Quantum Correlations and Fundamental Conservation Laws

Quantum Correlations from Classical Insights: Implications to entanglement and nonlocality

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4 results for discussion:

1) Correlation functions of quantum mechanics are direct consequence of the CLASSICAL conservation laws arising in space-time symmetries (fundamental conservation laws), applied to ensembles.

2) Any theory that has a correlation function different from the ones in QM is <u>incompatible with the fundamental conservation</u> laws and space-time symmetries, and therefore it is unphysical. Local hidden variable theories fall in this class. Bell's inequalities can be obeyed (in the general case) only by violating a fundamental conservation law, making them redundant in physics.

3) The origin of Bell's inequalities can be traced unambiguously to the single step of ignoring wave-particle duality and has nothing to do with the violation of Einstein locality (Indeed, they can be obeyed in those situations where wave-particle duality can be ignored).

4) The logical implication of the experimental result that Bell's inequalities are violated is that a classical statistical theory can reproduce quantum correlations (or any arbitrary correlation for that matter!) only if it violates Einstein locality, and NOT that QM is nonlocal!

Correlation functions of quantum mechanics are direct consequence of the CLASSICAL conservation laws arising in space-time symmetries (fundamental conservation laws), applied to ensembles

Conservation laws for energy, momentum and angular momentum with generalization to symmetries of internal spaces…

A theory independent correlation function derived with conservation laws as the only input is identical to the quantum correlation function!

The case of two 'spin-half' particles:

$$\Psi_S = \frac{1}{\sqrt{2}}\left(\left|+1\right\rangle_1 \left|-1\right\rangle_2 - \left|-1\right\rangle_1 \left|+1\right\rangle_2\right)$$

$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i \quad : A_i, B_i = \pm 1$$

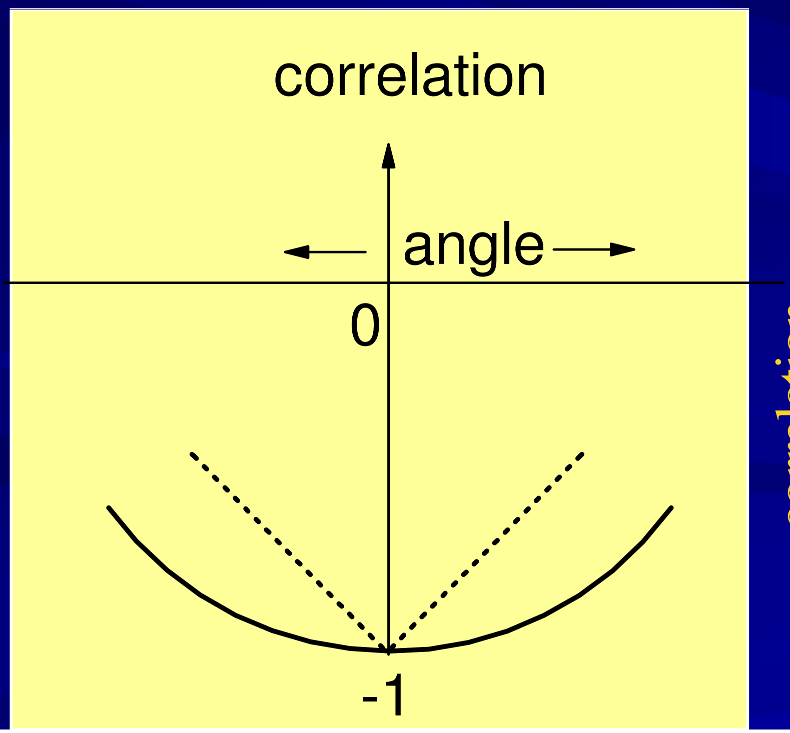
Important input

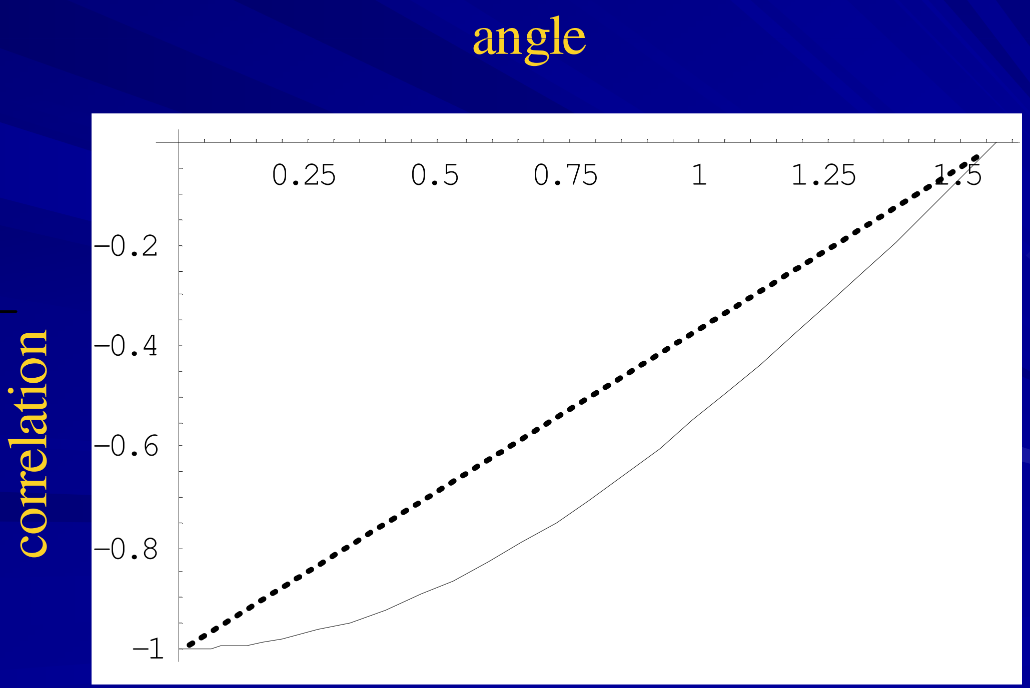
Quantum Mechanics: $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} = -\cos\theta$

$$P(\vec{a},\vec{b})_{QM} = \left\langle \Psi_S \middle| \sigma_1 \cdot \vec{a} \otimes \sigma_2 \cdot \vec{b} \middle| \Psi_s \right\rangle = -\vec{a} \cdot \vec{b}$$

$$P(\vec{a},\vec{b})_{Bell} = \int A(\vec{a},h) B(\vec{b},h) \rho(h) dh$$

The essence of Bell's theorem is that these two correlation functions have distinctly different dependences on the angle between the settings of the apparatus (difference of about 30% at specific angles).





± 1 ± 1

$$\sum_i A_i = \sum_i B_i = 0$$

Aim: Derive a unique <u>theory-independent</u> correlation function from the conservation of total angular momentum, demanding CONSERVATION ONLY ON THE AVERAGE

Random

A	B
-1	+1
-1	-1
+1	+1
-1	+1
+1	-1
-1	+1
-1	+1
+1	-1
-1	+1
+1	+1
-1	+1
+1	-1
+1	+1
+1	-1
+1	+1
-1	+1
+1	-1

Random

$$\sum_i A_i = \sum_i B_i = 0$$

$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i \neq 0$$

A (reordered)	B (reordered)
+1	+1
+1	-1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
-1	+1
-1	-1
-1	-1
-1	+1
-1	+1
-1	+1
-1	+1
-1	+1

A (reordered)	B (reordered)
+1	+1
+1	-1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
-1	+1
-1	-1
-1	-1
-1	+1
-1	+1
-1	+1
-1	+1
-1	+1

Average Angular Momentum/($\hbar$/2)

$$= \frac{1}{N_{A+}} \sum_i +1 = +1$$

Average Angular Momentum/($\hbar$/2)

$$= \frac{1}{N_{A-}} \sum_i -1 = -1$$

What are the AVERAGE angular momenta at B for the two sub-ensembles?

Conservation of angular momentum on the average implies that if the apparatus angles are equal, (a=b), then the average L vectors are opposite:

$$L_B = -L_A,\ for\ \vec{a} = \vec{b}$$

Therefore, for distinct $\vec{\mathrm{a}}$ and $\vec{\mathrm{b}}$, $L_B = -L_A \cos(\theta)$

$$\text{For } L_A = +1, \quad L_B = -\cos(\theta)$$

$$\text{For } L_A = -1, \quad L_B = +\cos(\theta)$$

$$\text{For } L_A = +1, \quad L_B = -\cos(\theta)$$

$$\text{For } L_A = -1, \quad L_B = +\cos(\theta)$$

Correlation functions for the sub-ensembles:

$$P(\vec{a},\vec{b})_{A_i=+1} = \frac{1}{N}\sum_i A_i B_i = \frac{+1}{N}\sum_i B_i \equiv L_B$$

$$P(\vec{a},\vec{b})_{A_i=-1} = \frac{1}{N}\sum_i A_i B_i = \frac{-1}{N}\sum_i B_i$$

$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i = \frac{1}{2}\left(P(\vec{a},\vec{b})_{A=+1} + P(\vec{a},\vec{b})_{A=-1}\right)$$

$$= \frac{1}{2}\left(+1 \times \frac{1}{N/2}\sum_{i=1}^{N/2} B_i + (-1) \times \frac{1}{N/2}\sum_{i=1+N/2}^{N} B_i\right)$$

$$= \frac{1}{2}\left(\langle B_i \rangle_{A_i=+1} - \langle B_i \rangle_{A_i=-1}\right) = \frac{1}{2}\left(L_{B(A=+1)} + L_{B(A=-1)}\right) = -\cos(\theta)$$

$$P(\vec{a},\vec{b})_{CL} = \left(L_{B(A=+1} - L_{B(A=-1}\right)/2 = -\cos(\theta)$$

This is the causally necessary consequence of the conservation law. We have the theory independent correlation function.

Fundamental Conservation Laws $\{F(p,q,s...)=0\} \Rightarrow$

Quantum Mechanical Correlation Functions $\{C_{QM}(\theta_i)\}$

$$\neg\{C_{QM}(\theta_i)\} \Rightarrow \neg\{F(p,q,s...)=0\}$$

A correlation function with a different functional form is incompatible with the conservation laws: they can be physically realized only by violating a fundamental conservation law!

Ref: Unnikrishnan (2005), Europhys. Lett. **69**, 489 , Pramana, **65**, 359

Any expectation that the experimental tests might have supported a correlation function different from $P(a,b)_{QM}$ certainly had not appreciated the fact that in order to get such a deviation, the conservation law for angular momentum has to be grossly violated, even on the average.

The requirement of conservation laws is <u>much stronger than the demarcating criteria based on Bell's inequalities</u> since the slightest deviation from the QM correlation function signals the fundamental incompatibility. The important consequence is that one cannot anymore cite "loopholes in experiments".

Obviously, the entire exercise of testing the Bell's inequalities, accepting the possibility of a violation, was (and is) a futile exercise based on inadequate understanding of the conflict with the conservation laws, just as futile as trying to build a perpetual motion machine.

Before we go ahead some usual questions have to be answered:

1) Classical spins obey the conservation laws – why are you not getting a cos(theta) correlation function there?

2) Bell's inequality is obeyed in experiment with classical spin, which also obeys conservation laws on the average. But the correlation function is different from that in QM. You say that the inequalities can be obeyed only by violating the conservation laws. Isn't there a glaring discrepancy?

3) Why the hell is the assumption of locality required in Bell's derivation?

1) The crucial point is the discrete-valued observable. There is no way to obey the conservation laws, but on the average.

2) If locality is allowed to be violated, then ANY arbitrary correlation can be supplied on demand in a classical theory

Higher Spins, Triplet state, GHZ etc…

Spin-S singlet: $+S, +(S-1), \ldots 0, \ldots -(S-1), -S$

are the possible values

1) Create 2S+1 sub-ensembles at A

2) For sub-ensemble with average (and individual) value (S-n), the average in the direction rotated at an angle is $(S-n)\cos\theta$

3) Then the average angular momentum at B for the matching sub-ensemble is $-(S-n)\cos\theta$ $\Rightarrow$ Correlation function

$$Av(A_i B_i) = -(S-n)^2 \cos\theta$$

Full Correlation function

$$P(\vec{a},\vec{b}) = \frac{2\sum_{n=0}^{S} -(S-n)^2 \cos\theta}{2S+1} = -\cos(\theta) S(S+1)/3$$

(Same as the QM correlation function!)

A correlation function with a different functional form is incompatible with the conservation laws: they can be physically realized only by violating a fundamental conservation law!

Local hidden variable theories of quantum correlations are incompatible with the conservation laws and therefore the Bell's inequalities can be obeyed (in the general case) only by violating a fundamental conservation law.

Alchemists are considered crackpots by standard scientists, but alchemy was considered important from the point of view of practical chemistry – as the historical seed of detailed chemistry, albeit with incorrect scientific foundations.

The field of quantum computation and information has benefited a lot from the 'tests of Bell's inequalities'. But if there was the realization in the seventies that the Bell correlation functions were grossly incompatible with the fundamental conservation laws, then people who wanted to test the inequalities would have been considered as…

It is really high time to stop glorifying experimental tests that were done because of inadequate understanding of basic facts.

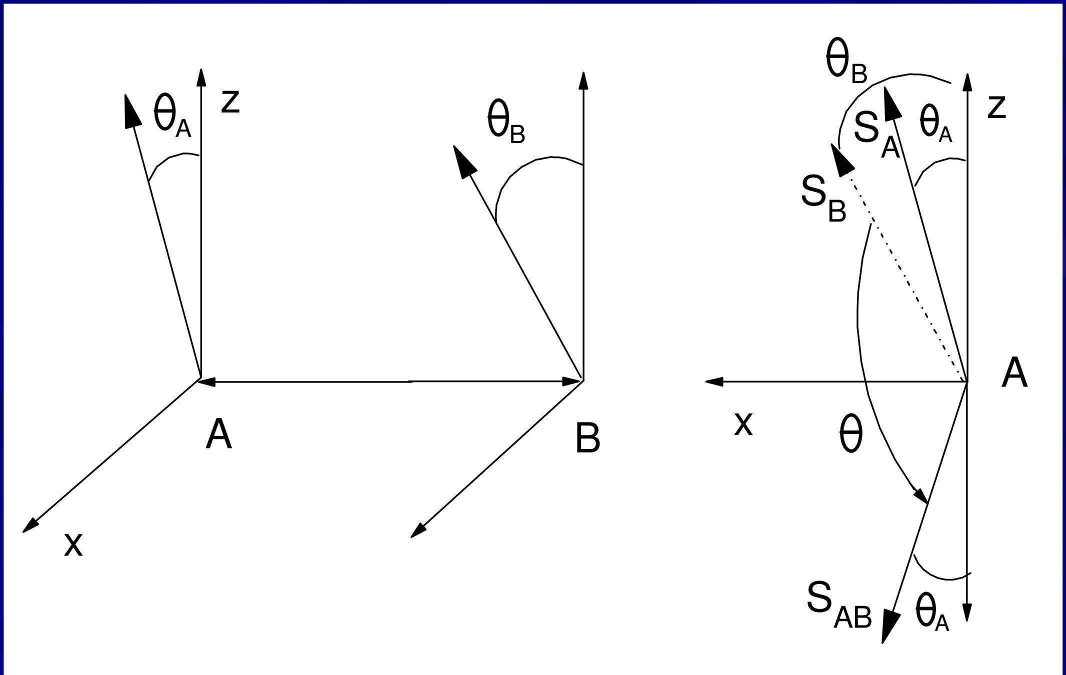
Spin-1/2 triplet

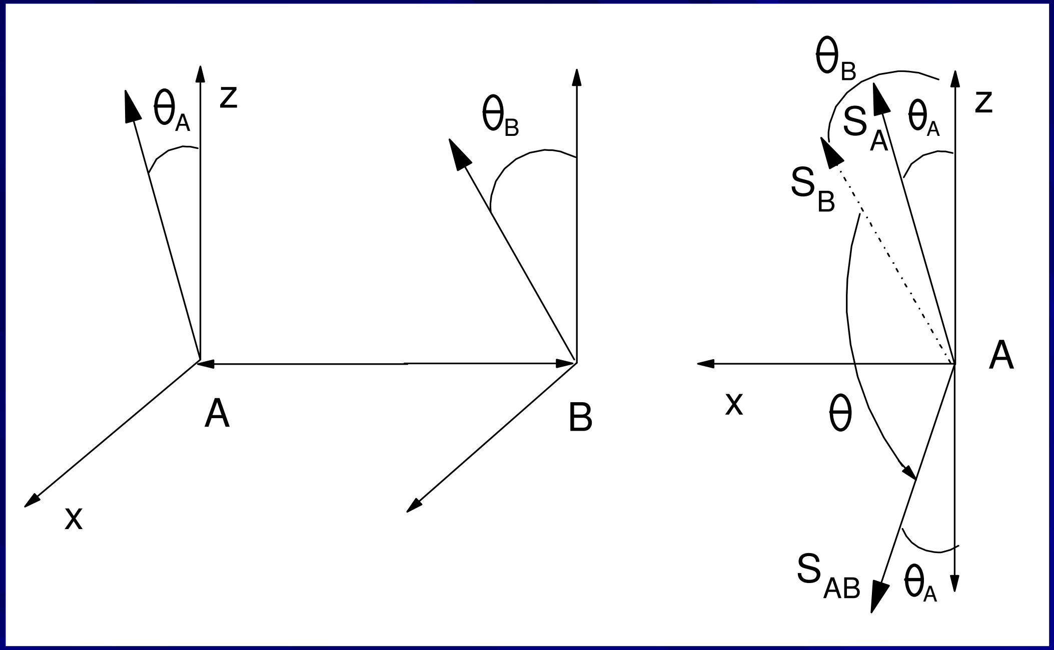
Total Angular momentum

$\sqrt{S(S+1)}$; S=1, with values of projection $m = \pm 1, 0$

Consider the m=0 case: Classically, this means that the average angular momentum along the z axis is zero, and in any direction in the x-y plane is 1 (aligned spins).

Let S_A be the average angular momentum of the +1 sub-ensemble at A. What is the average angular momentum of the correlated sub-ensemble at B?





$$P(\vec{a},\vec{b})_{A=+1} = +1_A \times (+1)\cos\theta = \cos\left(180-(\theta_A+\theta_B)\right) = -\cos(\theta_A+\theta_B)$$

$$P(\vec{a},\vec{b})_{A=-1} = (-1)\times(-1)\cos\theta = \cos\left(180-(\theta_A+\theta_B)\right) = -\cos(\theta_A+\theta_B)$$

$$P(\vec{a},\vec{b})_{S=1,m=0} = -\cos(\theta_A+\theta_B)$$

Ref: Unnikrishnan (2005), Europhys. Lett. **69**, 489 , Pramana, **65**, 359

From the conservation law

$$P(\vec{a},\vec{b})_{S=1,m=0} = -\cos(\theta_A + \theta_B)$$

From quantum mechanics:

$$\Psi_{TZ} = \frac{1}{\sqrt{2}}\left(|+1\rangle_1 |-1\rangle_2 + |-1\rangle_1 |+1\rangle_2\right)$$

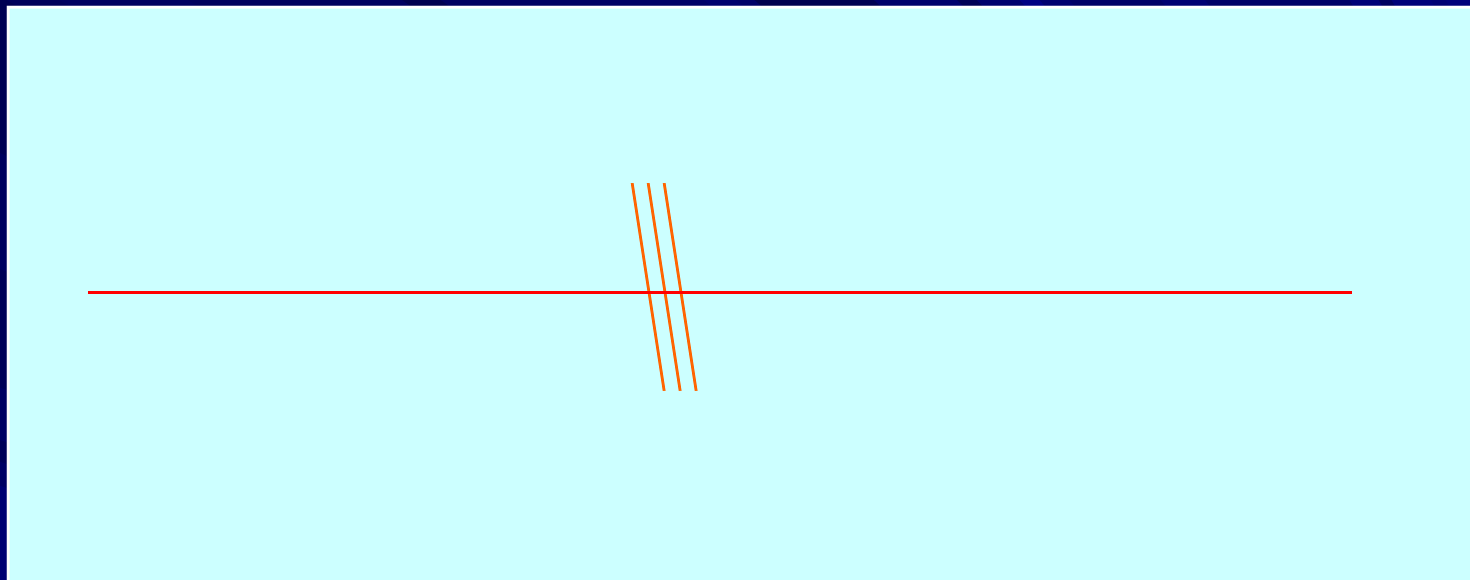
$$P(\vec{a},\vec{b})_{QM} = \langle \Psi_T | (\sigma_1 \cdot \vec{a})(\sigma_2 \cdot \vec{b}) | \Psi_T \rangle$$

$$\sigma \cdot n = \begin{bmatrix} n_3 & n_1 - in_2 \\ n_1 + in_2 & -n_3 \end{bmatrix}, \quad \Psi_{Tz} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_A \begin{pmatrix}0\\1\end{pmatrix}_B + \begin{pmatrix}0\\1\end{pmatrix}_A \begin{pmatrix}1\\0\end{pmatrix}_B\right]$$

$$P(\vec{a},\vec{b})^{QM}_{S=1,m=0} = -\cos(\theta_A + \theta_B)$$

Conservation law implies the Quantum Mechanical Correlation Function

The case of continuous variables:



Single particle two-slit interference pattern with mean at x=0:

$$P(x_B) = \left[1 + \cos\left(\alpha k x_B\right)\right]/2$$

Conservation of momentum: $\theta_B = -\theta_A$

Therefore, the two-particle correlation is simply the same interference pattern shifted by $x_B = -\theta_A \times d_A = -\alpha x_A$

$$P(x_A, x_B) = \left[1 + \cos k\alpha(x_A + x_B)\right]/2$$ 100% visibility

Two-particle QM correlation function for EPR, double-slit experiment:

$$P(x_A, x_B)_{QM} = [1 + \cos k\alpha(x_A + x_B)]/2$$

Agrees completely with the one derived from conservation law

(100% visibility can be affected by source size – reduction for both small size AND big size (quantum uncertainty + spatial coherence)

Typically, testable local hidden variable theories predict fringe patterns with much lower visibility, 70%, for example.

This can happen only by either violating the conservation of linear momentum, or by ignoring the wave-particle duality (ability to interfere completely) in the theory. Either way, such theories are unphysical in a basic way and are worthless for serious experimental tests.

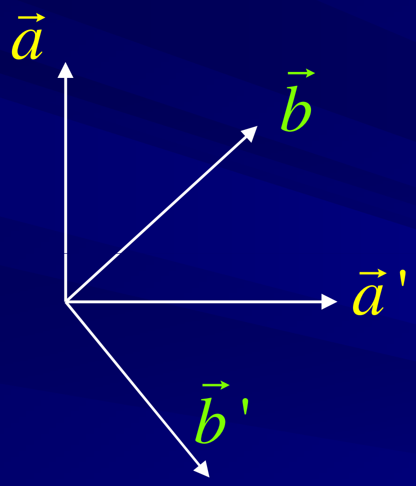
Mixed states:

Since the pure state correlations are completely equivalent to the conservation constraints as we have shown, the correlations in a mixed state is simply the statistical average of the pure state correlations, and therefore the statistical average of the conservation constraints. There is a direct linear relation between the correlations of the constituent pure states and that of the mixed state.

Therefore, we can quantify quantum entanglement in terms of the fidelity with conservation laws are obeyed – in terms of the fidelity of the correlations along the lines demonstrated here. The prescription is to subtract out the classically expected correlations expected from conservation laws, and the rest is from quantum entanglement.

Implication to Bell's inequalities:

Consider a theory of correlations that obeys the conservation laws on the average, where individual values are always ± 1 (with no reference to locality or otherwise).



$$P(\vec{a},\vec{b}) = -\cos(\vec{a},\vec{b})$$

$$P(\vec{a},\vec{b}) + P(\vec{a}\,',\vec{b}) + P(\vec{a}\,',\vec{b}\,') - P(\vec{a},\vec{b}\,') = 2\sqrt{2}$$

Therefore, a theory of correlations that respect the conservation laws violates the Bell's inequality, even with no reference to quantum mechanics.

A reanalysis of what Bell did to get the inequalities:

$$P_B(\vec{a},\vec{b}) = \int \rho(h)dh\ A(\vec{a},h)B(\vec{b},h), \quad \int \rho(h)dh = 1$$

Since $A(\vec{a}) = -B(\vec{a})$ and $P_B(\vec{a},\vec{a}) = -1,$ Bell wrote

$$P_B(\vec{a},\vec{b}) = -\int \rho(h)dh\ A(\vec{a},h)A(\vec{b},h)$$

$$P_B(\vec{a},\vec{b}) - P_B(\vec{a},\vec{c}) = -\int \rho(h)dh\ [A(\vec{a},h)A(\vec{b},h) - A(\vec{a},h)A(\vec{c},h)]$$

$$= -\int \rho(h)dh\ [A(\vec{a},h)A(\vec{b},h) - A(\vec{a},h)A(\vec{b},h)A(\vec{b},h)A(\vec{c},h)]$$

$$= \int \rho(h)dh\ A(\vec{a},h)A(\vec{b},h)[A(\vec{b},h)A(\vec{c},h) - 1]$$

$$\left|P_B(\vec{a},\vec{b}) - P_B(\vec{a},\vec{c})\right| \le \int \rho(h)dh\ [1 + A(\vec{b},h)B(\vec{c},h)] = 1 + P_B(\vec{b},\vec{c})$$

$$P_B(\vec{a},\vec{b}) = \int \rho(h)dh\ A(\vec{a},h)B(\vec{b},h), \quad \int \rho(h)dh = 1$$

Since $A(\vec{a}) = -B(\vec{a})$ and $P_B(\vec{a},\vec{a}) = -1,$ Bell wrote

$$P_B(\vec{a},\vec{b}) = -\int \rho(h)dh\ A(\vec{a},h)A(\vec{b},h)$$

Simultaneous definite values for quantum mechanically non-commuting observables!

Since $\sigma_A(\vec{z}) = -\sigma_B(\vec{z})$ and $P(\vec{z}_A,\vec{z}_B) = -1,$ we write

$$P(\sigma_A(\vec{x}),\sigma_B(\vec{z})) = \int \rho(h)dh\sigma_A(\vec{x})\ \sigma_B(\vec{z}) = -\int \rho(h)dh\sigma_A(\vec{x})\ \sigma_A(\vec{z})!$$

CSU, Proc. SPIE Photonics 2007, ESA Galileo Conf. 2007

Widespread beliefs: Experiments prove that there is non-locality, and that there is some superluminal, and perhaps instantaneous, influence passing between spatially separated and entangled particles (even though it cannot be used by us to send signals faster than light.)

The reason for this belief: Bell assumed that the hidden variable theories are local and that the measurement result at one location depends only on the settings of the local apparatus. But Bell's inequalities are violated in experiments.

FACT: The assumption of locality is not used in the DERIVATION of the inequality!

The need for such an assumption was simply that if not, ANY result can be simulated by sending an appropriate superluminal signal. But the inequality itself did not use this assumption. Instead, it used a back-step of assuming the possibility of SIMULTANEOUS values for incompatible (non-commuting) observables. This is the root of the inequality, and its violation in experiments just implies that certain observables cannot have simultaneous values – that is all. Experimental result has absolutely nothing to say about nonlocal superluminal influences.

Lack of both logical and empirical rigour

CSU, Proc. SPIE Photonics 2007, ESA Galileo Conf. 2007

If nonlocal influence are allowed then any classical theory (of the coin tossing type) can be made to reproduce whatever correlations one demands!

Hence the strict logical implication of the experimental results is that a classical theory of the type Bell considered can be a valid theory of microscopic phenomena IF one allows nonlocality as an additional feature.

This then takes away the uniqueness of quantum theory, contrary to the common belief.

Conclusions:

1. Fundamental Conservation laws uniquely implies quantum mechanical correlation functions.

2. Bell's inequalities can be obeyed (or even approached) in an experimental test with discrete observables only by violating a fundamental conservation law – therefore the inequalities are physically redundant. The experiments so far had been testing whether quantum mechanics violates conservation laws grossly on the average, without realizing it.

3. No further test of the Bell's inequalities are worth pursuing (unless one also believes in the certainty of gross violation of conservation laws and perpetual motion in nonrelativistic quantum mechanics!)

4. The real cause of the inequalities is ignoring wave-particle duality

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