Teleportation, Quantum Information Splitting and Dense Coding through Cluster and Brown States

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Teleportation

- Existence of Entanglement assist in information transfer by a sender, unaware of the information to be sent, as well as the destination.
- As is well known, the Bell state for example $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ can be used for the teleportation of an unknown qubit given by : $\alpha|0\rangle + \beta|1\rangle$
- Measurement leads to state collapse
- Classical communication and Local unitary transformations, teleports this state to Bob.

- Explicit protocol is as follows,
  Suppose, Alice and Bob share an entangled Bell state.
  Alice has an unknown qubit (which she wants to teleport to Bob) with her particle as follows :

$$(|00\rangle + |11\rangle)(\alpha|0\rangle + \beta|1\rangle) + (|00\rangle - |11\rangle)(\alpha|0\rangle - \beta|1\rangle)$$
$$+(|10\rangle + |01\rangle)(\beta|0\rangle + \alpha|1\rangle) + (|10\rangle - |01\rangle)(\beta|0\rangle - \alpha|1\rangle)$$

- Alice encodes, the outcome of her measurement in classical bits and sends it to Bob.
- Bob performs an unitary operation from the set $I, \sigma_1, i\sigma_2, \sigma_3$ and obtains the unknown qubit information.
- For example $(\alpha|0\rangle + \beta|1\rangle) \xrightarrow{\sigma_1} (\beta|0\rangle + \alpha|1\rangle)$

- In general, Suppose Alice and Bob share a maximally entangled state $|\varphi\rangle_{AB}$, where A and B respectively refer to the subsystems of Alice and Bob.
- Alice wants to teleport $|\psi\rangle_a$, in her possession, to Bob. Thus, she prepares the combined state,

$$|\psi\rangle_a|\varphi\rangle_{AB} = \frac{1}{\sqrt{D}}\sum_{x=1}^{D}|\phi_x\rangle_{aA}U_x|\psi\rangle_B.$$

- Here, U_x are unitary operators on subsystem B and $|\phi_x\rangle_{aA}$ are mutually orthogonal states of the joint system.
- Certain entangled states cannot be used for teleportation eg. W state cannot be used, but the GHZ state can be used for teleporting a single qubit state.

- But the Modified W state given by :

$$\frac{1}{\sqrt{2}}(|100\rangle + |010\rangle + \sqrt{2}|001\rangle),$$

  can be used for the same purpose.
- These are Unitarily connected with GHZ states
- Note that under LOCC , the W and the GHZ states belong to two different classes
- Note that Entangled two qubit system, like the Bell pair can also be teleported .

Arbitrary two qubit state

- An arbitrary two qubit state given by ,

$$|\psi\rangle = \alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle,$$

  satisfying $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\mu|^2 = 1$, can also be teleported.

- Carried out initally using product of two Bell pairs.
- Later a genuinely entangled four qubit states were used for this purpose (For eg: Yeo-Chua state and the Cluster state)

- We illustrate the usefulness of the five qubit Brown state for this purpose, which is as follows:

$$|\psi_5\rangle = \frac{1}{2}(|001\rangle|\phi_-\rangle + |010\rangle|\psi_-\rangle + |100\rangle|\phi_+\rangle + |111\rangle|\psi_+\rangle),$$

  where, $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are Bell states.
- Found by two different Numerical search procedures
- This state exhibits genuine multi-partite entanglement according to both negative partial transpose measure, as well as von Neumann entropy measure

Entanglement properties

- There is 1 ebit of entanglement between (1234|5) and two ebits between (123|45)
- hese are the maximum possible entanglement values between the respective subsets
- It is also to be noted that $Tr(\rho^2_{i_1}) = \frac{1}{2}$ and $Tr(\rho^2_{i_1,i_2})... = \frac{1}{4}$, where i_1, i_2... refer to the subsystems respectively
  it has Multiple entropy measures (MEMS) of $S_1 = 1$ and $S_2 = 2$ respectively.
- This is more than the entanglement exhibited by the GHZ, W and the cluster states.

Entanglement properties

- Even after tracing out one/two qubits from the state, entanglement sustains in the resulting subsystem and thus, is highly ‘robust’.
- Also, the state is maximally mixed, after we trace out any possible number of qubits, which is an indication of genuine multi-particle entanglement
- the above state assumes the same form for all 10 splits as (3+2). Thus , Alice can have any pair of 3 qubits in the above state to teleport to Bob.
- This is not possible with the four-qubit states known before.

Physical realization

- We start with two photons in the Bell state given by

$$\phi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle).$$

- We need to prepare another photon in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.
- One can combine both these states and perform a $UCNOT$ operation on the last two qubits and get a W class of states as follows :

$$\frac{1}{2}(|01\rangle + |10\rangle)(|0\rangle + |1\rangle) \stackrel{UCNOT(3,2)}{\longrightarrow}$$
$$\frac{1}{2}(|100\rangle + |010\rangle + |001\rangle + |111\rangle).$$

- We now take two photons in another Bell state,

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

The Brown state can be obtained by applying an unitary transformation U_b to their combined state as follows :

$$|W\rangle\ |\psi_+\rangle \xrightarrow{U_b} |\psi_5\rangle$$

- The unitary transform U_b is given by a 32×32 matrix, which can can be further decomposed into known gates in Quantum information.

Single qubit teleportation

- Let us first consider the situation in which Alice possesses qubits $1, 2, 3, 4$ and particle 5 belongs to Bob.
- Alice wants to teleport $(\alpha|0\rangle + \beta|1\rangle)$ to Bob. So, Alice prepares the combined state,

$$(\alpha|0\rangle + \beta|1\rangle)|\psi_5\rangle = |\phi_1\rangle_{a_{1+}}(\alpha|0\rangle + \beta|1\rangle) + |\phi_2\rangle_{a_{1-}}(\alpha|0\rangle - \beta|1$$
$$+|\phi_3\rangle_{a_{2+}}(\beta|0\rangle + \alpha|1\rangle) + |\phi_4\rangle_{a_{2-}}(\beta|0\rangle - \alpha|1\rangle$$

where, the $|\phi_x\rangle_{a_i\pm}$ are mutually orthogonal states of the measurement basis.

Single qubit teleportation

- The states $|\phi_x\rangle_{a_i\pm}$ are given as,

$$\begin{aligned}|\phi_x\rangle_{a_1\pm} &= (-|00011\rangle + |00100\rangle + |01001\rangle + |01110\rangle) \\ &\quad \pm(|10010\rangle - |10101\rangle + |11000\rangle + |11111\rangle), \\ |\phi_x\rangle_{a_2\pm} &= (-|10011\rangle + |10100\rangle + |11001\rangle + |11110\rangle) \\ &\quad \pm(|00010\rangle - |00101\rangle + |01000\rangle + |01111\rangle).\end{aligned}$$

- Alice can now make a five-particle measurement using $|\phi_x\rangle_{a_i\pm}$ and convey the outcome of her measurement to Bob via two classical bits.
- Bob can apply suitable unitary operations given by $(1, \sigma_1, i\sigma_2, \sigma_3)$ to recover the original state $(\alpha|0\rangle + \beta|1\rangle)$.
- This completes the teleportation protocol for the teleportation of a single qubit state using the state $|\psi_5\rangle$.

Arbitrary two qubit teleportation

- Alice has an arbitrary two qubit state,

$$|\psi\rangle = \alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle,$$

- which she has to teleport to Bob. Here α, β, γ and μ are any set of complex numbers satisfying $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\mu|^2 = 1.$
- Qubits 1, 2, 3 and 4, 5 respectively, belong to Alice and Bob.

Arbitrary two qubit teleportation

- Alice prepares the combined state: $|\psi\rangle|\psi_5\rangle =$

$$\frac{1}{4}[|\psi_5\rangle_1(\alpha|01\rangle + \gamma|00\rangle + \mu|11\rangle + \beta|10\rangle) + |\psi_5\rangle_2(\alpha|01\rangle + \gamma|00\rangle - \mu|11\rangle - \beta|10$$

$$+|\psi_5\rangle_3(\alpha|01\rangle - \gamma|00\rangle + \mu|11\rangle - \beta|10\rangle) + |\psi_5\rangle_4(\alpha|01\rangle - \gamma|00\rangle - \mu|11\rangle + \beta|10$$

$$+|\psi_5\rangle_5(\alpha|11\rangle + \gamma|10\rangle + \mu|01\rangle + \beta|00\rangle) + |\psi_5\rangle_6(\alpha|11\rangle - \gamma|10\rangle + \mu|01\rangle - \beta|00$$

$$+|\psi_5\rangle_7(\alpha|11\rangle + \gamma|10\rangle - \mu|01\rangle - \beta|00\rangle) + |\psi_5\rangle_8(\alpha|11\rangle - \gamma|10\rangle - \mu|01\rangle + \beta|00$$

$$+|\psi_5\rangle_9(\alpha|00\rangle + \gamma|01\rangle + \mu|10\rangle + \beta|11\rangle) + |\psi_5\rangle_{10}(\alpha|00\rangle - \gamma|01\rangle + \mu|10\rangle - \beta|11$$

$$+|\psi_5\rangle_{11}(\alpha|00\rangle + \gamma|01\rangle - \mu|10\rangle - \beta|11\rangle) + |\psi_5\rangle_{12}(\alpha|00\rangle - \gamma|01\rangle - \mu|10\rangle + \beta|11$$

$$+|\psi_5\rangle_{13}(\alpha|10\rangle + \gamma|11\rangle + \mu|00\rangle + \beta|01\rangle) + |\psi_5\rangle_{14}(\alpha|10\rangle - \gamma|11\rangle + \mu|00\rangle - \beta|01$$

$$+|\psi_5\rangle_{15}(\alpha|10\rangle + \gamma|11\rangle - \mu|00\rangle - \beta|01\rangle) + |\psi_5\rangle_{16}(\alpha|10\rangle - \gamma|11\rangle - \mu|00\rangle + \beta|01\rangle$$

- Here, $|\psi_5\rangle_i$'s forming the mutual orthogonal basis of measurement are given by :

$$|\psi_5\rangle_1 = \frac{1}{2}[|\phi_-\rangle|010\rangle + |\phi_+\rangle|111\rangle + |\psi_-\rangle|001\rangle + |\psi_+\rangle|100\rangle];$$

$$|\psi_5\rangle_2 = \frac{1}{2}[|\phi_+\rangle|010\rangle + |\phi_-\rangle|111\rangle + |\psi_+\rangle|001\rangle + |\psi_-\rangle|100\rangle];$$

$$|\psi_5\rangle_3 = \frac{1}{2}[|\psi_-\rangle|001\rangle + |\psi_+\rangle|100\rangle - |\phi_+\rangle|010\rangle - |\phi_-\rangle|111\rangle];$$

$$|\psi_5\rangle_4 = \frac{1}{2}[|\psi_-\rangle|001\rangle + |\psi_+\rangle|100\rangle - |\phi_-\rangle|010\rangle - |\phi_+\rangle|111\rangle];$$

$$|\psi_5\rangle_5 = \frac{1}{2}[|\psi_+\rangle|111\rangle - |\psi_-\rangle|010\rangle - |\phi_-\rangle|001\rangle + |\phi_+\rangle|100\rangle];$$

$$|\psi_5\rangle_6 = \frac{1}{2}[|\psi_-\rangle|111\rangle - |\psi_+\rangle|010\rangle + |\phi_+\rangle|001\rangle - |\phi_-\rangle|100\rangle];$$

$$|\psi_5\rangle_7 = \frac{1}{2}[|\psi_-\rangle|111\rangle - |\psi_+\rangle|010\rangle - |\phi_+\rangle|001\rangle + |\phi_-\rangle|100\rangle];$$

$$|\psi_5\rangle_8 = \frac{1}{2}[|\psi_+\rangle|111\rangle - |\psi_-\rangle|010\rangle + |\phi_-\rangle|001\rangle - |\phi_+\rangle|100\rangle];$$

$$|\psi_5\rangle_9 = \frac{1}{2}[|\psi_+\rangle|111\rangle + |\psi_-\rangle|010\rangle + |\phi_-\rangle|001\rangle + |\phi_+\rangle|000\rangle];$$

$$|\psi_5\rangle_{10} = \frac{1}{2}[|\psi_-\rangle|111\rangle + |\psi_+\rangle|010\rangle - |\phi_-\rangle|001\rangle - |\psi_+\rangle|100\rangle];$$

$$|\psi_5\rangle_{11} = \frac{1}{2}[|\psi_-\rangle|111\rangle + |\psi_+\rangle|010\rangle + |\phi_+\rangle|001\rangle + |\phi_-\rangle|100\rangle];$$

$$|\psi_5\rangle_{12} = \frac{1}{2}[|\psi_+\rangle|111\rangle + |\psi_-\rangle|010\rangle - |\phi_+\rangle|100\rangle - |\phi_-\rangle|000\rangle];$$

$$|\psi_5\rangle_{13} = \frac{1}{2}[|\psi_+\rangle|100\rangle - |\psi_-\rangle|001\rangle - |\phi_-\rangle|010\rangle + |\phi_+\rangle|111\rangle];$$

$$|\psi_5\rangle_{14} = \frac{1}{2}[|\psi_-\rangle|100\rangle - |\psi_+\rangle|001\rangle + |\phi_+\rangle|010\rangle - |\phi_-\rangle|111\rangle];$$

$$|\psi_5\rangle_{15} = \frac{1}{2}[|\psi_-\rangle|100\rangle - |\psi_+\rangle|001\rangle - |\phi_+\rangle|010\rangle + |\phi_-\rangle|111\rangle];$$

$$|\psi_5\rangle_{16} = \frac{1}{2}[|\psi_+\rangle|100\rangle - |\psi_-\rangle|001\rangle + |\phi_-\rangle|010\rangle - |\phi_+\rangle|111\rangle].$$

- Alice can make a five-particle measurement and then convey her results to Bob.
- Bob retrieves the original state $|\psi\rangle_b$ by applying an unitary transform to the respective states.
- This successfully completes the teleportation protocol of a two qubit state using $|\psi_5\rangle$.

Information splitting

- The first scheme for the information splitting of a single qubit state was demonstrated by Hillery *et al.* using the three and the four particle GHZ states.
- Information splitting of an arbitrary two qubit state, was carried out using four Bell pairs (eight particles)
- No four qubit state could be used for this purpose
- We demonstrate the usefulness of the five particle Brown state for Information splitting of an arbitrary single and two qubit states.

Proposal I

- Alice possesses qubit 1, Bob possess qubit 2, 3, 4 and Charlie 5.
- Alice has a unknown qubit ($\alpha|0\rangle + \beta|1\rangle$) which she wants Bob and Charlie to share.
- Alice combines the unknown qubit with the Brown state and performs a Bell measurement and convey her outcome to Charlie by two cbits.
- For instance, if Alice measures in the basis $|\psi_+\rangle$, then the Bob-Charlie system evolves into the entangled state:

$$\alpha(|01\rangle|\phi_-\rangle + |10\rangle|\psi_-\rangle) + \beta(|00\rangle|\phi_+\rangle + |11\rangle|\psi_+\rangle).$$

- Now Bob can perform a three partite measurement and convey his outcome to Charlie by two cbits.

- Having known the outcome of both their measurements, Charlie can obtain the state by performing appropriate unitary transformations.
- The outcome of the measurement performed by Bob and the state obtained by Charlie are shown in the table below:

Outcome of the measurement	**State obtained**
$\frac{1}{2}(\|010\rangle - \|101\rangle + \|001\rangle + \|110\rangle)$	$\alpha\|1\rangle + \beta\|0\rangle$
$\frac{1}{2}(\|100\rangle - \|011\rangle + \|000\rangle + \|111\rangle)$	$\alpha\|0\rangle + \beta\|1\rangle$
$\frac{1}{2}(\|010\rangle - \|101\rangle - \|001\rangle - \|110\rangle)$	$\alpha\|1\rangle - \beta\|0\rangle$
$\frac{1}{2}(\|100\rangle - \|000\rangle - \|111\rangle - \|011\rangle)$	$\alpha\|0\rangle - \beta\|1\rangle$

- Here, Bob can also perform a single particle measurement followed by a two particle measurement instead of a three particle measurement.
- However, this would consume an extra cbit of information.

Arbitrary two qubit state

- We let Alice posses particles 1,2, Bob have particle 3, and Charlie has particles 4 and 5 in the Brown state respectively.
- Alice first, combines the state $|\psi\rangle$ with the Brown state and makes a four - particle measurement.
- The outcome of the measurement made by Alice and the entangled state obtained by Bob and Charlie are shown in the table(the coefficient $\frac{1}{4}$ is removed for convenience).

Table

Outcome of the measurement	**State obtained**
$(\|0000\rangle + \|1001\rangle + \|0110\rangle + \|1111\rangle)$	$\alpha\|\Omega_1\rangle + \mu\|\Omega_2\rangle + \gamma\|\Omega_3\rangle + \beta\|\Omega_4\rangle$
$(\|0000\rangle - \|1001\rangle + \|0110\rangle - \|1111\rangle)$	$\alpha\|\Omega_1\rangle - \mu\|\Omega_2\rangle + \gamma\|\Omega_3\rangle - \beta\|\Omega_4\rangle$
$(\|0000\rangle + \|1001\rangle - \|0110\rangle - \|1111\rangle)$	$\alpha\|\Omega_1\rangle + \mu\|\Omega_2\rangle - \gamma\|\Omega_3\rangle - \beta\|\Omega_4\rangle$
$(\|0000\rangle - \|1001\rangle - \|0110\rangle + \|1111\rangle)$	$\alpha\|\Omega_1\rangle - \mu\|\Omega_2\rangle - \gamma\|\Omega_3\rangle + \beta\|\Omega_4\rangle$
$(\|0010\rangle + \|0101\rangle + \|1000\rangle + \|1101\rangle)$	$\alpha\|\Omega_3\rangle + \gamma\|\Omega_4\rangle + \mu\|\Omega_1\rangle + \beta\|\Omega_2\rangle$
$(\|0010\rangle - \|0101\rangle + \|1000\rangle - \|1101\rangle)$	$\alpha\|\Omega_3\rangle - \gamma\|\Omega_4\rangle + \mu\|\Omega_1\rangle - \beta\|\Omega_2\rangle$
$(\|0010\rangle + \|0101\rangle - \|1000\rangle - \|1101\rangle)$	$\alpha\|\Omega_3\rangle + \gamma\|\Omega_4\rangle - \mu\|\Omega_1\rangle - \beta\|\Omega_2\rangle$
$(\|0010\rangle - \|0101\rangle - \|1000\rangle + \|1101\rangle)$	$\alpha\|\Omega_3\rangle - \gamma\|\Omega_4\rangle - \mu\|\Omega_1\rangle + \beta\|\Omega_2\rangle$
$(\|0001\rangle + \|0100\rangle + \|1011\rangle + \|1110\rangle)$	$\alpha\|\Omega_2\rangle + \gamma\|\Omega_1\rangle + \mu\|\Omega_4\rangle + \beta\|\Omega_3\rangle$

Table(continued)

Outcome of the measurement	State obtained
$(\|0001\rangle - \|0100\rangle + \|1011\rangle - \|1110\rangle)$	$\alpha\|\Omega_2\rangle - \gamma\|\Omega_1\rangle + \mu\|\Omega_4\rangle - \beta\|\Omega_3\rangle$
$(\|0001\rangle + \|0100\rangle - \|1011\rangle - \|1110\rangle)$	$\alpha\|\Omega_2\rangle + \gamma\|\Omega_1\rangle - \mu\|\Omega_4\rangle - \beta\|\Omega_3\rangle$
$(\|0001\rangle - \|0100\rangle - \|1011\rangle + \|1110\rangle)$	$\alpha\|\Omega_2\rangle - \gamma\|\Omega_1\rangle - \mu\|\Omega_4\rangle + \beta\|\Omega_3\rangle$
$(\|0011\rangle + \|1010\rangle + \|0101\rangle + \|1100\rangle)$	$\alpha\|\Omega_4\rangle + \mu\|\Omega_3\rangle + \gamma\|\Omega_2\rangle + \beta\|\Omega_1\rangle$
$(\|0011\rangle - \|1010\rangle + \|0101\rangle - \|1100\rangle)$	$\alpha\|\Omega_4\rangle - \mu\|\Omega_3\rangle + \gamma\|\Omega_2\rangle - \beta\|\Omega_1\rangle$
$(\|0011\rangle + \|1010\rangle - \|0101\rangle - \|1100\rangle)$	$\alpha\|\Omega_4\rangle + \mu\|\Omega_3\rangle - \gamma\|\Omega_2\rangle - \beta\|\Omega_1\rangle$
$(\|0011\rangle - \|1010\rangle - \|0101\rangle + \|1100\rangle)$	$\alpha\|\Omega_4\rangle - \mu\|\Omega_3\rangle - \gamma\|\Omega_2\rangle + \beta\|\Omega_1\rangle$

Here,

$$|\Omega_1\rangle = \frac{1}{2}(|101\rangle - |110\rangle), |\Omega_2\rangle = \frac{1}{2}(|000\rangle - |011\rangle),$$
$$|\Omega_3\rangle = \frac{1}{2}(|001\rangle + |010\rangle), |\Omega_4\rangle = \frac{1}{2}(|100\rangle + |111\rangle).$$

- Neither Bob nor Charlie can reconstruct the original state $|\psi\rangle$ from the above states by local operations.
- Now Bob performs a measurement on his particle in the basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$.
- For instance if Bob gets the state $(\alpha|\Omega_1\rangle + \mu|\Omega_2\rangle + \gamma|\Omega_3\rangle + \beta|\Omega_4\rangle)$ then Charlie's particle evolves into any one of the following states:
- $(\pm\alpha|\phi_-\rangle + \mu|\psi_-\rangle + \gamma|\phi_+\rangle \pm \beta|\psi_+\rangle)$.
- If Charlie gets the above states, then he performs the unitary transformations $(\pm|11\rangle\langle\psi_+| + |10\rangle\langle\phi_+|) \pm (|00\rangle\langle\phi_-| + |01\rangle\langle\psi_-|)$ on his two particles to get back $|\psi\rangle$.
- The Brown state could also be used for information splitting of a three partite GHZ type state given by : $|GHZ\rangle = \alpha|000\rangle + \beta|111\rangle$.

Superdense coding

- Entanglement is quite handy in communicating information efficiently, in a quantum channel.
- Alice and Bob share an entangled state, namely $|\psi\rangle_{AB}$.
- Then Alice can convert her state into different orthogonal states by applying suitable unitary transforms on her particle
- Bob then does appropriate Bell measurements on his qubits to retrieve the encoded information.
- It is known that two classical bits per qubit can be exchanged by sending information through a Bell state.
- We shall discuss the suitability of $|\psi_5\rangle$, as a resource for superdense coding.

Superdense coding

- Let us assume that Alice has first three qubits, and Bob has last two qubits.
- Alice can apply the set of unitary transforms on her particle and generate 64 states out of which 32 are mutually orthogonal as shown below:

$$U_x^3 \otimes I \otimes I \longrightarrow |\psi_5\rangle_{x_i}.$$

- Bob can then perform a five-partite measurement in the basis of $|\psi_5\rangle_{x_i}$ and distinguish these states.

Capacity of dense coding

- The capacity of superdense coding is defined as,

$$X(\rho^{AB}) = \log_2 d_A + S(\rho^B) - S(\rho^{AB}),$$

  where d_A is the dimension of Alice's system, $S(\rho)$ is von-Neumann entropy.
- For the state $|\psi_5\rangle$, $X(\rho^{AB}) = 3 + 2 - 0 = 5$.
- Thus, the super dense coding reaches the "Holevo bound" allowing five classical bits to be transmitted through three quantum bits consuming only two ebits of entanglement.

- One can also send four classical bits by sending two qubits consuming two ebits of entanglement.
- Thus the Brown state could also be used instead of four partite cluster state.
- All the applications considered here could also be carried out using a state of the type :

$$|\psi_5\rangle = A_1|001\rangle|\phi_-\rangle + A_2|010\rangle|\psi_-\rangle \\ +A_3|100\rangle|\phi_+\rangle + A_4|111\rangle|\psi_+\rangle,$$

- where A_i is an integer, if the following relations are satisfied :

$$-\sum_{n=1}^{4} A_i^2(1+\log_2 A_i^2) = 1,$$

$$-(A_3^2+A_4^2)\log_2(A_3^2+A_4^2) = \frac{1}{2}.$$

- This is a necessary but not sufficient condition.

Cluster states

- Unlike the Brown state, cluster states have been experimentally realized in laboratory conditions.
- These states were first introduced for linear optics one way quantum computation.
- They show a strong violation of local reality and are shown to be robust against decoherence
- They exhibit remarkably different entanglement properties from the GHZ states.
- The four partite cluster state has been proven to be of immense use in one way Quantum computation and also for Quantum error correction.
- Recently, one way quantum computing with four qubit cluster states has been experimentally realized.

The five particle cluster state

- The five particle cluster state is given by :

$$|C_5\rangle = \frac{1}{2}(|00000\rangle + |00111\rangle + |11101\rangle + |11010\rangle).$$

- These states have been identified as Task-oriented Maximally Entangled States.
- We let Alice posses particles 1,5, Bob posses particle 2, and Charlie posses particles 3 and 4 in the $|C_5\rangle$ respectively.

- Alice first, combines the state $|\psi\rangle$ with $|C_5\rangle$ and performs a four - particle measurement and conveys the outcome of her measurement to Charlie by four cbits of information.
- The outcome of the measurement made by Alice and the entangled state obtained by Bob and Charlie are shown in the table(the coefficient $\frac{1}{2}$ is removed for convenience).

Table

Outcome of the Measurement	State obtained
$\|0000\rangle + \|1011\rangle + \|0111\rangle + \|1110\rangle$	$\alpha\|000\rangle + \mu\|011\rangle + \gamma\|110\rangle + \beta\|101\rangle$
$\|0000\rangle + \|1011\rangle - \|0111\rangle - \|1110\rangle$	$\alpha\|000\rangle + \mu\|011\rangle - \gamma\|110\rangle - \beta\|101\rangle$
$\|0000\rangle - \|1011\rangle + \|0111\rangle - \|1110\rangle$	$\alpha\|000\rangle - \mu\|011\rangle + \gamma\|110\rangle - \beta\|101\rangle$
$\|0000\rangle - \|1011\rangle - \|0111\rangle + \|1110\rangle$	$\alpha\|000\rangle - \mu\|011\rangle - \gamma\|110\rangle + \beta\|101\rangle$
$\|0001\rangle + \|1000\rangle + \|0110\rangle + \|1111\rangle$	$\alpha\|011\rangle + \mu\|000\rangle + \gamma\|101\rangle + \beta\|110\rangle$
$\|0001\rangle + \|1000\rangle - \|0110\rangle - \|1111\rangle$	$\alpha\|011\rangle + \mu\|000\rangle - \gamma\|101\rangle - \beta\|110\rangle$
$\|0001\rangle - \|1000\rangle + \|0110\rangle - \|1111\rangle$	$\alpha\|011\rangle - \mu\|000\rangle + \gamma\|101\rangle - \beta\|110\rangle$
$\|0001\rangle - \|1000\rangle - \|0110\rangle + \|1111\rangle$	$\alpha\|011\rangle - \mu\|000\rangle - \gamma\|101\rangle + \beta\|110\rangle$
$\|0011\rangle + \|1010\rangle + \|0100\rangle + \|1101\rangle$	$\alpha\|110\rangle + \mu\|101\rangle + \gamma\|000\rangle + \beta\|011\rangle$

Table(continued)

Outcome of the Measurement	State obtained
$\|0011\rangle + \|1010\rangle - \|0100\rangle - \|1101\rangle$	$\alpha\|110\rangle + \mu\|101\rangle - \gamma\|000\rangle - \beta\|011\rangle$
$\|0011\rangle - \|1010\rangle + \|0100\rangle - \|1101\rangle$	$\alpha\|110\rangle - \mu\|101\rangle + \gamma\|000\rangle - \beta\|011\rangle$
$\|0011\rangle - \|1010\rangle - \|0100\rangle + \|1101\rangle$	$\alpha\|110\rangle - \mu\|101\rangle - \gamma\|000\rangle + \beta\|011\rangle$
$\|0010\rangle + \|1011\rangle + \|0101\rangle + \|1100\rangle$	$\alpha\|101\rangle + \mu\|110\rangle + \gamma\|011\rangle + \beta\|000\rangle$
$\|0010\rangle + \|1011\rangle - \|0101\rangle - \|1100\rangle$	$\alpha\|101\rangle + \mu\|110\rangle - \gamma\|011\rangle - \beta\|000\rangle$
$\|0010\rangle - \|1011\rangle + \|0101\rangle - \|1100\rangle$	$\alpha\|101\rangle - \mu\|110\rangle + \gamma\|011\rangle - \beta\|000\rangle$
$\|0010\rangle - \|1011\rangle - \|0101\rangle + \|1100\rangle$	$\alpha\|101\rangle - \mu\|110\rangle - \gamma\|011\rangle + \beta\|000\rangle$

- Bob can perform a measurement in the basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, and communicate the outcome of his measurement to Charlie who then, performs unitary transformations to get the state $|\psi\rangle$.
- For instance, had the Bob-Charlie system been the first state, then, after Bob's measurement, Charlie's state collapses to $(\alpha|00\rangle \pm \mu|11\rangle \pm \gamma|10\rangle + \beta|01\rangle)$, which can be converted to $|\psi\rangle$, by performing an appropriate Unitary operation.
- This completes the protocol for QIS of an arbitrary two qubit state using $|C_5\rangle$.

The six particle cluster state

- The six particle cluster state is given by :

$$|C_6\rangle = \frac{1}{2}(|000000\rangle + |000111\rangle + |111000\rangle - |111111\rangle)$$

- Recently, the exact state has been realized in laboratory conditions.
- It can be shown that one can devise four protocols for QIS of a single qubit state and two protocols for the QIS of an arbitrary two qubit state using this state.
- All the schemes discussed are fully technologically feasible.

Conjecture

- The above detailed analysis suggests the following conjecture:

  The maximum number of protocols that can be constructed for the Quantum information splitting of an arbitrary n qubit information, using a genuinely entangled channel of N qubits, among two parties is $(N-2n)$.

- By substituting $N=4$ and $n=2$, in this formula, we can deduce that four qubit states cannot be used for the QIS of an arbitrary two qubit state.

References

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- Thank you for your patience
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