Bell’s inequalities. Lecture II

Adán Cabello
Universidad de Sevilla, Spain.

International School on Quantum Information
(Bhubaneswar, India).
March 7th, 2008.

Plan

- GHZ: Beating EPR using their own weapons (perfect correlations)
- Mermin inequality: Violation that grows exponentially
- Bipartite AVN
- Bipartite AVN with only single-qubit measurements

Plan

- GHZ: Beating EPR using their own weapons (perfect correlations)
- Mermin inequality: Violation that grows exponentially
- Bipartite AVN
- Bipartite AVN with only single-qubit measurements

Greenberger, Horne and Zeilinger

GHZ's proof of Bell's theorem

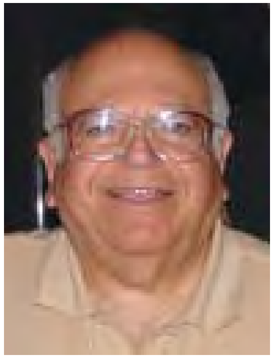
- "Opened a new chapter on the hidden variables problem"", and made "the strongest case against local realism since Bell's work".

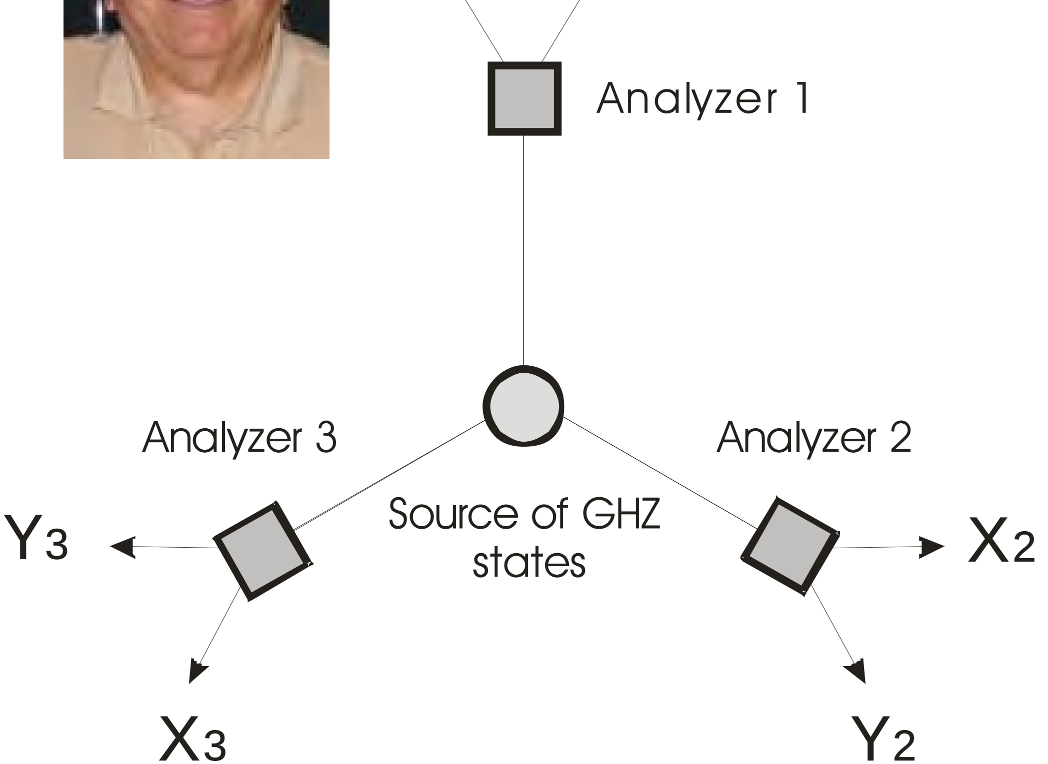
- Quantum reduction of the communication complexity.

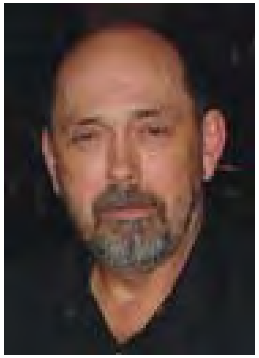
- Quantum secret sharing.

- Multipartite entanglement.

GHZ’s proof of Bell’s theorem







GHZ's proof of Bell's theorem

$$|GHZ\rangle = |HHH\rangle - |VVV\rangle$$

$$X_1 Y_2 Y_3 |GHZ\rangle = |GHZ\rangle$$

$$Y_1 X_2 Y_3 |GHZ\rangle = |GHZ\rangle$$

$$Y_1 Y_2 X_3 |GHZ\rangle = |GHZ\rangle$$

$$X_1 X_2 X_3 |GHZ\rangle = -|GHZ\rangle$$

Notation for single photon observables

Polarization observables:

$$X = |H\rangle\langle V| + |V\rangle\langle H|$$

$$Y = i\,(|V\rangle\langle H| - |H\rangle\langle V|)$$

$$Z = |H\rangle\langle H| - |V\rangle\langle V|$$

GHZ's proof of Bell's theorem: X_i and Y_i are ER

GHZ:

$$|HHH\rangle - |VVV\rangle$$

$$v(X_1)\,v(Y_2) = v(Y_3)$$

$$v(Y_1)\,v(X_2) = v(Y_3)$$

$$v(Y_1)\,v(Y_2) = v(X_3)$$

$$v(X_1)\,v(X_2) = -\,v(X_3)$$

GHZ's proof of Bell's theorem: Contradiction!

GHZ:

$$|HHH\rangle - |VVV\rangle$$

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -v(X_3)$$

Why “all-versus-nothing”?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

Why "all-versus-nothing"?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = +v(X_3)$$

Why "all-versus-nothing"?

$$v(X_1)\,v(Y_2) = v(Y_3)$$

$$v(Y_1)\,v(X_2) = v(Y_3)$$

$$v(Y_1)\,v(Y_2) = v(X_3)$$

$$v(X_1)\,v(X_2) = -\,v(X_3)$$

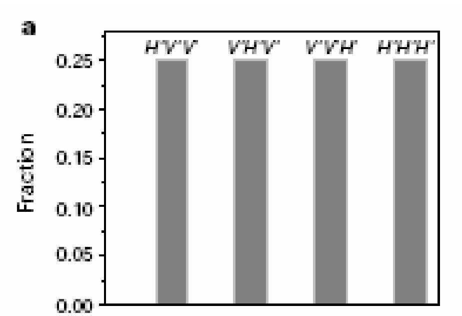
Experimental GHZ

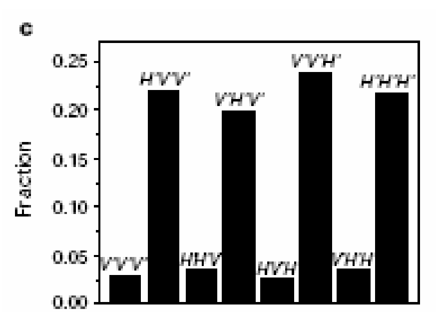
letters to nature

Experimental test of quantum nonlocality in three-photon Greenberger–Horne–Zeilinger entanglement

Jian-Wei Pan*, Dik Bouwmeester†, Matthew Daniell*, Harald Weinfurter‡ & Anton Zeilinger*

** Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria*
† Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, UK
‡ Sektion Physik, Ludwig-Maximilians-Universität of München, Schellingstrasse 4/III, D-80799 München, Germany





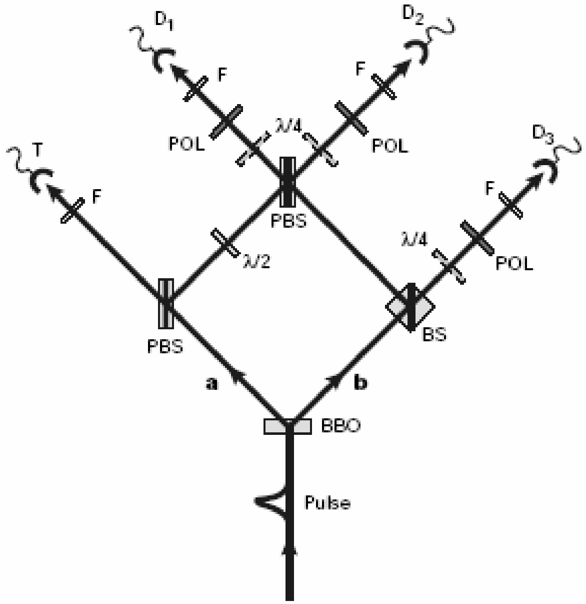


Figure 1 Experimental set-up for Greenberger–Horne–Zeilinger (GHZ) tests of quantum nonlocality. Pairs of polarization-entangled photons[28] (one photon *H* polarized and the other *V*) are generated by a short pulse of ultraviolet light (~ 200 fs, $\lambda = 394$ nm). Observation of the desired GHZ correlations requires fourfold coincidence and therefore two pairs[29]. The photon registered at T is always *H* and thus its partner in **b** must be *V*. The photon reflected at the polarizing beam-splitter (PBS) in arm **a** is always *V*, being turned into equal superposition of *V* and *H* by the $\lambda/2$ plate, and its partner in arm **b** must be *H*. Thus if all four detectors register at the same time, the two photons in D_1 and D_2 must either both have been *VV* and reflected by the last PBS or *HH* and transmitted. The photon at D_3 was therefore *H* or *V*, respectively. Both possibilities are made indistinguishable by having equal path lengths via **a** and **b** to D_1 (D_2) and by using narrow bandwidth filters (*F* $\approx$ 4 nm) to stretch the coherence time to about 500 fs, substantially larger than the pulse length[30]. This effectively erases the prior correlation information and, owing to indistinguishability, the three photons registered at D_1, D_2 and D_3 exhibit the desired GHZ correlations predicted by the state of equation (1), where for simplicity we assume the polarizations at D_3 to be defined at right angles relative to the others. Polarizers oriented at 45° and $\lambda/4$ plates in front of the detectors allow measurement of linear *H'*/*V'* (circular *R*/*L*) polarization.

Plan

- GHZ: Beating EPR using their own weapons (perfect correlations)
- Mermin inequality: Violation that grows exponentially
- Bipartite AVN
- Bipartite AVN with only single-qubit measurements

The Mermin inequality

VOLUME 65, NUMBER 15 PHYSICAL REVIEW LETTERS 8 OCTOBER 1990

Extreme Quantum Entanglement in a Superposition of Macroscopically Distinct States

N. David Mermin

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

(Received 29 May 1990)

A Bell inequality is derived for a state of n spin-$\frac{1}{2}$ particles which superposes two macroscopically distinct states. Quantum mechanics violates this inequality by an amount that grows exponentially with n.

The CHSH inequality

$$\boxed{\left| \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \right| \leq 2}$$

$$\boxed{\beta_{\mathrm{QM}} = 2\sqrt{2} \approx 2.8}$$

The Mermin inequality

$$\boxed{\left|\langle A_1 B_0 C_0 \rangle + \langle A_0 B_1 C_0 \rangle + \langle A_0 B_0 C_1 \rangle - \langle A_1 B_1 C_1 \rangle\right| \leq 2}$$

$$\boxed{\beta_{\mathrm{QM}} = 4}$$

The *n*-qubit Mermin inequality

$$\boxed{\frac{\beta_{\mathrm{QM}}}{\beta_{\mathrm{Local\ models}}} = 2^{(n-1)/2}}$$

The violation grows exponentially with *n*!!!

$$\boxed{\frac{\beta_{\mathrm{QM}}}{\beta_{\mathrm{Local\ models}}} = 2^{(n\text{-}1)/2}}$$

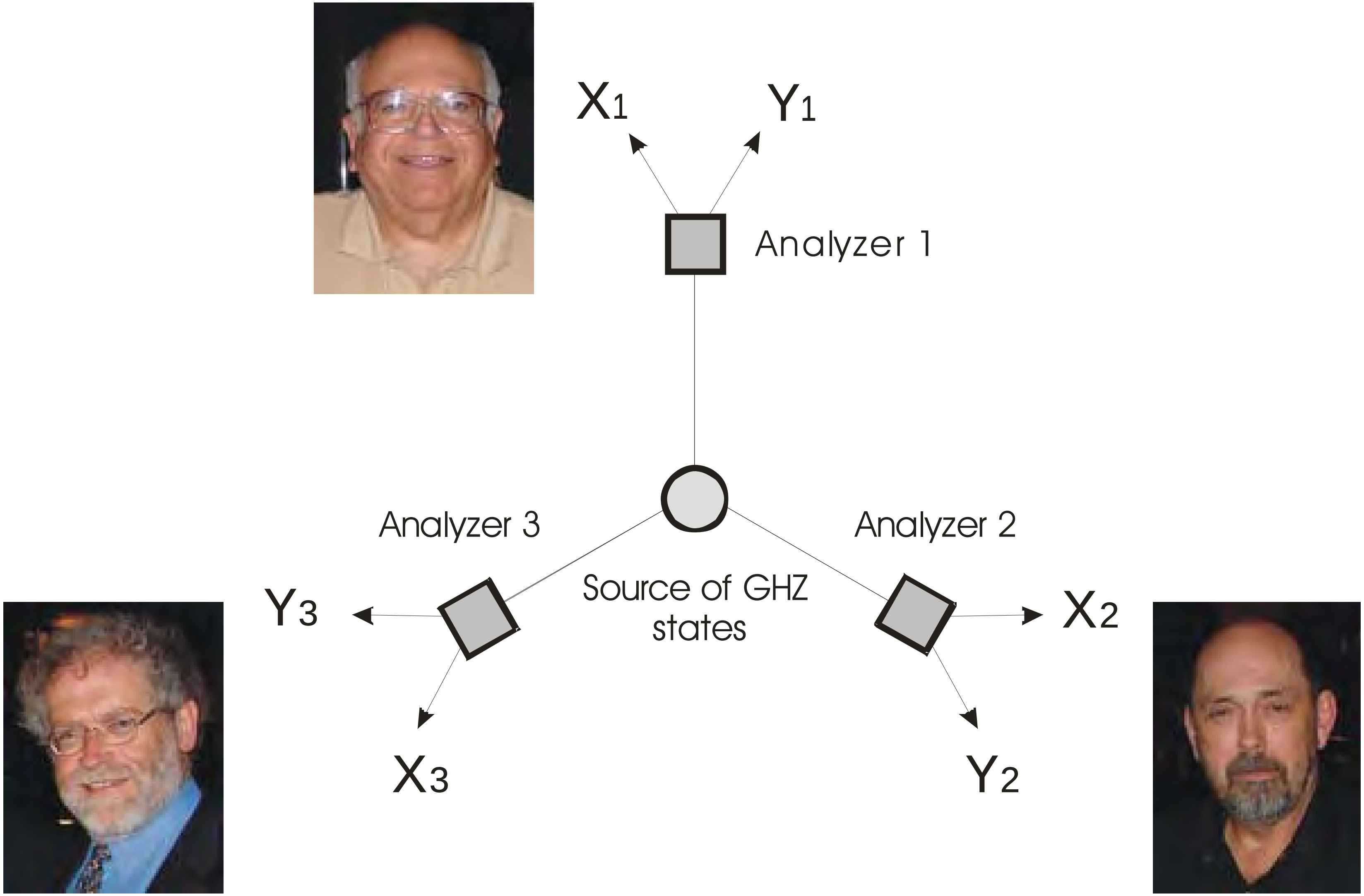
Problems for an experimental demonstration

A nonlocality proof using *n*-qubit GHZ states requires *n* space-like separated observers.

For GHZ states decoherence also grows with *n*.

The minimum detection efficiency for a loophole-free test is 0.5 (when *n* goes to infinity).

GHZ’s requires three observers



Why GHZ's requires three observers?

$$v(X_1)\,v(Y_2) = v(Y_3)$$

$$v(Y_1)\,v(X_2) = v(Y_3)$$

$$v(Y_1)\,v(Y_2) = v(X_3)$$

$$v(X_1)\,v(X_2) = -\,v(X_3)$$

Why GHZ's requires three observers?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

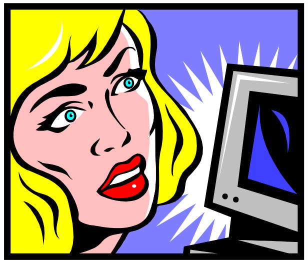
$$v(X_1)v(X_2) = -v(X_3)$$

Plan

- GHZ: Beating EPR using their own weapons (perfect correlations)
- Mermin inequality: Violation that grows exponentially
- Bipartite AVN
- Bipartite AVN with only single-qubit measurements

Problem

- Two-observer AVN proofs?



The first two-observer AVN proof

Double Bell:

$$(|HV\rangle - |VH\rangle)\,(|ud\rangle - |du\rangle)$$

$$v(Z_1) = -\,v(Z_2)$$

$$v(z_1) = -\,v(z_2)$$

$$v(X_1) = -\,v(X_2)$$

$$v(x_1) = -\,v(x_2)$$

$$v(Z_1 z_1) = v(Z_2)v(z_2)$$

$$v(X_1 x_1) = v(X_2)v(x_2)$$

$$v(Z_1)v(x_1) = v(Z_2 x_2)$$

$$v(X_1)v(z_1) = v(X_2 z_2)$$

$$v(Z_1 z_1)v(X_1 x_1) = -\,v(Z_2 x_2)v(X_2 z_2)$$

AC, PRL **86**, 1911 (2001); **87**, 010403 (2001).

Notation for single photon observables

Polarization observables:

$$X = |H\rangle\langle V| + |V\rangle\langle H|$$

$$Y = i\,(|V\rangle\langle H| - |H\rangle\langle V|)$$

$$Z = |H\rangle\langle H| - |V\rangle\langle V|$$

Path observables:

$$x = |u\rangle\langle d| + |d\rangle\langle u|$$

$$y = i\,(|d\rangle\langle u| - |u\rangle\langle d|)$$

$$z = |u\rangle\langle u| - |d\rangle\langle d|$$

Four qubits in two photons

VOLUME 90, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 25 APRIL 2003

All-Versus-Nothing Violation of Local Realism for Two Entangled Photons

Zeng-Bing Chen,[1] Jian-Wei Pan,[1,2] Yong-De Zhang,[1] Časlav Brukner,[2] and Anton Zeilinger[2]
[1]Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230027, China
[2]Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria
(Received 18 November 2002; published 24 April 2003)

It is shown that the Greenberger-Horne-Zeilinger theorem can be generalized to the case with only two entangled particles. The reasoning makes use of two photons which are maximally entangled both in polarization and in spatial degrees of freedom. In contrast to Cabello's argument of "all versus nothing" nonlocality with four photons [Phys. Rev. Lett. **87**, 010403 (2001)], our proposal to test the theorem can be implemented with linear optics and thus is well within the reach of current experimental technology.

$$z_1 \cdot z_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \qquad z_1' \cdot z_2'|\Psi\rangle_{12} = -|\Psi\rangle_{12},$$

$$x_1 \cdot x_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \qquad x_1' \cdot x_2'|\Psi\rangle_{12} = -|\Psi\rangle_{12},$$

$$z_1 z_1' \cdot z_2 \cdot z_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$x_1 x_1' \cdot x_2 \cdot x_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$z_1 \cdot x_1' \cdot z_2 x_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$x_1 \cdot z_1' \cdot x_2 z_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$z_1 z_1' \cdot x_1 x_1' \cdot z_2 x_2' \cdot x_2 z_2'|\Psi\rangle_{12} = -|\Psi\rangle_{12}.$$

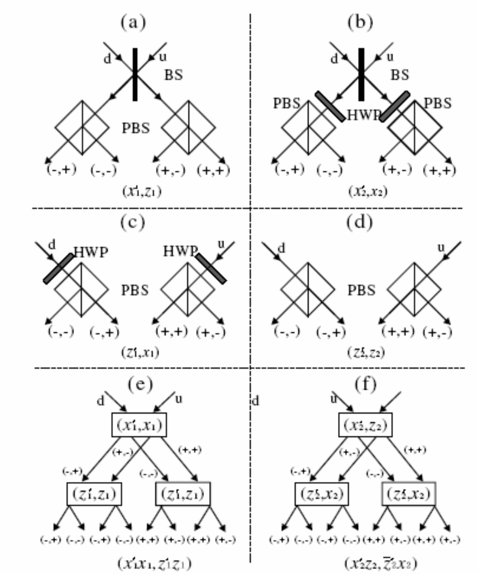


FIG. 2. Six apparatuses for measuring z_1, x_1', and $z_1 \cdot x_1'$ (a); x_2, x_2', and $x_2 \cdot x_2'$ (b); z_1', x_1, and $x_1 \cdot z_1'$ (c); z_2, z_2', and $z_2 \cdot z_2'$ (d); $z_1 z_1'$, $x_1 x_1'$, and $z_1 z_1' \cdot x_1 x_1'$ (e); $z_2 x_2'$, $x_2 z_2'$, and $z_2 x_2' \cdot x_2 z_2'$ (f). By $\pm$, we mean ± 1.

Rome and Hefei experiments

PRL 95, 240405 (2005) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2005

All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement

C. Cinelli, M. Barbieri, R. Perris, P. Mataloni, and F. De Martini

Dipartimento di Fisica dell' Università "La Sapienza"
and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy

(Received 27 April 2005; published 9 December 2005)

We report the experimental realization and the characterization of polarization and momentum hyperentangled two-photon states, generated by a new parametric source of correlated photon pairs. By adoption of these states an "all-versus-nothing" test of quantum mechanics was performed. The two-photon hyperentangled states are expected to find at an increasing rate a widespread application in state engineering and quantum information.

PRL 95, 240406 (2005) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2005

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

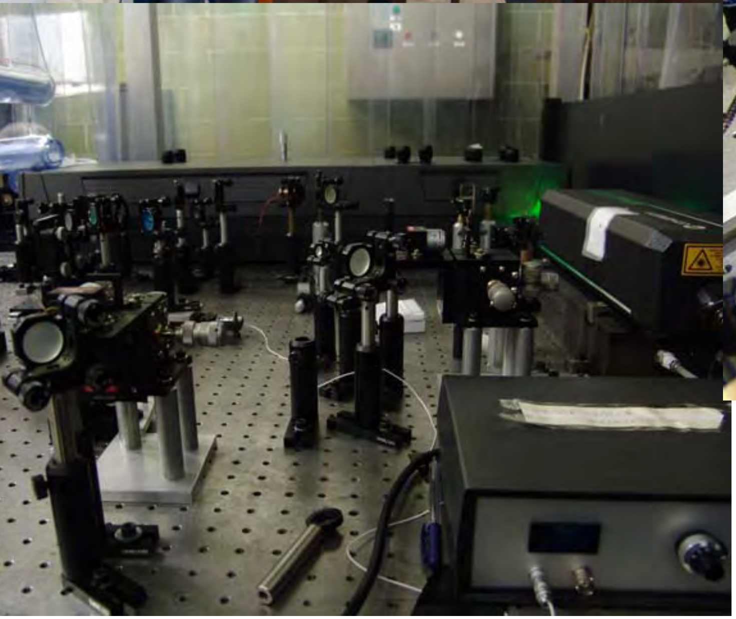
Tao Yang,[1] Qiang Zhang,[1] Jun Zhang,[1] Juan Yin,[1] Zhi Zhao,[1,2] Marek Żukowski,[3]
Zeng-Bing Chen,[1,2,*] and Jian-Wei Pan[1,2,†]

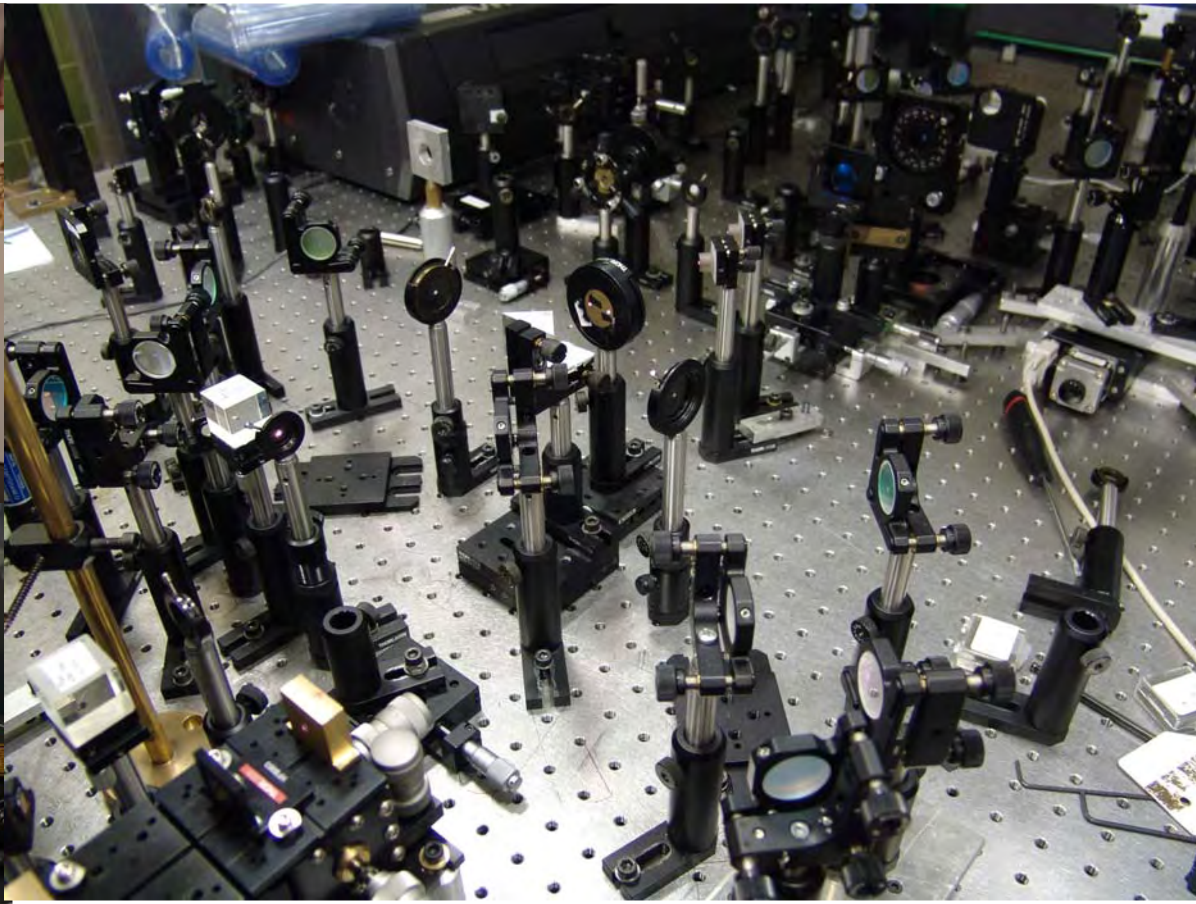
[1]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics,*
University of Science and Technology of China, Hefei, Anhui 230026, China
[2]*Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, D-69120 Heidelberg, Germany*
[3]*Instytut Fizyki Teoretycznej i Astrofizyki Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*

(Received 4 June 2005; published 9 December 2005)

We develop and exploit a source of two-photon, four-dimensional entanglement to report the first two-particle all-versus-nothing test of local realism with a linear optics setup, but without resorting to a noncontextuality assumption. Our experimental results are in good agreement with quantum mechanics while in extreme contradiction to local realism. Potential applications of our experiment are briefly discussed.

Rome experiment 2005



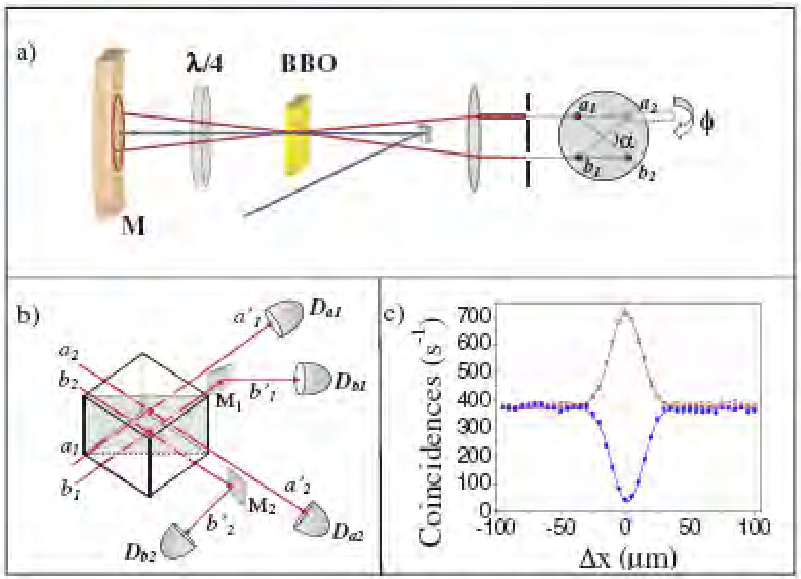


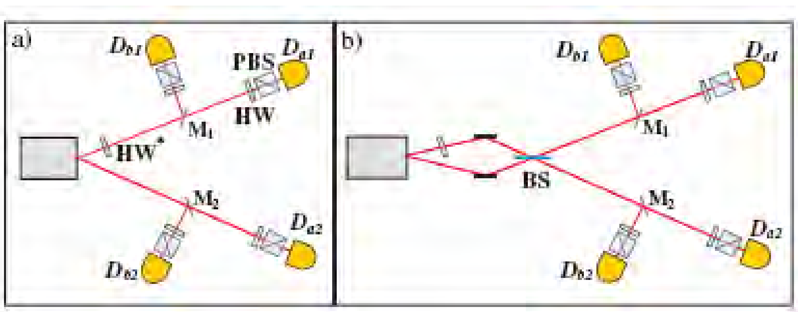
Rome experiment 2005

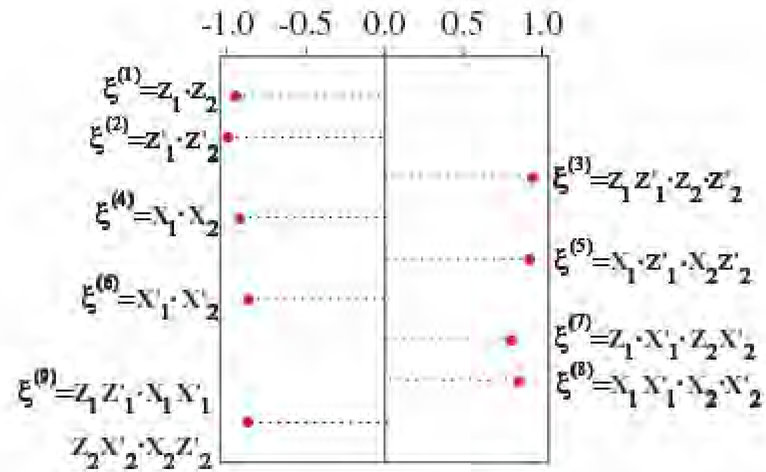
All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement

C. Cinelli, M. Barbieri, R. Perris, P. Mataloni, and F. De Martini

Dipartimento di Fisica dell' Università "La Sapienza"
and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy



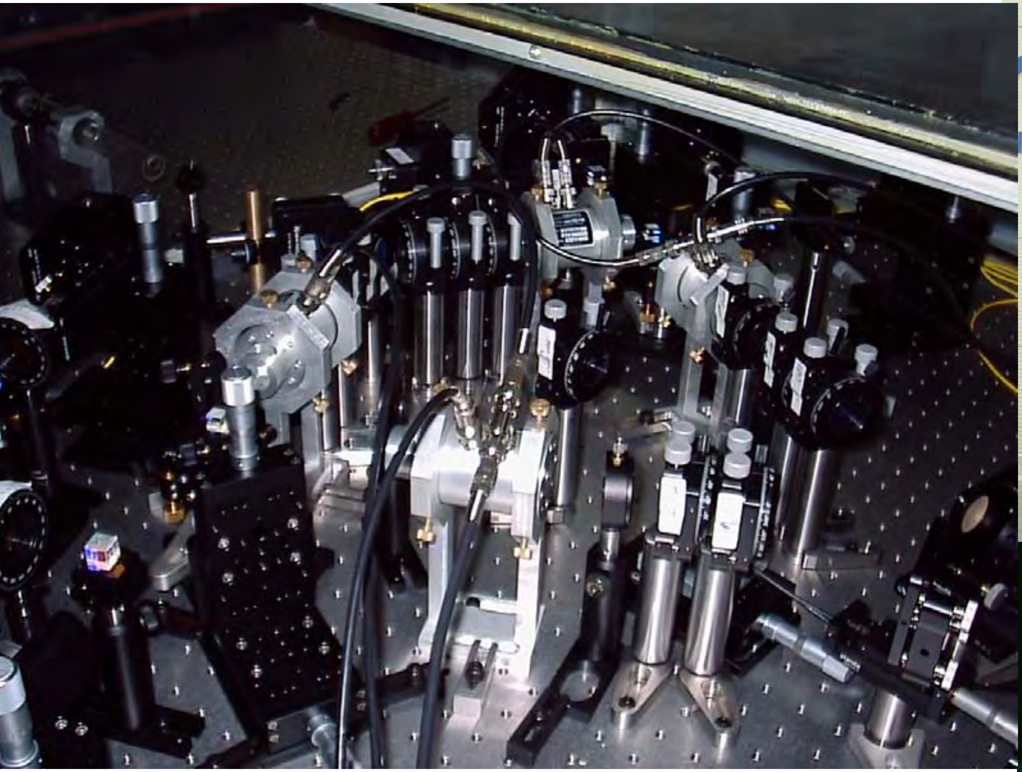




$$7 < \sum_{i=1}^{9} |\xi^{(i)}| = (8.114 \pm 0.011) < 9$$

FIG. 3 (color online). Barchart of expectation values for the nine operators involved in the experiment. The following results have been obtained: $z_1 \cdot z_2 = -0.9428 \pm 0.0030$, $z'_1 \cdot z'_2 = -0.9953 \pm 0.0033$, $z_1 z'_1 \cdot z_2 \cdot z'_2 = 0.9424 \pm 0.0030$, $x_1 \cdot x_2 = -0.9215 \pm 0.0033$, $x_1 \cdot z'_1 \cdot x_2 z'_2 = 0.9217 \pm 0.0033$, $x'_1 \cdot x'_2 = -0.8642 \pm 0.0043$ $z_1 \cdot x'_1 \cdot z_2 x'_2 = 0.8039 \pm 0.0040$, $x_1 x'_1 \cdot x_2 \cdot x'_2 = 0.8542 \pm 0.0040$, $z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2 = -0.8678 \pm 0.0043$.

Hefei experiment 2005



Hefei experiment 2005

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

Tao Yang,[1] Qiang Zhang,[1] Jun Zhang,[1] Juan Yin,[1] Zhi Zhao,[1,2] Marek Żukowski,[3]
Zeng-Bing Chen,[1,2,*] and Jian-Wei Pan[1,2,†]

[1]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*

[2]*Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, D-69120 Heidelberg, Germany*

[3]*Instytut Fizyki Teoretycznej i Astrofizyki Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*

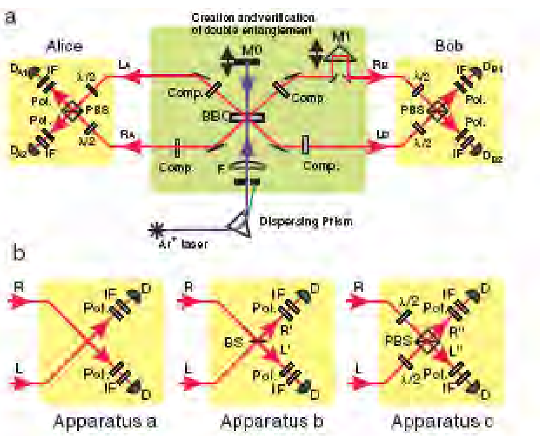


FIG. 1 (color online). Experimental setups. (a) An ultraviolet beam from Argon ion laser (351.1 nm, 120 mW) is directed into the BBO crystal from opposite directions, and thus can create photon pairs (with wavelength 702.2 nm) in $|\Psi\rangle$. Four compensators (Comp.) are used to offset the birefringent effect caused by the BBO crystal during parametric down-conversion. The reflection mirrors M0 and M1 are mounted on translation stages, to balance each arm of the interferometer and to optimize the entanglement in path. (b) Apparatuses to measure all necessary observables of doubly entangled states. D is single-photon count module, with collection and detection efficiency 26%; IF is interference filter with a bandwidth of 2.88 nm and a center wavelength of 702.2 nm; Pol. is polarizer. Apparatus c has been included in (a) at the locations of Alice and Bob.

$$z_A \cdot z_B|\Psi\rangle = -|\Psi\rangle, \qquad z'_A \cdot z'_B|\Psi\rangle = -|\Psi\rangle, \qquad (1)$$

$$x_A \cdot x_B|\Psi\rangle = -|\Psi\rangle, \qquad x'_A \cdot x'_B|\Psi\rangle = -|\Psi\rangle, \qquad (2)$$

$$z_A z'_A \cdot z_B \cdot z'_B|\Psi\rangle = |\Psi\rangle, \qquad x_A x'_A \cdot x_B \cdot x'_B|\Psi\rangle = |\Psi\rangle, \quad (3)$$

$$z_A \cdot x'_A \cdot z_B x'_B|\Psi\rangle = |\Psi\rangle, \qquad x_A \cdot z'_A \cdot x_B z'_B|\Psi\rangle = |\Psi\rangle, \quad (4)$$

$$z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B|\Psi\rangle = -|\Psi\rangle. \qquad (5)$$

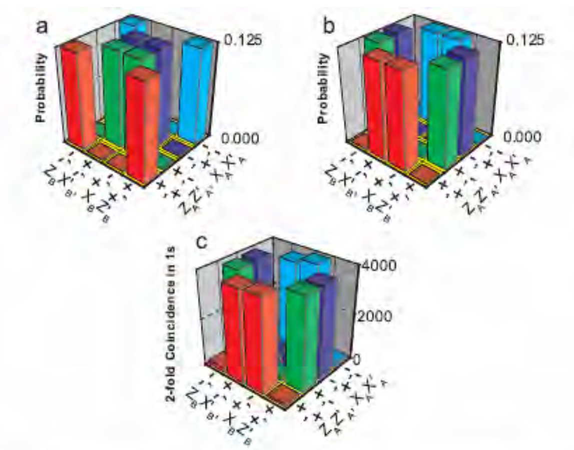


FIG. 3 (color online). Predictions of LR (a) and of QM (b), and observed results (c) for the $z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B$ experiment.

Requires two-qubit measurements!

$$v(Z_1) = -v(Z_2)$$

$$v(z_1) = -v(z_2)$$

$$v(X_1) = -v(X_2)$$

$$v(x_1) = -v(x_2)$$

$$v(Z_1 z_1) = v(Z_2)v(z_2)$$

$$v(X_1 x_1) = v(X_2)v(x_2)$$

$$v(Z_1)v(x_1) = v(Z_2 x_2)$$

$$v(X_1)v(z_1) = v(X_2 z_2)$$

$$v(Z_1 z_1)v(X_1 x_1) = -v(Z_2 x_2)v(X_2 z_2)$$

Requires two-qubit measurements!

$$v(Z_1) = -v(Z_2)$$

$$v(z_1) = -v(z_2)$$

$$v(X_1) = -v(X_2)$$

$$v(x_1) = -v(x_2)$$

$$v(Z_1 z_1) = v(Z_2)v(z_2)$$

$$v(X_1 x_1) = v(X_2)v(x_2)$$

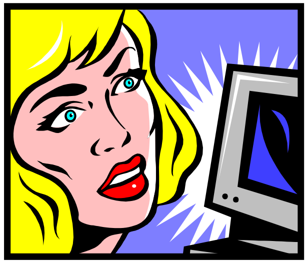
$$v(Z_1)v(x_1) = v(Z_2 x_2)$$

$$v(X_1)v(z_1) = v(X_2 z_2)$$

$$v(Z_1 z_1)v(X_1 x_1) = -v(Z_2 x_2)v(X_2 z_2)$$

Problem

- Two-observer AVN proof with single-qubit observables?



Plan

- GHZ: Beating EPR using their own weapons (perfect correlations)
- Mermin inequality: Violation that grows exponentially
- Bipartite AVN
- Bipartite AVN with only single-qubit measurements

Two-observer AVN proof with single qubit observables

Hyperentangled cluster:

$$|HuHu\rangle + |HdHd\rangle + |VuVu\rangle - |VdVd\rangle$$

$$v(Z_1) = v(Z_2)$$
$$v(z_1) = v(z_2)$$
$$v(X_1) = v(X_2)v(z_2)$$
$$v(x_1) = v(Z_2)v(x_2)$$
$$v(Y_1) = -v(Y_2)v(z_2)$$
$$v(y_1) = -v(Z_2)v(y_2)$$
$$v(X_2) = v(X_1)v(z_1)$$
$$v(x_2) = v(Z_1)v(x_1)$$
$$v(Y_2) = -v(Y_1)v(z_1)$$
$$v(y_2) = -v(Z_1)v(y_1)$$

Two-observer AVN proof with single qubit observables

$$v(X_1) = v(X_2)\,v(z_2)$$

$$v(Y_1) = -\,v(Y_2)\,v(z_2)$$

$$v(X_1)\,v(x_1) = v(Y_2)\,v(y_2)$$

$$v(Y_1)\,v(x_1) = v(X_2)\,v(y_2)$$

AC, PRL **95**, 210401 (2005).

Rome experiment 2007

Realization and characterization of a 2-photon 4-qubit linear cluster state

Giuseppe Vallone[1,*], Enrico Pomarico[1,*], Paolo Mataloni[1,*], Francesco De Martini[1,*], Vincenzo Berardi[2]
[1]Dipartimento di Fisica dell'Universitá "La Sapienza" and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy
[2]Dipartimento Interateneo di Fisica, Università e Politecnico di Bari and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Bari, 70126 Italy

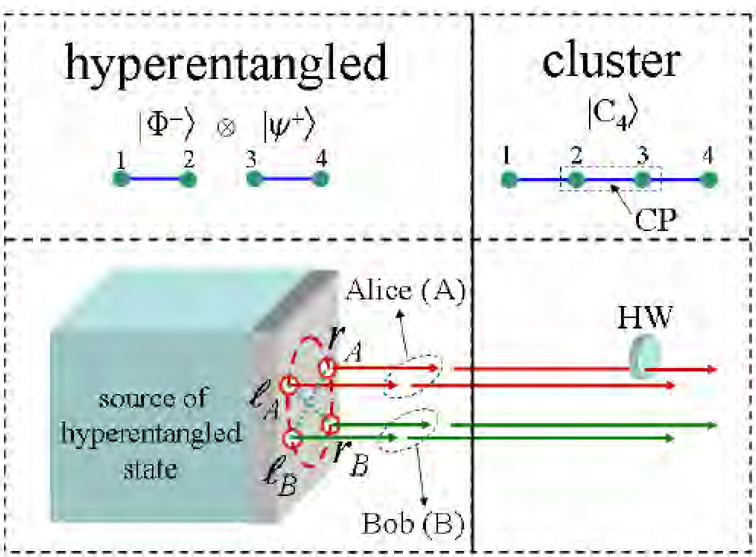


FIG. 1: Generation of the linear cluster state by a source of polarization-momentum hyperentangled 2-photon state. The state $|\Xi\rangle = |\Phi^-\rangle \otimes |\psi^+\rangle$ corresponds to two separate 2-qubit clusters. The *HW* acts as a Controlled-Phase (CP) thus generating the 4-qubit linear cluster $|C_4\rangle$.

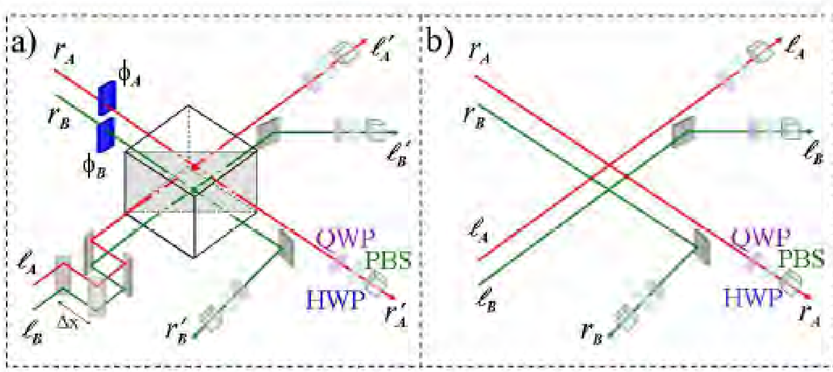


FIG. 2: Interferometer and measurement apparatus. a) The mode pairs r_A-ℓ_B and ℓ_A-r_B are matched on the BS. The phase shifters ϕ_A and ϕ_B (thin glass plates) are used for the measurement of momentum observables. The polarization analyzers on each of BS output modes are shown (QWP/HWP=Quarter/Half-Wave Plate, PBS=Polarized Beam Splitter). b) Same configuration as in a) with BS and glasses removed.

PRL **98**, 180502 (2007).

Rome experiment 2007

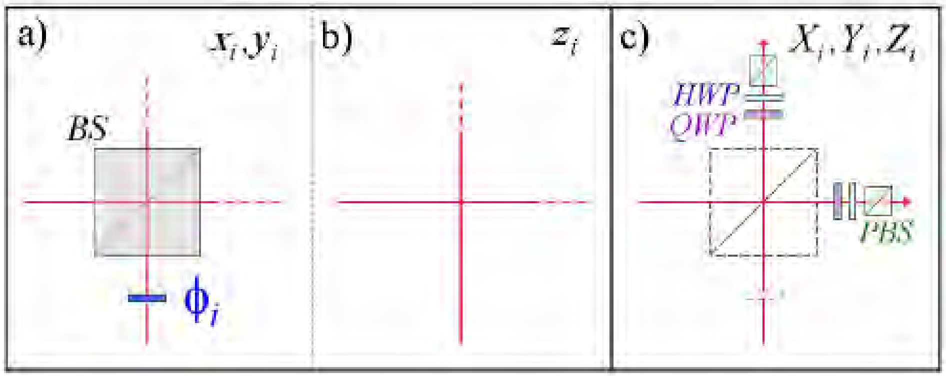


FIG. 4: Measurement setup for momentum (a),b)) and polarization (c)) observables for photon i (i=A, B). By the a) setup we measure x_i ($\phi_i = 0$) and y_i ($\phi_i = \frac{\pi}{2}$), while the b) setup is used for measuring z_i. By the c) setup we measure X_i ($\theta_Q = \frac{\pi}{4}$; $\theta_H = \frac{1}{8}\pi, \frac{3}{8}\pi$), Y_i ($\theta_Q = 0$; $\theta_H = \frac{1}{8}\pi, \frac{3}{8}\pi$) and Z_i ($\theta_Q = 0$; $\theta_H = 0, \frac{\pi}{4}$), where $\theta_{H(Q)}$ is the angle between the $HWP(QWP)$ optical axis and the vertical direction. The polarization analysis is performed contextually to x_i, y_i (i.e. with BS and glass) or z_i (without BS and glass), as shown by the dotted lines for BS and glass in c).

Observable	Value	$\mathcal{W}$	S	C
$Z_A Z_B$	$+0.9283 \pm 0.0032$	✓		
$Z_A x_A x_B$	$+0.8194 \pm 0.0049$	✓		
$X_A z_A X_B$	-0.9074 ± 0.0037	✓		✓
$z_A z_B$	-0.9951 ± 0.0009	✓		✓
$x_A Z_B x_B$	$+0.8110 \pm 0.0050$	✓		✓
$Z_A y_A y_B$	$+0.8071 \pm 0.0050$			✓
$Y_A z_A Y_B$	$+0.8948 \pm 0.0040$			✓
$X_A X_B z_B$	$+0.9074 \pm 0.0037$	✓	✓	✓
$Y_A Y_B z_B$	-0.8936 ± 0.0041		✓	✓
$X_A x_A Y_B y_B$	$+0.8177 \pm 0.0055$		✓	
$Y_A x_A X_B y_B$	$+0.7959 \pm 0.0055$		✓	

TABLE I: Experimental values of the observables used for measuring the entanglement witness $\mathcal{W}$ and the expectation value of S on the cluster state $|C_4\rangle$. The third column (C) refers to the control measurements needed to verify that X_A, Y_A, x_A, X_B, Y_B, y_B and z_B can be considered as elements of reality. Each experimental value corresponds to a measure lasting an average time of 10 sec. In the experimental errors we considered the poissonian statistic and the uncertainties due to the manual setting of the polarization analysis wave plates.

$$\mathrm{Tr}[S\rho_{exp}] = 3.4145 \pm 0.0095$$

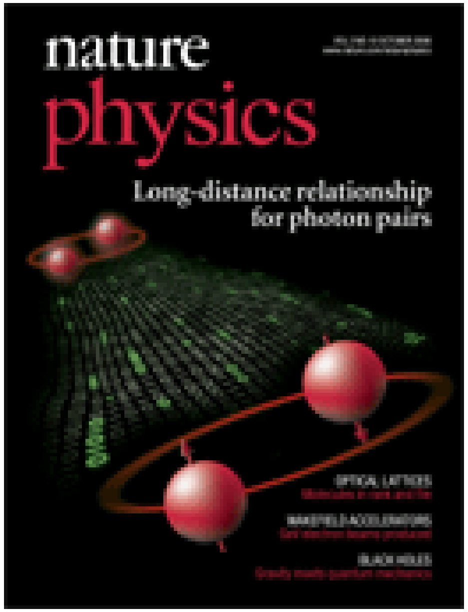
PRL **98**, 180502 (2007).

Motivation #1: Six-photon six-qubit states

• Q. Zhang, A. Goebel, C. Wagenknecht, Y.-A. Chen, B. Zhao, T. Yang, A. Mair, J. Schmiedmayer, and J.-W. Pan, “Experimental quantum teleportation of a two-qubit composite system”,
Nature Physics **2**, 678 (2006).

• C.-Y. Lu, X.-Q. Zhou, O. Gühne, W.-B. Gao, J. Zhang, Z.-S. Yuan, A. Goebel, T. Yang, and J.-W. Pan,
“Experimental entanglement of six photons in graph states‘”,
Nature Physics **3**, 91 (2007).

• Other groups are preparing six-photon six-qubit states.



Motivation #2: Two-photon six-qubit states

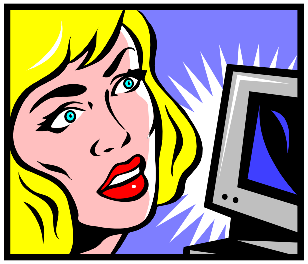
• J. T. Barreiro, N. K. Langford, N. A. Peters, and P. G. Kwiat,
"Hyper-entangled photons",
Phys. Rev. Lett. **95**, 260501 (2005).

$$\underbrace{(|HH\rangle + |VV\rangle)}_{\text{polarization}} \otimes \underbrace{(|rl\rangle + \alpha|gg\rangle + |lr\rangle)}_{\text{spatial modes}} \otimes \underbrace{(|ss\rangle + |ff\rangle)}_{\text{energy time}}.$$

• Other groups are preparing six-qubit two-photon hyper-entangled states.

Problem

- If we distribute *n* qubits between two parties, what quantum pure states and distributions of qubits allow AVN proofs using only single-qubit measurements?



First ingredient: Bipartite elements of reality

- Enough number of perfect correlations to define bipartite EPR's elements of reality. Every single-qubit observable involved in the proof must satisfy EPR's criterion; i.e., the result of measuring any of Alice's (Bob's) single-qubit observables must be possible to be predicted with certainty using the results of spacelike separated single-qubit measurements on Bob's (Alice's) qubits.

Second ingredient: Algebraic contradiction

- Enough number of perfect correlations to reach into a contradiction with EPR's elements of reality. Any of the observables satisfying EPR's condition *cannot* have predefined results, because it is impossible to assign them values -1 or 1 satisfying all the perfect correlations predicted by QM.

Perfect correlations, stabilizer and graph states

- Perfect correlations are needed to establish elements of reality and to prove that they are incompatible with QM.

- Simultaneous eigenstates of a sufficiently large set of tensor operators products of single-qubit operators.

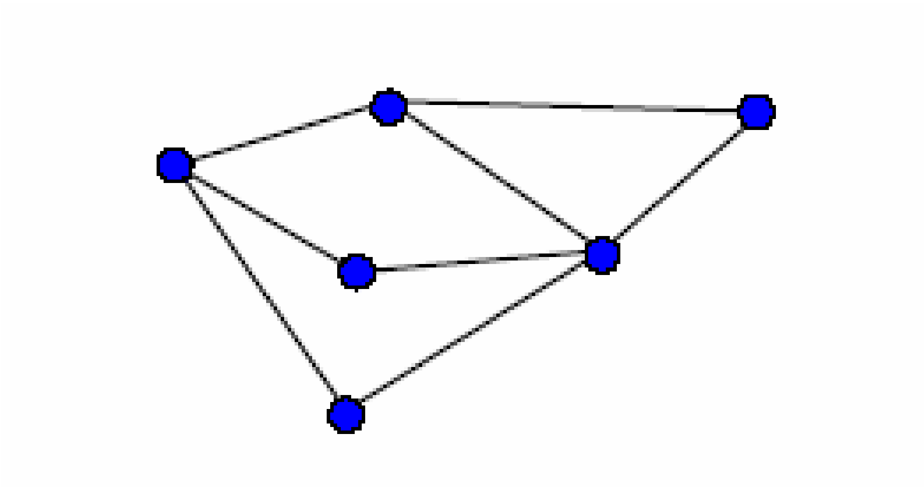
- We can restrict our attention to X, Y, Z. Stabilizer states!

- Any stabilizer state is local Clifford equivalent to a graph state. Graph states!!

Graph states

Graph states are a family of multiqubit pure entangled states.

Each graph state is associated to a graph



Vertices: qubits.

Edges: entanglement between the connected qubits.

Graph states are useful

- Initial states for measurement-based quantum computation (some of them are universal resources)
- Quantum error correction (stabilizer states)
- All-versus-nothing (AVN) nonlocality proofs
- Exponentially-growing-with size nonlocality

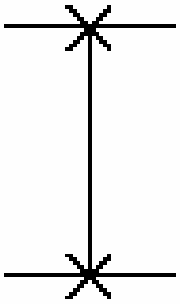
Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*

$$C_Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} =$$

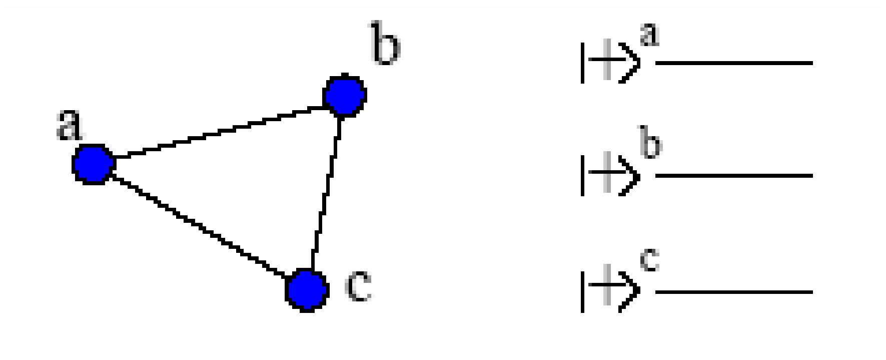


Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*

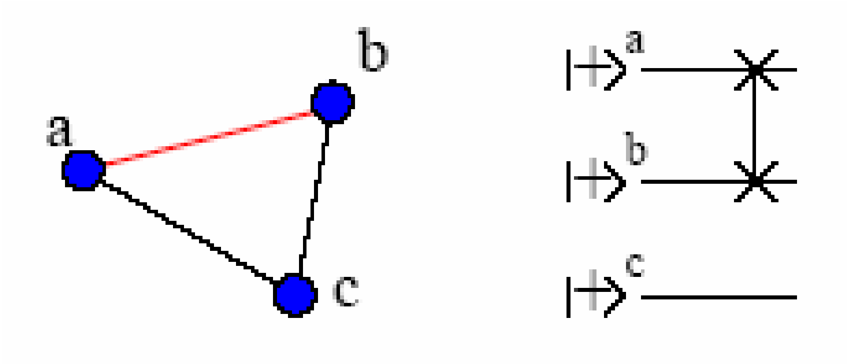


Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*

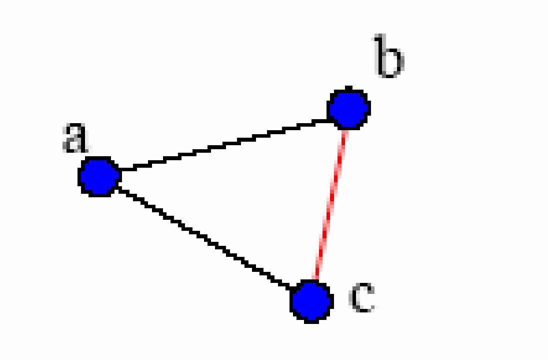


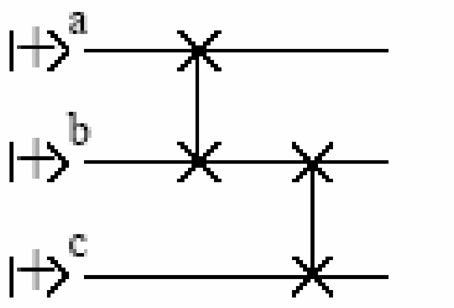
Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*



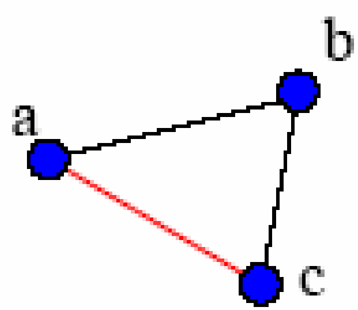


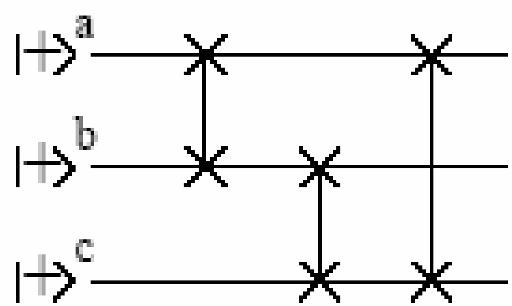
Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*



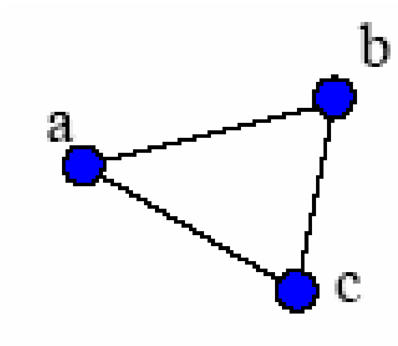


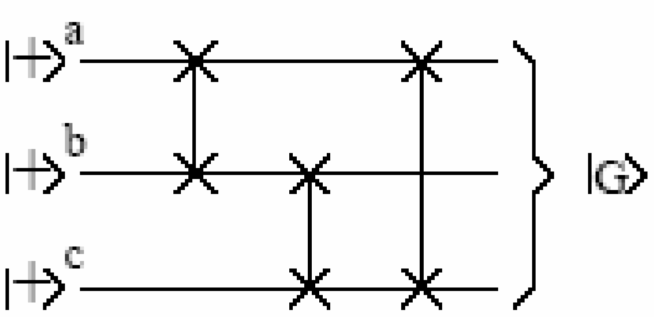
Graph states: Constructive definition

For a given graph *G*, a preparation of the corresponding graph state $|G\rangle$ consists:

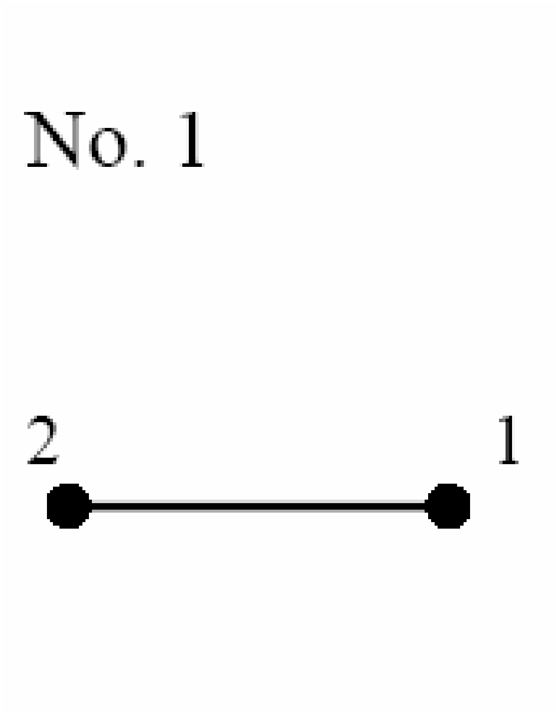
• In associating wich each vertex a qubit in the state $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, then

• In applying, for each edge between two qubits *a* and *b*, the unitary transformation C_Z on the qubits *a* and *b*





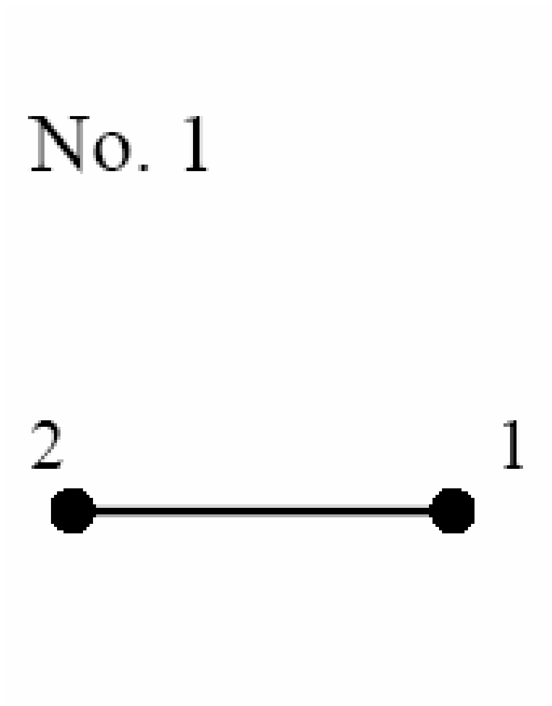
Graph states: Algebraic definition

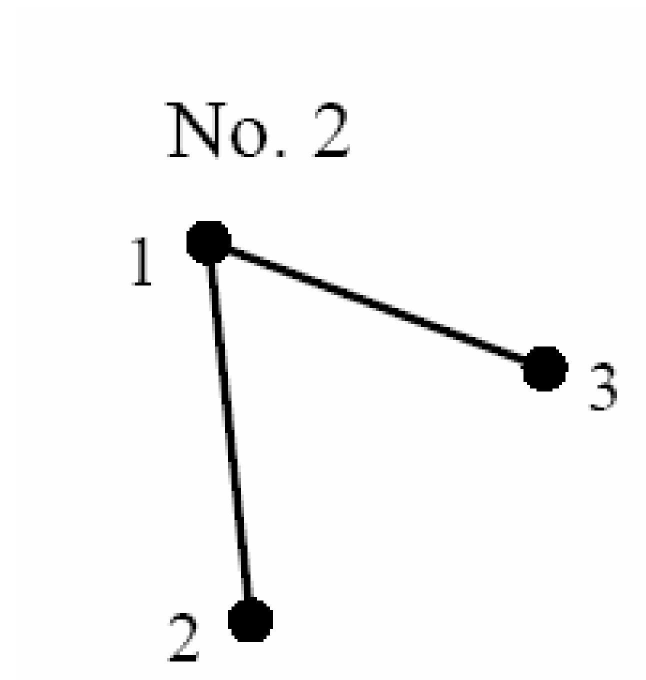


Eq. associated to qubit 1: $XZ\,|\phi_1\rangle = |\phi_1\rangle$,

" " " " 2: $ZX\,|\phi_1\rangle = |\phi_1\rangle$.

Graph states: Algebraic definition





Eq. associated to qubit 1: $XZ\,|\phi_1\rangle = |\phi_1\rangle$,

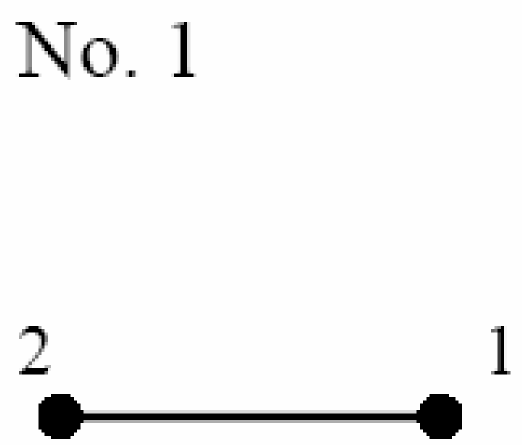
“ “ “ “ 2: $ZX\,|\phi_1\rangle = |\phi_1\rangle$.

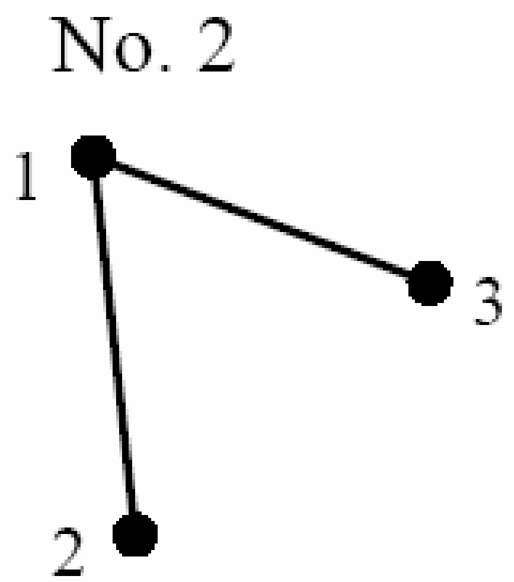
“ “ “ “ 1: $XZZ\,|\phi_2\rangle = |\phi_2\rangle$,

“ “ “ “ 2: $ZXI\,|\phi_2\rangle = |\phi_2\rangle$,

“ “ “ “ 3: $ZIX\,|\phi_2\rangle = |\phi_2\rangle$.

Graph states: Algebraic definition





Eq. associated to qubit 1: $XZ\,|\phi_1\rangle = |\phi_1\rangle$,
“ “ “ “ 2: $ZX\,|\phi_1\rangle = |\phi_1\rangle$.

“ “ “ “ 1: $XZZ\,|\phi_2\rangle = |\phi_2\rangle$,
“ “ “ “ 2: $ZXI\,|\phi_2\rangle = |\phi_2\rangle$,
“ “ “ “ 3: $ZIX\,|\phi_2\rangle = |\phi_2\rangle$.

Graph states: Entanglement

Property: Two quantum states "have the same entanglement" *iff* they are LU-equivalent.

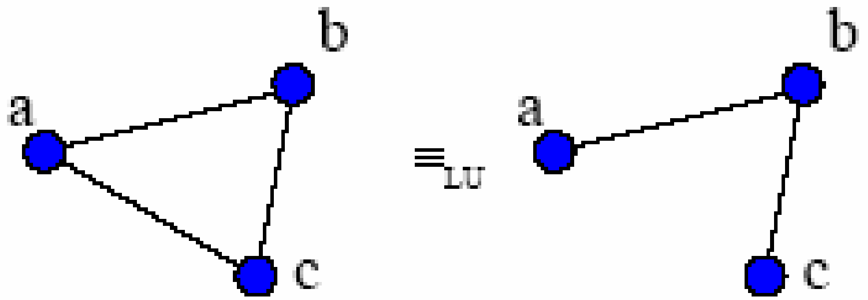
Definition [LU-equivalence] : $|\phi\rangle \equiv_{LU} |\psi\rangle$ *iff* there exists a local unitary transformation U (i.e $U = U_1 \otimes \ldots \otimes U_n$, where each U_i is a 1-qubit unitary) such that $|\phi\rangle = U\,|\psi\rangle$.

Definition [LC-equivalence] : $|\phi\rangle \equiv_{LC} |\psi\rangle$ *iff* there exists a local Clifford transformation C (i.e. $C = C_1 \otimes \ldots \otimes C_n$, where $C_i \in \langle H, S\rangle$) such that $|\phi\rangle = C\,|\psi\rangle$.

Graph states: Entanglement

Graph-based representation of entanglement is not unique:

$$\exists G, G' \,/\, |G\rangle \equiv_{LU} |G'\rangle \; and \; G \neq G'$$

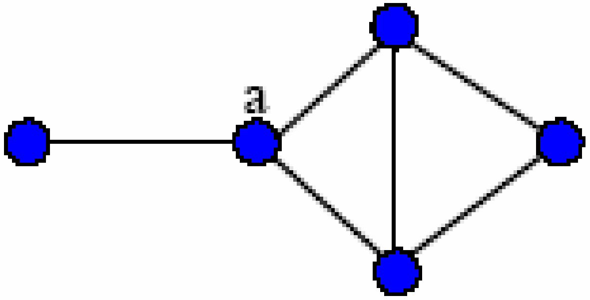


Graph states: Entanglement

Theorem [Van den Nest, 2004]:

$|G\rangle \equiv_{LC} |G'\rangle$ iff there exists a sequence of *local complementations* which transforms G into G'.

Local Complementation according to a: $G * a = G \Delta K_{N(a)}$

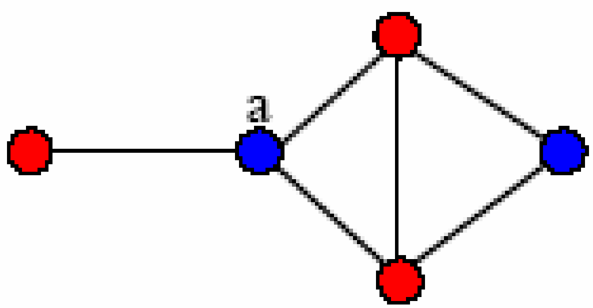


Graph states: Entanglement

Theorem [Van den Nest, 2004]:

$|G\rangle \equiv_{LC} |G'\rangle$ iff there exists a sequence of *local complementations* which transforms G into G'.

Local Complementation according to a: $G * a = G \Delta K_{N(a)}$

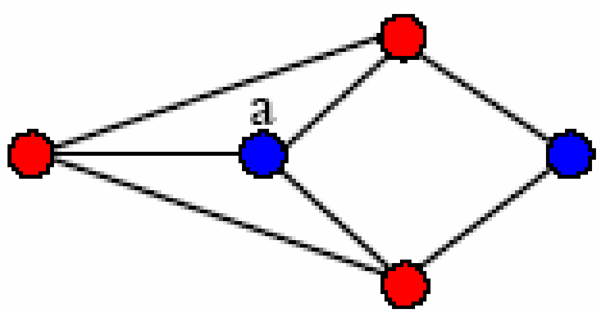


Graph states: Entanglement

Theorem [Van den Nest, 2004]:

$|G\rangle \equiv_{LC} |G'\rangle$ iff there exists a sequence of *local complementations* which transforms G into G'.

Local Complementation according to a: $G * a = G \Delta K_{N(a)}$

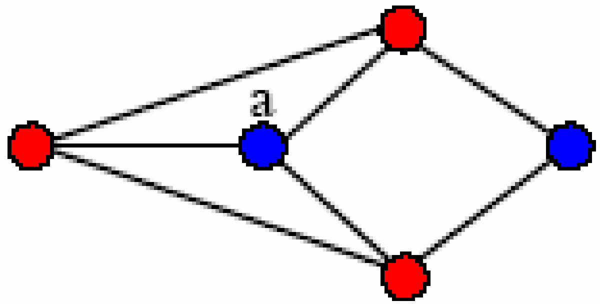


Graph states: Entanglement

Theorem [Van den Nest, 2004]:

$|G\rangle \equiv_{LC} |G'\rangle$ iff there exists a sequence of *local complementations* which transforms G into G'.

Local Complementation according to a: $G * a = G \Delta K_{N(a)}$

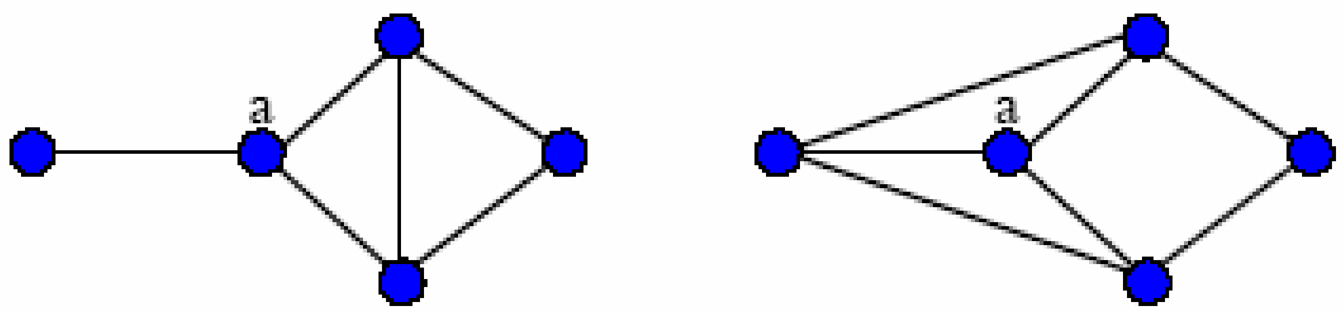


Graph states: Entanglement

Theorem [Van den Nest, 2004]:

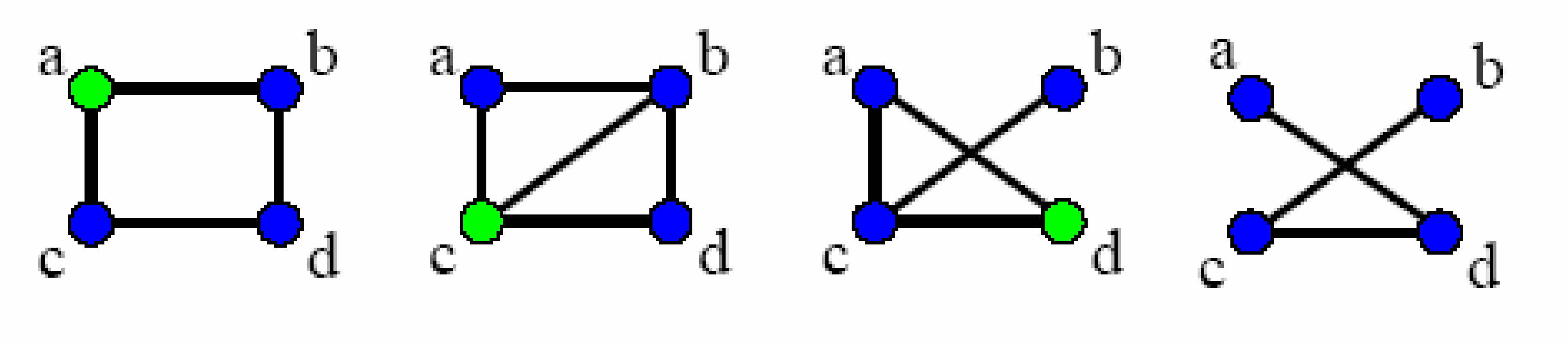
$|G\rangle \equiv_{LC} |G'\rangle$ iff there exists a sequence of *local complementations* which transforms G into G'.

Local Complementation according to a: $G * a = G \Delta K_{N(a)}$

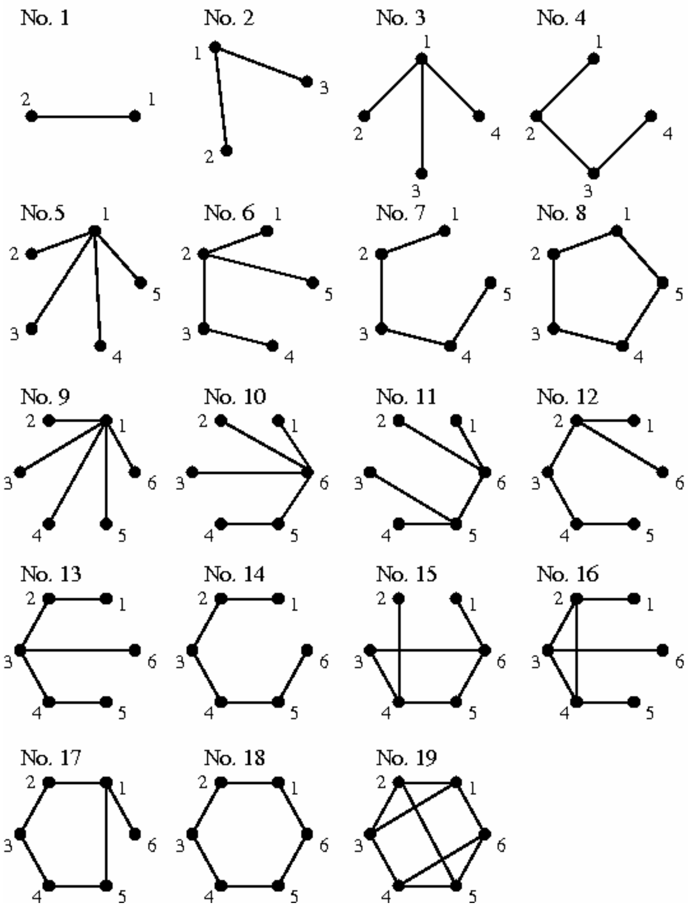


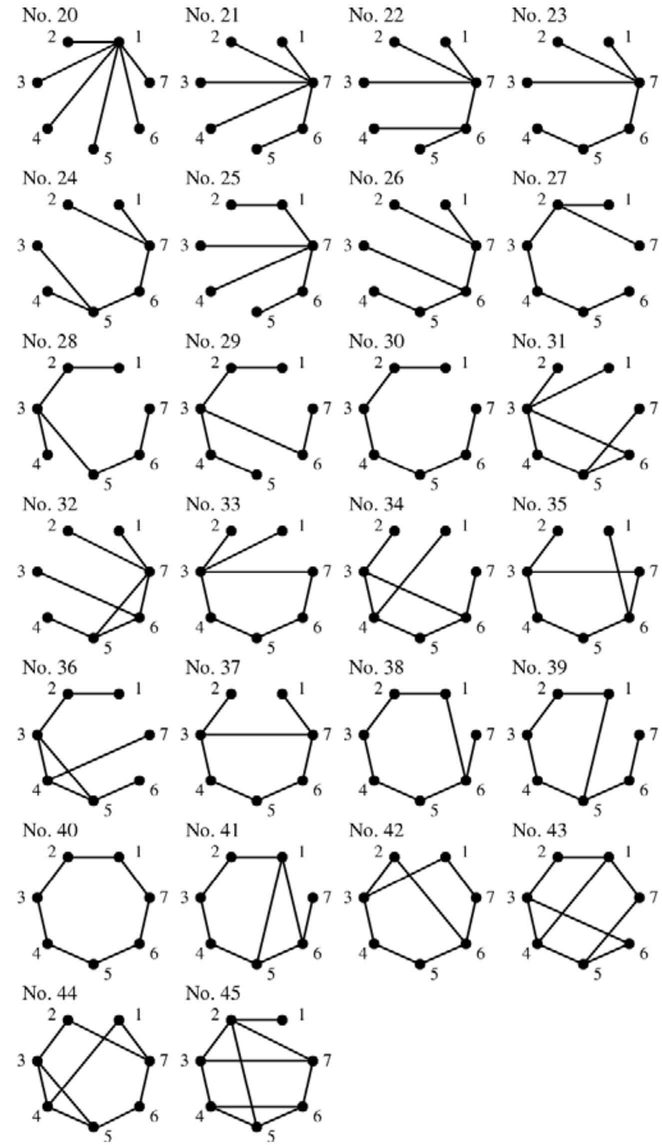
Graph states: Entanglement

The following graphs represent *LC*-equivalent graph states.
Therefore, they represent *LU*-equivalent states.
Therefore, they have the same entanglement.



All graph states up to seven qubits

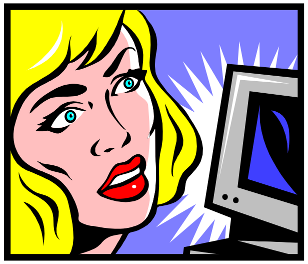




Graphs corresponding to all possible graph states, up to seven qubits, which are not equivalent under single-qubit transformations and graph isomorphism, M. Hein, J. Eisert, and H. J. Briegel, PRA **69**, 062311 (2004).

Problem

• If we distribute *n* qubits between two parties,
what quantum graph states and distributions of qubits
allow AVN proofs using only single-qubit measurements?



First ingredient: Bipartite elements of reality

Lemma: A distribution of n qubits between Alice (who is given n_A qubits) and Bob (who is given n_B qubits) permits bipartite elements of reality if and only if $n_A = n_B$, and the reduced stabilizer of Alice's (Bob's) qubits contains *all* possible variations with repetition of the four elements, $\mathbb{1}$, X, Y, and Z, choose n_A (n_B), and none of them repeated.

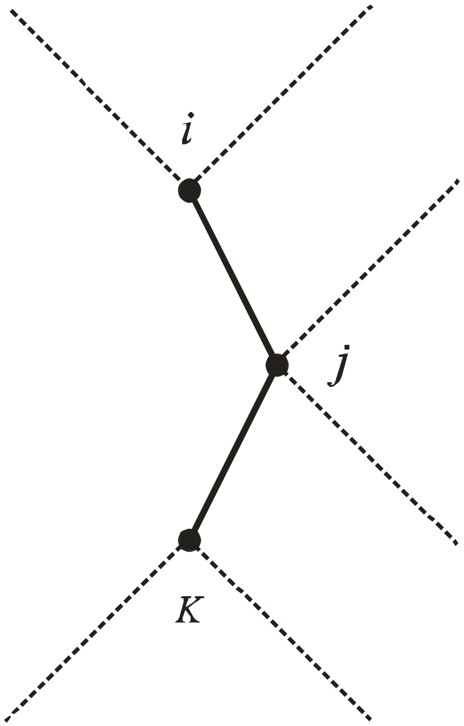
AC & P. Moreno, PRL **99**, 220402 (2007).

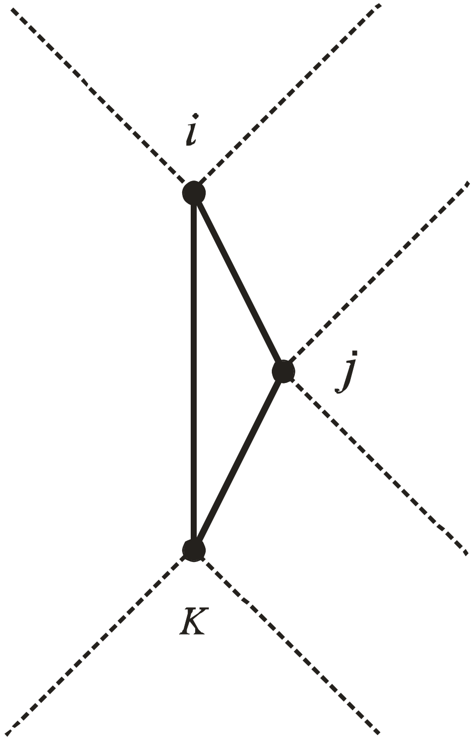
Second ingredient: Algebraic contradiction

Lemma: Any graph state associated to a connected graph of three or more vertices leads to an algebraic contradictions with the concept of elements of reality (when each qubit is distributed to a different party).

AC & P. Moreno, PRL **99**, 220402 (2007).

Second ingredient: Examples of contradictions





$$
\begin{aligned}
g_i g_j |G\rangle &= |G\rangle \\
g_j |G\rangle &= |G\rangle \\
g_j g_k |G\rangle &= |G\rangle \\
g_i g_j g_k |G\rangle &= |G\rangle
\end{aligned}
$$

$$
\begin{aligned}
g_i |G\rangle &= |G\rangle \\
g_j |G\rangle &= |G\rangle \\
g_k |G\rangle &= |G\rangle \\
g_i g_j g_k |G\rangle &= |G\rangle
\end{aligned}
$$

Four-qubit cluster state

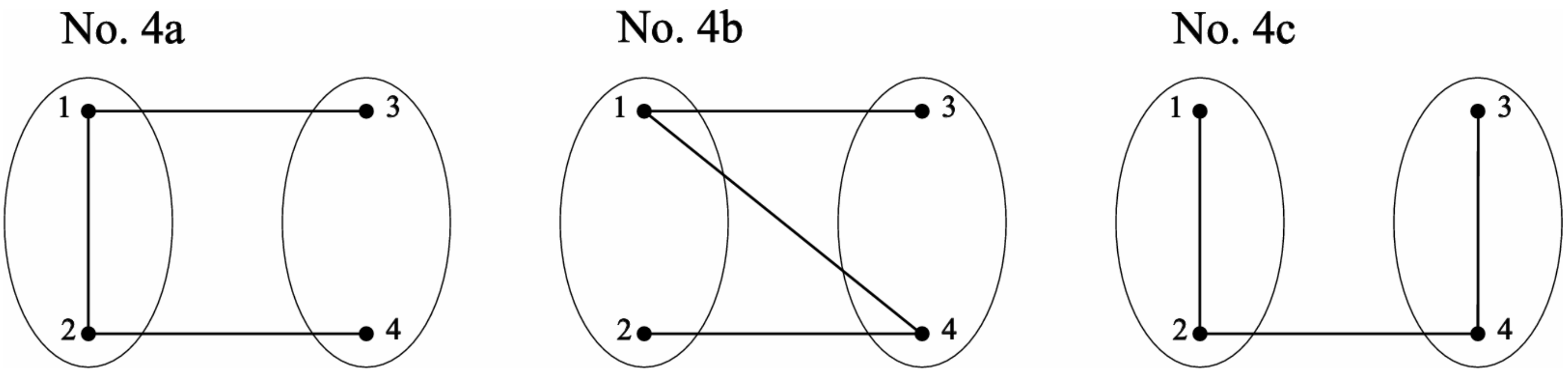
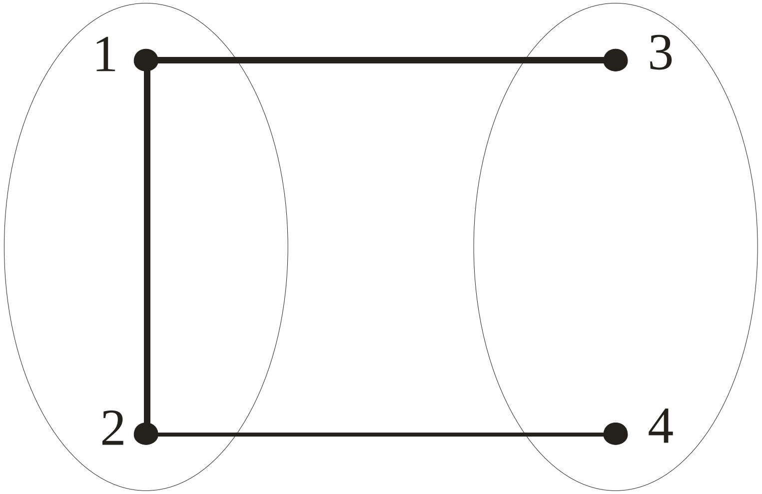


FIG. 1: Bipartite distributions of the 4-qubit linear cluster state (graph state no. 4 according to Hein *et al.* [28]). The distribution no. 4a permits bipartite elements of reality. The distribution 4b is physically equivalent (it is just relabeling the basis of the qubits). The distribution 4c is not equivalent to the other two, but does not permit bipartite elements of reality.

Four-qubit graph state allowing bipartite AVN proofs

No. 4a



$$|\psi_{4a}\rangle = \frac{1}{2}(|00\rangle|\bar{0}\bar{0}\rangle + |01\rangle|\bar{0}\bar{1}\rangle + |10\rangle|\bar{1}\bar{0}\rangle - |11\rangle|\bar{1}\bar{1}\rangle).$$

Alice's elements of reality	Bob's elements of reality
$X_1 = Z_3X_4$	$X_3 = Z_1$
$Y_1 = Y_3X_4$	$Y_3 = Y_1Z_2$
$Z_1 = X_3$	$Z_3 = X_1Z_2$
$X_2 = X_3Z_4$	$X_4 = Z_2$
$Y_2 = X_3Y_4$	$Y_4 = Z_1Y_2$
$Z_2 = X_4$	$Z_4 = Z_1X_2$

Bipartite AVN proof with single qubit observables…

$$v(X_1) = v(X_2)v(z_2)$$

$$v(Y_1) = -\,v(Y_2)v(z_2)$$

$$v(X_1)v(x_1) = v(Y_2)v(y_2)$$

$$v(Y_1)v(x_1) = v(X_2)v(y_2)$$

AC, PRL **95**, 210401 (2005).

… the only one with four qubits!!!!

$$v(X_1) = v(X_2)\,v(z_2)$$

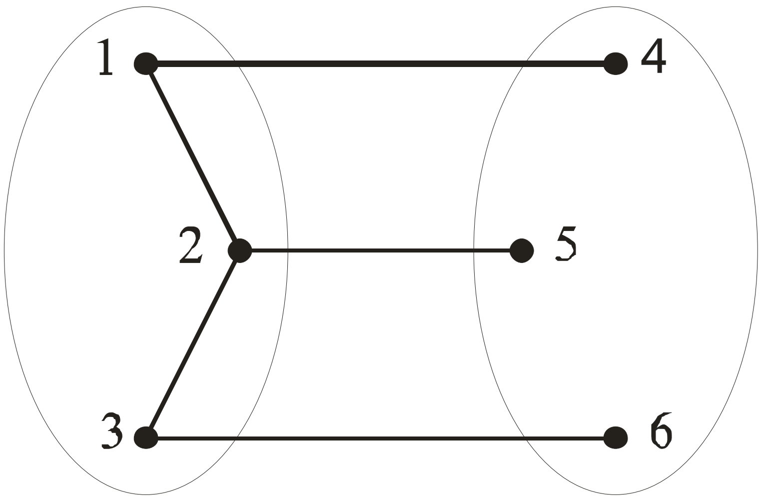
$$v(Y_1) = -\,v(Y_2)\,v(z_2)$$

$$v(X_1)\,v(x_1) = v(Y_2)\,v(y_2)$$

$$v(Y_1)\,v(x_1) = v(X_2)\,v(y_2)$$

Six-qubit graph states allowing bipartite AVN proofs

No. 13a

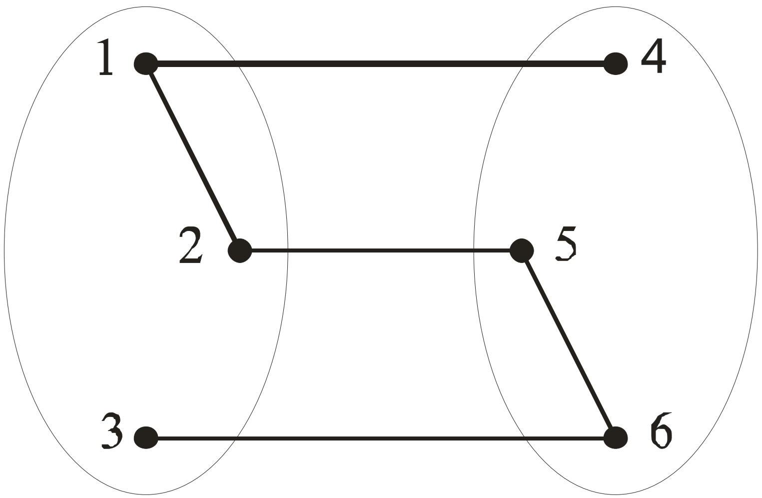


$$
\begin{aligned}
|\psi_{13a}\rangle = & \frac{1}{2\sqrt{2}}(|\bar{0}\bar{0}\bar{0}\bar{0}\bar{0}\bar{0}\rangle + |\bar{0}\bar{0}\bar{1}\bar{0}\bar{1}\bar{1}\rangle + |\bar{0}\bar{1}\bar{0}\bar{0}\bar{1}\bar{0}\rangle \\
& + |\bar{0}\bar{1}\bar{1}\bar{0}\bar{0}\bar{1}\rangle + |\bar{1}\bar{0}\bar{0}\bar{1}\bar{1}\bar{0}\rangle + |\bar{1}\bar{0}\bar{1}\bar{1}\bar{0}\bar{1}\rangle \\
& + |\bar{1}\bar{1}\bar{0}\bar{1}\bar{0}\bar{0}\rangle + |\bar{1}\bar{1}\bar{1}\bar{1}\bar{1}\bar{1}\rangle).
\end{aligned}
$$

Alice's elements of reality	Bob's elements of reality
$X_1 = Z_4 X_5$	$X_4 = Z_1$
$Y_1 = Y_4 X_5$	$Y_4 = Y_1 Z_2$
$Z_1 = X_4$	$Z_4 = X_1 Z_2$
$X_2 = X_4 Z_5 X_6$	$X_5 = Z_2$
$Y_2 = X_4 Y_5 X_6$	$Y_5 = Z_1 Y_2 Z_3$
$Z_2 = X_5$	$Z_5 = Z_1 X_2 Z_3$
$X_3 = X_5 Z_6$	$X_6 = Z_3$
$Y_3 = X_5 Y_6$	$Y_6 = Z_2 Y_3$
$Z_3 = X_6$	$Z_6 = Z_2 X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 14a

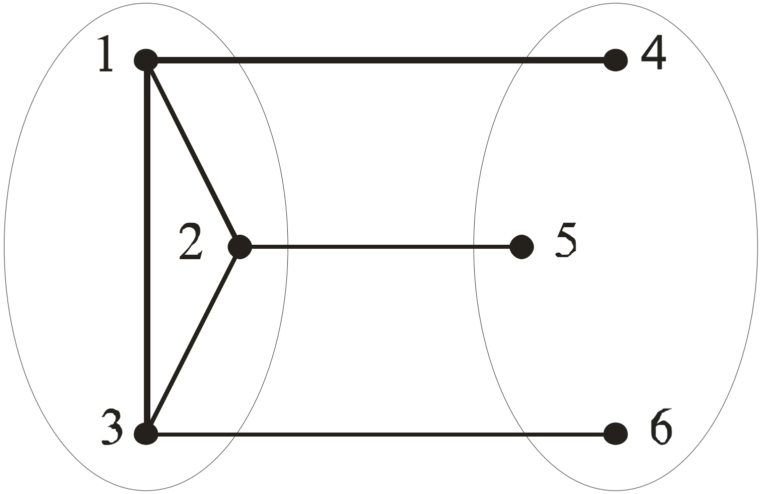


$$
\begin{aligned}
|\psi_{14a}\rangle \;=\; & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}0\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{1}\rangle \\
& +|0\bar{1}1\bar{0}1\bar{0}\rangle + |1\bar{0}0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{1}1\bar{0}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}0\bar{1}\rangle).
\end{aligned}
$$

Alice′s elements of reality	Bob′s elements of reality
$X_1 = Z_4X_5Z_6$	$X_4 = Z_1$
$Y_1 = Y_4X_5Z_6$	$Y_4 = Y_1Z_2$
$Z_1 = X_4$	$Z_4 = X_1Z_2$
$X_2 = X_4Z_5$	$X_5 = Z_2X_3$
$Y_2 = X_4Y_5Z_6$	$Y_5 = Z_1Y_2X_3$
$Z_2 = X_5Z_6$	$Z_5 = Z_1X_2$
$X_3 = Z_6$	$X_6 = Z_1X_2Z_3$
$Y_3 = Z_5Y_6$	$Y_6 = Z_1X_2Y_3$
$Z_3 = X_6$	$Z_6 = X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 16a

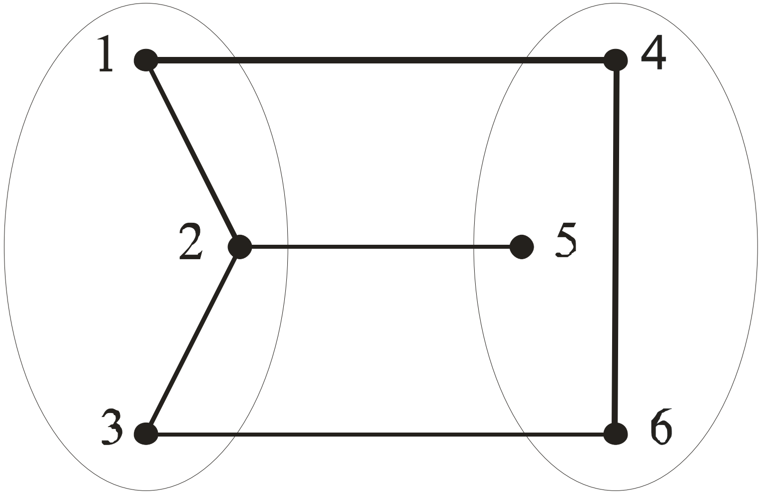


$$|\psi_{16a}\rangle = \frac{1}{2\sqrt{2}}(|\bar{0}\bar{0}\bar{0}\bar{0}\bar{0}\bar{0}\rangle + |\bar{0}\bar{0}\bar{1}\bar{0}\bar{1}\bar{1}\rangle + |\bar{0}\bar{1}\bar{0}\bar{0}\bar{1}\bar{0}\rangle + |\bar{0}\bar{1}\bar{1}\bar{0}\bar{0}\bar{1}\rangle + |\bar{1}\bar{0}\bar{0}\bar{1}\bar{1}\bar{0}\rangle - |\bar{1}\bar{0}\bar{1}\bar{1}\bar{0}\bar{1}\rangle + |\bar{1}\bar{1}\bar{0}\bar{1}\bar{0}\bar{0}\rangle - |\bar{1}\bar{1}\bar{1}\bar{1}\bar{1}\bar{1}\rangle).$$

Alice's elements of reality	Bob's elements of reality
$X_1 = Z_4X_5X_6$	$X_4 = Z_1$
$Y_1 = Y_4X_5X_6$	$Y_4 = Y_1Z_2Z_3$
$Z_1 = X_4$	$Z_4 = X_1Z_2Z_3$
$X_2 = X_4Z_5X_6$	$X_5 = Z_2$
$Y_2 = X_4Y_5X_6$	$Y_5 = Z_1Y_2Z_3$
$Z_2 = X_5$	$Z_5 = Z_1X_2Z_3$
$X_3 = X_4X_5Z_6$	$X_6 = Z_3$
$Y_3 = X_4X_5Y_6$	$Y_6 = Z_1Z_2Y_3$
$Z_3 = X_6$	$Z_6 = Z_1Z_2X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 17a

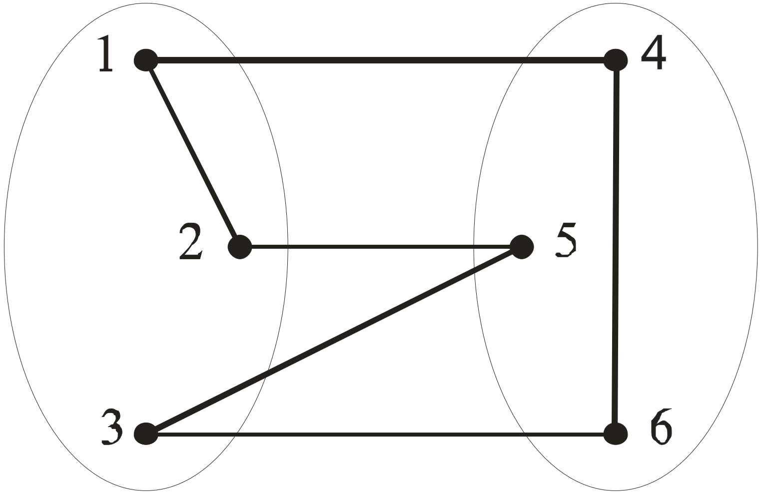


$$
\begin{aligned}
|\psi_{17a}\rangle \;=\; & \frac{1}{2\sqrt{2}}(|00\bar{0}\bar{0}\bar{0}0\rangle + |00\bar{1}\bar{1}\bar{0}1\rangle + |01\bar{0}\bar{1}\bar{1}1\rangle \\
& +|01\bar{1}\bar{0}\bar{1}0\rangle + |10\bar{0}\bar{1}\bar{0}0\rangle + |10\bar{1}\bar{0}\bar{0}1\rangle \\
& -|11\bar{0}\bar{0}\bar{1}1\rangle - |11\bar{1}\bar{1}\bar{1}1\rangle).
\end{aligned}
$$

Alice′s elements of reality	Bob′s elements of reality
$X_1 = Z_4 X_5$	$X_4 = Z_1 Z_2 X_3$
$Y_1 = Y_4 X_5 Z_6$	$Y_4 = Y_1 X_3$
$Z_1 = X_4 Z_6$	$Z_4 = X_1 Z_2$
$X_2 = Y_4 Z_5 Y_6$	$X_5 = Z_2$
$Y_2 = Y_4 Y_5 Y_6$	$Y_5 = Z_1 Y_2 Z_3$
$Z_2 = X_5$	$Z_5 = Z_1 X_2 Z_3$
$X_3 = X_5 Z_6$	$X_6 = X_1 Z_2 Z_3$
$Y_3 = Z_4 X_5 Y_6$	$Y_6 = X_1 Y_3$
$Z_3 = Z_4 X_6$	$Z_6 = Z_2 X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 18a

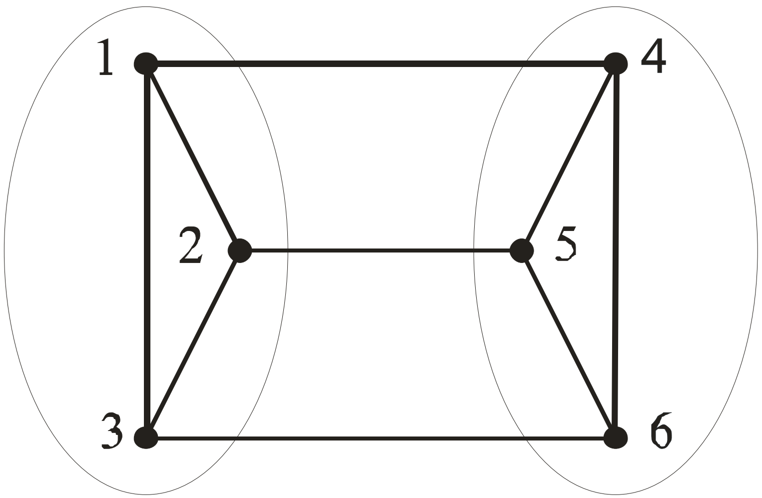


$$
\begin{aligned}
|\psi_{18a}\rangle &= \frac{1}{2\sqrt{2}}(|\bar{0}00\bar{0}0\bar{0}\rangle + |\bar{0}01\bar{0}1\bar{1}\rangle + |\bar{0}10\bar{1}1\bar{1}\rangle \\
&+|\bar{0}11\bar{1}0\bar{0}\rangle + |\bar{1}00\bar{1}0\bar{1}\rangle + |\bar{1}01\bar{1}1\bar{0}\rangle \\
&+|\bar{1}10\bar{0}1\bar{0}\rangle + |\bar{1}11\bar{0}0\bar{1}\rangle).
\end{aligned}
$$

Alice's elements of reality	Bob's elements of reality
$X_1 = X_5 X_6$	$X_4 = X_2 X_3$
$Y_1 = -X_4 X_5 Y_6$	$Y_4 = -X_1 Y_2 X_3$
$Z_1 = X_4 Z_6$	$Z_4 = X_1 Z_2$
$X_2 = X_4 Z_5 Z_6$	$X_5 = Z_2 Z_3$
$Y_2 = Y_4 Y_5 Y_6$	$Y_5 = Z_1 Y_2 Z_3$
$Z_2 = Z_4 X_5 X_6$	$Z_5 = Z_1 X_2$
$X_3 = Z_5 Z_6$	$X_6 = X_1 Z_2 Z_3$
$Y_3 = Z_4 Z_5 Y_6$	$Y_6 = Y_1 Y_2 Y_3$
$Z_3 = Z_4 X_6$	$Z_6 = Z_1 X_2 X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 19a

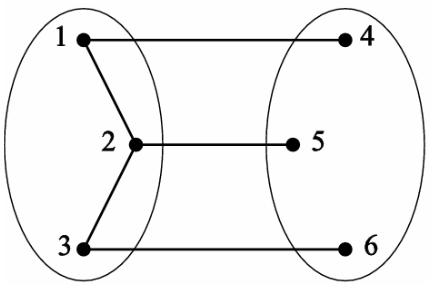


$$
\begin{aligned}
|\psi_{19a}\rangle = \frac{1}{4}(&|\bar{0}0000\bar{0}\rangle + |\bar{0}0001\bar{1}\rangle + |\bar{0}0110\bar{0}\rangle \\
&-|\bar{0}0111\bar{1}\rangle + |\bar{0}1010\bar{1}\rangle + |\bar{0}1011\bar{0}\rangle \\
&-|\bar{0}1100\bar{1}\rangle + |\bar{0}1101\bar{0}\rangle + |\bar{1}0010\bar{1}\rangle \\
&-|\bar{1}0011\bar{0}\rangle + |\bar{1}0100\bar{1}\rangle + |\bar{1}0101\bar{0}\rangle \\
&+|\bar{1}1000\bar{0}\rangle - |\bar{1}1001\bar{1}\rangle - |\bar{1}1110\bar{0}\rangle \\
&-|\bar{1}1111\bar{1}\rangle).
\end{aligned}
$$

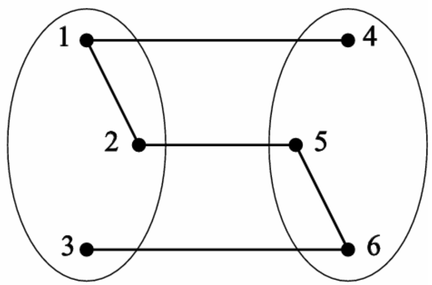
Alice's elements of reality	Bob's elements of reality
$X_1 = Z_4Y_5Y_6$	$X_4 = Z_1Y_2Y_3$
$Y_1 = -Y_4X_5X_6$	$Y_4 = -Y_1X_2X_3$
$Z_1 = X_4Z_5Z_6$	$Z_4 = X_1Z_2Z_3$
$X_2 = Y_4Z_5Y_6$	$X_5 = Y_1Z_2Y_3$
$Y_2 = -X_4Y_5X_6$	$Y_5 = -X_1Y_2X_3$
$Z_2 = Z_4X_5Z_6$	$Z_5 = Z_1X_2Z_3$
$X_3 = Y_4Y_5Z_6$	$X_6 = Y_1Y_2Z_3$
$Y_3 = -X_4X_5Y_6$	$Y_6 = -X_1X_2Y_3$
$Z_3 = Z_4Z_5X_6$	$Z_6 = Z_1Z_2X_3$

Six-qubit graph states allowing bipartite AVN proofs

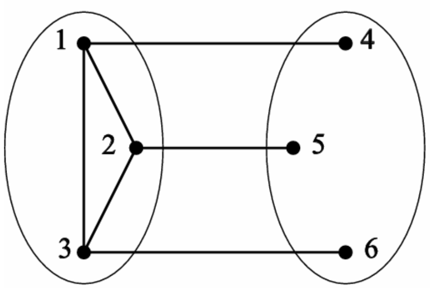
No. 13a



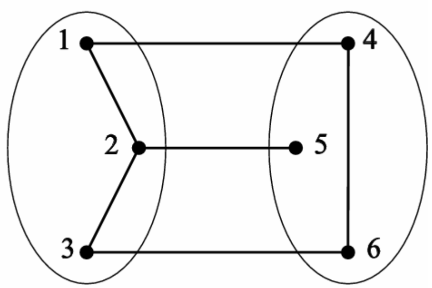
No. 14a



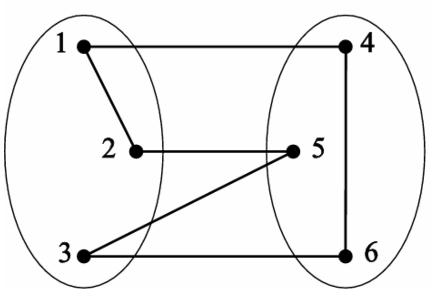
No. 16a



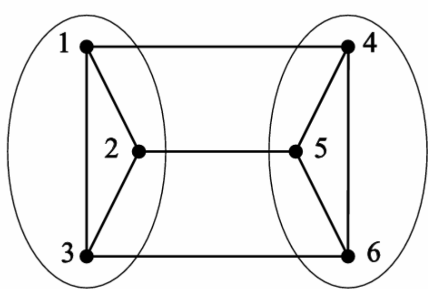
No. 17a



No. 18a



No. 19a



$$
\begin{aligned}
|\psi_{13a}\rangle = {} & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
& +|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle + |1\bar{0}1\bar{1}0\bar{1}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}1\bar{1}\rangle), \\
|\psi_{14a}\rangle = {} & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}0\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{1}\rangle \\
& +|0\bar{1}1\bar{0}1\bar{0}\rangle + |1\bar{0}0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{1}1\bar{0}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}0\bar{1}\rangle), \\
|\psi_{16a}\rangle = {} & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
& +|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle - |1\bar{0}1\bar{1}0\bar{1}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle - |1\bar{1}1\bar{1}1\bar{1}\rangle), \\
|\psi_{17a}\rangle = {} & \frac{1}{2\sqrt{2}}(|00\bar{0}\bar{0}\bar{0}0\rangle + |00\bar{1}\bar{1}\bar{0}1\rangle + |01\bar{0}\bar{1}\bar{1}1\rangle \\
& +|01\bar{1}\bar{0}\bar{1}0\rangle + |10\bar{0}\bar{1}\bar{0}0\rangle + |10\bar{1}\bar{0}\bar{0}1\rangle \\
& -|11\bar{0}\bar{0}\bar{1}1\rangle - |11\bar{1}\bar{1}\bar{1}1\rangle), \\
|\psi_{18a}\rangle = {} & \frac{1}{2\sqrt{2}}(|\bar{0}000\bar{0}\bar{0}\rangle + |\bar{0}010\bar{1}\bar{1}\rangle + |\bar{0}101\bar{1}\bar{1}\rangle \\
& +|\bar{0}111\bar{0}\bar{0}\rangle + |\bar{1}001\bar{0}\bar{1}\rangle + |\bar{1}011\bar{1}\bar{0}\rangle \\
& +|\bar{1}100\bar{1}\bar{0}\rangle + |\bar{1}110\bar{0}\bar{1}\rangle), \\
|\psi_{19a}\rangle = {} & \frac{1}{4}(|\bar{0}0000\bar{0}\rangle + |\bar{0}0001\bar{1}\rangle + |\bar{0}0110\bar{0}\rangle \\
& -|\bar{0}0111\bar{1}\rangle + |\bar{0}1010\bar{1}\rangle + |\bar{0}1011\bar{0}\rangle \\
& -|\bar{0}1100\bar{1}\rangle + |\bar{0}1101\bar{0}\rangle + |\bar{1}0010\bar{1}\rangle \\
& -|\bar{1}0011\bar{0}\rangle + |\bar{1}0100\bar{1}\rangle + |\bar{1}0101\bar{0}\rangle \\
& +|\bar{1}1000\bar{0}\rangle - |\bar{1}1001\bar{1}\rangle - |\bar{1}1110\bar{0}\rangle \\
& -|\bar{1}1111\bar{1}\rangle).
\end{aligned}
$$

Problems

- Which is the maximum degree of nonlocality *D* for a six-qubit graph state allowing bipartite elements of reality?
- Which is the maximum *D* for the perfect correlations of a *n*-qubit graph state?
- Which is the relation between *D* and η?
- Can these results help us to make a loophole-free experiment?

(*D* is defined as the ratio between the QM value and the bound of the Bell inequality. It is related to the minimum overall detection efficiency η required for a loophole-free experiment.)

Answers?

Next talk