Basic introduction to quantum theory

Consider an electron

Measure its spin angular momentum

Direction	Value
x	$\frac{h}{2}$
y	$\frac{h}{2}$
z	$\frac{h}{2}$

Certainly

$$\sqrt{\frac{h}{4}^2 + \frac{h}{4}^2 + \frac{h}{4}^2}$$

$$= \frac{\sqrt{3}\,h}{2}$$

Now you measure along a direction which makes equal angle with all the axes.

What result do you expect?

But this common sense idea of vector and its components really does not work for spin angular momentum of an electron.

In whichever direction you measure, the result is

$$\boxed{\begin{array}{ll} \text{either} & +\dfrac{h}{2} \\[2em] \text{or} & -\dfrac{h}{2} \end{array}}$$

- Classical physics were unable to explain this and some other peculiarities.

- Through trial and error process a new physical theory arose which is

Quantum Mechanics

System $\Rightarrow$ **Hilbert space**

State $\Rightarrow$ **Density operator**

If ρ is a density operator, then

i) $\rho^\dagger = \rho$ **(self adjoint)**

ii) ρ **is positive (eigen values are non-negetive)**

iii) $Tr[\rho] = 1$

Observable $\Rightarrow$ **Self adjoint operator**

A is a self adjoint operator

$$A \to a_1, a_2, \ldots, a_n \qquad |\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_n\rangle$$

$$\boxed{A|\psi_r\rangle = a_r|\psi_r\rangle}$$

$|\psi_r\rangle\langle\psi_r|$ **is a projection operator**

Spectral representation

$$A = \sum a_r |\psi_r\rangle\langle\psi_r|$$

— More about density operator —

If $\rho^2 = \rho$, then there exists a vector such that

$$\boxed{\rho = |\psi\rangle\langle\psi|}$$

$|\psi\rangle\langle\psi|$ being one dimensional projection operator.

- ρ Being a self adjoint $\rho = \Sigma p_i |\psi_i\rangle\langle\psi_i|,\ p_i \geq 0$

- $\Sigma q_j |\phi_j\rangle\langle\phi_j|$, with $q_j \geq 0$

  is a density operator for any set $\{|\phi_j\rangle\}$.

Collection of all density operators form a convex set, the extremal points being one dimensional projection operator.

Measurement rules

Initial state = ρ

Measurement of A

Possible results	Probabilities	Final State
a_1	$Tr[\rho\|\psi_1\rangle\langle\psi_1\|]$	$\|\psi_1\rangle\langle\psi_1\|$
a_2	$Tr[\rho\|\psi_2\rangle\langle\psi_2\|]$	$\|\psi_2\rangle\langle\psi_2\|$
$\vdots$	$\vdots$	$\vdots$
a_n	$Tr[\rho\|\psi_n\rangle\langle\psi_n\|]$	$\|\psi_n\rangle\langle\psi_n\|$

Dynamics

H is the Hamiltonian acting on the system.

At $t = t_1$ the state is $\rho_{t=t_1}$

then at $t = t_2$

$$\boxed{\rho_{t=t_2} = U \rho_{t=t_1} U^\dagger}$$

where

$$U = e^{-\frac{i}{\hbar}\int_{t_1}^{t_2} H\, dt}$$

Quantum mechanical description of spin

Pure State $\longrightarrow$ Normalised vector $\begin{bmatrix} a \\ b \end{bmatrix}$

a, b complex and

$$|a|^2 + |b|^2 = 1$$

Observable $\longrightarrow$ 2×2 self adjoint matrix

$$\begin{bmatrix} m & p \\ n & q \end{bmatrix} = \begin{bmatrix} m^* & n^* \\ p^* & q^* \end{bmatrix}$$

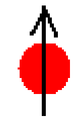
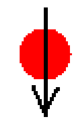
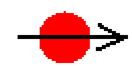
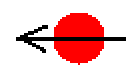
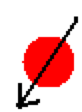
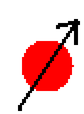
S.A. operator

	Eigen values	Eigen vector
A	a_1	$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$
	a_2	$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

- **Eigen values are real.**
- **Eigen vectors are orthogonal, When** $a_1 \neq a_2$

$$\begin{bmatrix} x_1^* & y_1^* \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

Some examples

Observable	Eigen values	Eigen vector		Symbol
$\sigma_z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$+1$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$		$\vert 0\rangle$
	-1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$		$\vert 1\rangle$
$\sigma_x \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$+1$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$		$\vert 0_x\rangle = \frac{1}{\sqrt{2}}(\vert 0\rangle + \vert 1\rangle)$
	-1	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$		$\vert 1_x\rangle = \frac{1}{\sqrt{2}}(\vert 0\rangle - \vert 1\rangle)$
$\sigma_y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$+1$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$		
	-1	$\frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ i \end{bmatrix}$		

Measurement rules

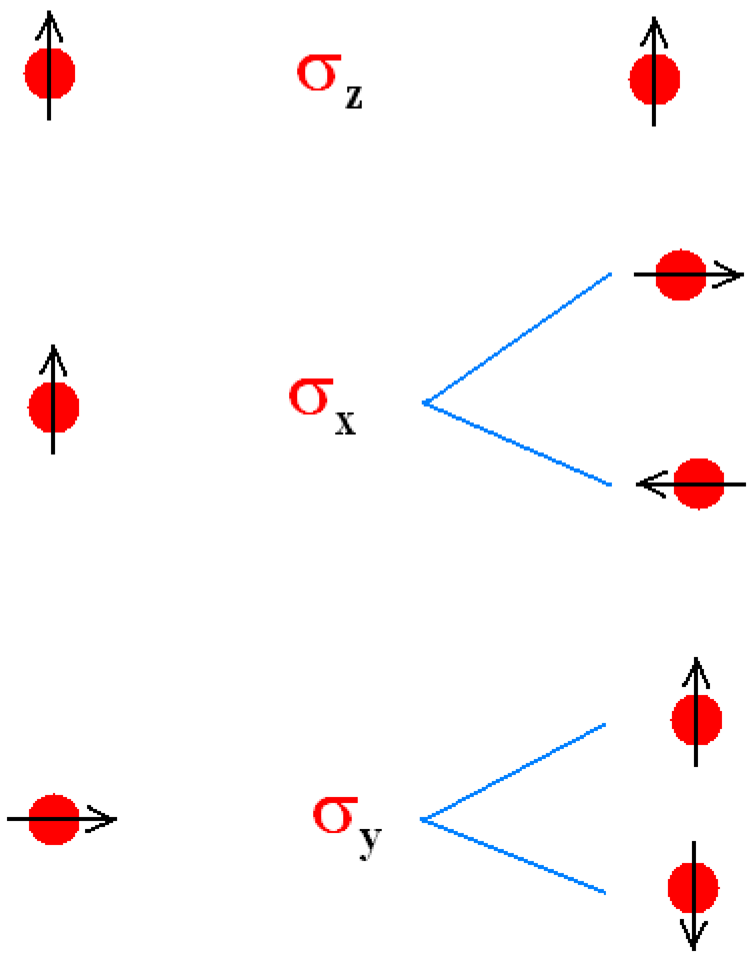
$$\text{Initial state} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Measurement of A

Possible results	Probabilities	Final State
a_1	$\left\| \begin{bmatrix} x^* & y^* \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \right\|^2$	$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$
a_2	$\left\| \begin{bmatrix} x^* & y^* \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \right\|^2$	$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

Uncertainty relation

State **Measure** **Result** **Probability**



$$1$$

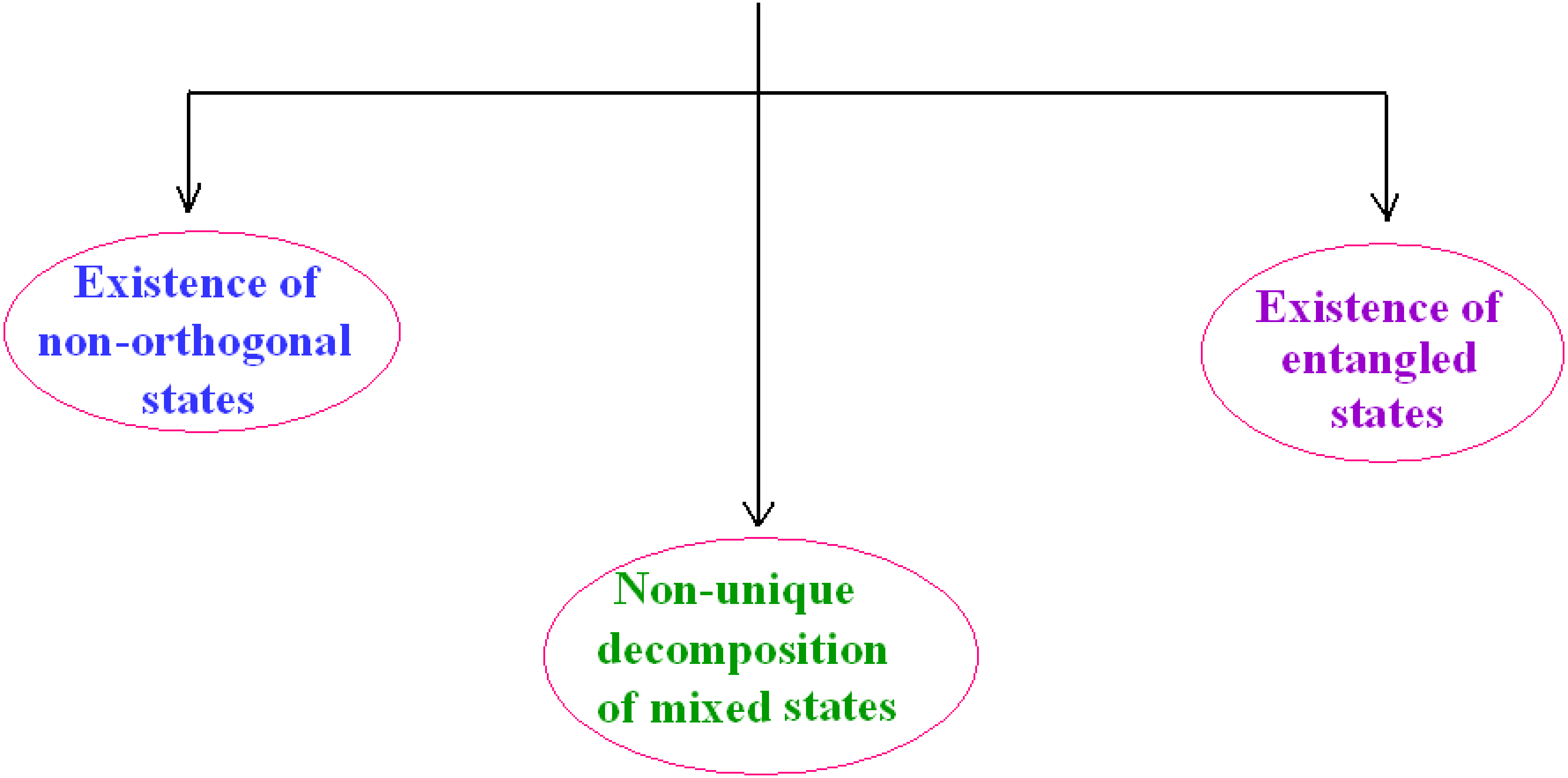
$$\frac{1}{2} = \left| \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right|^2$$

$$\frac{1}{2} = \left| \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right|^2$$

$$\frac{1}{2}$$

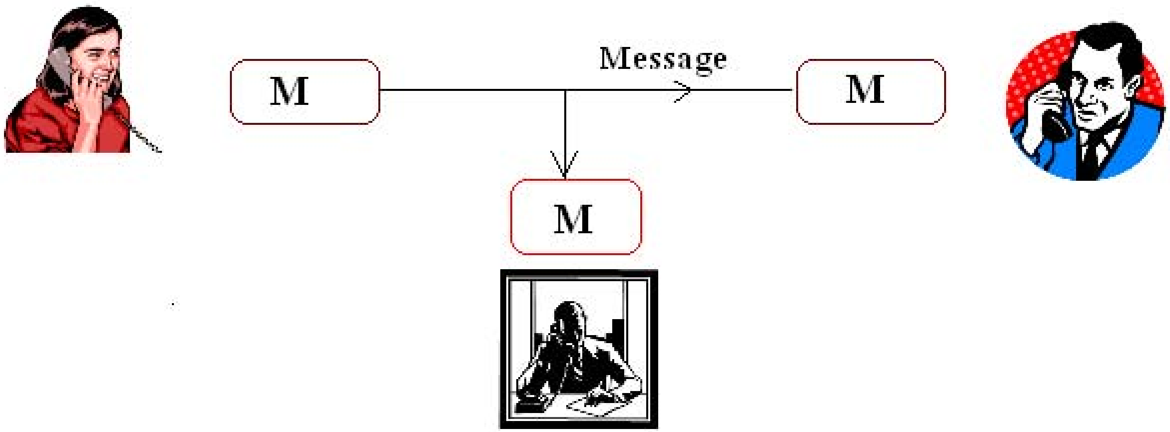
$$\frac{1}{2}$$

Three outstanding features of quantum mechanics

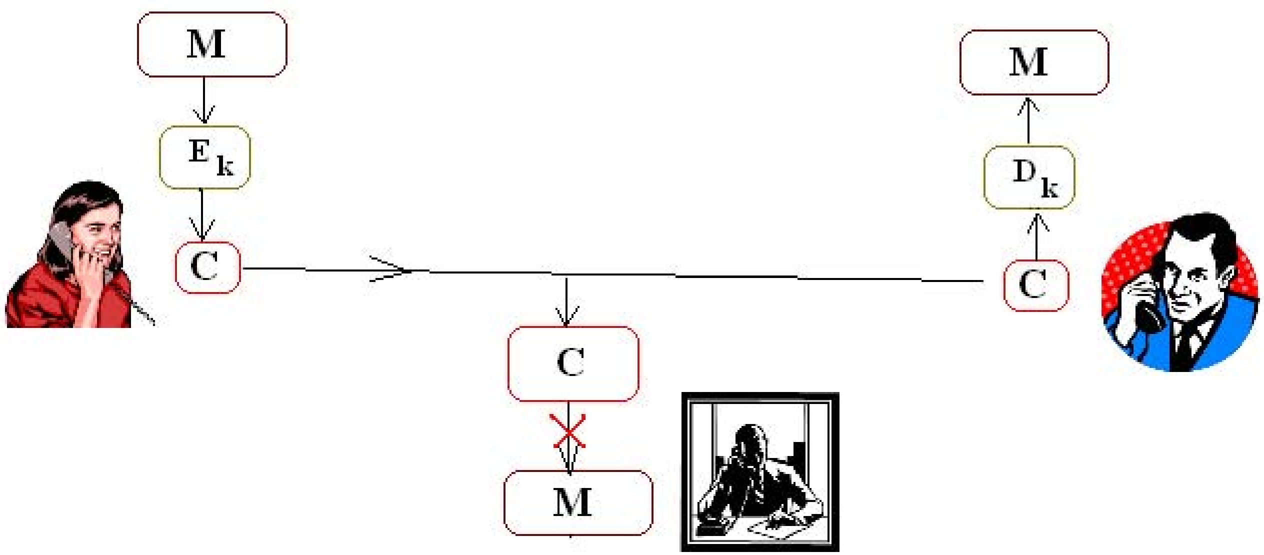


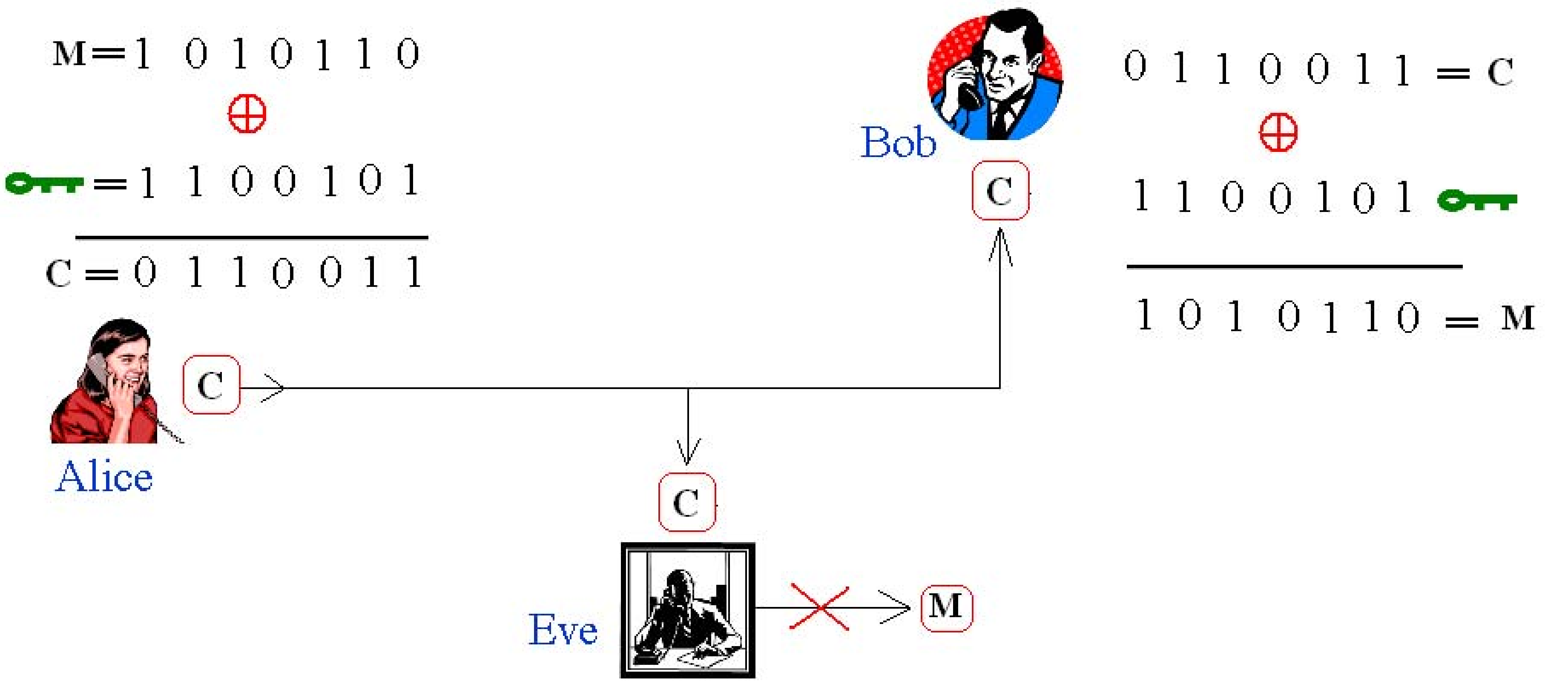
Quantum key generation

Prblem of secrecy



Art of cryptography





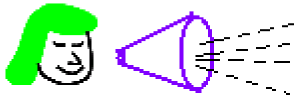
- **How to generate the key when Alice and Bob are far apart.**
- **Classical laws provide no solution.**
- **Quantum laws provide a secure protocol.**

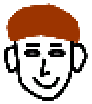
Alice selects qubits randomly and sends them to Bob one by one.

Bob randomly selects one of the measurements

σ_z σ_x

and records the basis and results.

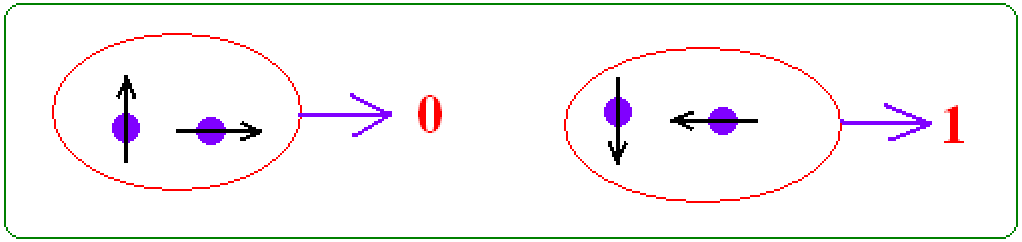




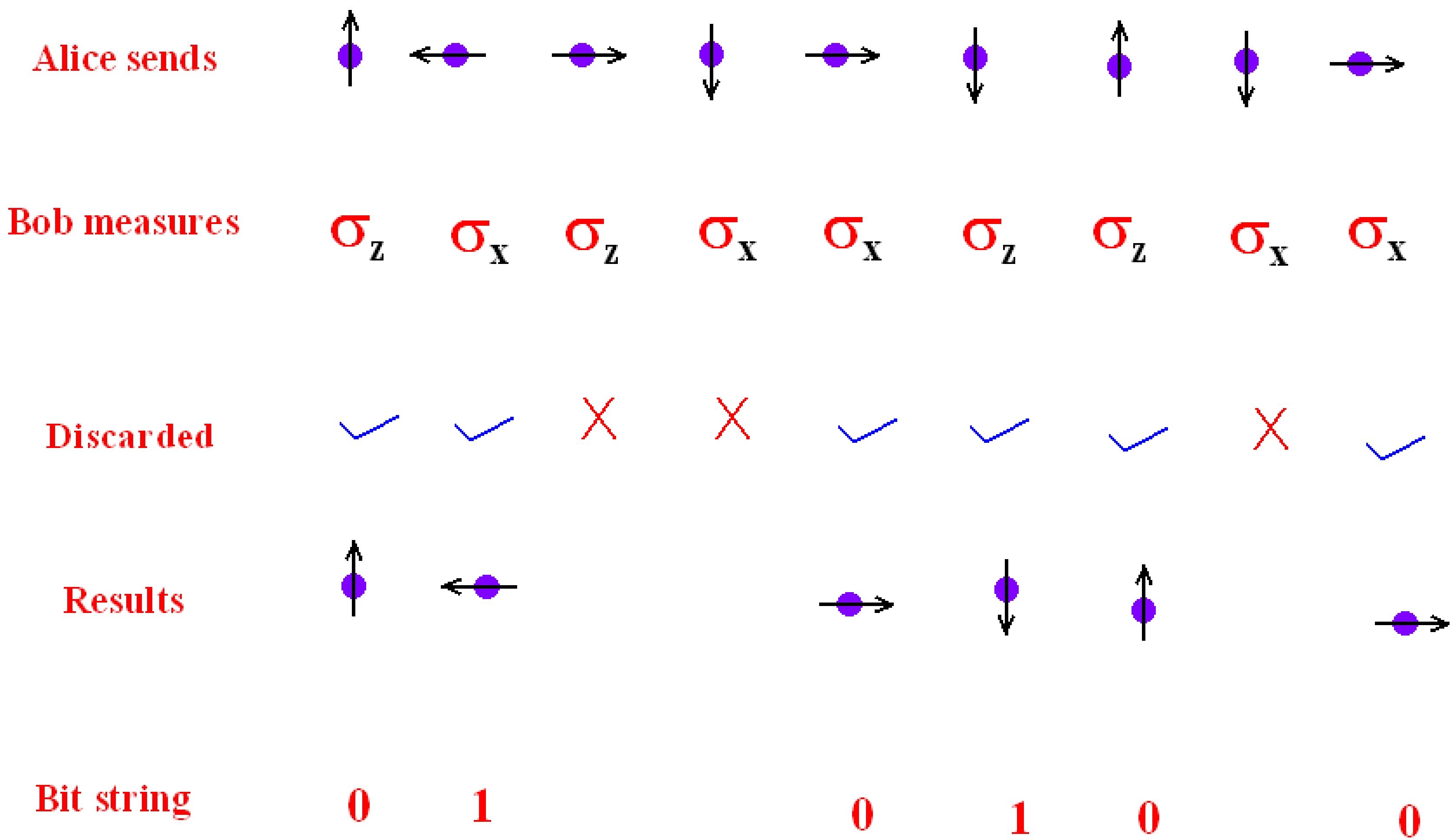
Alice announces the basis but
not the polarization (up or down)

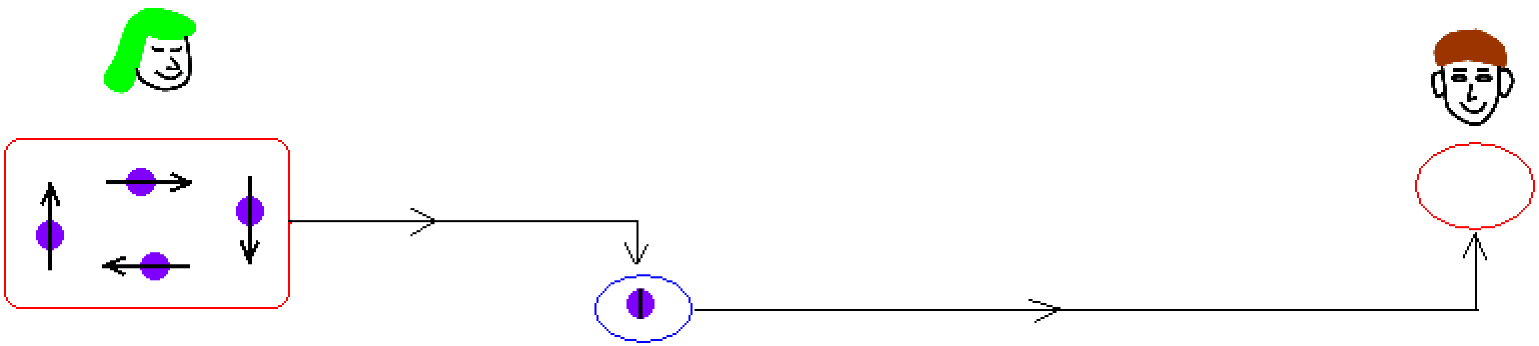
Bob discards the cases when
the basis do not match

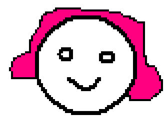
For the rest they assign bit according to



And generate the key


Alice sends
Bob measures
σ_z σ_x σ_z σ_x σ_x σ_z σ_z σ_x σ_x
Discarded
Results
Bit string
0 1 0 1 0 0





Aim:

to learn the polarization without being deyected

	Eve	Results	Bob	Results	Eve's status
↑	σ_z	↑	σ_z	↑	Eve gains
↑	σ_x	→ (50%)	σ_z	↑ (50%)	No gain No detection
		← (50%)		↓ (50%)	Eve detected

Impossibility of bit commitment in quantum mechanics

Before the game starts, Alice has to commit one of the result.

India will win
Or
India will lose

If final results comes true, Bob has to pay
Otherwise Alice has to pay

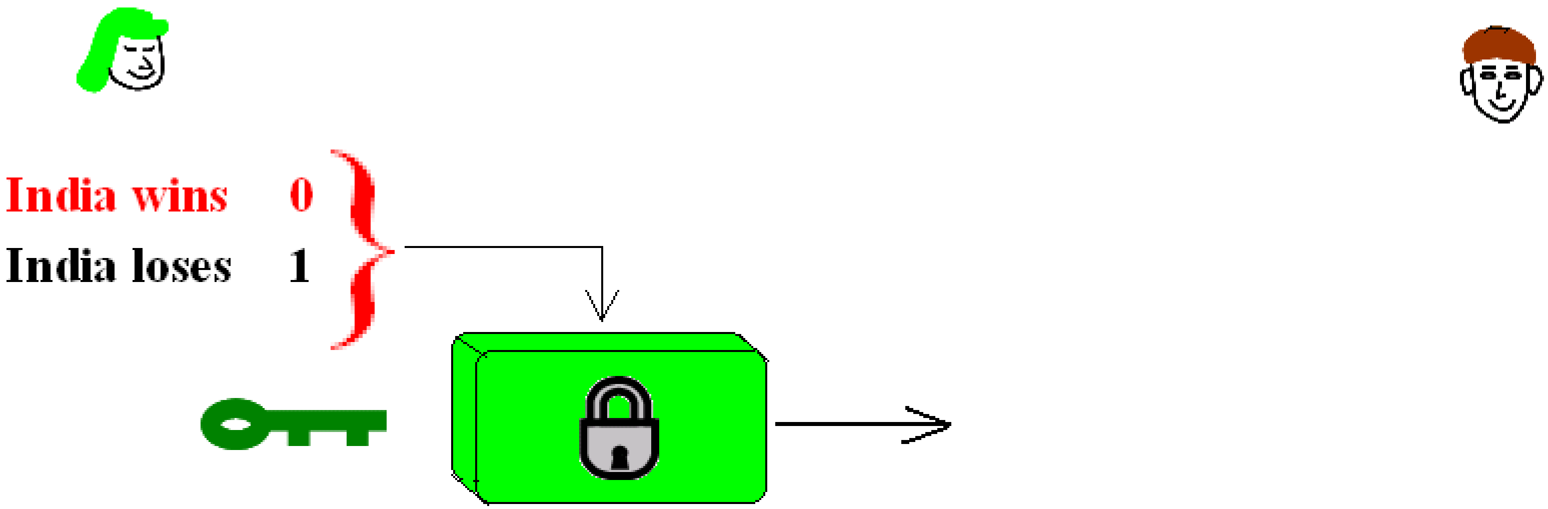
Condition :

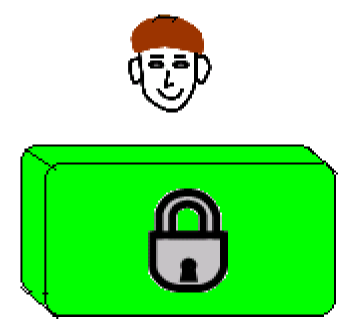
Alice would not be able to change her commitment.

Bob would not be able to learn the Alice's commitment before she reveals it after the match ends.

Can it be made possible ?

Commit phase

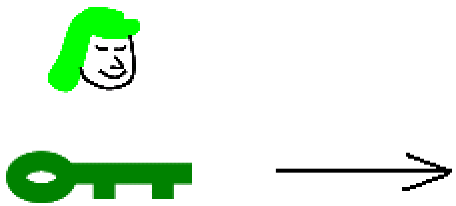


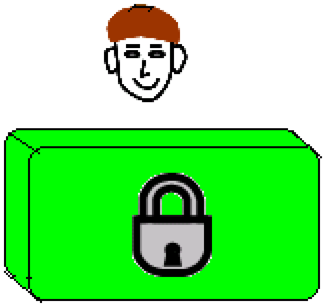


The arrangement is such that :

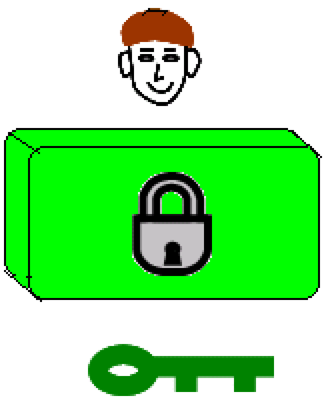
- **Alice can not change her commitment.**
- **Bob can not learn the commitment.**

Opening phase





Alice tells her bit

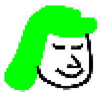


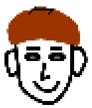
Bob checks

In classical physics there is no law to make it successful.

Why not try with quantum laws ?

Non-unique decomposition of mixed states provide an opportunity.





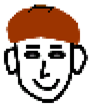
Prepare n no. of spin states

$$\Psi_y^{1} \otimes \Psi_y^{2} \otimes \Psi_y^{3} \cdots \otimes \Psi_y^{n}$$

Bit – 0 Measure σ_z

Bit - 1 Measure σ_x

Note down the result and send particles to Bob



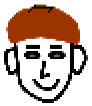
$$\tfrac{1}{2} I^{1} \otimes \tfrac{1}{2} I^{2} \otimes \tfrac{1}{2} I^{3} \cdots \otimes \tfrac{1}{2} I^{n}$$

Opening phase

Alice announces
her committment

and

spin polarization of the n
particles in the basis according
to the announced bit



Bit	measures
0	σ_z
1	σ_x

and

verifies Alice's answers

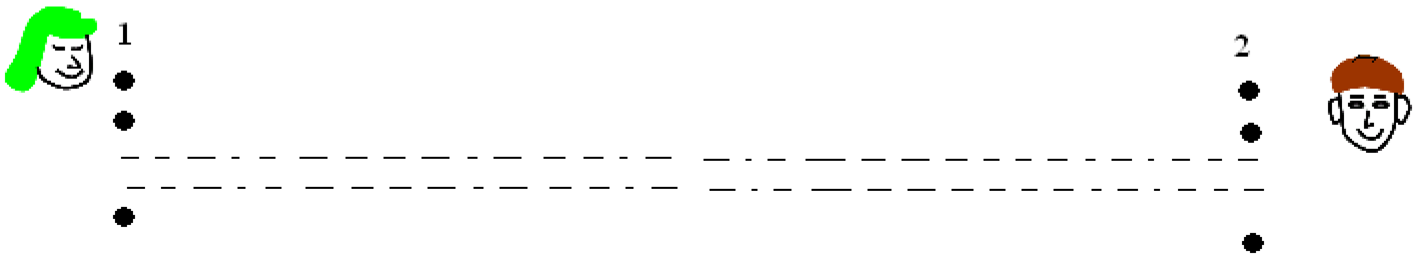
If Alice wants to cheat,
her probability of success

$$= \left(\frac{1}{2}\right)^n$$

But entanglement makes the protocol insecure.

Commit phase

Commit no bit



Alice prepares n pairs of spin particles in the state

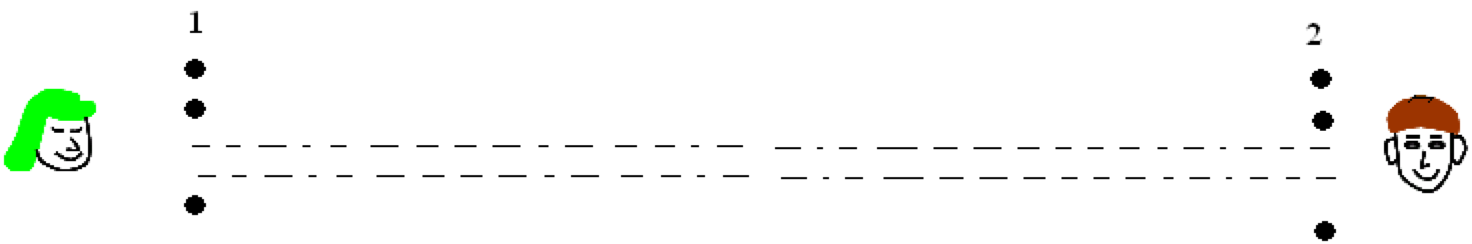
$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2\right)$$

and

sends one of each pair.

The density matris is same as before

Opening phase



Alice decides her bit

and

choses the measurement accordingly.

Due to anti-correlation of the singlet state, Bob will find no fault.

Bit	measures on 1
0	σ_z
1	σ_x

But announces opposite result.

Quantum dense coding

Two level system

Allowed state

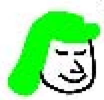
Classical

$$\left.\begin{array}{c} \bullet \\ \bullet \end{array}\right\}$$ **Two different states**

Quantum
(Two dimensional Hilbert space)

$$\left.\begin{array}{c} |0\rangle \\ |1\rangle \\ a|0\rangle + b|1\rangle \end{array}\right\}$$ **Infinitely many different states**

So one two level quantum system can be used to encode enormous classical information.



Sending one two level quantum system is enough

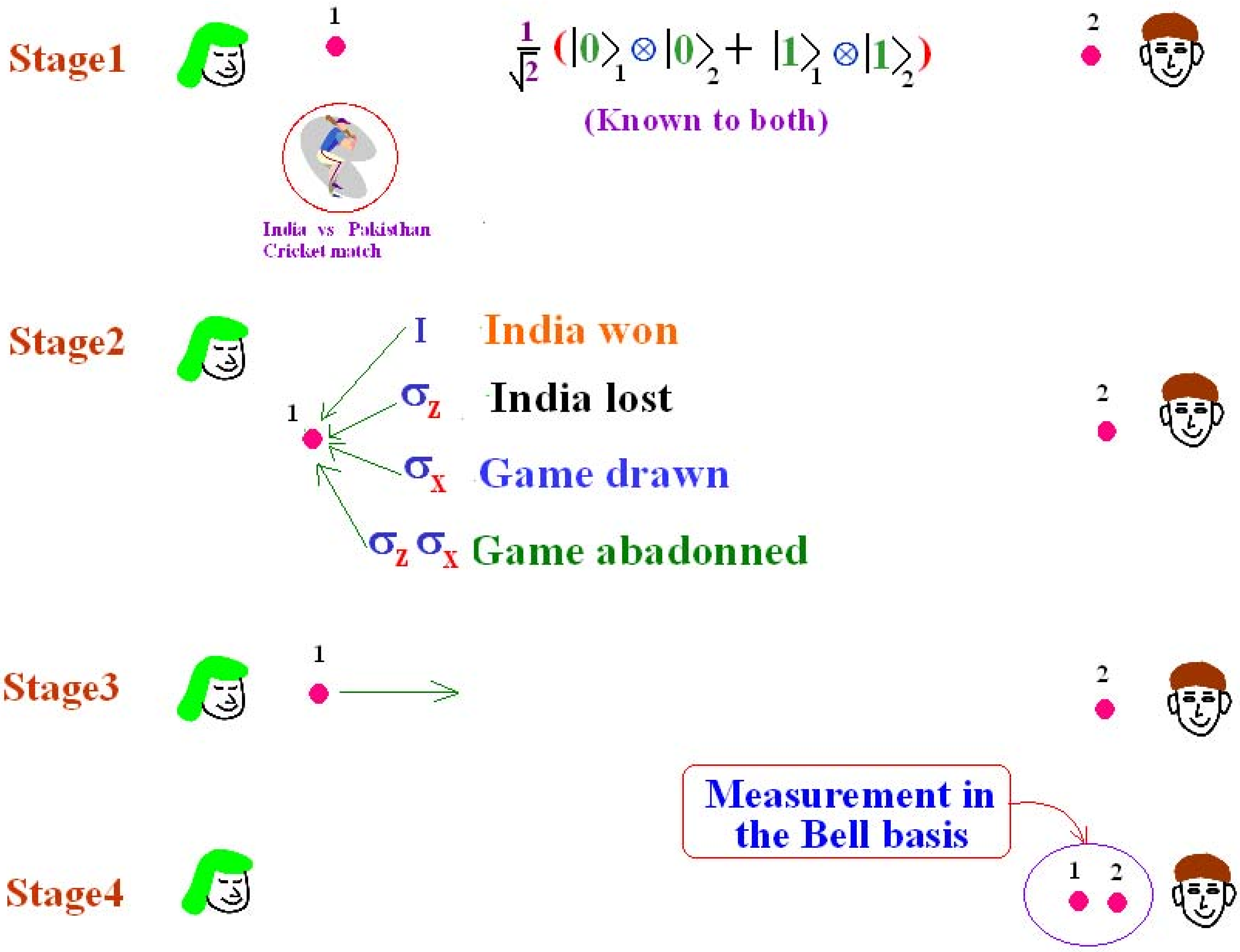
But how to distinguish the four (non-orthogonal) states

India vs Pakisthan Cricket match

	Classical encoding	quantum encoding
India won	● ●	$\|0\rangle$
India lost	● ●	$\|1\rangle$
Game drawn	● ●	$\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$
Game abadonned	● ●	$\frac{1}{\sqrt{2}}\|0\rangle - \frac{1}{\sqrt{2}}\|1\rangle$

	quantum encoding product states	quantum encoding entangled states
India won	$\vert 0\rangle_1 \otimes \vert 0\rangle_2$	$\frac{1}{\sqrt{2}}(\vert 0\rangle_1 \otimes \vert 0\rangle_2 + \vert 1\rangle_1 \otimes \vert 1\rangle_2)$
India lost	$\vert 1\rangle_1 \otimes \vert 1\rangle_2$	$\frac{1}{\sqrt{2}}(\vert 0\rangle_1 \otimes \vert 0\rangle_2 - \vert 1\rangle_1 \otimes \vert 1\rangle_2)$
Game drawn	$\vert 0\rangle_1 \otimes \vert 1\rangle_2$	$\frac{1}{\sqrt{2}}(\vert 0\rangle_1 \otimes \vert 1\rangle_2 + \vert 1\rangle_1 \otimes \vert 0\rangle_2)$
Game abadonned	$\vert 1\rangle_1 \otimes \vert 0\rangle_2$	$\frac{1}{\sqrt{2}}(\vert 0\rangle_1 \otimes \vert 1\rangle_2 - \vert 1\rangle_1 \otimes \vert 0\rangle_2)$

- So any way two two level systems are required.
- But surprisingly entanglement makes a difference.
- One system can be sent much before the game starts.


Stage1
1
$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right)$
(Known to both)
2
India vs Pakisthan
Cricket match
Stage2
1
I India won
σ_z India lost
σ_x Game drawn
$\sigma_z \sigma_x$ Game abadonned
2
Stage3
1
2
Measurement in the Bell basis
Stage4
1 2

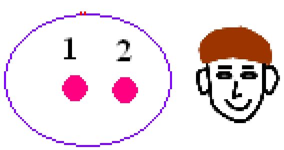
– How it works –

$$I^1 \otimes I^2 \, \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right)$$

$$\sigma_z^1 \otimes I^2 \, \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2\right)$$

$$\sigma_x^1 \otimes I^2 \, \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 + |1\rangle_1 \otimes |0\rangle_2\right)$$

$$\sigma_x^1 \sigma_z^1 \otimes I^2 \, \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2\right)$$



Measurement in the Bell basis → **Learn the Bell state** → **Learn Alice's operation** → **Learn the result of the match**

Quantum teleportation

$|\psi\rangle$

Physical transfer of particle is not allowed

$|\psi\rangle$

Preparing quantum state at distant location

State unknown

Quantum teleportation

State known

Remote state preparation

– Quantum teleportation –

Share maximally entangled state

1 2

$|\psi\rangle$

3

$$\frac{1}{\sqrt{2}}\left(|0\rangle_2 \otimes |0\rangle_3 + |1\rangle_2 \otimes |1\rangle_3\right)$$

1 2

3

Measurement in Bell basis

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 + |1\rangle_1 \otimes |0\rangle_2\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2\right)$$

$\overset{1}{\bullet}$ $\overset{2}{\bullet}$

Collapses on

Through

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right) \dashrightarrow \overset{3}{\bullet} \leftarrow I$$

or

2 bits of

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2\right) \dashrightarrow \overset{3}{\bullet} \leftarrow \sigma_z$$

or

classical

$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 + |1\rangle_1 \otimes |0\rangle_2\right) \dashrightarrow \overset{3}{\bullet} \leftarrow \sigma_x$$

or

communication

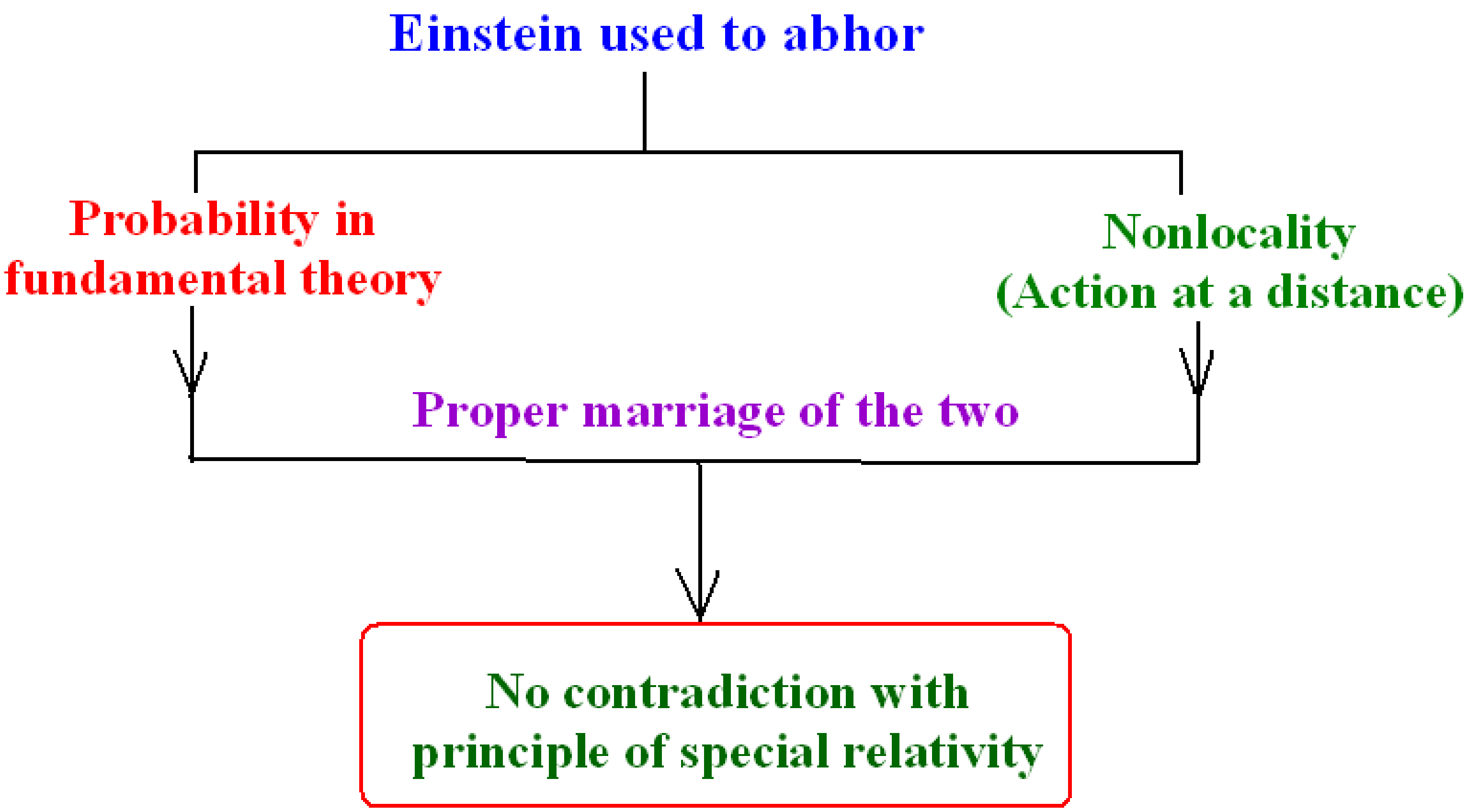
$$\frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2\right) \dashrightarrow \overset{3}{\bullet} \leftarrow \sigma_x \sigma_z$$

$\overset{1}{\bullet}$ $\overset{2}{\bullet}$

$\overset{3}{\bullet}$

$|\psi\rangle$

Nonlocality and quantum communication


Einstein used to abhor
Probability in fundamental theory
Nonlocality (Action at a distance)
Proper marriage of the two
No contradiction with principle of special relativity

$a_1 = ?$

or

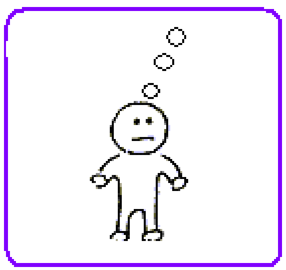
$a_2 = ?$

They are not allowed
to communicate
after the game starts.

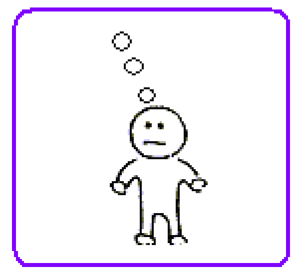
$b_1 = ?$

or

$b_2 = ?$



Alice

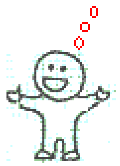


Bob

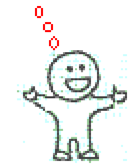
Answers can be 1 or -1

Winning condition

Alice



Bob



There answers have to satisfy

a_1	$V(a_1)\ V(b_1) = +1$	b_1
a_1	$V(a_1)\ V(b_2) = +1$	b_2
a_2	$V(a_2)\ V(b_1) = +1$	b_1
a_2	$V(a_2)\ V(b_2) = -1$	b_2

Obviously if they can win this game without communication, they can win it even if separated by space like distance.

Alice and Bob can not win this game by any strategy which decides the answers for both locally.

Question	Alices's answers	Question	Bob's answers
a_1	$V_{Alice}(a_1)$	b_1	$V_{Bob}(b_1)$
a_2	$V_{Alice}(a_2)$	b_2	$V_{Bob}(b_2)$

Now the answers have to satisfy all the winning conditions as pair of question in each turn are random.

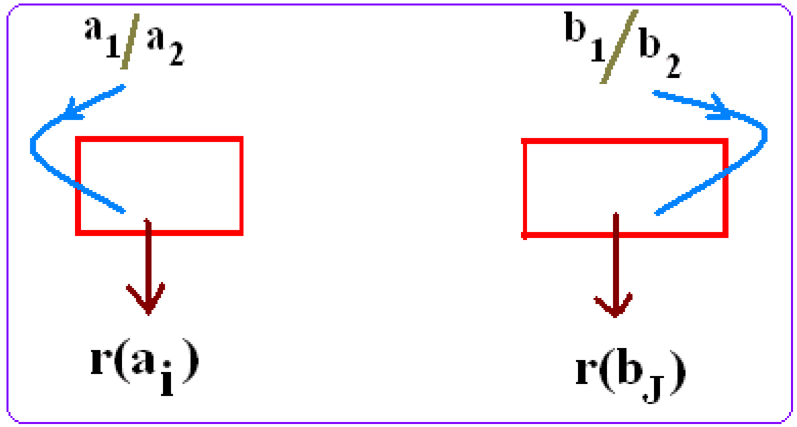
$$V_{Alice}(a_1)\,V_{Bob}(b_1) = +1$$

$$V_{Alice}(a_1)\,V_{Bob}(b_2) = +1$$

$$V_{Alice}(a_2)\,V_{Bob}(b_1) = +1$$

$$V_{Alice}(a_2)\,V_{Bob}(b_2) = -1$$

Existence of deterministic non-local correlation helping win this game would imply signaling (violation of special relativity).



Possible correlation 1

$r(a_1)$	$r(b_1)$
+1	+1
$r(a_1)$	$r(b_2)$
+1	+1
$r(a_2)$	$r(b_1)$
+1	+1
$r(a_2)$	$r(b_2)$
+1	−1

Possible correlation 2

$r(a_1)$	$r(b_1)$
−1	−1
$r(a_1)$	$r(b_2)$
−1	−1
$r(a_2)$	$r(b_1)$
−1	−1
$r(a_2)$	$r(b_2)$
+1	−1

Possible correlation which does not imply signalling

$r(a_1)$	$r(b_1)$	Probability
+1	+1	$\frac{1}{2}$
-1	-1	$\frac{1}{2}$

$r(a_1)$	$r(b_2)$	Probability
+1	+1	$\frac{1}{2}$
-1	-1	$\frac{1}{2}$

$r(a_2)$	$r(b_1)$	Probability
+1	+1	$\frac{1}{2}$
-1	-1	$\frac{1}{2}$

$r(a_2)$	$r(b_2)$	Probability
-1	+1	$\frac{1}{2}$
+1	-1	$\frac{1}{2}$

There is no physical theory which provides this kind of correlation.

A three party game

a_1 or a_2

Alice

$V(a_i)\ (=\pm 1)$

b_1 or b_2

Bob

$V(b_j)\ (=\pm 1)$

c_1 or c_2

Charl

$V(c_k)\ (=\pm 1)$

Pattern of questions

Alice	Bob	Charlie	Winning condition
a_1	b_2	c_2	$V(a_1)\,V(b_2)\,V(c_2) = +1$
a_2	b_1	c_2	$V(a_2)\,V(b_1)\,V(c_2) = +1$
a_2	b_2	c_1	$V(a_2)\,V(b_2)\,V(c_1) = +1$
a_1	b_1	c_1	$V(a_1)\,V(b_1)\,V(c_1) = -1$

One possible correlation

$r(a_1)$	$r(b_2)$	$r(c_2)$
+1	+1	+1
$r(a_2)$	$r(b_1)$	$r(c_2)$
+1	+1	+1
$r(a_2)$	$r(b_2)$	$r(c_1)$
+1	+1	+1
$r(a_1)$	$r(b_1)$	$r(c_1)$
+1	+1	-1

But again this implies signalling

Possible no-signalling correlation

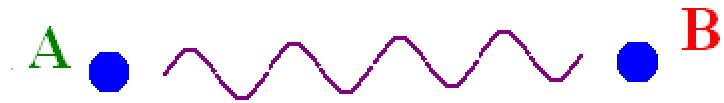
$r(a_1)$	$r(b_2)$	$r(c_2)$	Probability
+1	+1	+1	$\frac{1}{4}$
-1	-1	+1	$\frac{1}{4}$
-1	+1	-1	$\frac{1}{4}$
+1	-1	-1	$\frac{1}{4}$

$r(a_2)$	$r(b_2)$	$r(c_1)$	Probability
+1	+1	+1	$\frac{1}{4}$
-1	-1	+1	$\frac{1}{4}$
-1	+1	-1	$\frac{1}{4}$
+1	-1	-1	$\frac{1}{4}$

$r(a_2)$	$r(b_1)$	$r(c_2)$	Probability
+1	+1	+1	$\frac{1}{4}$
-1	-1	+1	$\frac{1}{4}$
-1	+1	-1	$\frac{1}{4}$
+1	-1	-1	$\frac{1}{4}$

$r(a_1)$	$r(b_1)$	$r(c_1)$	Probability
-1	+1	+1	$\frac{1}{4}$
+1	-1	+1	$\frac{1}{4}$
+1	+1	-1	$\frac{1}{4}$
-1	-1	-1	$\frac{1}{4}$

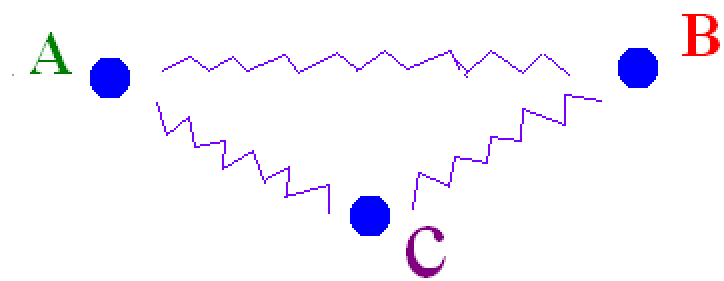
Surprisingly quantum mechanical
world provides this type of correlation



Two party state : $\begin{bmatrix}1\\0\end{bmatrix}_A \otimes \begin{bmatrix}0\\1\end{bmatrix}_B - \begin{bmatrix}0\\1\end{bmatrix}_A \otimes \begin{bmatrix}1\\0\end{bmatrix}_B$

Measure on A	Measure on B	Result	Probability
$\sigma_{\theta,\phi}$	$\sigma_{\theta,\phi}$	$\left.\begin{matrix}+1(\text{up})_A\\ -1(\text{down})_B\end{matrix}\right\}$	$\frac{1}{2}$
		$\left.\begin{matrix}+1(\text{up})_B\\ -1(\text{down})_A\end{matrix}\right\}$	$\frac{1}{2}$
$\mathbf{n}_1$	$\mathbf{n}_2$	Perfectly Anti-correlated	$\frac{1}{2}(1 + \mathbf{n}_1 \cdot \mathbf{n}_2)$

The correlation has no classical analog but still it does not help win the two party game.



Three party state:

$$V_{ABC} = \begin{bmatrix}1\\0\end{bmatrix}_A \otimes \begin{bmatrix}1\\0\end{bmatrix}_B \otimes \begin{bmatrix}1\\0\end{bmatrix}_C - \begin{bmatrix}0\\1\end{bmatrix}_A \otimes \begin{bmatrix}0\\1\end{bmatrix}_B \otimes \begin{bmatrix}0\\1\end{bmatrix}_C$$

$$\sigma_X^A \otimes \sigma_Y^B \otimes \sigma_Y^C \; V_{ABC} = +1 \; V_{ABC}$$

$$\sigma_Y^A \otimes \sigma_X^B \otimes \sigma_Y^C \; V_{ABC} = +1 \; V_{ABC}$$

$$\sigma_Y^A \otimes \sigma_Y^B \otimes \sigma_X^C \; V_{ABC} = +1 \; V_{ABC}$$

$$\sigma_X^A \otimes \sigma_X^B \otimes \sigma_X^C \; V_{ABC} = -1 \; V_{ABC}$$

$$\sigma_X^A \otimes \sigma_Y^B \otimes \sigma_Y^c \; \Psi_{ABC} = +1 \; \Psi_{ABC}$$

$$P\,(+1,+1,+1) = \left| \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}_A \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}_B \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}_c ,\; \Psi_{ABC} \right|^2 = \frac{1}{4}$$

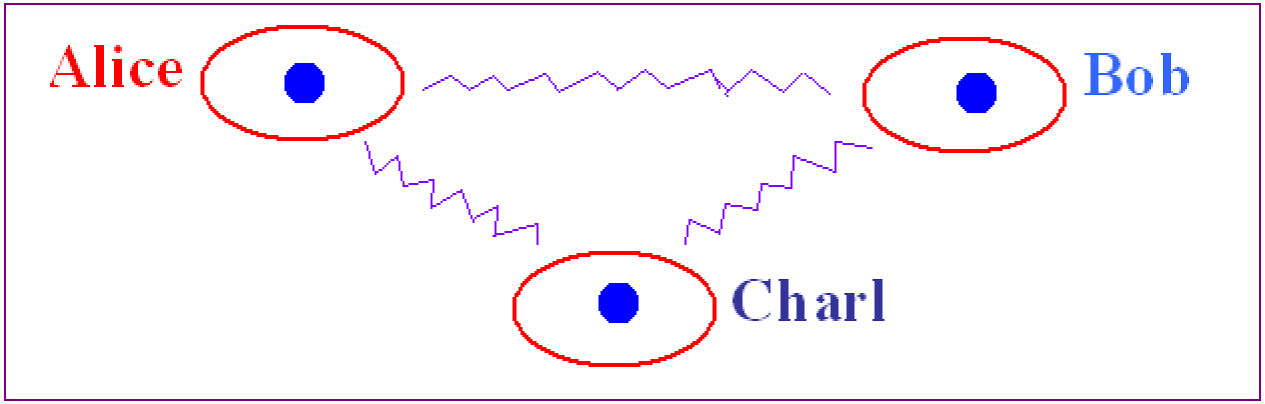
$$P\,(+1,-1,-1) = \left| \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}_A \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\i\end{bmatrix}_B \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\i\end{bmatrix}_c ,\; \Psi_{ABC} \right|^2 = \frac{1}{4}$$

$$P\,(-1,+1,-1) = \left| \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}_A \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}_B \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\i\end{bmatrix}_c ,\; \Psi_{ABC} \right|^2 = \frac{1}{4}$$

$$P\,(-1,-1,+1) = \left| \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}_A \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\i\end{bmatrix}_B \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}_c ,\; \Psi_{ABC} \right|^2 = \frac{1}{4}$$

So for this measurement set up, the product of results is always 1.

— Strategy to win the three party game —



Question	Measurement	Outcome	Answer
a_1, b_1, c_1	σ_X	+1 (up)	+1
		-1 (down)	-1
a_2, b_2, c_2	σ_Y	+1 (up)	+1
		-1 (down)	-1

Important feature :

Though they win this game, no one can learn the questions put to others. So no real information flows.

Local realistic theory $<$ quantum non-locality $<$ Signalling