mermin inequality

$$\left| \langle A_1 B_0 C_0 \rangle + \langle A_0 B_1 C_0 \rangle + \langle A_0 B_0 C_1 \rangle - \langle A_1 B_1 C_1 \rangle \right| \leq 2$$

$$\beta_{\mathrm{QM}} = 4$$

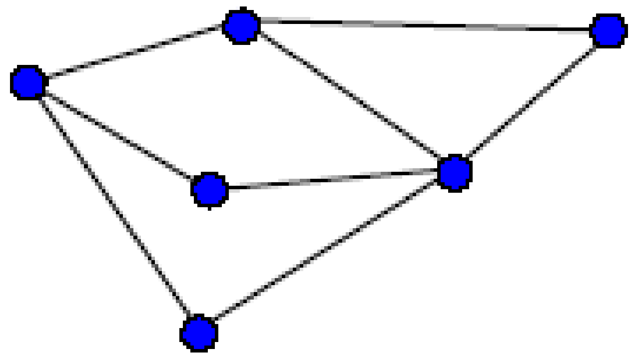
The *n*-qubit Mermin inequality

$$\frac{\beta_{\text{QM}}}{\beta_{\text{Local models}}} = 2^{(n\text{-}1)/2}$$

Graph states

Graph states are a family of multiqubit pure entangled states.

Each graph state is associated to a graph



Vertices: qubits.

Edges: entanglement between the connected qubits.

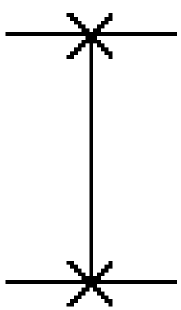
Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*

$$C_Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} =$$



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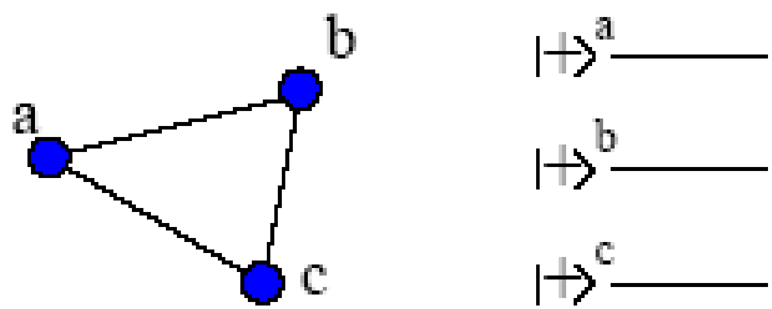

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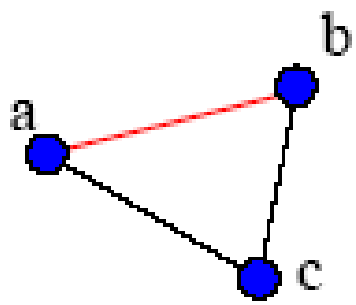

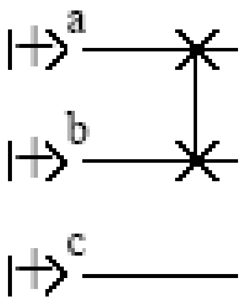

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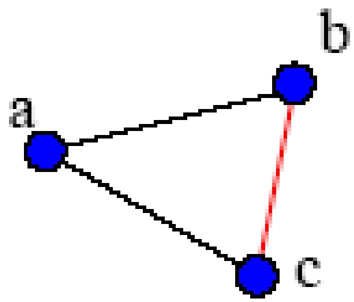

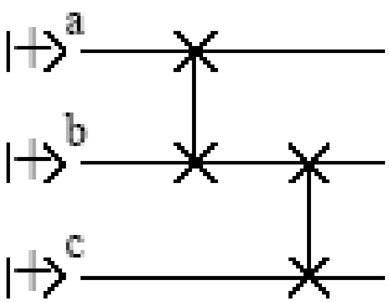

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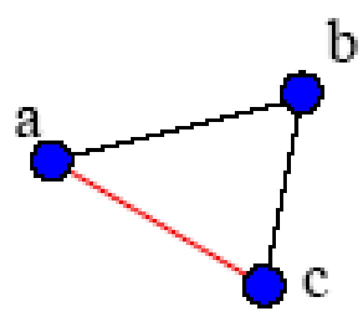

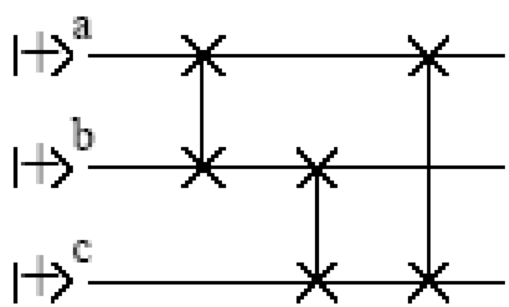

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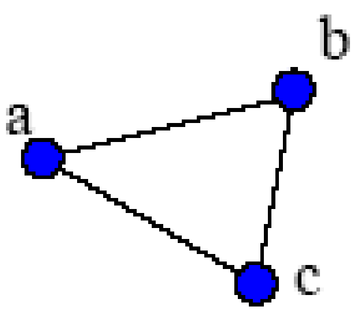
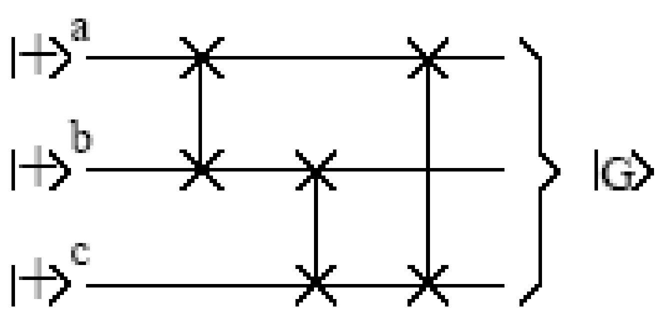
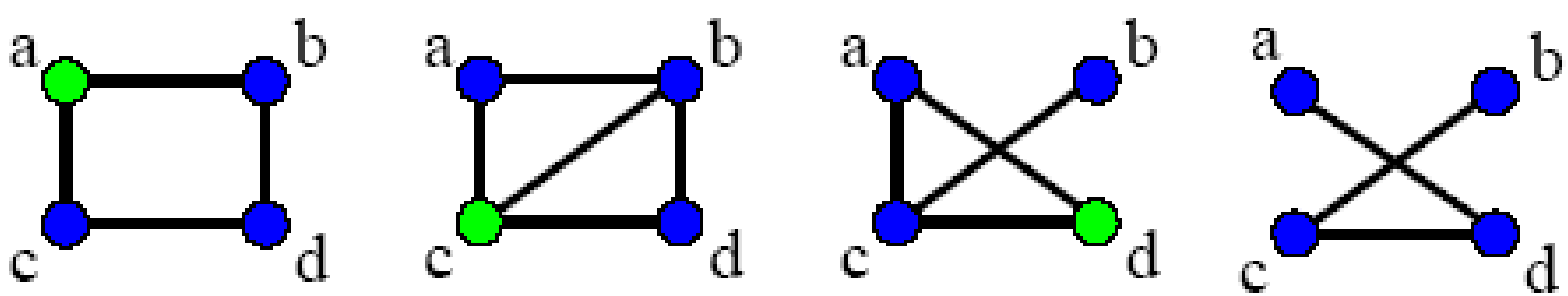


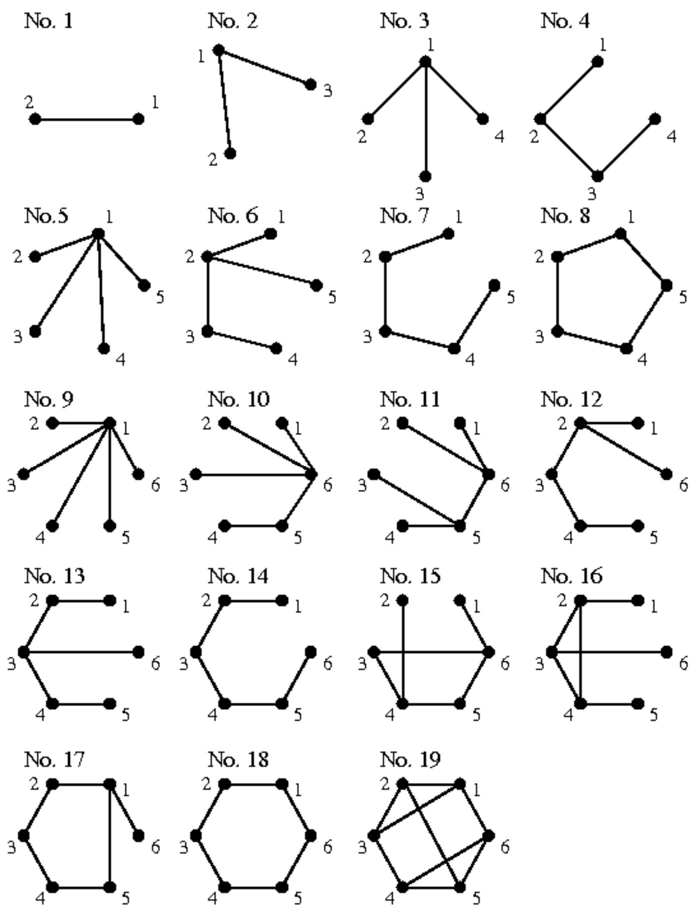


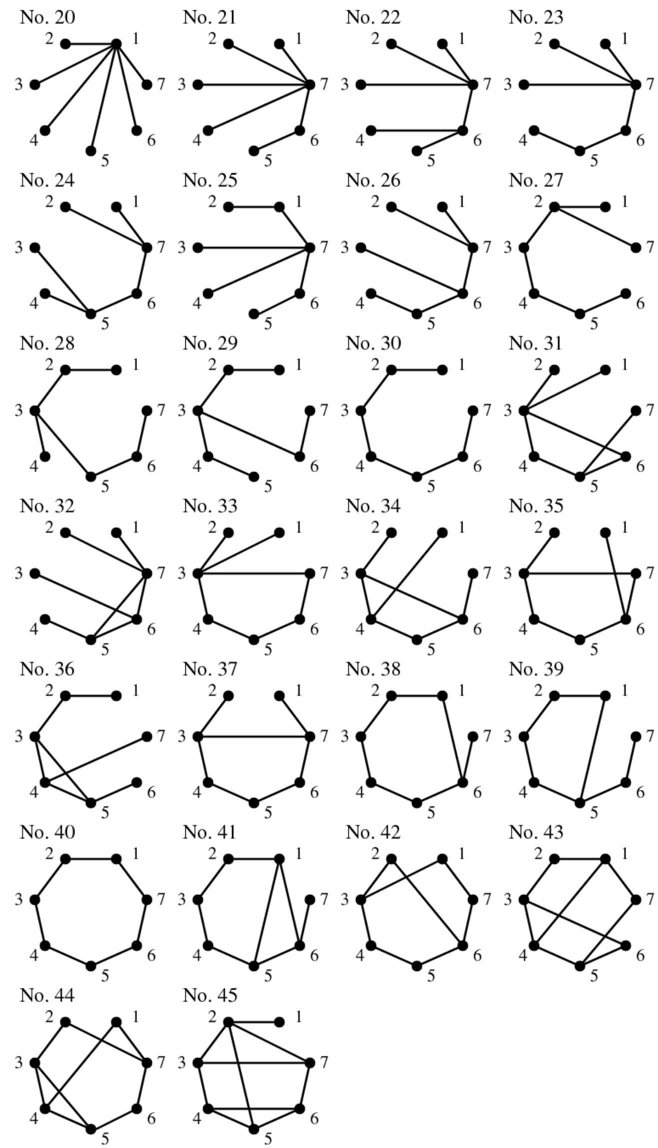
Graph states: Entanglement

The following graphs represent *LC*-equivalent graph states.
Therefore, they represent *LU*-equivalent states.
Therefore, they have the same entanglement.



All graph states up to seven qubits





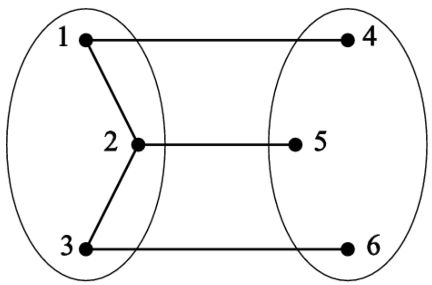
Graphs corresponding to all possible graph states, up to seven qubits, which are not equivalent under single-qubit transformations and graph isomorphism, M. Hein, J. Eisert, and H. J. Briegel, PRA **69**, 062311 (2004).

Problem

- If we distribute *n* qubits between two parties, what quantum graph states and distributions of qubits allow AVN proofs using only single-qubit measurements?

Six-qubit graph states allowing bipartite AVN proofs

No. 13a



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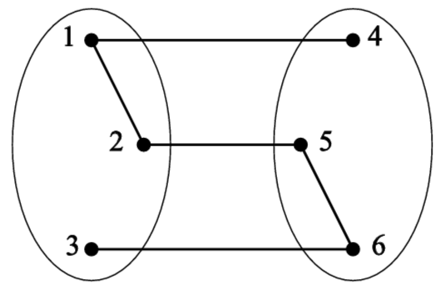

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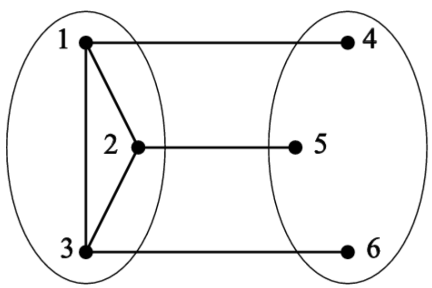

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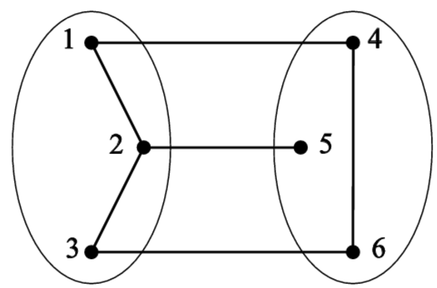

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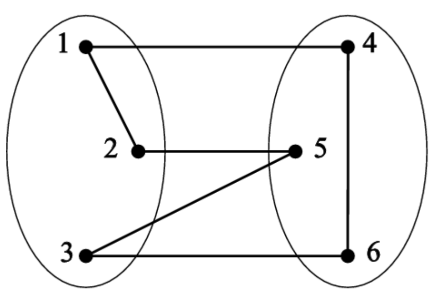

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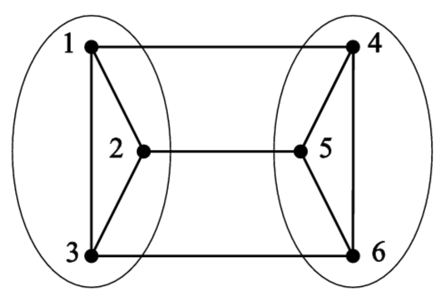

$$
\begin{aligned}
|\psi_{13a}\rangle &= \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
&+|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle + |1\bar{0}1\bar{1}0\bar{1}\rangle \\
&+|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}1\bar{1}\rangle), \\
|\psi_{14a}\rangle &= \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}0\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{1}\rangle \\
&+|0\bar{1}1\bar{0}1\bar{0}\rangle + |1\bar{0}0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{1}1\bar{0}\rangle \\
&+|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}0\bar{1}\rangle), \\
|\psi_{16a}\rangle &= \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
&+|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle - |1\bar{0}1\bar{1}0\bar{1}\rangle \\
&+|1\bar{1}0\bar{1}0\bar{0}\rangle - |1\bar{1}1\bar{1}1\bar{1}\rangle), \\
|\psi_{17a}\rangle &= \frac{1}{2\sqrt{2}}(|00\bar{0}\bar{0}\bar{0}0\rangle + |00\bar{1}\bar{1}\bar{0}1\rangle + |01\bar{0}\bar{1}\bar{1}1\rangle \\
&+|01\bar{1}\bar{0}\bar{1}0\rangle + |10\bar{0}\bar{1}\bar{0}0\rangle + |10\bar{1}\bar{0}\bar{0}1\rangle \\
&-|11\bar{0}\bar{0}\bar{1}1\rangle - |11\bar{1}\bar{1}\bar{1}1\rangle), \\
|\psi_{18a}\rangle &= \frac{1}{2\sqrt{2}}(|\bar{0}000\bar{0}\bar{0}\rangle + |\bar{0}010\bar{1}\bar{1}\rangle + |\bar{0}101\bar{1}\bar{1}\rangle \\
&+|\bar{0}111\bar{0}\bar{0}\rangle + |\bar{1}001\bar{0}\bar{1}\rangle + |\bar{1}011\bar{1}\bar{0}\rangle \\
&+|\bar{1}100\bar{1}\bar{0}\rangle + |\bar{1}110\bar{0}\bar{1}\rangle), \\
|\psi_{19a}\rangle &= \frac{1}{4}(|\bar{0}0000\bar{0}\rangle + |\bar{0}0001\bar{1}\rangle + |\bar{0}0110\bar{0}\rangle \\
&-|\bar{0}0111\bar{1}\rangle + |\bar{0}1010\bar{1}\rangle + |\bar{0}1011\bar{0}\rangle \\
&-|\bar{0}1100\bar{1}\rangle + |\bar{0}1101\bar{0}\rangle + |\bar{1}0010\bar{1}\rangle \\
&-|\bar{1}0011\bar{0}\rangle + |\bar{1}0100\bar{1}\rangle + |\bar{1}0101\bar{0}\rangle \\
&+|\bar{1}1000\bar{0}\rangle - |\bar{1}1001\bar{1}\rangle - |\bar{1}1110\bar{0}\rangle \\
&-|\bar{1}1111\bar{1}\rangle).
\end{aligned}
$$

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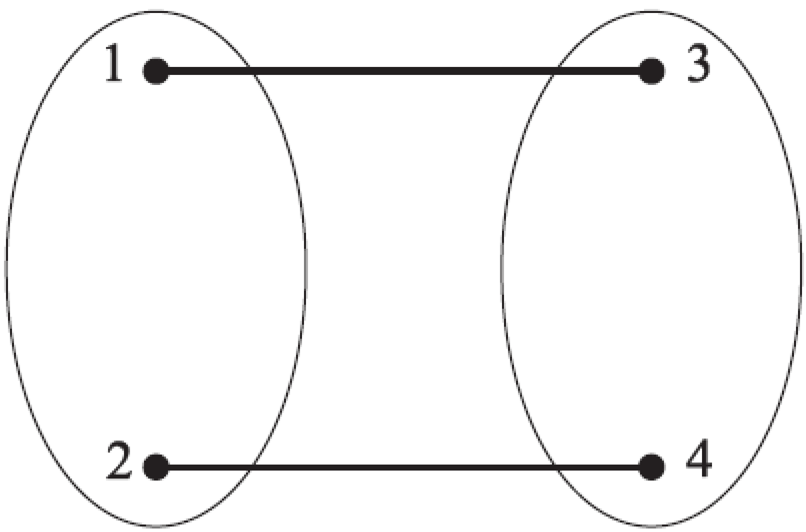
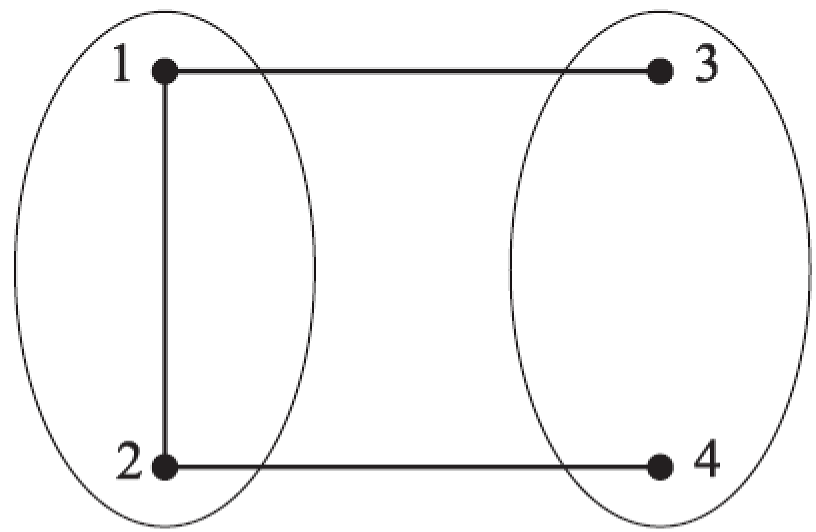
Two-photon four-qubit experiments

No. 1 twice



M. Barbieri, F. De Martini, P. Mataloni, G. Vallone, and AC, PRL **97**, 140407 (2006).

No. 4a



G. Vallone, E. Pomarico, P. Mataloni, F. De Martini, and V. Berardi, PRL **98**, 180502 (2007).

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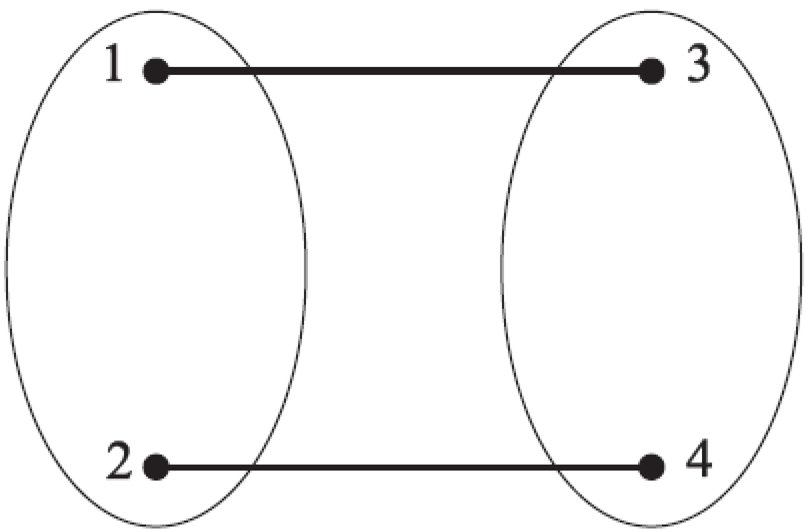
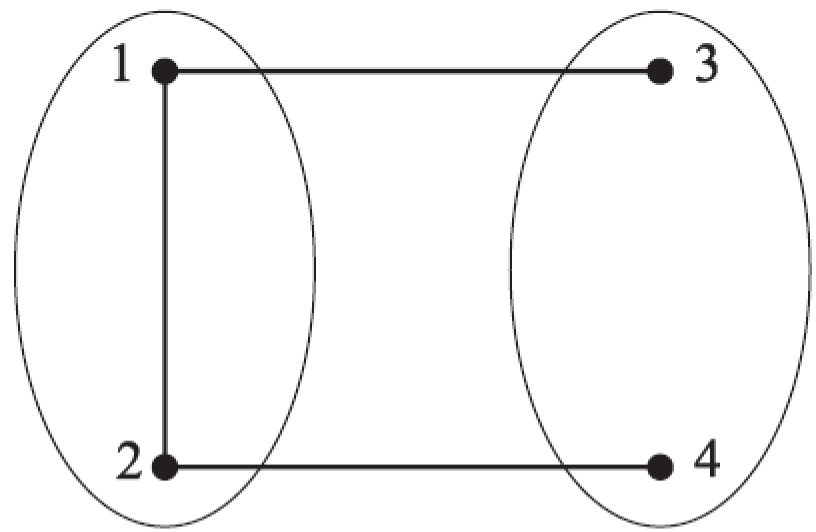


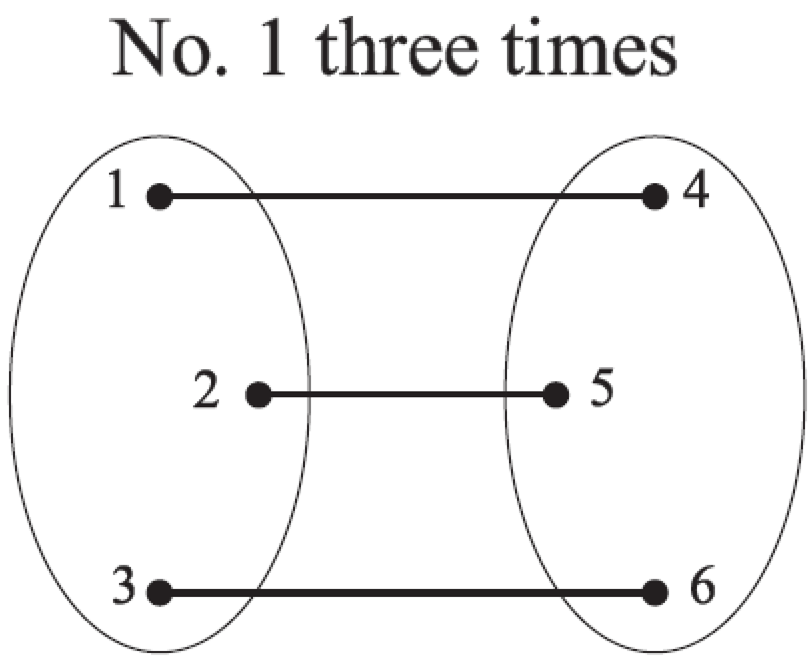
$D = 2$, 16 terms

No. 4a



$D = 2$, 4 terms

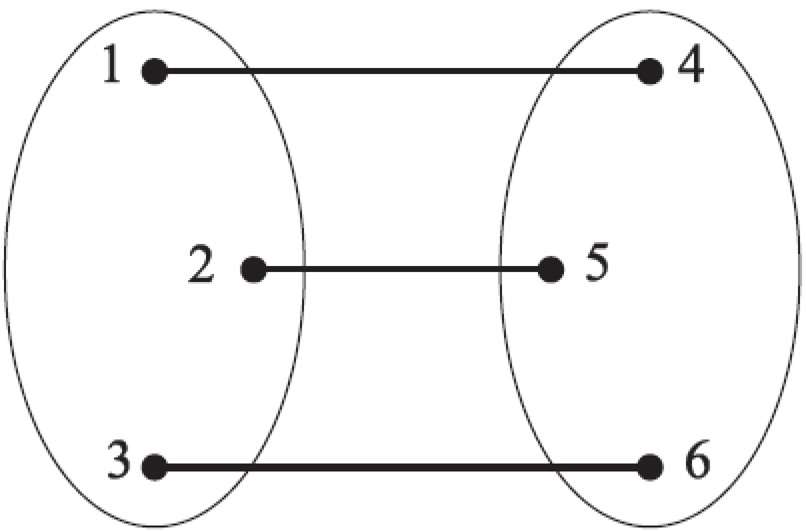
Two-photon six-qubit experiments



$D = 2.8$, 64 terms

Two-photon six-qubit experiments

No. 1 three times



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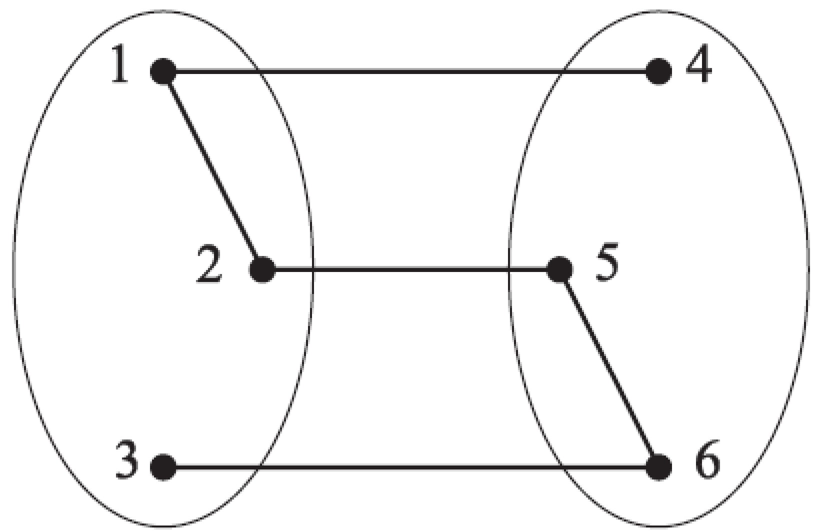

$D = 4$, 16 terms

Two-photon six-qubit experiments

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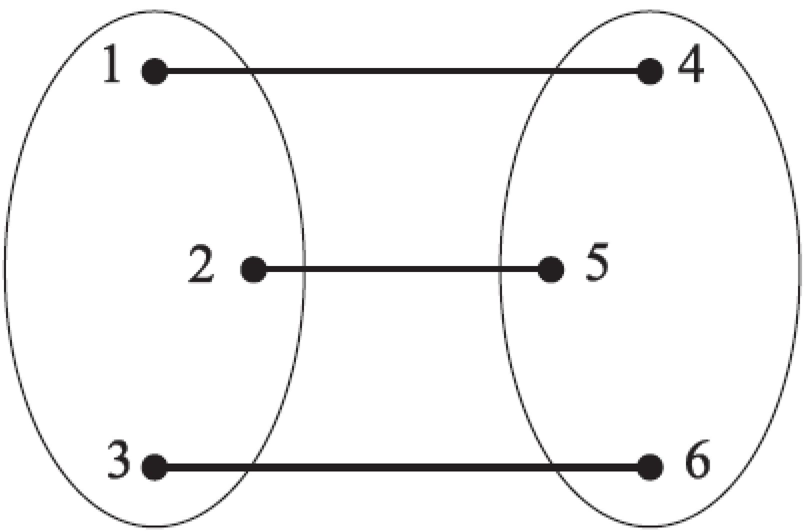

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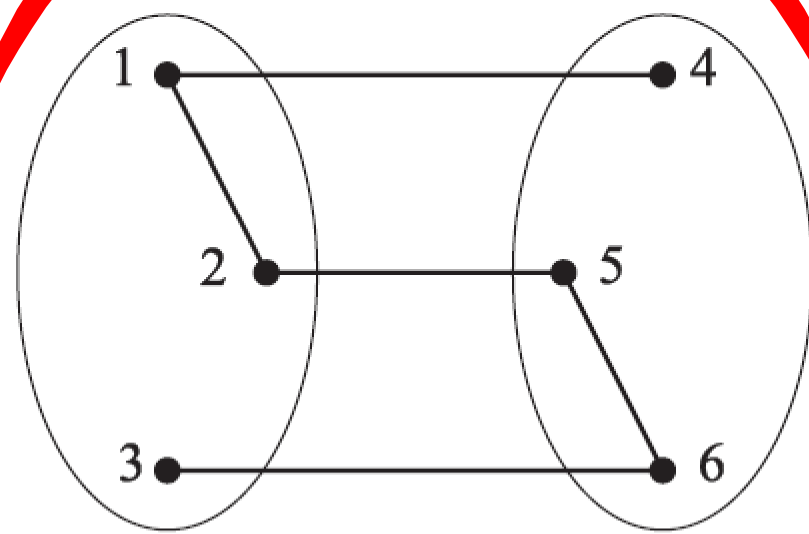

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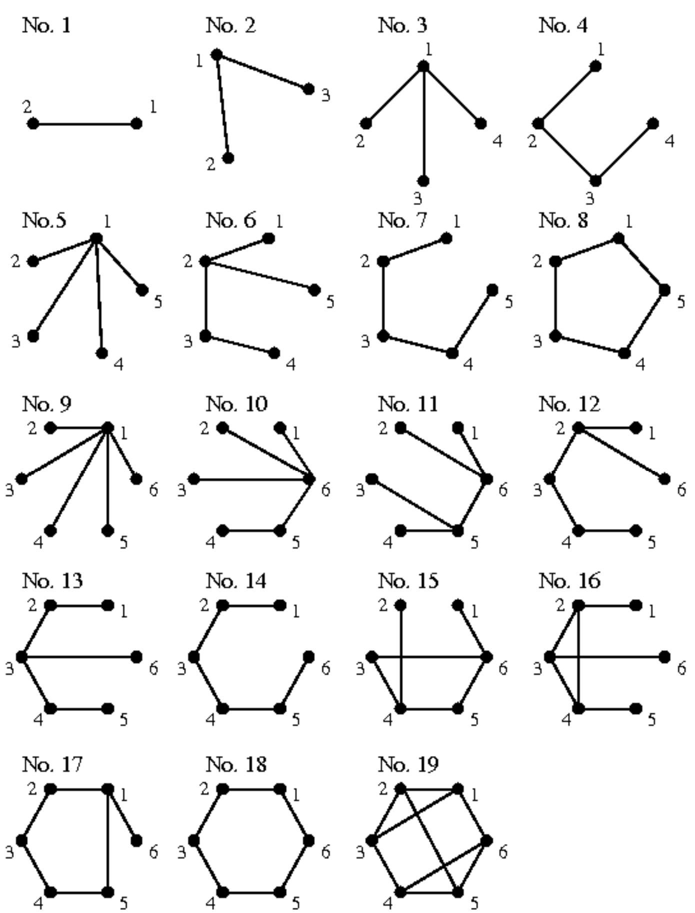
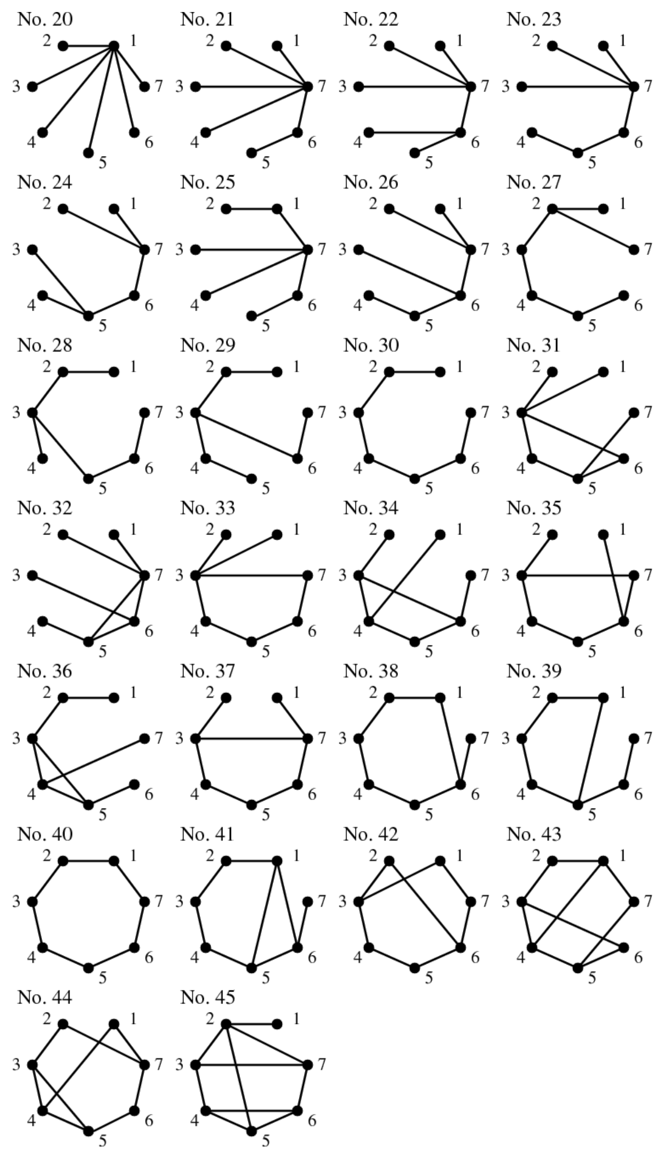
All graph states up to seven qubits





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Motivation

PRL 95, 120405 (2005) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2005

Bell Inequalities for Graph States

Otfried Gühne,[1] Géza Tóth,[2] Philipp Hyllus,[3,4] and Hans J. Briegel[1,5]

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[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, D-85748 Garching, Germany*
[3]*Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany*
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(Received 13 October 2004; published 14 September 2005)

We investigate the nonlocal properties of graph states. To this aim, we derive a family of Bell inequalities which require three measurement settings for each party and are maximally violated by graph states. In turn, for each graph state there is an inequality maximally violated only by that state. We show that for certain types of graph states the violation of these inequalities increases exponentially with the number of qubits. We also discuss connections to other entanglement properties such as the positivity of the partial transpose or the geometric measure of entanglement.

$$\begin{aligned}\mathcal{B}(FC_3) = {} & \mathbb{1}^{(1)}\mathbb{1}^{(2)}\mathbb{1}^{(3)} + X^{(1)}Z^{(2)}Z^{(3)} + Z^{(1)}X^{(2)}Z^{(3)} \\ & + Z^{(1)}Z^{(2)}X^{(3)} + Y^{(1)}Y^{(2)}\mathbb{1}^{(3)} + Y^{(1)}\mathbb{1}^{(2)}Y^{(3)} \\ & + \mathbb{1}^{(1)}Y^{(2)}Y^{(3)} - X^{(1)}X^{(2)}X^{(3)}.\end{aligned}$$

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[3]Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany
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$$\mathcal{B}(FC_3) = X^{(1)}Z^{(2)}Z^{(3)} + Z^{(1)}X^{(2)}Z^{(3)} + Z^{(1)}Z^{(2)}X^{(3)} - X^{(1)}X^{(2)}X^{(3)}.$$

The Mermin inequality

$$\left|\langle A_1B_0C_0\rangle+\langle A_0B_1C_0\rangle+\langle A_0B_0C_1\rangle-\langle A_1B_1C_1\rangle\right|\leq 2$$

$$\beta_{\mathrm{QM}}=4$$

Mermin inequalities for graph states

- Gühne et al., 2005:

$$\beta = \sum_{j=1}^{2^N} s_j \qquad s_j|\psi_N\rangle = |\psi_N\rangle$$

- Now: Find the one with the largest degree of nonlocality of the family

$$\beta_k = \sum_{j=1}^{2^N-1} a_{kj} s_j, \qquad a_{kj} \in \{0, 1\}$$

Mermin inequalities for graph states

TABLE I: Mermin inequalities for all graph states of $n < 6$ qubits.

Graph state	g_i	$\beta \leq 2p - q$	Settings	$\mathcal{D}$
2 (GHZ$_3$)	$g_1 = X_1Z_2Z_3$	$g_1(\mathbb{1}+g_2)(\mathbb{1}+g_3) \leq 2$	2-2-2	2
	$g_i = Z_1X_i$ for $i \neq 1$			
3 (GHZ$_4$)	$g_1 = X_1Z_2Z_3Z_4$	$g_1(\mathbb{1}+g_2g_3+g_2g_4+g_3g_4) \leq 2$ and $g_1 \to g_1g_2$	1-2-2-2	2
	$g_i = Z_1X_i$ for $i \neq 1$	$g_1(\mathbb{1}+g_i)(\mathbb{1}+g_j) \leq 2$ and $g_1 \to g_1g_k$	2-2(i)-2(j)-1(k)	
4 (LC$_4$)	$g_1 = X_1Z_2$, $g_4 = Z_3X_4$	$(\mathbb{1}+g_1)g_2(\mathbb{1}+g_3) \leq 2$ and $g_3 \to g_3g_4$	2-2-2-1	2
	$g_i = Z_{i-1}X_iZ_{i+1}$ for $i = 2, 3$	$(\mathbb{1}+g_1)g_2(g_3+g_4) \leq 2$ and $g_3 \to g_3g_4$	2-2-1-2	
		$g_i \to g_{i+1}$		
5 (GHZ$_5$)	$g_1 = X_1Z_2Z_3Z_4Z_5$,	$g_1(\mathbb{1}+g_2)(\mathbb{1}+g_3)(\mathbb{1}+g_4)(\mathbb{1}+g_5) \leq 4$	2-2-2-2-2	4
	$g_i = Z_1X_i$ for $i \neq 1$			
6 (Y$_5$)	$g_1 = X_1Z_2$, $g_5 = Z_2X_5$	$g_2\big[(\mathbb{1}+g_1+g_5)(\mathbb{1}+g_3+g_3g_4)+(\mathbb{1}+g_1g_5)g_4\big]$		
	$g_2 = Z_1X_2Z_5$	$+(g_1+g_5)g_3(\mathbb{1}+g_4) \leq 7$	3-3-3-3-2	$\frac{15}{7}$
	$g_3 = Z_2X_3Z_4$	$g_2 \to g_2g_4$	3-3-3-3-3	
	$g_4 = Z_3X_4$	$\beta \to g_4\beta$ and 32 nonsymmetric more		
7 (LC$_5$)	$g_1 = X_1Z_2$, $g_5 = Z_4X_5$	$(\mathbb{1}+g_1)\big[(\mathbb{1}+g_2)g_3(\mathbb{1}+g_4)+g_2g_4\big](\mathbb{1}+g_5) \leq 8$	3-3-3-3-3	$\frac{5}{2}$
	$g_i = Z_{i-1}X_iZ_{i+1}$ for $i = 2, 3, 4$			
8 (RC$_5$)	$g_i = Z_{i-1}X_iZ_{i+1}$	$\gamma + \sum_{i=1}^{5} g_ig_{i+1} \leq 9$	3-3-3-3-3	$\frac{7}{3}$
		$\gamma + g_jg_{i+1} + g_ig_{i+2} + g_{i-1}g_{i+1}$		
		$+g_{i-2}g_ig_{i+1}g_{i+2} + g_{i-2}g_{i-1}g_ig_{i+1} \leq 9$	3-3-3-3-3	
		$\gamma = \frac{1}{2}\left[\prod_{i=1}^{5}(\mathbb{1}+g_i) - \prod_{i=1}^{5}(\mathbb{1}-g_i)\right]$		
		and 105 more		

Mermin inequalities for graph states

TABLE II: Symmetric Mermin inequalities for all graph states of $n = 6$ qubits.

Graph state	g_i	$\beta \leq 2p - q$	Settings	$\mathcal{D}$
9 (GHZ_6)	$g_1 = X_1 Z_2 Z_3 Z_4 Z_5 Z_6$	$g_1(\mathbb{1} + \sum_{i \neq j \neq 1} g_i g_j + \sum_{i \neq j \neq k \neq l \neq 1} g_i g_j g_k g_l) \leq 4$	1-2-2-2-2-2	4
	$g_i = Z_1 X_i$ for $i \neq 1$	and $g_1 \to g_1 g_2$	1-2-2-2-2-2	
10	$g_i = X_i Z_6$ for $i = 1, 2, 3$	$(\mathbb{1} + g_1)(\mathbb{1} + g_2)(\mathbb{1} + g_3)(\mathbb{1} + g_5) g_6 \leq 4$	2-2-2-1-2-2	4
	$g_4 = X_4 Z_5$	$g_5 \to g_4 g_5$	2-2-2-1-2-2	
	$g_5 = Z_4 X_5 Z_6$	$(\mathbb{1} + g_1)(\mathbb{1} + g_2)(\mathbb{1} + g_3)(g_4 + g_5) g_6 \leq 4$	2-2-2-2-1-2	
	$g_6 = Z_1 Z_2 Z_3 Z_5 X_6$	$g_5 \to g_4 g_5$	2-2-2-2-1-2	
11 (H_6)	$g_1 = X_1 Z_6$, $g_2 = X_2 Z_6$	$g_1(\mathbb{1} + g_2)(\mathbb{1} + g_3)(\mathbb{1} + g_4)(\mathbb{1} + g_5) g_6 \leq 4$	1-2-3-3-3-2	4
	$g_3 = X_3 Z_5$, $g_4 = X_4 Z_5$	$g_1 \leftrightarrow g_2$ (i.e., permute them)	2-1-3-3-3-2	
	$g_5 = Z_3 Z_4 X_5$, $g_6 = Z_1 Z_2 X_6$	$(\mathbb{1} + g_1)(\mathbb{1} + g_2) g_3 (\mathbb{1} + g_4) g_5 (\mathbb{1} + g_6) \leq 4$	3-3-1-2-2-3	
		$g_3 \leftrightarrow g_4$	3-3-2-1-2-3	
12 (Y_6)	$g_1 = X_1 Z_2$, $g_6 = Z_2 X_6$	$(\mathbb{1} + g_1) g_2 (\mathbb{1} + g_3) g_4 (\mathbb{1} + g_5)(\mathbb{1} + g_6) \leq 4$	2-2-1-2-2-2	4
	$g_2 = Z_1 X_2 Z_3 Z_6$, $g_3 = Z_2 X_3 Z_4$			
	$g_4 = Z_3 X_4 Z_5$, $g_5 = Z_4 X_5$			
13 (E_6)	$g_1 = X_1 Z_2$, $g_5 = Z_4 X_5$	$(\mathbb{1} + g_3 + g_3 g_6)[(\mathbb{1} + g_1) g_2 + g_4(\mathbb{1} + g_5)$		
	$g_2 = Z_1 X_2 Z_3$, $g_4 = Z_3 X_4 Z_5$	$+(\mathbb{1} + g_1) g_2 g_4 (\mathbb{1} + g_5)] \leq 8$	2-3-3-3-2-2	3
	$g_3 = Z_2 X_3 Z_4 Z_6$, $g_6 = Z_3 X_6$	and 37 more		
14 (LC_6)	$g_1 = X_1 Z_2$, $g_6 = Z_5 X_6$	$(\mathbb{1} + g_1) g_2 (\mathbb{1} + g_3)(\mathbb{1} + g_4) g_5 (\mathbb{1} + g_6) \leq 4$	2-2-3-3-2-2	4
	$g_i = Z_{i-1} X_i Z_{i+1}$ for $i = 2, 3, 4, 5$			
15	$g_1 = X_1 Z_6$, $g_2 = X_2 Z_4$	$(g_3 + g_5)(\mathbb{1} + g_1)(\mathbb{1} + g_2)(\mathbb{1} + g_4)(\mathbb{1} + g_6)$		
	$g_3 = X_3 Z_4 Z_6$, $g_5 = Z_4 X_5 Z_6$	$+(\mathbb{1} + g_3 g_5)(g_4 + g_2 g_4 + g_6 + g_1 g_6) \leq 16$	3-3-3-3-3-3	$\frac{5}{2}$
	$g_4 = Z_2 Z_3 X_4 Z_5$, $g_6 = Z_1 Z_3 Z_5 X_6$	and 6 more		
16	$g_1 = X_1 Z_2$, $g_5 = Z_4 X_5$	$g_3(\mathbb{1} + g_1 + g_2 + g_1 g_2 + g_4 + g_5 + g_4 g_5)(\mathbb{1} + g_6)$		
	$g_2 = Z_1 X_2 Z_3 Z_4$, $g_4 = Z_2 Z_3 X_4 Z_5$	$+(\mathbb{1} + g_1) g_2 (\mathbb{1} + g_5 + g_6) + g_4(\mathbb{1} + g_5)(\mathbb{1} + g_1 + g_6)$		
	$g_3 = Z_2 X_3 Z_4 Z_6$	$+(\mathbb{1} + g_1) g_2 g_4 (\mathbb{1} + g_5) \leq 12$	3-3-3-3-3-3	3
	$g_6 = Z_3 X_6$	and 3 more		
17	$g_1 = X_1 Z_2 Z_6$	$[g_1(\mathbb{1} + g_2 g_5)(g_3 + g_4)$		
	$g_2 = Z_1 X_2 Z_3$, $g_5 = Z_1 Z_4 X_5$	$+(\mathbb{1} + g_1)(g_2 + g_5)(\mathbb{1} + g_3 g_4)](\mathbb{1} + g_6) \leq 8$	3-3-3-3-3-3	3
	$g_3 = Z_2 X_3 Z_4$, $g_4 = Z_3 X_4 Z_5$	$\beta \to g_3 \beta$	3-3-3-3-3-3	
	$g_6 = Z_1 X_6$			
18 (RC_6)	$g_i = Z_{i-1} X_i Z_{i+1}$	$\sum_{i=1}^{64} s_i - \mathbb{1} - \sum_{i=1}^{6} g_i - g_1 g_3 g_5 - g_2 g_4 g_6 \leq 19$	3-3-3-3-3-3	$\frac{55}{19}$
19	$g_1 = X_1 Z_2 Z_3 Z_6$, $g_4 = Z_3 X_4 Z_5 Z_6$	$g_1 g_4 + g_3 g_6 + g_1 g_3 g_4 g_6 + g_2(g_4 + g_6 + g_4 g_6)$		
	$g_2 = Z_1 X_2 Z_3 Z_5$, $g_5 = Z_2 Z_4 X_5 Z_6$	$+g_5(g_1 + g_3 + g_1 g_3) + (g_2 + g_5)(g_3 g_4 + g_1 g_6 + g_1 g_3 g_4 g_6)$		
	$g_3 = Z_1 Z_2 X_3 Z_4$, $g_6 = Z_1 Z_4 Z_5 X_6$	$+g_2 g_5 [g_1 g_4 (\mathbb{1} + g_3 + g_6) + g_3 g_6 (\mathbb{1} + g_1 + g_4)] \leq 9$	3-3-3-3-3-3	$\frac{7}{3}$

Problems

- Which is the maximum degree of nonlocality *D* for a six-qubit graph state allowing bipartite elements of reality?
- Which is the maximum *D* for the perfect correlations of a *n*-qubit graph state?
- Which is the relation between *D* and η?
- Can these results help us to make a loophole-free experiment?

(*D* is the ratio between the QM value and the bound of the Bell inequality. η is the minimum overall detection efficiency required for a loophole-free experiment.)

For GHZ states and the Mermin inequalities

$$\frac{\beta_{\text{QM}}}{\beta_{\text{Local models}}} = 2^{(n\text{-}1)/2}$$

$$\eta = \frac{n}{2n-2}$$

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