OPTIMAL DISCRIMINATION OF QUANTUM OPERATIONS

Massimiliano Federico Sacchi
CNR-INFM and CNISM, Unità di Pavia
University of Pavia, ITALY
www.qubit.it

QUit quantum information theory group

cnism consorzio nazionale interuniversitario per le scienze fisiche della materia

International School and Conference on Quantum Information 2008

4th - 12th March 2008, Bhubaneswar (India)

OUTLINE

0. review of optimal discrimination protocol for two quantum states

1. minimum-error discrimination of two unitary transformations: entanglement is useless!

2. minimum-error discrimination of two quantum operations: entanglement can improve the discrimination !!

3. complete solution for two Pauli channels

4. entanglement can improve the discrimination of EBC

5. minimax discrimination

MINIMUM-ERROR DISCRIMINATION BETWEEN TWO QUANTUM STATES

ρ_1 and ρ_2 with prior probability p_1 and $p_2 = 1 - p_1$

Look for the two-valued POVM $\{\Pi_1 , \Pi_2\}$ with $\Pi_1 + \Pi_2 = I$

that minimizes the error probability

$$\boxed{p_E = p_1 \mathrm{Tr}[\rho_1 \Pi_2] + p_2 \mathrm{Tr}[\rho_2 \Pi_1]}$$

Optimal POVM: Π_1 and Π_2
are the orthogonal projectors on the support
of the positive and negative part of the operator

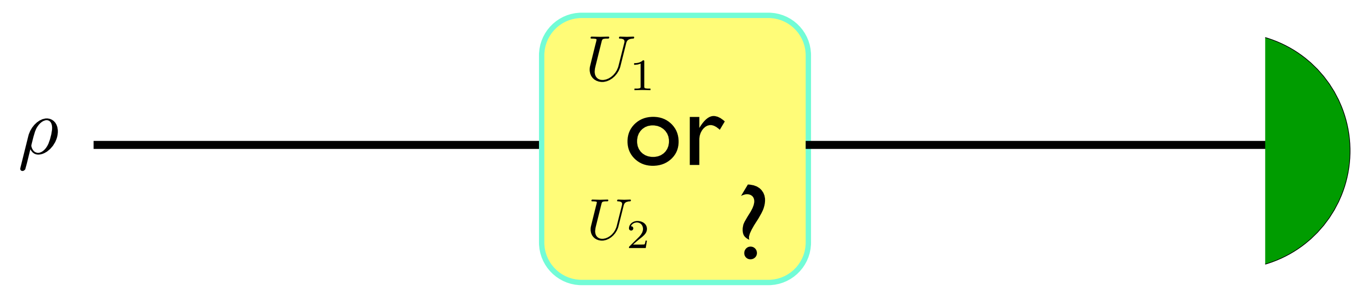
$p_1\rho_1 - p_2\rho_2$ (Helstrom, 1976)

$$\boxed{p_E = \frac{1}{2}\left(1 - \|p_1\rho_1 - p_2\rho_2\|_1\right)}$$ with $\|A\|_1 = \mathrm{Tr}\sqrt{A^\dagger A} = \max_U |\mathrm{Tr}[UA]| = \sum_i s_i(A)$

... for pure states

$$\boxed{p_E = \frac{1}{2}\left[1 - \sqrt{1 - 4p_1p_2|\langle\psi_1|\psi_2\rangle|^2}\right]}$$

MINIMUM-ERROR DISCRIMINATION OF TWO UNITARY TRANSFORMATIONS



Choose ρ to minimize the error probability

$$p_E = \frac{1}{2}\left(1 - \|p_1 U_1 \rho U_1^\dagger - p_2 U_2 \rho U_2^\dagger\|_1\right)$$

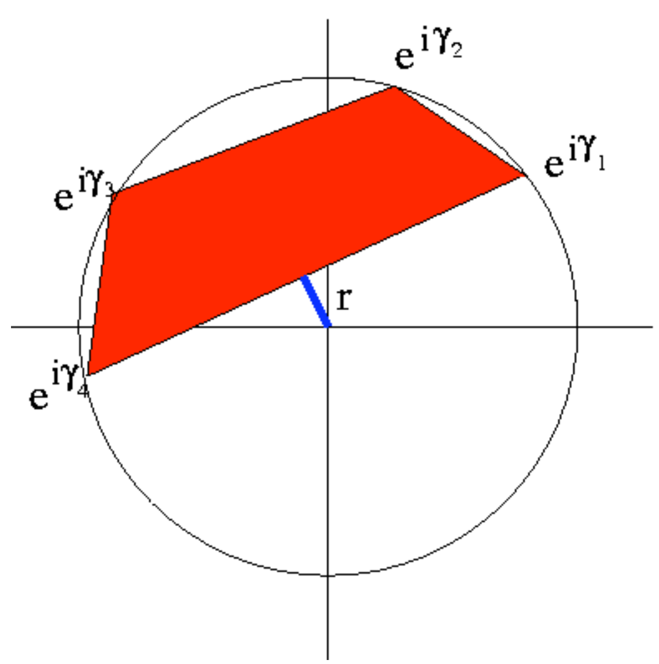
Concavity ➡ $\rho = |\psi\rangle\langle\psi|$ pure

$$p_E = \frac{1}{2}\left[1 - \sqrt{1 - 4p_1 p_2 |\langle\psi|U_2^\dagger U_1|\psi\rangle|^2}\right]$$

Diagonalize $W \equiv U_2^\dagger U_1 = \sum_j e^{i\gamma_j}|\phi_j\rangle\langle\phi_j|$ and write $|\psi\rangle = \sum_j c_j|\phi_j\rangle$

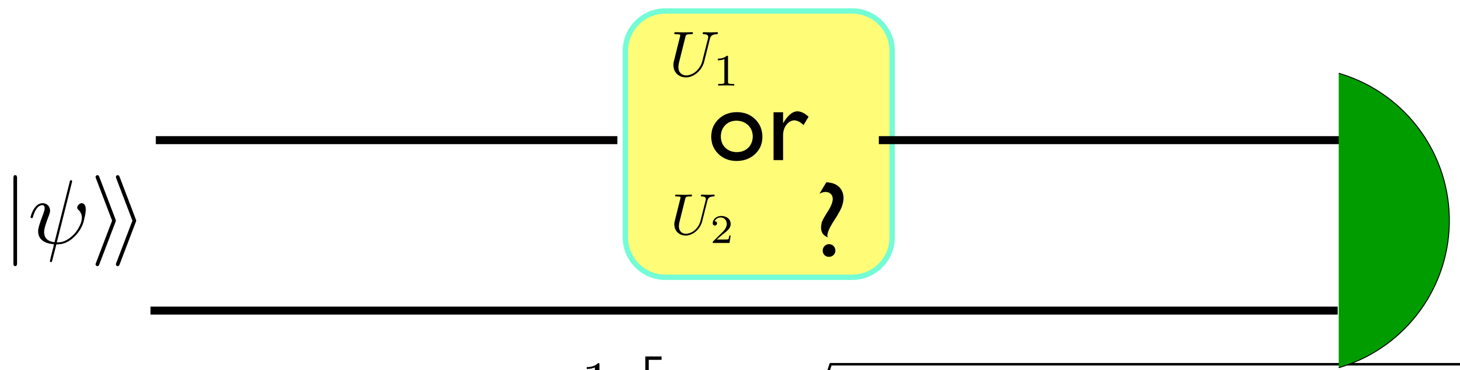
$$p_E = \frac{1}{2}\left[1 - \sqrt{1 - 4p_1 p_2 \left|\sum_j e^{i\gamma_j}|c_j|^2\right|^2}\right]$$

$$\min_{\{c_j\}} \left|\sum_j e^{i\gamma_j}|c_j|^2\right|^2 = r(U_2^\dagger U_1)^2$$



(Acín et al., D'Ariano et al., 2001)

DOES ENTANGLEMENT IMPROVE THE DISCRIMINATION ?



error probability: $$p_E = \frac{1}{2}\left[1 - \sqrt{1 - 4p_1p_2|\langle\langle\psi|U_2^\dagger U_1 \otimes I|\psi\rangle\rangle|^2}\right]$$

NO !

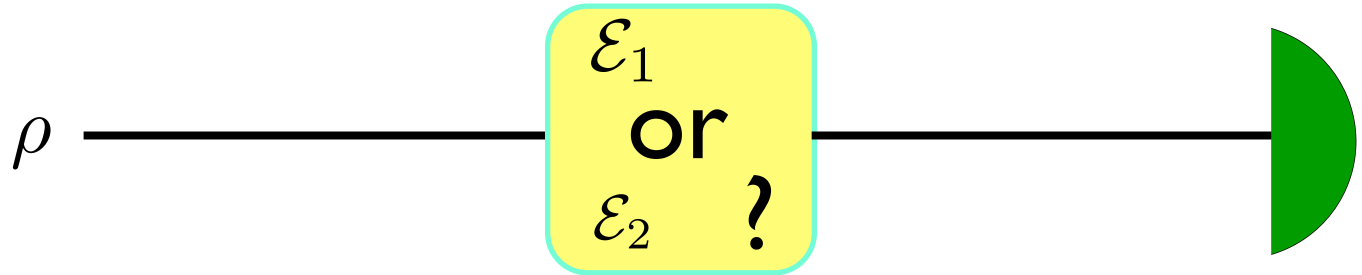
$U_2^\dagger U_1$ and $U_2^\dagger U_1 \otimes I$ have the same spectrum !

however...

entanglement can improve the discrimination among unitaries in a larger set

Example: $\{I\,, \sigma_x\,, \sigma_y\,, \sigma_z\}$ can be perfectly distinguished using a maximally entangled state and a Bell measurement

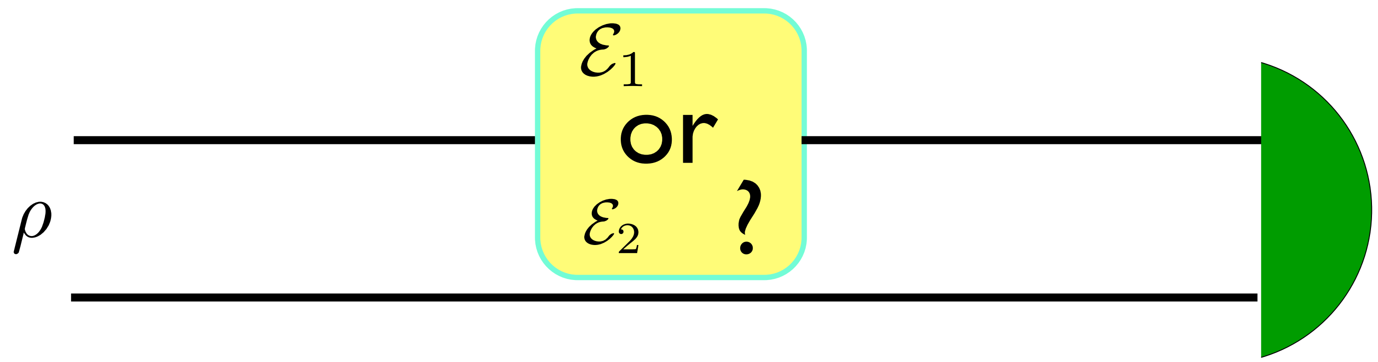
WHAT ABOUT DISCRIMINATING BETWEEN TWO QUANTUM CHANNELS ?



Choose ρ to minimize the error probability

$$p'_E = \frac{1}{2}\left(1 - \max_{\rho \in \mathcal{H}} \|p_1 \mathcal{E}_1(\rho) - p_2 \mathcal{E}_2(\rho)\|_1\right)$$

Can entanglement be useful ?



$$p_E = \frac{1}{2}\left(1 - \max_{\rho \in \mathcal{H} \otimes \mathcal{K}} \|p_1(\mathcal{E}_1 \otimes \mathcal{I})\rho - p_2(\mathcal{E}_2 \otimes \mathcal{I})\rho\|_1\right)$$

CB-norm

$\mathcal{K} = \mathcal{H}$

In both scenarios, the optimal ρ is pure

Separable states are useless

Example

$p_1 \qquad \mathcal{E}_1(\rho) = \rho \qquad\qquad \dim(\mathcal{H}) = 2$

$p_2 \qquad \mathcal{E}_2(\rho) = \frac{1}{3}(\sigma_x \rho \sigma_x + \sigma_y \rho \sigma_y + \sigma_z \rho \sigma_z) = \frac{1}{3}(2I - \rho)$

$$\begin{aligned}
&\mathcal{E}_1(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi| \qquad |\psi\rangle \in \mathcal{H} \, , \\
&\mathcal{E}_2(|\psi\rangle\langle\psi|) = \frac{1}{3}(2I - |\psi\rangle\langle\psi|) \qquad |\psi\rangle \in \mathcal{H} \, , \\
&(\mathcal{E}_1 \otimes \mathcal{I})(|\psi\rangle\langle\psi|) = |\psi\rangle\langle\psi| \qquad |\psi\rangle \in \mathcal{H} \otimes \mathcal{H} \, , \\
&(\mathcal{E}_2 \otimes \mathcal{I})(|\psi\rangle\langle\psi|) = \frac{1}{3}(2I \otimes \mathrm{Tr}_1[|\psi\rangle\langle\psi|] - |\psi\rangle\langle\psi|) \qquad |\psi\rangle \in \mathcal{H} \otimes \mathcal{H} \, ,
\end{aligned}$$

Without entanglement $\quad p'_E = \frac{1}{2}\left(1 - \left|p_1 - \frac{p_2}{3}\right| - \frac{2}{3}p_2\right) \quad \begin{cases} \frac{1}{3}(1 - p_1) & \text{for} \quad p_1 > \frac{1}{4} \\ p_1 & \text{for} \quad p_1 \leq \frac{1}{4} \end{cases}$

Using a maximally entangled state $|\phi\rangle$

$$p_E = 0$$

perfect discrimination

WHEN IS ENTANGLEMENT USEFUL ?

Isomorphism between operators on $\mathcal{H}$ and bipartite vectors on $\mathcal{H}\otimes\mathcal{H}$

$$A \longleftrightarrow |A\rangle\!\rangle \equiv \sum_{n,m}\langle n|A|m\rangle\,|n\rangle\otimes|m\rangle = A\otimes I|I\rangle\!\rangle = I\otimes A^\tau|I\rangle\!\rangle \qquad \mathrm{Tr}[A^\dagger B] = \langle\!\langle A|B\rangle\!\rangle$$

$$|A\rangle\!\rangle = |UDV\rangle\!\rangle = U\otimes V^\tau|D\rangle\!\rangle = U\otimes V^\tau\sum_{n=1}^{r} d_n|n\rangle\otimes|n\rangle \Longrightarrow \mathrm{rank}(A) = \text{Schmidt number of } |A\rangle\!\rangle$$

maximally entangled state $= \frac{1}{\sqrt{d}}|U\rangle\!\rangle$, with U unitary and $d=\dim(\mathcal{H})$

$p_1 \quad \mathcal{E}_1 \quad , \quad p_2 \quad \mathcal{E}_2$

$$\boxed{\Delta = [p_1(\mathcal{E}_1\otimes I) - p_2(\mathcal{E}_2\otimes I)]|I\rangle\!\rangle\langle\!\langle I|}$$

$$\max_{|\xi\rangle\!\rangle}\|[p_1(\mathcal{E}_1\otimes\mathcal{I}) - p_2(\mathcal{E}_2\otimes\mathcal{I})]|\xi\rangle\!\rangle\langle\!\langle\xi|\|_1 = \max_{\mathrm{Tr}[\xi^\dagger\xi]=1}\|I\otimes\xi^\tau\Delta I\otimes\xi^*\|_1$$

minimum-error probability

$$p_E = \frac{1}{2}\left(1 - \max_{\mathrm{Tr}[\xi^\dagger\xi]=1}\|I\otimes\xi^\tau\Delta I\otimes\xi^*\|_1\right) = \boxed{\frac{1}{2}\left(1 - \max_{P\geq 0\,,\,\mathrm{Tr}[P^2]=1}\|I\otimes P\Delta I\otimes P\|_1\right)}$$

No need of entanglement IFF
the maximum can be achieved by a rank-one P (MFS, 2005)

DISCRIMINATING TWO QUANTUM CHANNELS :
a relevant case

Δ diagonal on a "Bell basis"

$$\Delta = \sum_n r_n |U_n\rangle\!\rangle\langle\!\langle U_n| \qquad \mathrm{Tr}[U_m^\dagger U_n] = d\delta_{n,m}$$

$$\max_{\mathrm{Tr}[\xi^\dagger\xi]=1} \|I \otimes \xi^\tau \Delta I \otimes \xi^*\|_1 \leq \sum_n |r_n| \max_{\mathrm{Tr}[\xi^\dagger\xi]=1} \left\|U_n \otimes I|\xi\rangle\!\rangle\langle\!\langle\xi|U_n^\dagger \otimes I\right\|_1 = \sum_n |r_n|$$

Any maximally entangled state saturates the bound

example:
<u>two generalized Pauli channels</u>

$$\mathcal{E}_i(\rho) = \sum_n q_n^{(i)} U_n \rho U_n^\dagger \,, \qquad \sum_n q_n^{(i)} = 1$$

$$r_n = p_1 q_n^{(1)} - p_2 q_n^{(2)}$$

DISCRIMINATION FOR PAULI CHANNELS

$$p_1 \quad \mathcal{E}^{(1)}(\rho) = \sum_{i=0}^{3} q_i^{(1)} \sigma_i \rho \sigma_i \ , \qquad\qquad p_2 \quad \mathcal{E}^{(2)}(\rho) = \sum_{i=0}^{3} q_i^{(2)} \sigma_i \rho \sigma_i$$

$$\Delta = \sum_{i=0}^{3} r_i |\sigma_i\rangle\rangle\langle\langle\sigma_i| \qquad\qquad r_i = p_1 q_i^{(1)} - p_2 q_i^{(2)}$$

Any maximally entangled state is optimal

$$\boxed{p_E = \frac{1}{2}\left(1 - \sum_{i=0}^{3} |r_i|\right)}$$

When does entanglement strictly improve the discrimination ?

$$p'_E = \frac{1}{2}\left(1 - \max_{P'} \|I \otimes P' \, \Delta \, I \otimes P'\|_1\right) \ ;$$

$$P' = \begin{pmatrix} x & \sqrt{x(1-x)}e^{i\phi} \\ \sqrt{x(1-x)}e^{-i\phi} & 1-x \end{pmatrix} \ ,$$

$$0 \le x \le 1 \ , \qquad 0 \le \phi \le 2\pi \ .$$

$\mathrm{rank}(P') = 1$

$$p'_E = \frac{1}{2}(1 - M) \ , \qquad M = \max\{|r_0 + r_3| + |r_1 + r_2| \, , |r_0 + r_1| + |r_2 + r_3| \, , |r_0 + r_2| + |r_1 + r_3|\}$$

$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$

input an eigenstate of $\quad \sigma_z \qquad\qquad \sigma_x \qquad\qquad \sigma_y$

No need of entanglement IFF $\quad M = \sum_{i=0}^{3} |r_i| \quad$ **IFF** $\quad \det(\Delta) \ge 0$

ENTANGLEMENT CAN IMPROVE THE DISCRIMINATION OF ENTANGLEMENT-BREAKING CHANNELS

EBC IFF for any bipartite Γ $(\mathcal{E}\otimes I)\Gamma$ is separable

IFF $\mathcal{E}(\rho)=\sum_k \langle\phi_k|\rho|\phi_k\rangle|\psi_k\rangle\langle\psi_k|$ with $\sum_k |\phi_k\rangle\langle\phi_k| = I$

EBC can be simulated by a measure-and-prepare channel

Two depolarizing channels

$$\mathcal{E}_i^D(\rho) = q_i\,\rho + \frac{1-q_i}{3}\sum_{\alpha=1}^{3}\sigma_\alpha\,\rho\,\sigma_\alpha\ , \qquad q_1 \neq q_2\ , \qquad p_1 = p \text{ and } p_2 = 1-p$$

Entanglement improves the discrimination IFF $\Pi_{\alpha=0}^{3}\, r_\alpha < 0$

$$r_0 = p\,q_1 - (1-p)\,q_2\ ,$$
$$r_1 = r_2 = r_3 = p\,\frac{1-q_1}{3} - (1-p)\,\frac{1-q_2}{3}\ .$$

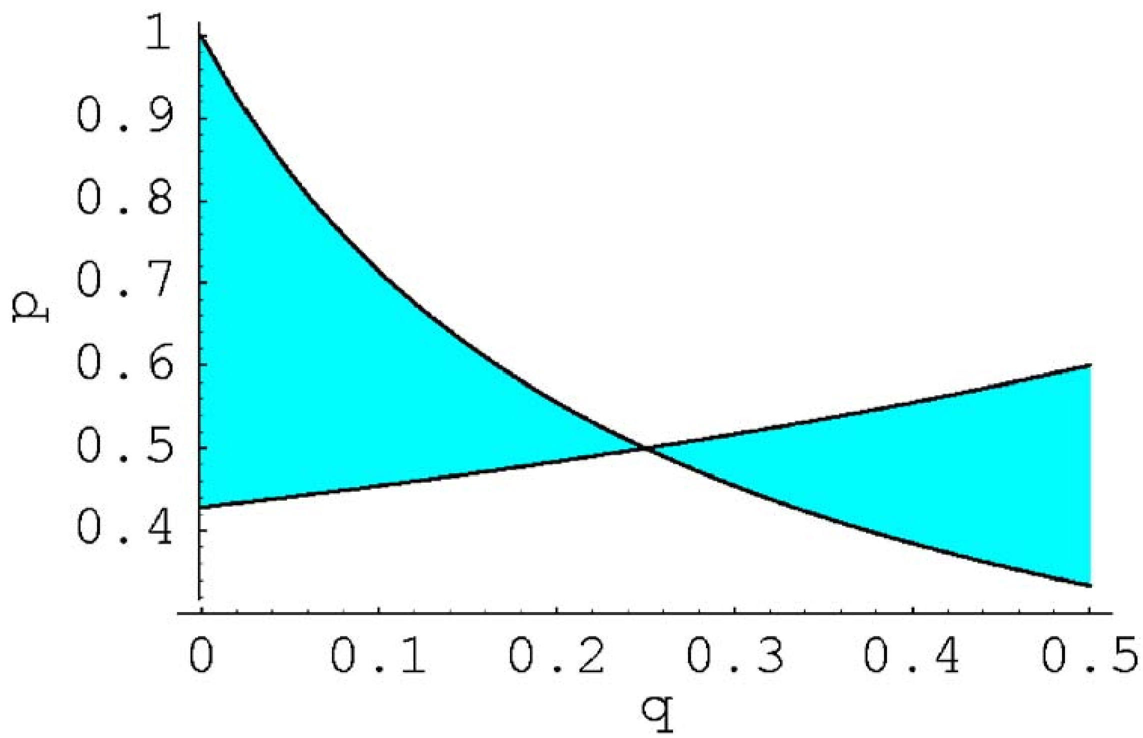
$$\frac{1-q_2}{2-q_1-q_2} < p < \frac{q_2}{q_1+q_2} \qquad \text{for} \quad q_1 < q_2\ ,$$
$$\frac{q_2}{q_1+q_2} < p < \frac{1-q_2}{2-q_1-q_2} \qquad \text{for} \quad q_1 > q_2\ .$$

For $q_1, q_2 \leq 1/2$ both channels are EBC p prior probability of a depolarizing channel with $q \leqslant 1/2$

(1-p) prior probability of a completely depolarizing channel

Both channels are EBC

(MFS, 2005)



In the blue region a maximally entangled state <u>strictly</u> improves the distinguishability

MINIMAX DISCRIMINATION BETWEEN TWO QUANTUM STATES

Two quantum states ρ_1 and ρ_2

No a priori probabilities

Look for the two-valued POVM $\{\Pi_1, \Pi_2\}$ that minimizes the largest of the probabilities of misidentification

$$\boxed{R_M(\rho_1,\rho_2) = \min_{\{\Pi_1,\Pi_2\}} \max(\mathrm{Tr}[\rho_1\Pi_2], \mathrm{Tr}[\rho_2\Pi_1])}$$

"minimum risk"

equiv. to maximize the smallest of the probabilities of correct detection $\max_{\{\Pi_1,\Pi_2\}} \min(\mathrm{Tr}[\rho_1\Pi_1], \mathrm{Tr}[\rho_2\Pi_2])$

Relation bewteen MINIMAX and MINIMUM-ERROR strategies

(D'Ariano, MFS, Kahn, 2005)

"Bayes risk" $$R_B(p) = \frac{1}{2}\left(1 - \|p\rho_1 - (1-p)\rho_2\|_1\right)$$

Thm 1: There is a measurement $\{\Pi_1, \Pi_2\}$ that is optimal in the Bayes scheme for some a priori probability $(p_*, 1-p_*)$ such that $\boxed{\mathrm{Tr}[\rho_1\Pi_1] = \mathrm{Tr}[\rho_2\Pi_2]}$.

This measurement is optimal in the minimax scheme as well, and one has $R_M(\rho_1, \rho_2) = R_B(p_*) = \mathrm{Tr}[\rho_1\Pi_2]$.

Thm 2: The minimum risk is given by $\boxed{R_M(\rho_1, \rho_2) = \max_p R_B(p)}$, and the a priori probability achieving the maximum is $p = p_*$ of Thm 1

Optimal minimax measurement given by a non-orthogonal POVM

Consider $\rho_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \rho_2 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}.$

$\mathcal{R}_B(p)$ is maximal for $p = \dfrac{1}{3}$

Imposing $\mathrm{Tr}[\rho_1 \Pi_1] = \mathrm{Tr}[\rho_2 \Pi_2] = \dfrac{1}{3}$

The optimal minimax POVM is unique and non-orthogonal $P_1 = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}.$

Conclusions & open problems

- The problems of discriminating between two unitary transformations and between two quantum operations have quite different solutions
- Entanglement can improve the discrimination
- Even if both QO's are entanglement-breaking
- Multiple copies of QO: serial, parallel, mixed schemes
- Suitable distance measure is still lacking
- Unambigous discrimination
- LOCC discrimination
- Minimax discrimination

ERASABLE AND UNERASABLE CORRELATIONS

Massimiliano Federico Sacchi
CNR-INFM and CNISM, Unità di Pavia
University of Pavia, ITALY
www.qubit.it

QUit quantum information theory group

cnism consorzio nazionale interuniversitario per le scienze fisiche della materia

Joint work with
G.M. D'Ariano, R. Demkowicz-Dobrzanski, and P. Perinotti

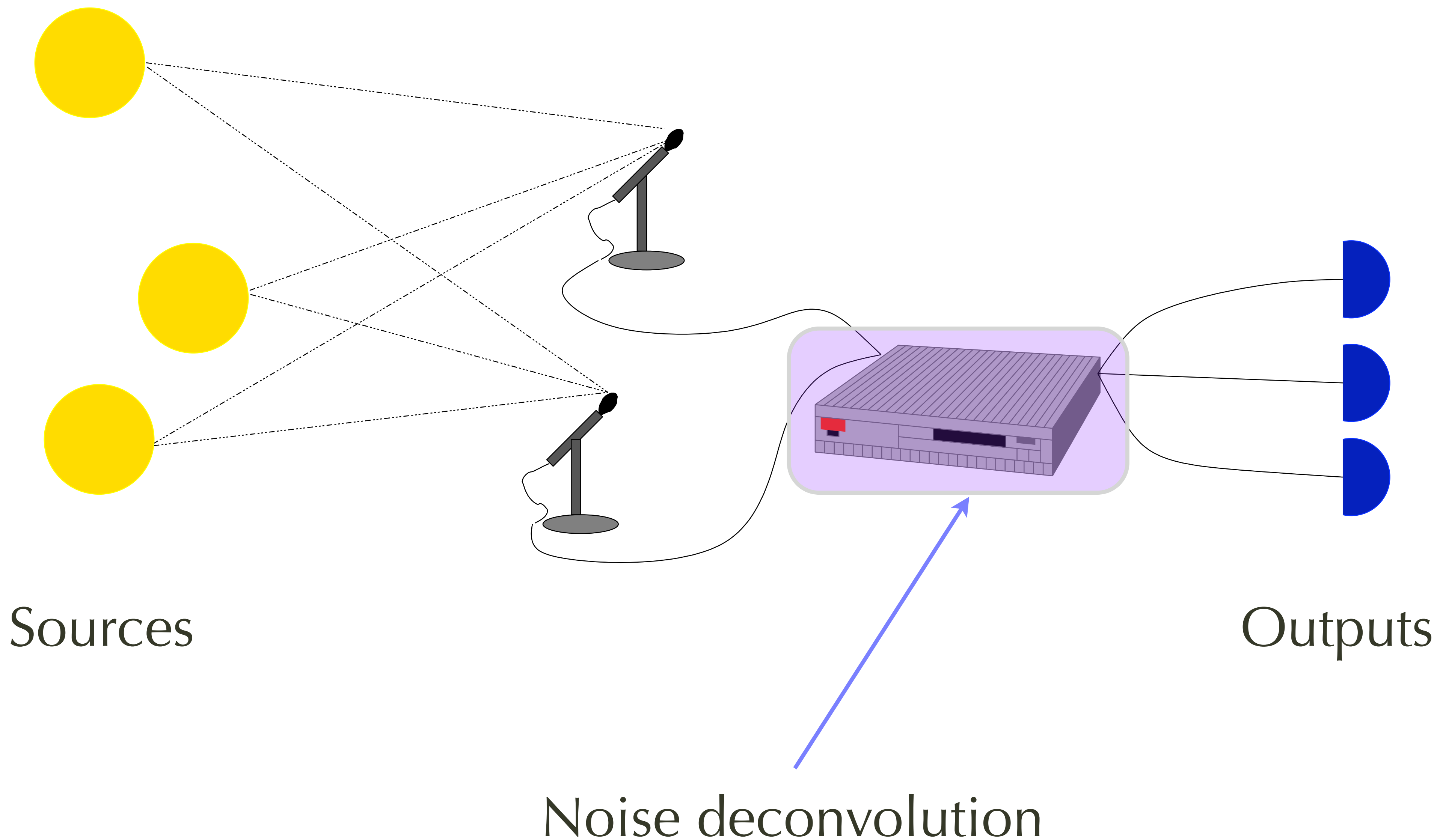
International School and Conference on Quantum Information 2008

4th - 12th March 2008, Bhubaneswar (India)

Intro

- No-go theorems → better understanding of Q.M.
- What about correlations ?
- Quantum/classical
- Beneficial/detrimental for specific tasks
- Correlation of optimal clones are the worst for state estimation (Demkowicz-Dobrzanzki, PRA 2005)
- Features of correlations between clones ?
- Cloning without correlations ?
- Can we erase correlations ? Qudits vs Continuous Variables
- Quantum version of the classical cocktail-party problem ?

Cocktail-party problem



Demixing by Independent Component Analysis:
the p.d. of the sum of independent random variables is "more Gaussian" than the p.d. of the independent random variables themselves

Quantum cocktail-party problem

courtesy by Tomasz Szkodziński

$$|\psi(t)\rangle_{ABE} = V(U_A(t) \otimes U_B(t) \otimes I)|0\rangle \otimes |0\rangle \otimes |E\rangle.$$

V, U_A, U_B unknown

quite hard...

Determine the SIGNALS U_A, U_B

Faithful decorrelation

N-partite quantum state $\rho \in S$

$$\boxed{\mathcal{D}(\rho) = \rho_1 \otimes \ldots \otimes \rho_N}$$

ρ_i is the ith party reduced density matrix of ρ

IMPOSSIBLE (nonlinear) if S = all density matrices

Terno, PRA 1999

IMPOSSIBLE if S contains ρ', ρ''
and a convex combination of them, and the reduced states of ρ', ρ'' are different at least for two parties

DDPS, PRL 2007

What about approximate decorrelation ?

Decorrelation for covariant set of states

§ N-partite "seed" quantum state ρ

§ Encode information via a UIR of a group G

$$\rho_{\boldsymbol{g}} := U_{g_1} \otimes \ldots \otimes U_{g_N} \rho U_{g_1}^{\dagger} \otimes \ldots \otimes U_{g_N}^{\dagger}$$

§ Look for a decorrelating map that maximizes the averaged single-site fidelity

$$\overline{F}[\rho, \mathscr{D}] = \frac{1}{N} \sum_{i=1}^{N} \int_{G^N} d\boldsymbol{g}\; F(U_{g_i} \mathrm{Tr}_{\bar{i}}[\rho] U_{g_i}^{\dagger}, \mathrm{Tr}_{\bar{i}}[\mathscr{D}(U_{\boldsymbol{g}} \rho U_{\boldsymbol{g}}^{\dagger})]),$$

§ w.l.o.g. look for covariant map $\boxed{\mathscr{D}\left(U_{\boldsymbol{g}} \rho U_{\boldsymbol{g}}^{\dagger}\right) = U_{\boldsymbol{g}} \mathscr{D}\left(\rho\right) U_{\boldsymbol{g}}^{\dagger} \quad \forall \rho.}$

that decorrelates the seed state $\mathscr{D}(\rho) = \tilde{\rho}_1 \otimes \ldots \tilde{\rho}_N$

§ Thanks to covariance, correlations in <u>all states</u> of the orbit will be erased

1. Two qubits with different signals

§ Permutation invariant seed state => ρ_{AB} block diagonal form w.r.t. singlet-triplet subspaces

$$\rho_{AB}(\alpha,\beta) = U(\alpha)\otimes U(\beta)\rho_{AB}U(\alpha)^\dagger \otimes U(\beta)^\dagger$$

§ Local states:

$$\rho_A(\alpha) = \mathrm{Tr}_B[\rho_{AB}(\alpha,\beta)] = \tfrac{1}{2}[\mathbb{1} + \eta \boldsymbol{n}_A(\alpha)\cdot\boldsymbol{\sigma}],$$
$$\rho_B(\beta) = \mathrm{Tr}_A[\rho_{AB}(\alpha,\beta)] = \tfrac{1}{2}[\mathbb{1} + \eta \boldsymbol{n}_B(\beta)\cdot\boldsymbol{\sigma}],$$

§ Group and permutational covariant maps:

$$\mathcal{D}(\rho_{AB}) = a\rho_{AB} + b\mathcal{D}_1(\rho_{AB}) + c\mathcal{D}_2(\rho_{AB}),$$

where
$$\mathcal{D}_1(\rho_{AB}) = \tfrac{1}{3}(\rho_A\otimes\mathbb{1} + \mathbb{1}\otimes\rho_B - \rho_{AB}),$$
$$\mathcal{D}_2(\rho_{AB}) = \tfrac{1}{9}(4\mathbb{1}\otimes\mathbb{1} - 2\rho_A\otimes\mathbb{1} - 2\mathbb{1}\otimes\rho_B + \rho_{AB}),$$
$$a+b+c=1$$

§ <u>Output</u> <u>states</u>:

$$\eta\to\tilde{\eta}$$

$$\tilde{\rho}_A(\alpha) = \tfrac{1}{2}[\mathbb{1} + \tilde{\eta}\boldsymbol{n}_A(\alpha)\cdot\boldsymbol{\sigma}],$$
$$\tilde{\rho}_B(\beta) = \tfrac{1}{2}[\mathbb{1} + \tilde{\eta}\boldsymbol{n}_B(\beta)\cdot\boldsymbol{\sigma}],$$

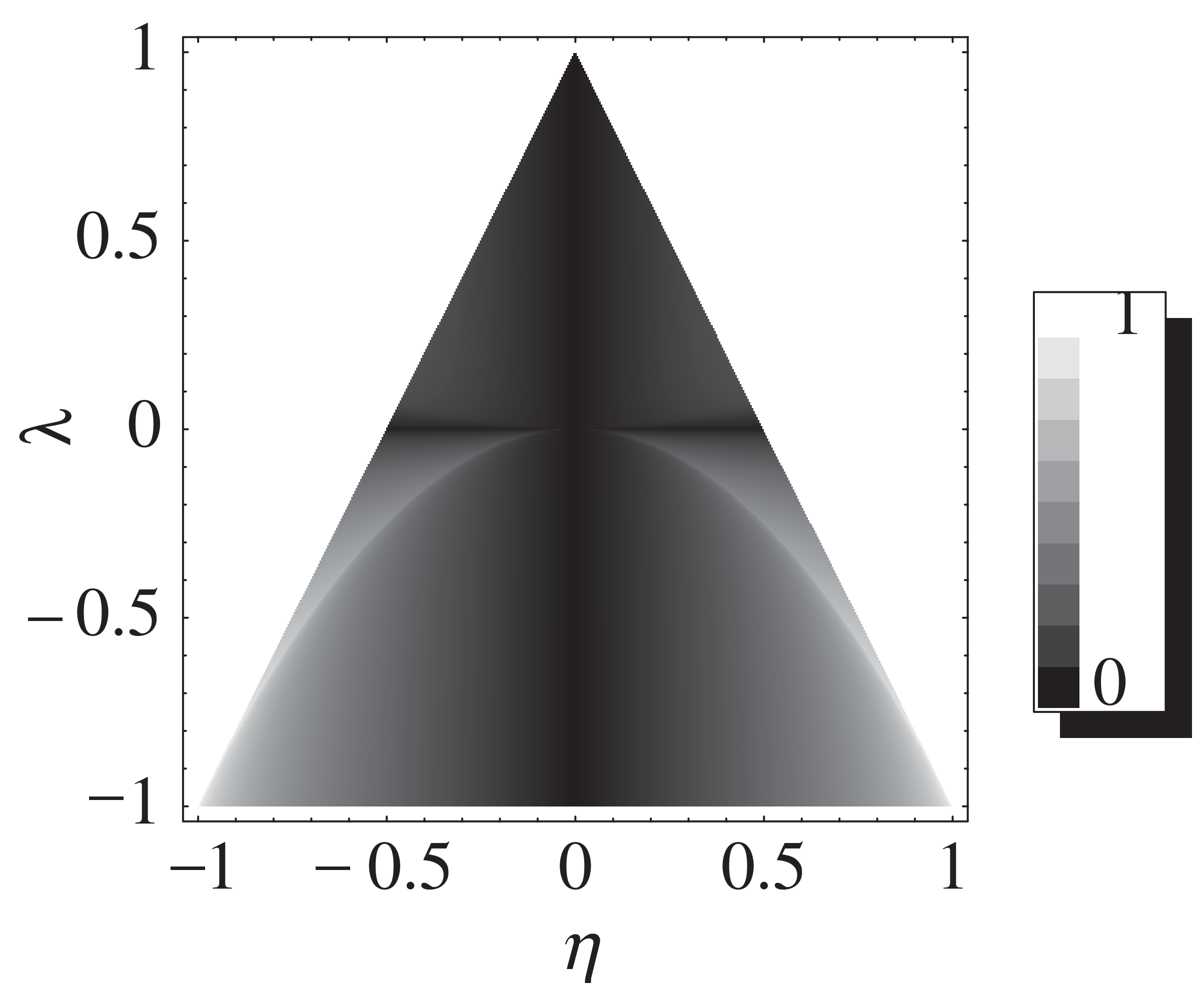
§ Imposing $\mathcal{D}(\rho_{AB}) = \tilde{\rho}^{\otimes 2} = [\frac{1}{2}(\mathbb{1} + \tilde{\eta}\sigma_z)]^{\otimes 2}$

nontrivial decorrelation ($\tilde{\eta} > 0$) is possible only when the seed state has the form

$$\rho_{AB} = \tfrac{1}{4}[\mathbb{1} \otimes \mathbb{1} + \eta(\sigma_z \otimes \mathbb{1} + \mathbb{1} \otimes \sigma_z) - \lambda\sigma_z \otimes \sigma_z].$$

§ All states are separable

§ Maximal achievable $\tilde{\eta}$ vs η and λ



II. Two qubits with identical signals

§ The decorrelation condition $\mathcal{D}(\rho_{AB}) = \tilde{\rho}^{\otimes 2}$ is nontrivially satisfied for ρ_{AB} diagonal in the singlet-triplet basis

$$\rho_{AB} = p|\Psi^-\rangle\langle\Psi^-| + (1-p)\rho_{\rm sym}$$ with

$$\rho_{\rm sym} = \tfrac{1}{4}[\mathbb{1}\otimes\mathbb{1} + \eta(\sigma_z\otimes\mathbb{1} + \mathbb{1}\otimes\sigma_z) + (1+\lambda)/2(\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y) - \lambda\sigma_z\otimes\sigma_z].$$

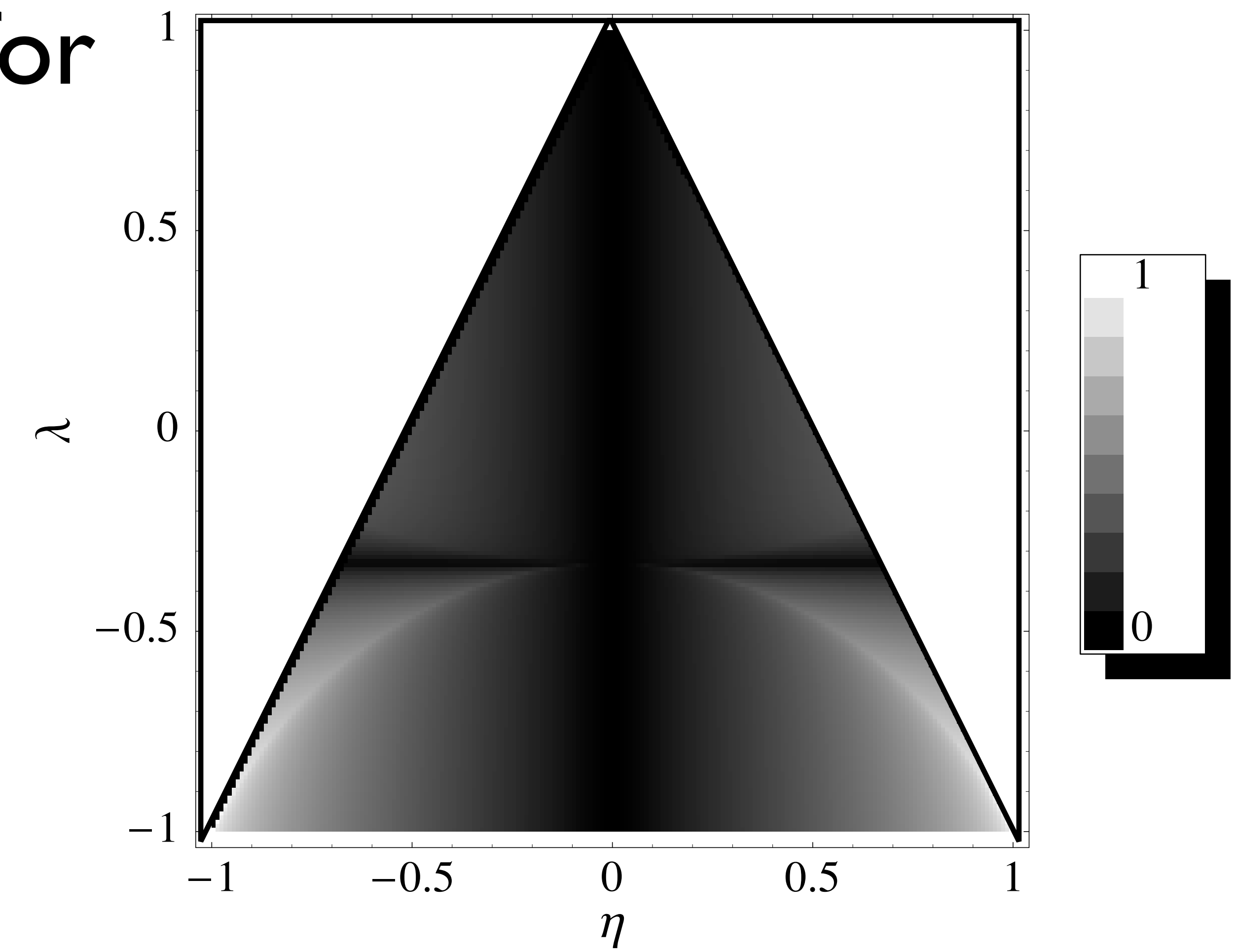
§ Correlation cannot be erased for

$p = 1$ maximally entangled

$\eta = 0$ diagonal on Bell basis

$\lambda = -1/3$

output clones of a universal cloning machine!!!



N to M universal cloning of qudits without correlations is impossible

§ w.l.o.g. M=N+1 and pure states
(use partial trace and depolarizing channels)

§ Universal covariance implies

$$\Lambda[(|\phi\rangle\langle\phi|)^{\otimes N}] = \left(\eta|\phi\rangle\langle\phi| + \frac{1-\eta}{2}\mathbb{1}\right)^{\otimes N+1}$$

§ Consider $|\phi\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$,

=> r.h.s. poly($e^{\pm i\phi}$) with degree N+1 and
l.h.s poly($e^{\pm i\phi}$) with degree at most N (for linearity)

§ Should hold for any ϕ => necessarily $\boxed{\eta = 0}$

§ The proof just uses linearity => impossible even for asymmetric and probabilistic cloning

more generally...

cloning with factorized clones is impossible for any set of pure states which contains a finite arch of states of the form

$$|\phi\rangle := \sqrt{p}|0\rangle + \sqrt{1-p}e^{i\phi}|1\rangle$$

What about discrete set of states ?

Conjecture: linearly dependent set of states cannot be cloned without correlations

Linear independent states can be probabilistically perfectly cloned via unambiguous state discrimination

III. Gaussian states

§ It is always possible to decorrelate any state in the set

$$D(\alpha) \otimes D(\beta)\rho_{AB} D(\alpha)^\dagger \otimes D(\beta)^\dagger$$

where ρ_{AB} is a two-mode Gaussian state

$$\boxed{\rho_{AB} = \frac{1}{\pi^2} \int d^4\boldsymbol{q}\, e^{-\frac{1}{2}\boldsymbol{q}^T \boldsymbol{M} \boldsymbol{q}} D(\boldsymbol{q})}$$

§ Just use a covariant Gaussian decorrelating channel

$$\mathscr{D}(\rho) = \frac{\sqrt{\det \boldsymbol{G}}}{(2\pi)^2} \int d^4\boldsymbol{x}\, e^{-\frac{1}{2}\boldsymbol{x}^T \boldsymbol{G} \boldsymbol{x}} D(\boldsymbol{x}) \rho D^\dagger(\boldsymbol{x})$$

with suitable positive matrix G.

§ $\mathscr{D}(\rho_{AB})$ will be still Gaussian, with new block-diagonal covariance matrix $\widetilde{M}$, i.e. decorrelated

§ The channel is covariant => decorrelation for any $\alpha,\ \beta$

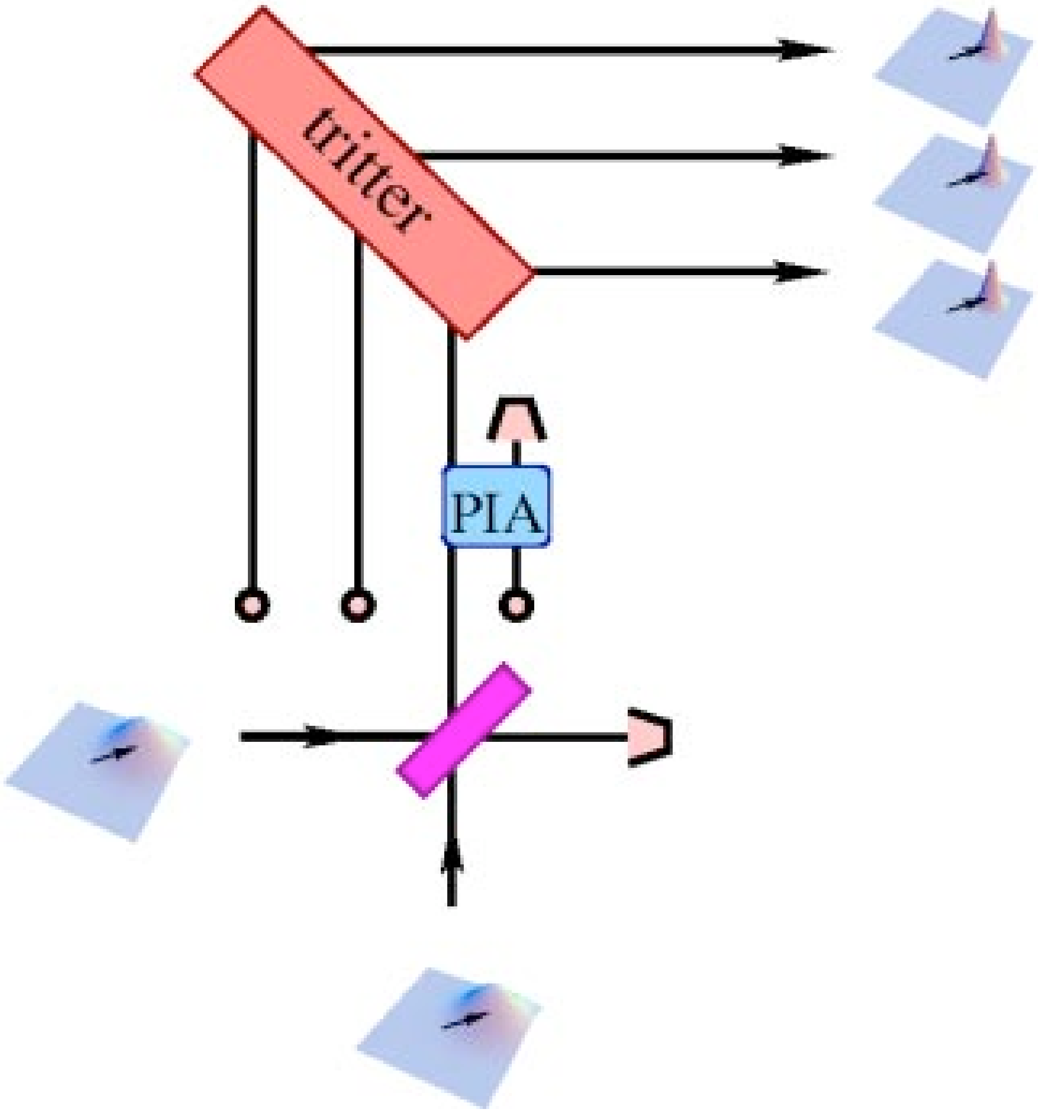
Cloning without correlations for GS

1. Use N-splitter to concentrate the signal in one mode

2. Amplify the signal by a PIA with power gain M/N

3. Distribute the amplified mode by a M-splitter with (M-1) ~~vacuum modes~~

thermal states with suitable photon number



Conclusions & open problems

- Only few states can be decorrelated if the covariance group is “large”
- <u>Any</u> joint Gaussian state can be decorrelated
- Covariant cloning without correlations:
  <u>NO for qudits,</u>   <u>YES for CV</u>
- ? Experimental set-up for covariant decorrelation for qudits
- ? Optimal decorrelators for CV
- ? Restriction to bilocal or LOCC operations
- ? No-cloning without correlations for <u>discrete set</u> of states

References

G.M. D'Ariano, R. Demkowicz-Dobrzanski, P. Perinotti, and M.F. Sacchi, PRL 2007

G.M. D'Ariano, R. Demkowicz-Dobrzanski, P. Perinotti, and M.F. Sacchi, PRA 2008

Thank you !