Quantum Communications & Entanglement Distribution With Spin Chains & Allied Systems

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Quantum Information at UCL:

UCL Physics & Astronomy Department (entirely theoretical):

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Postdocs: Jaeyoon Cho, Ho Tsang Ng, Janet Anders,
Ahsan Nazir (Jan 2008), **Abolfazl Bayat** (Jan 2008)
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Martina Avellino (LCN), Dara McCutcheon (LCN)

London Centre for Nanotechnology (theory & experiment):

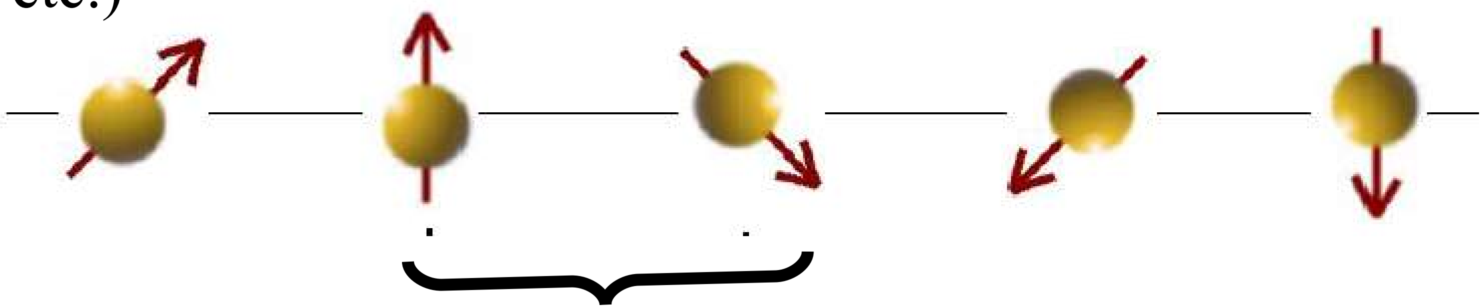
Faculty: Gabriel Aeppli, Andrew Fisher, Marshall Stoneham, …

Also several students & postdocs

Former Student: Daniel Burgarth
External: V. Giovannetti (Pisa), K. Jacobs (Boston), K. Shizume (Japan)

What are spin chains?

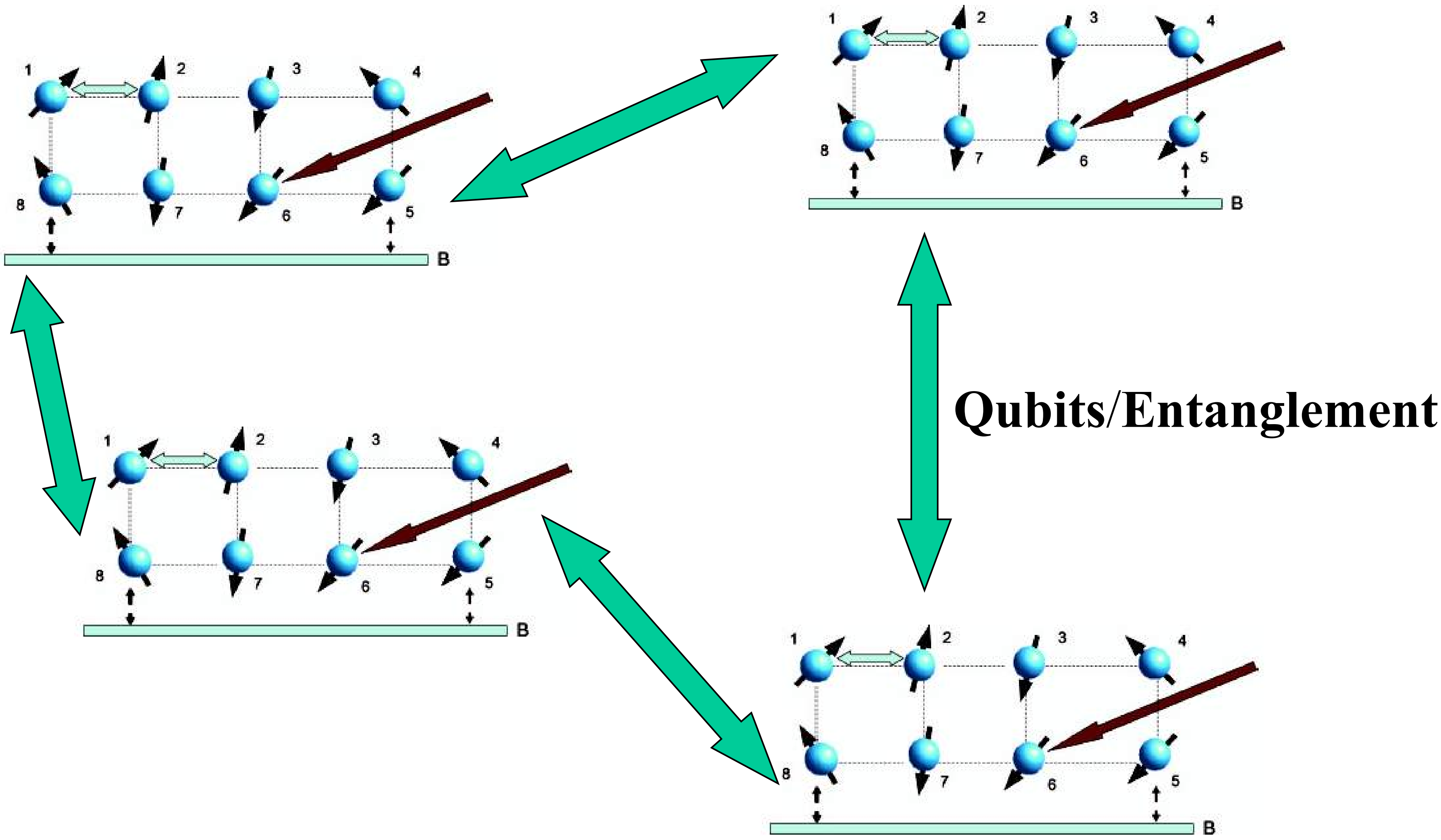
One dimensional array of spins (qubits, qutrits etc.)



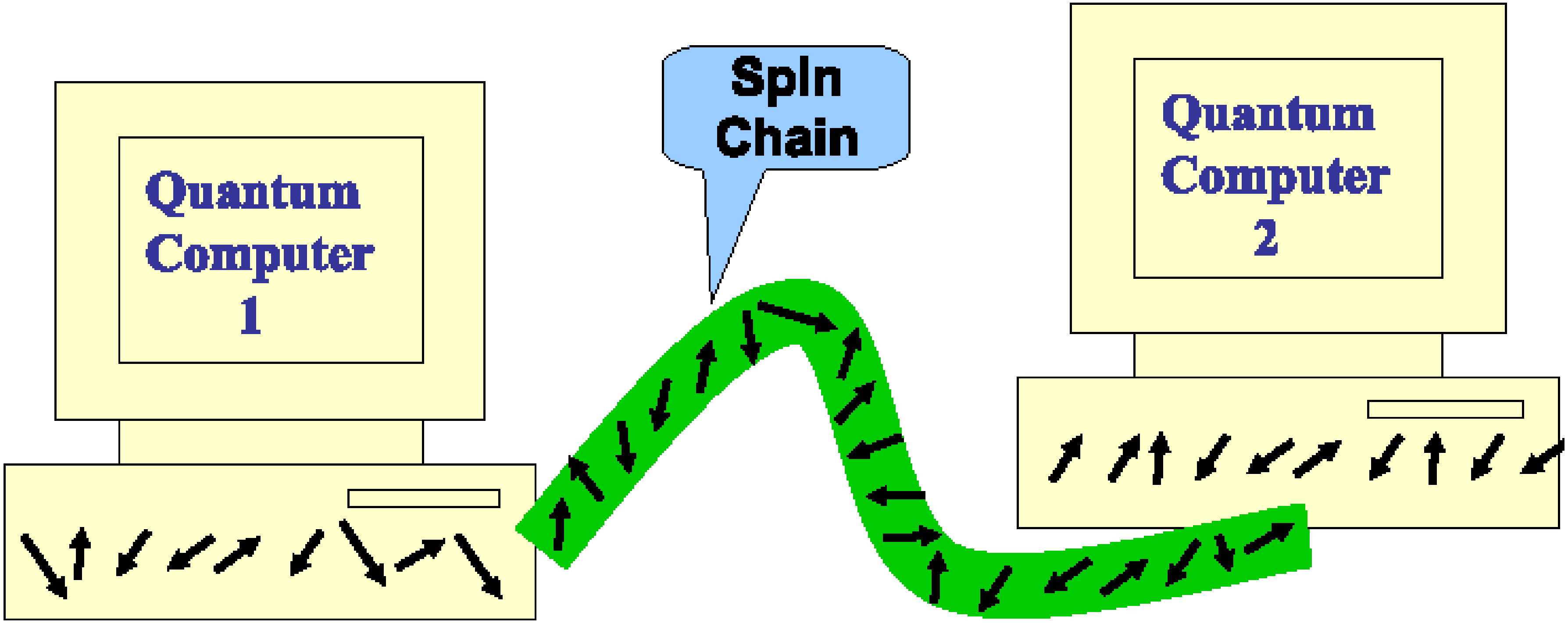
Perpetually interacting

Examples: Quasi 1D magnets, Josephson junction arrays, quantum dot arrays, optical lattices etc.

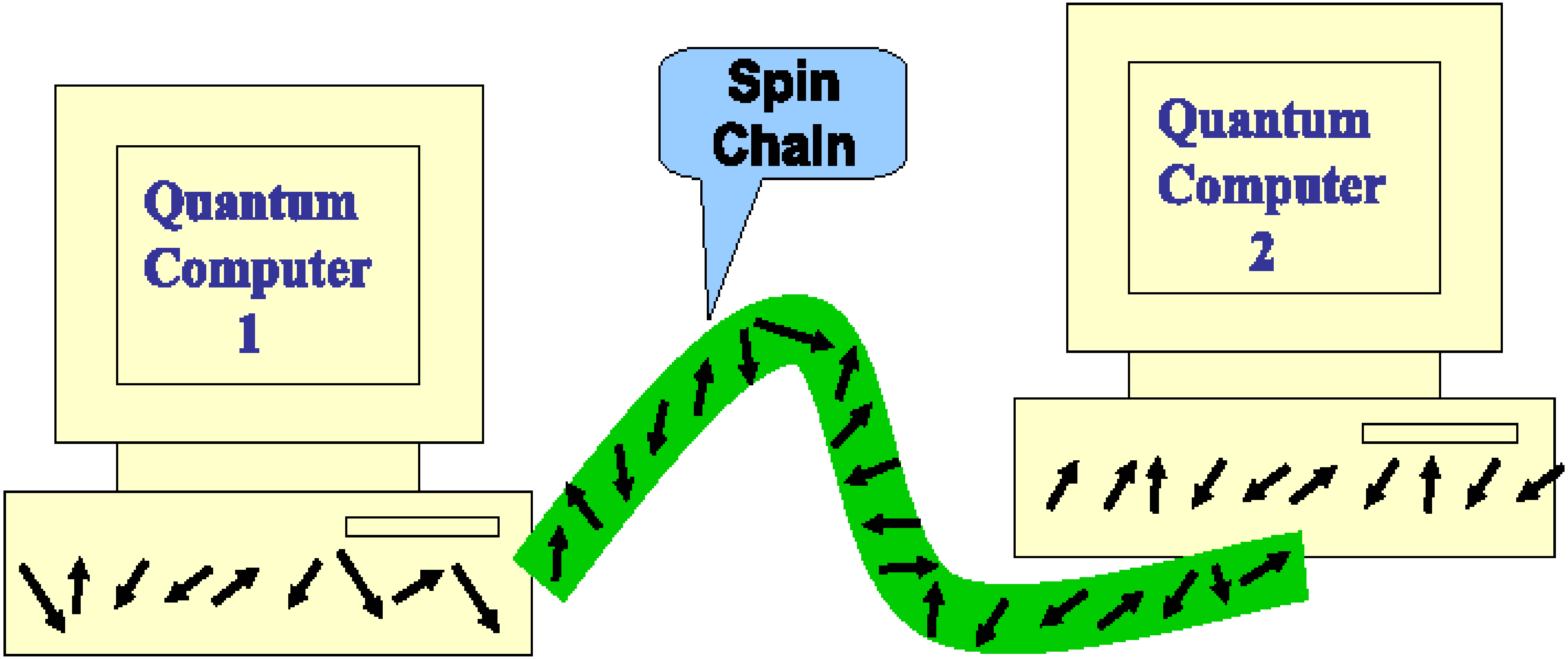
Quantum processors in the near future will be small!
--- bus mode, fabrication, efficiency



Usually the use of photons are envisaged for making the links

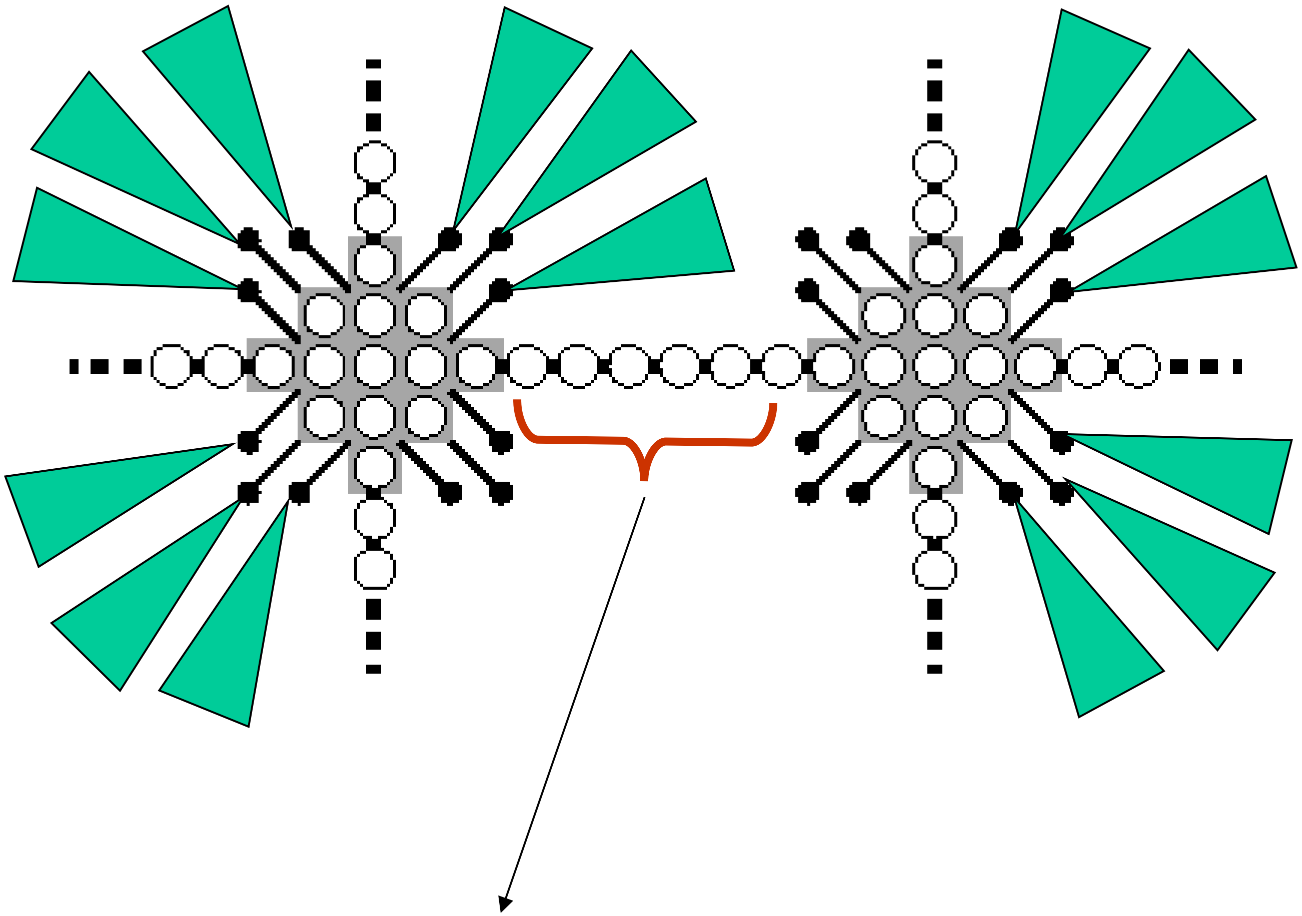


- Motivation 1:
  Avoiding inter-conversion between solid state qubits and photons for linking quantum registers.



Motivation 2: A quantum information perspective on a canonical condensed matter system (how various chains transmit information as opposed to how much entanglement is in them)

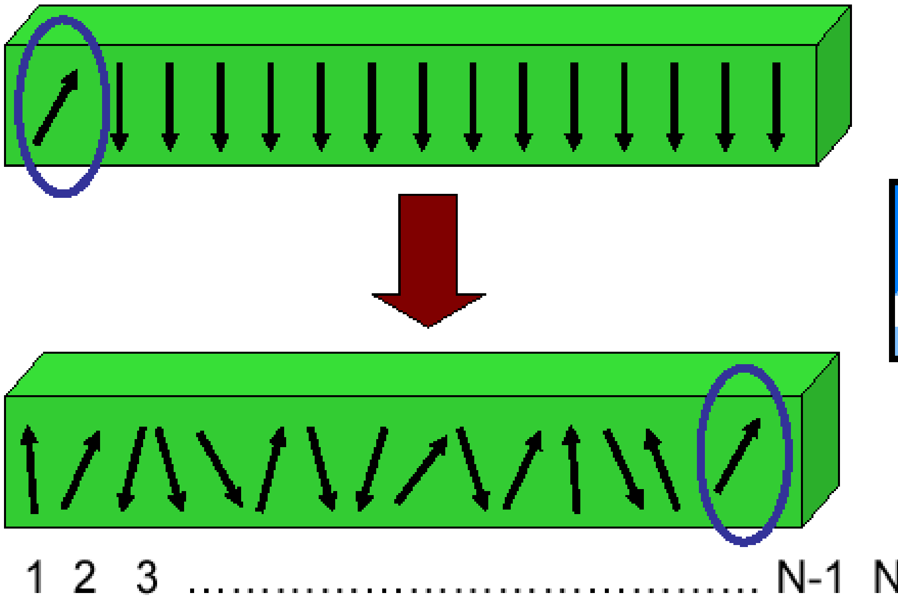
Macroscopic Control Gates cannot control too many qubits



Low or no control region (equivalent to a spin chain)

Motivation 3: Spin chains will naturally arise in low control parts of a quantum computer.

The simplest spin chain protocol one can imagine:



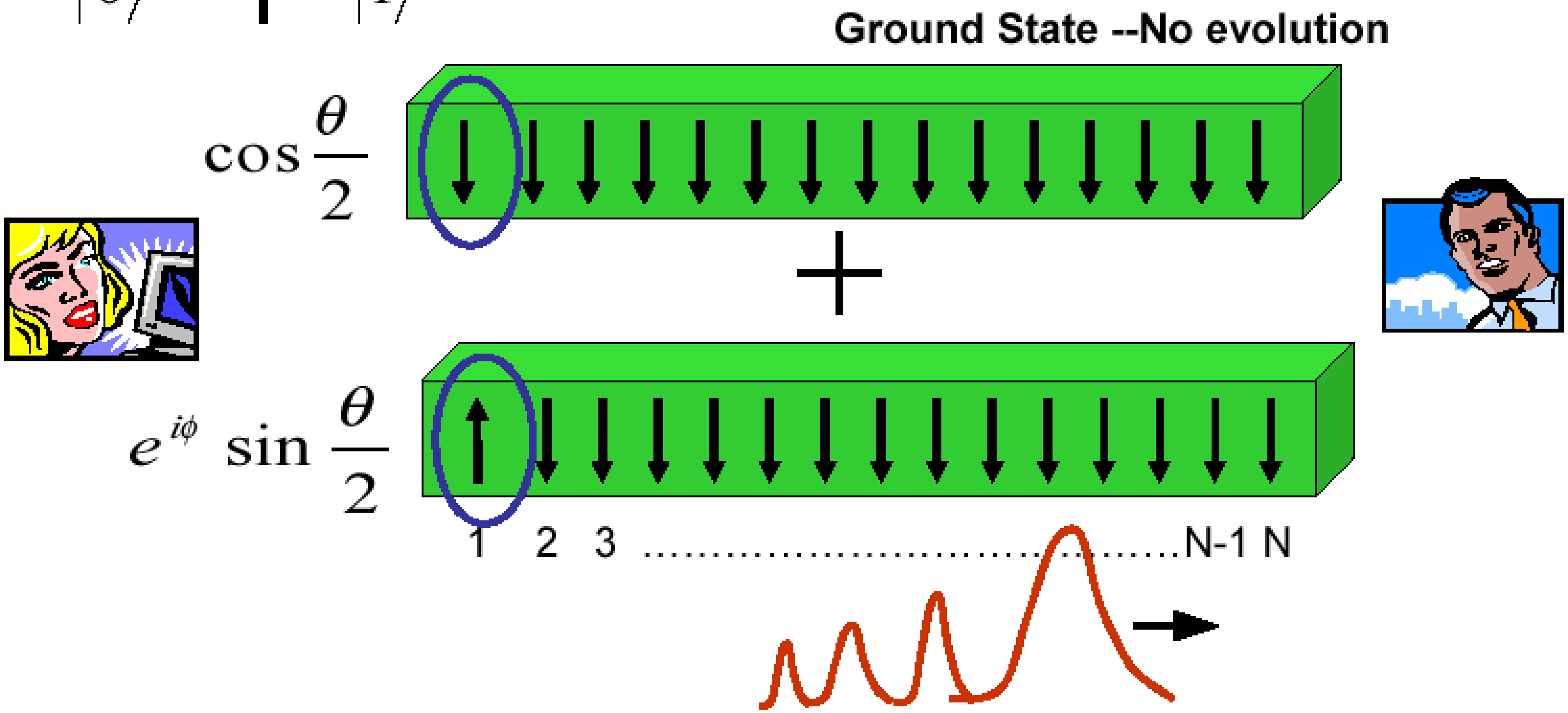
$$H = -\frac{J}{2}\sum_{j=1}^{N-1} \sigma_j \cdot \sigma_{j+1}$$

- Isotropic
- Uniform
- Ferromagnetic,
- Nearest Neighbor

S. Bose, Phys. Rev. Lett. 91, 207901 (2003).

Alice's input state: $|\phi\rangle_1 = \cos\frac{\theta}{2}|0\rangle_1 + e^{i\phi}\sin\frac{\theta}{2}|1\rangle_1$

$\downarrow \equiv |0\rangle \qquad \uparrow \equiv |1\rangle$



Optimize

Everything depends *only* on: $\boxed{f_{N,1}(t) = \langle \mathbf{N}|e^{-iHt}|\mathbf{1}\rangle}$

$|\mathbf{1}\rangle \equiv |100...00\rangle$

$|\mathbf{N}\rangle \equiv |000...01\rangle$

the transition amplitude: $\boxed{f_{N,1}(t) = \langle \mathbf{N} | e^{-iHt} | \mathbf{1} \rangle}$

$|\mathbf{1}\rangle \equiv |100...00\rangle$

$|\mathbf{N}\rangle \equiv |000...01\rangle$

At any general time t the output density matrix is:

$$\rho_N(t) = P(t)|\psi(t)\rangle\langle\psi(t)|_N + (1 - P(t))|0\rangle\langle 0|_N$$

Unavoidable mixing due to dispersion.

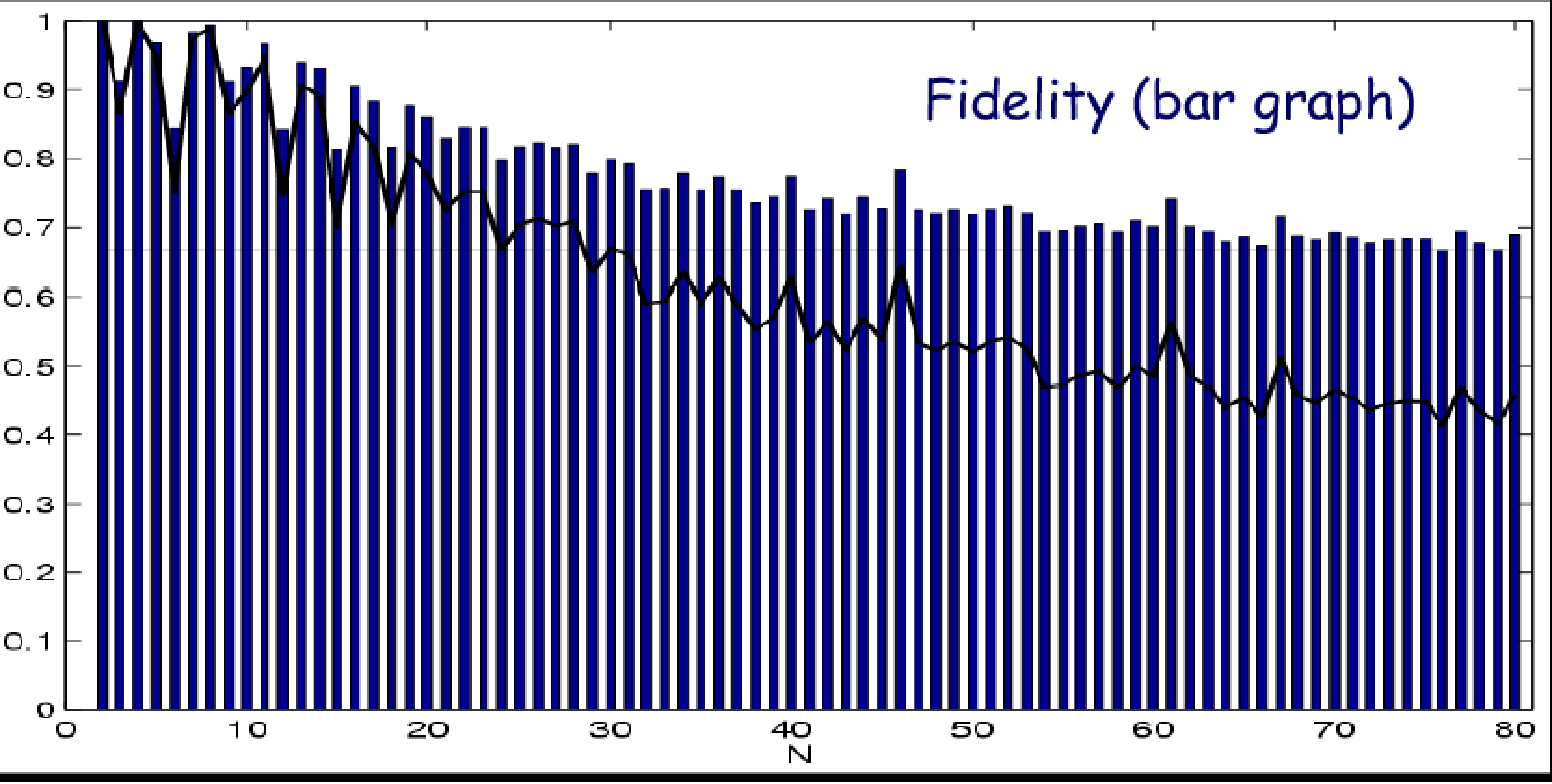
th $P(t) = \cos^2\frac{\theta}{2} + |f_{N,1}(t)|^2 \sin^2\frac{\theta}{2}$

Correctable phase factor

$$|\psi(t)\rangle_N = \frac{1}{\sqrt{P(t)}}\left(\cos\frac{\theta}{2}|0\rangle_N + e^{i\phi}|f_{N,1}(t)|\sin\frac{\theta}{2}|1\rangle_N\right)$$

Damping factor due to dispersion

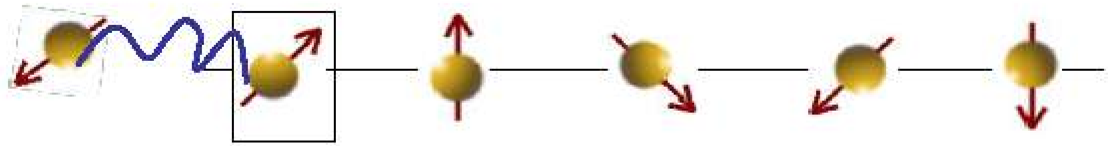
At any time t it behaves *as* an amplitude damping channel



Fidelity:

- function of time and imperfect in general
- near perfect for N=4 & 8
- better than classically achievable fidelity (2/3) in time 4000/J for up to N=80

Entanglement transfer through arbitrarily long chains:



$$E_C \approx |f_{N,1}(t)|$$

$$\text{At} \quad t \approx O(\frac{N}{J}), \quad E_C \approx \frac{1}{N^{1/3}}$$

This proves that a spin chain of any length can behave as a quantum channel

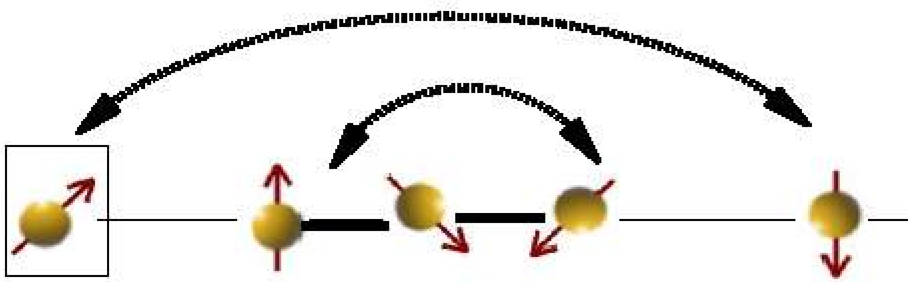
Practicality: *Several transmissions followed by entanglement distillation needed even for transmitting one ebit of entanglement !!!*

S.Bose, PRL 91, 207901 (2003).

For transfer of entangled states through spin chains see also:
V. Subrahmanyam, Phys. Rev. A 69, 034304 (2004)

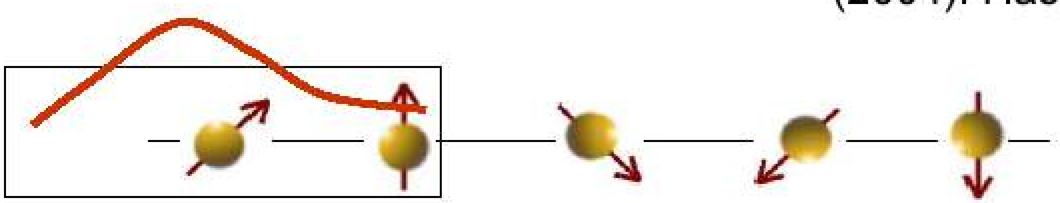
Some Really Clever Schemes for Perfect Transfer (By Other Groups):

Engineered Couplings: Christandl, Datta, Ekert, Landahl, PRL **92**, 187902 (2004).
Albanese, Christandl, Datta, Ekert, PRL **93,** 230502 (2004).



Could we do without engineering?

Encoding a qubit in a large number of spins: Osborne & Linden, PRA **69,** 052315 (2004). Haselgrove, PRA (2005).



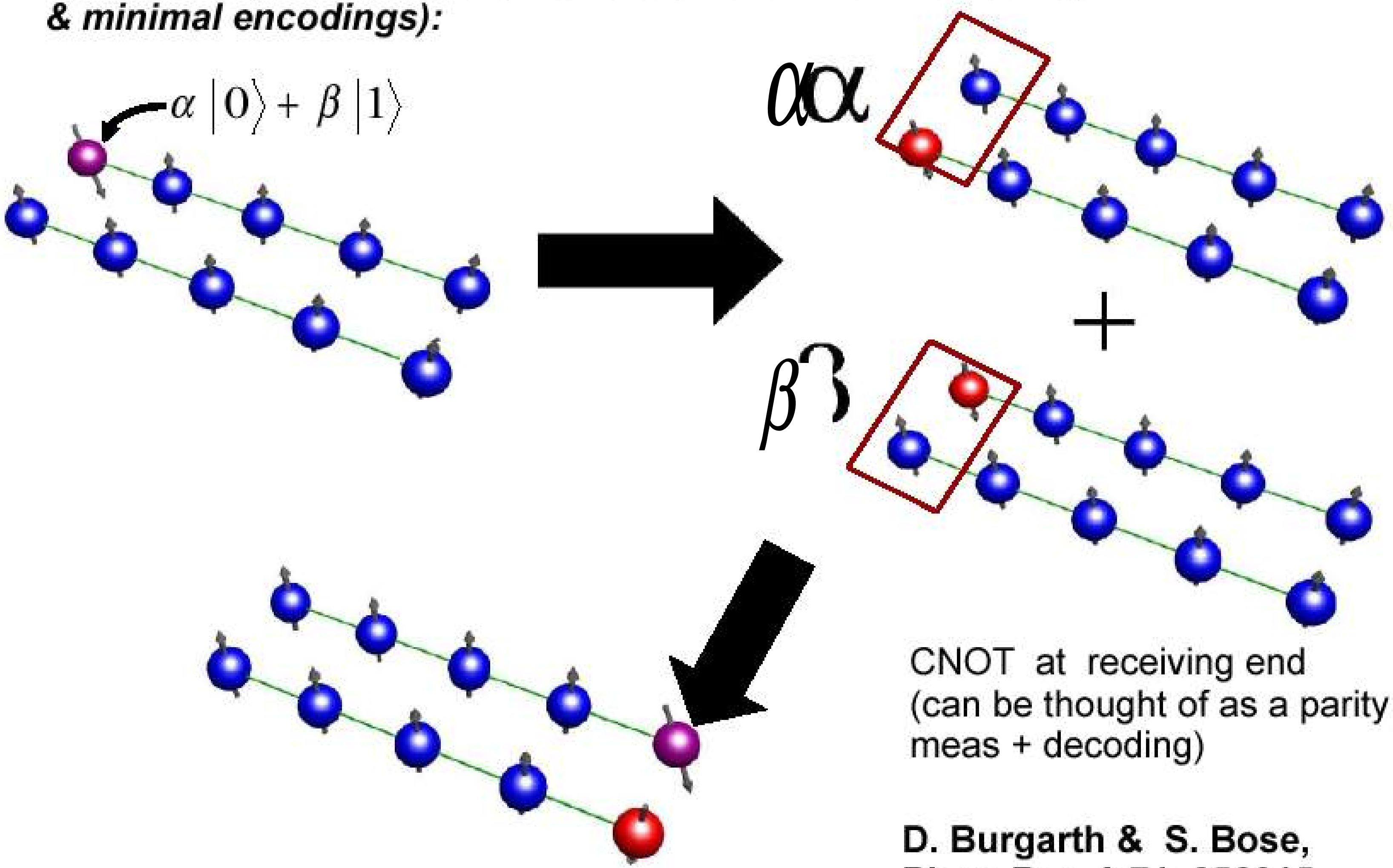
Could we do with coding on a lower number of spins?

At least two other broad classes of ideas:

Gapped system (Li *et. al.*, PRA (2005), Plenio & Semiao, NJP (2005), Wojcik, PRA, '05)

Modulated Ising Chain (Fitzsimons & Twamley, PRL, 2006).

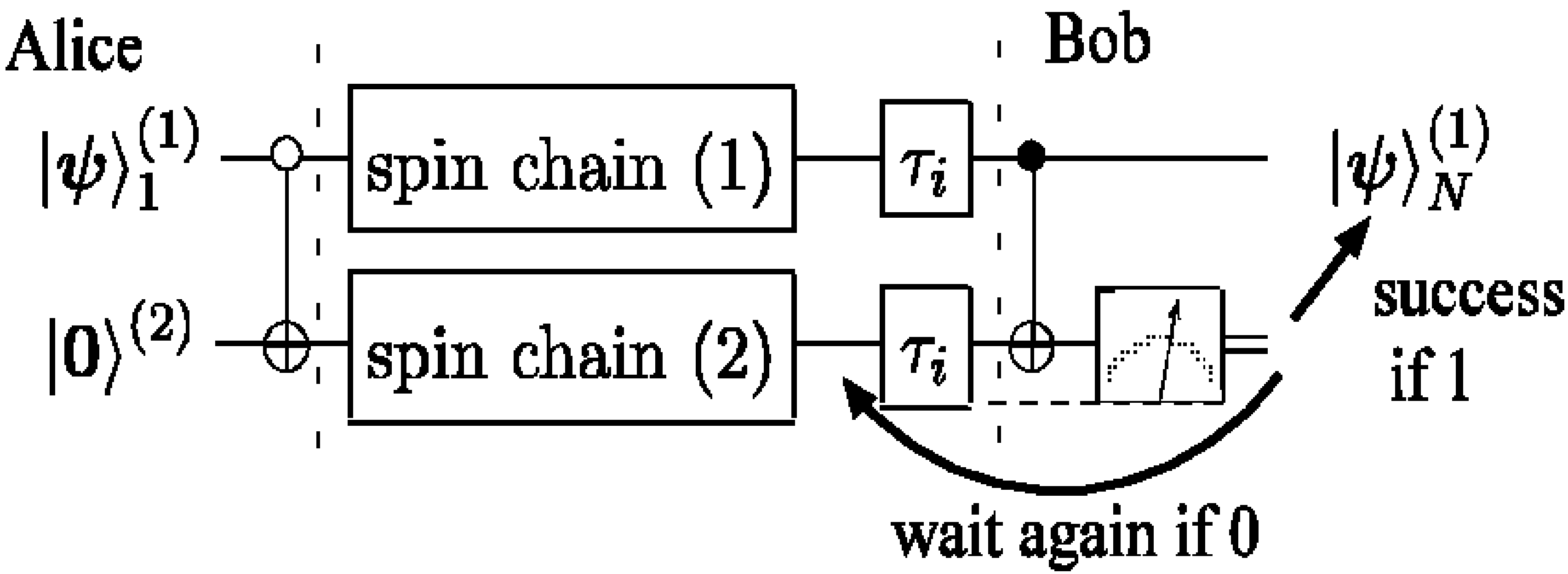
Conclusively Perfect State Transfer (with uniform couplings & minimal encodings):



CNOT at receiving end
(can be thought of as a parity meas + decoding)

D. Burgarth & S. Bose, Phys. Rev. A 71, 052315 (2005)

Arbitrarily Perfect State Transfer
Parallel Spin Chains for an amplitude delaying channel!



The initial and final gates are not even needed if the quantum computers being connected are themselves using dual rail encoding (such as double dot systems).

D. Burgarth & S. Bose, Phys. Rev. A 71, 052315 (2005)

Time-scale from heuristic argument:

Choose first measurement at $\tau_{\max} \approx \dfrac{N}{2J}$ so that the maxima of the

first Bessel wave $\left|f_{N,1}(\tau_{\max})\right|^2 \approx \dfrac{1.35}{N^{2/3}}$ reaches the *N* th site.

Then wait for $O(\tau_{return}) \approx \dfrac{N}{J}$ (for the wave packet to be reflected back from Alice's end and returned again to Bob)

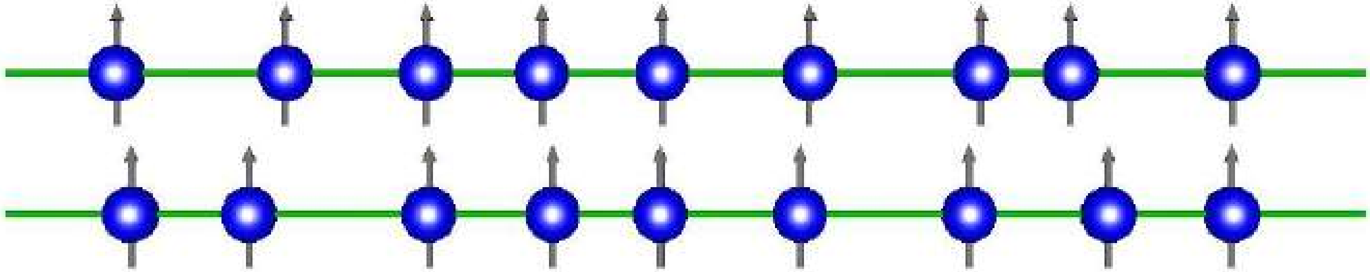
and perform the second measurement, and so on …

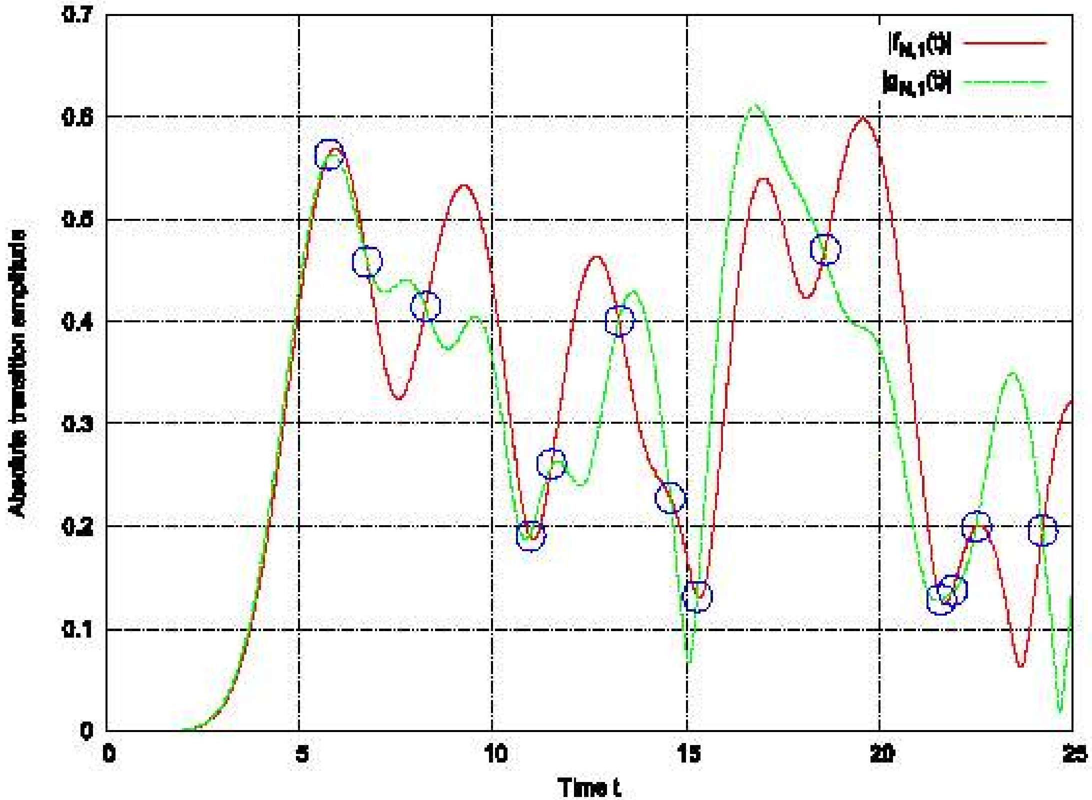
To make the joint probability of failure lower than δ we require time

$$\tau(\delta) \approx 0.51 \frac{N^{5/3}}{J} \left|\ln \delta\right|$$

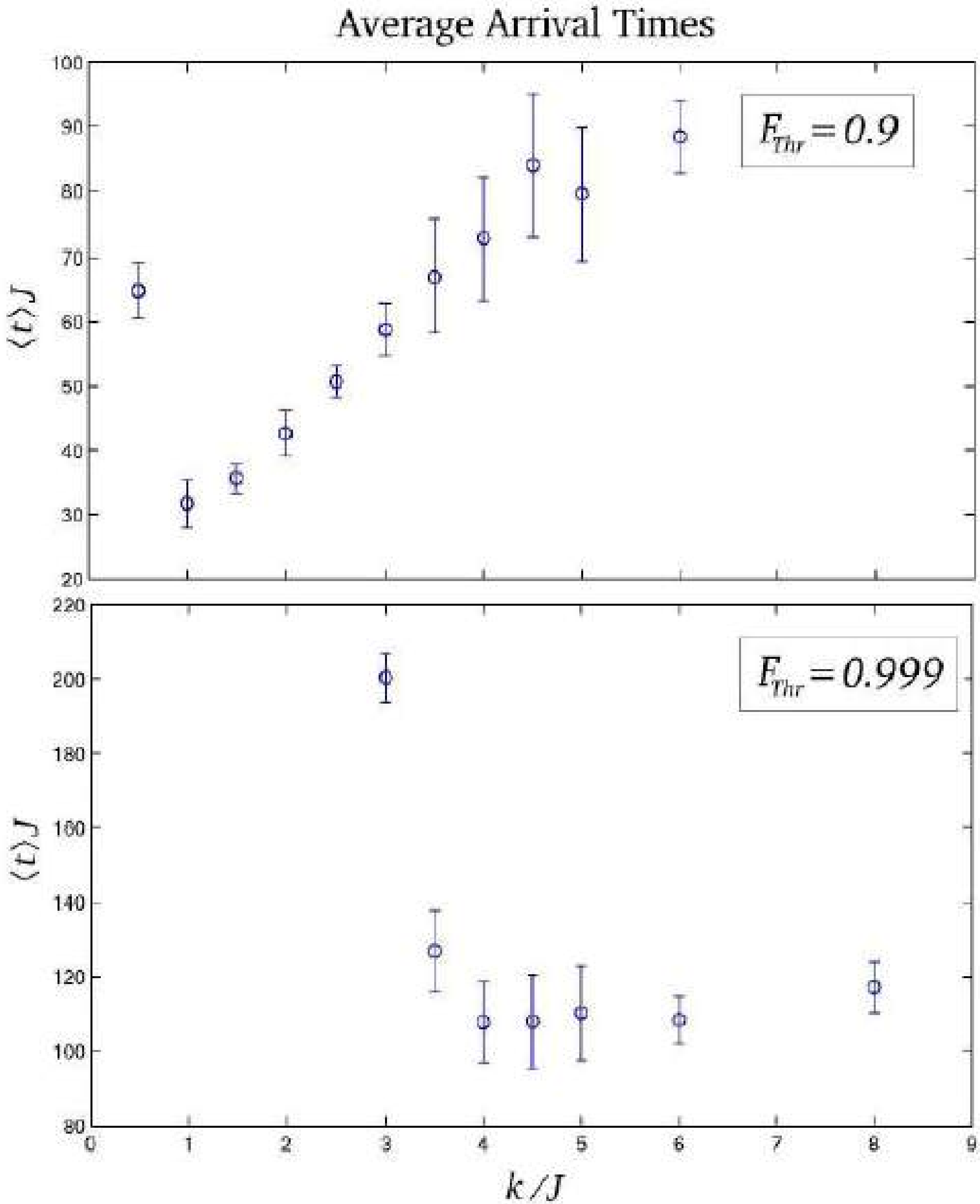
The optimized scheme performs better, of course!

What happens when the parallel chains are somewhat mismatched (random)?





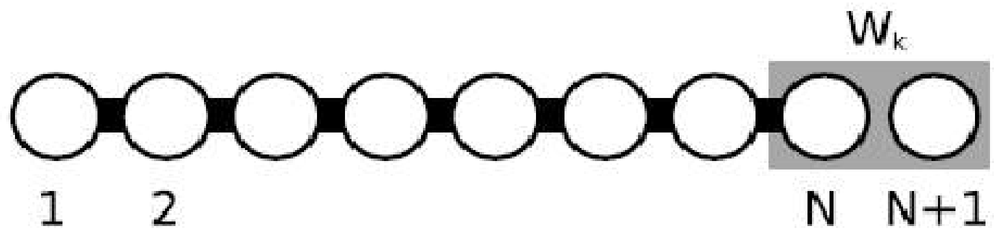
D. Burgarth & S. Bose, New J. Phys. **7** 135 (2005)



Realistic continuous measurements of strength k

K. Shizume, K. Jacobs, D. Burgarth and S. Bose, Phys. Rev. A **75,** 062328 (2007)

Another Trick:



$$W(c,d)\left[\{c|\mathbf{N}\rangle + d|\mathbf{N}+\mathbf{1}\rangle\}\right] = |\mathbf{N}+\mathbf{1}\rangle.$$

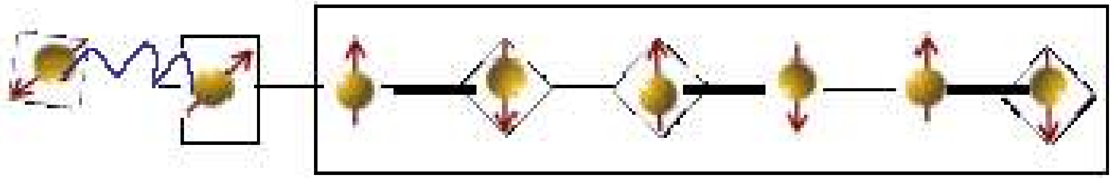
$$H(t) = J\sum_{n=1}^{N-1} \sigma_n^- \sigma_{n+1}^+ + \Delta(t)\sigma_N^- \sigma_{N+1}^+ + \mathrm{h.c.}$$

$$H(t) = J\sum_{n=1}^{N} \sigma_n^- \sigma_{n+1}^+ + \mathrm{h.c.} + B\Delta(t)\sigma_{N+1}^z$$

D. Burgarth, S. Bose and V. Giovannetti, Phys. Rev. A **75**, 062327 (2007).

Entanglement transfer through AFM

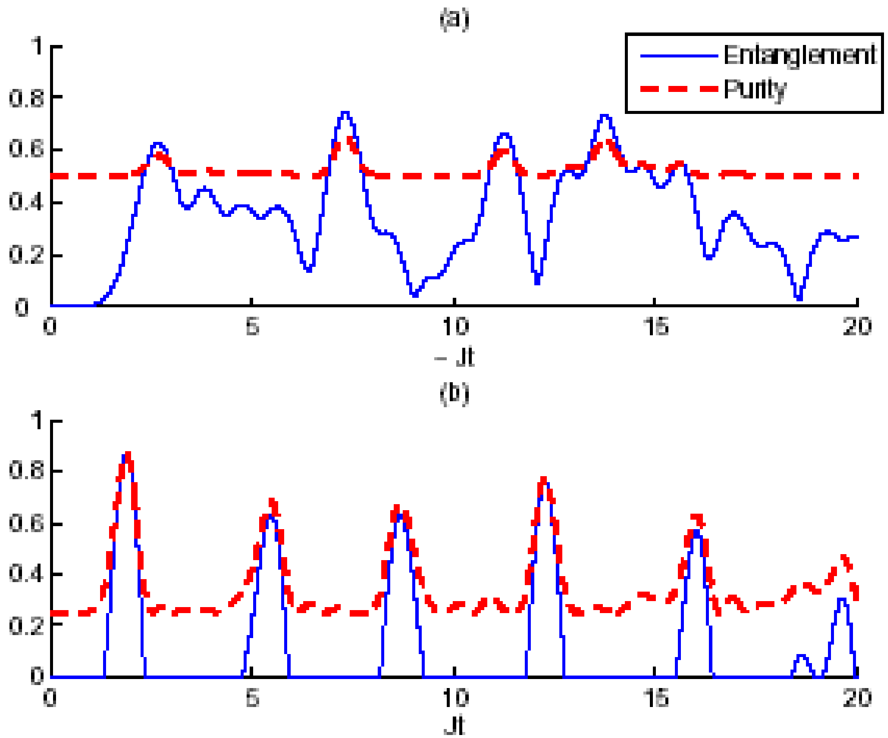
- We append one member of a singlet to one end of an open AFM chain in its ground state.



$$|\psi(0)\rangle = |\psi^-\rangle_{0'0} \otimes |\psi_g\rangle_{ch}$$

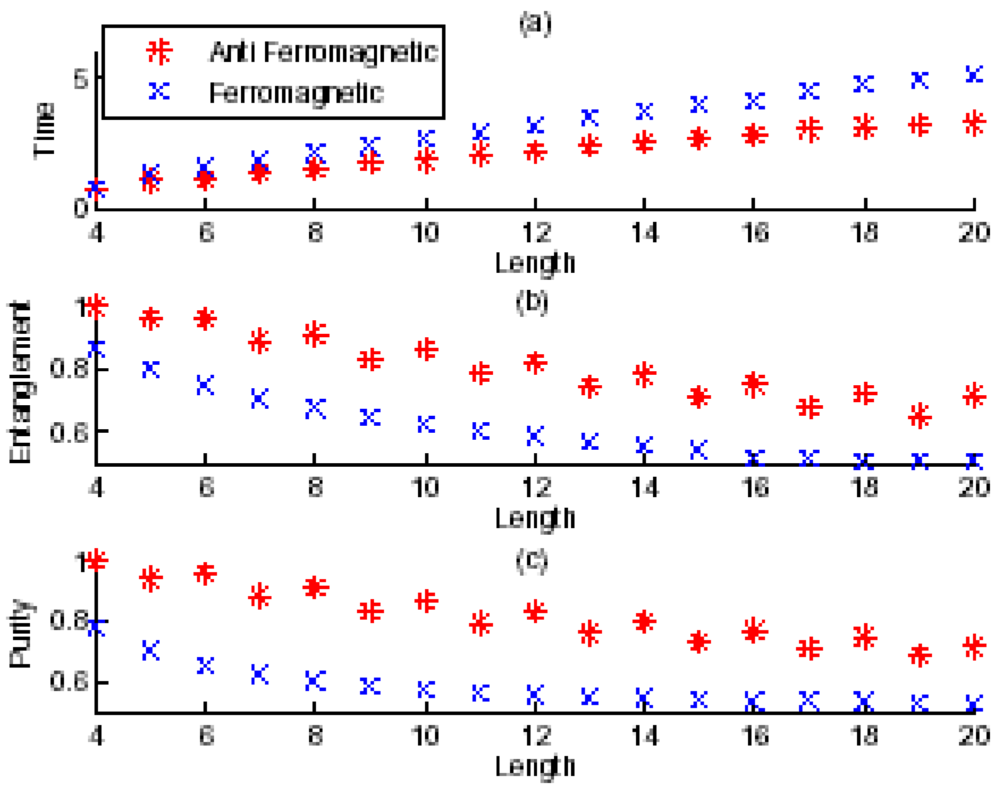
- Motivation 1: Approx half spins facing oppositely
- Motivation 2: Already entanglement inside
- Motivation 3: Symmetry: Depolarizing channel

Entanglement through FM vs AFM

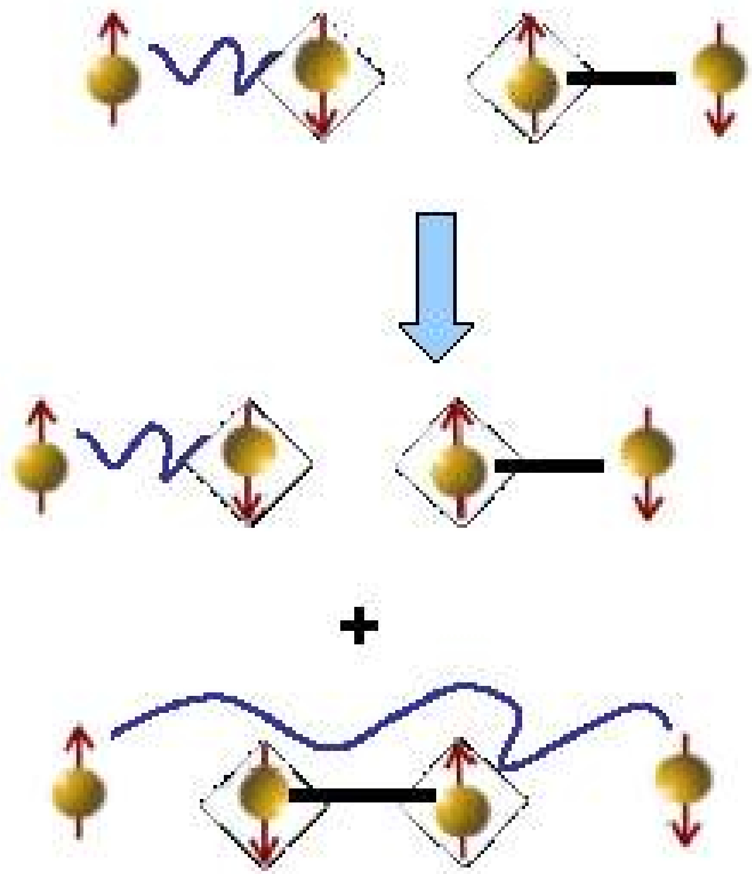


- Reason: Depolarizing channel, Werner state
  $\mathcal{W} = p|\psi^-\rangle\langle\psi^-| + (1-p)I$ entangled for $p > 1/3$

AFM vs FM for various Lengths



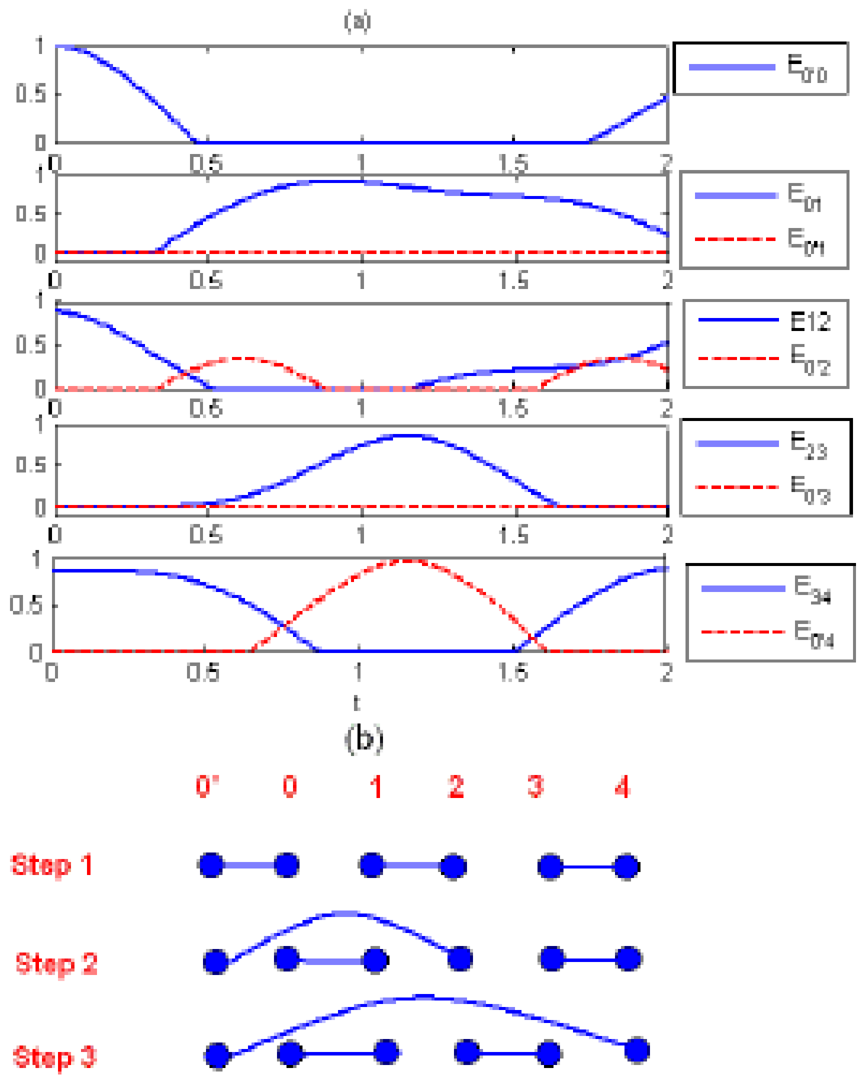
Simplest 2+2 case



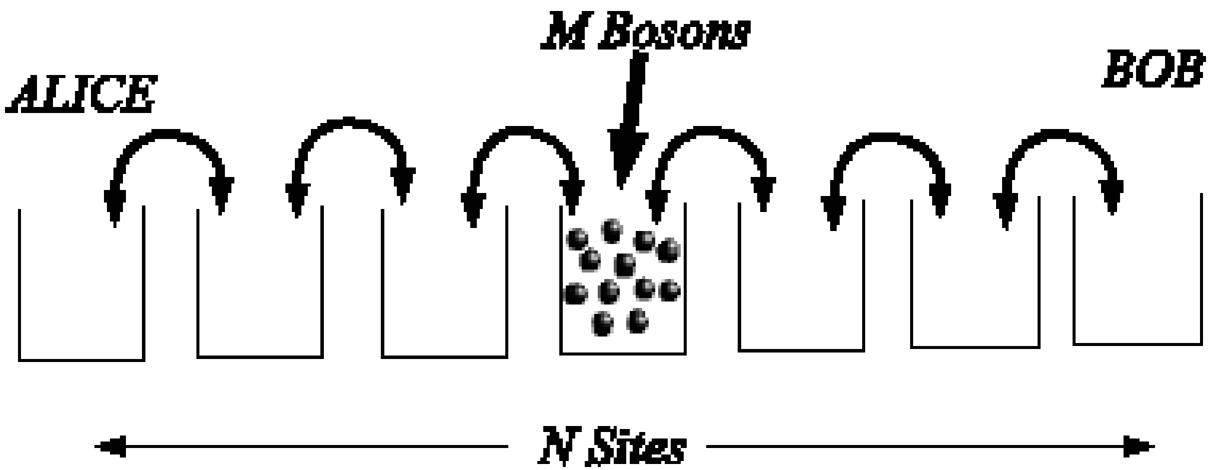
Cos 2Jt

Sin 2Jt

Mode of entanglement propagation



A different system as a quantum channel



$$|\Psi(0)\rangle = \frac{(a^\dagger_{\frac{N+1}{2}})^M}{\sqrt{M!}}|0\rangle.$$

$$H = J\sum_{j=1}^{N}(a^\dagger_j a_{j+1} + a_j a^\dagger_{j+1}).$$

Ideal Gas: Bosons hop independently

$$a^\dagger_{\frac{N+1}{2}} \rightarrow \sum_{j=1}^{N} f_j(t) a_j^\dagger$$

- $f_j(t) =$ amplitude of the transfer of a single boson from the $\frac{N+1}{2}$th site to the jth site in time t.

$$|\Psi(t)\rangle = \frac{(\sum_{j=1}^{N} f_j(t) a_j^\dagger)^M}{\sqrt{M!}} |0\rangle.$$

where $f_j(t)$ are identical to that of a XY spin chain:

$$f_j(t) = \frac{2}{N+1} \sum_{k=1}^{N} \{\sin \frac{\pi k}{2} \sin \frac{\pi k j}{N+1}\} e^{i2Jt\cos\frac{k\pi}{N+1}}.$$

State of the ends

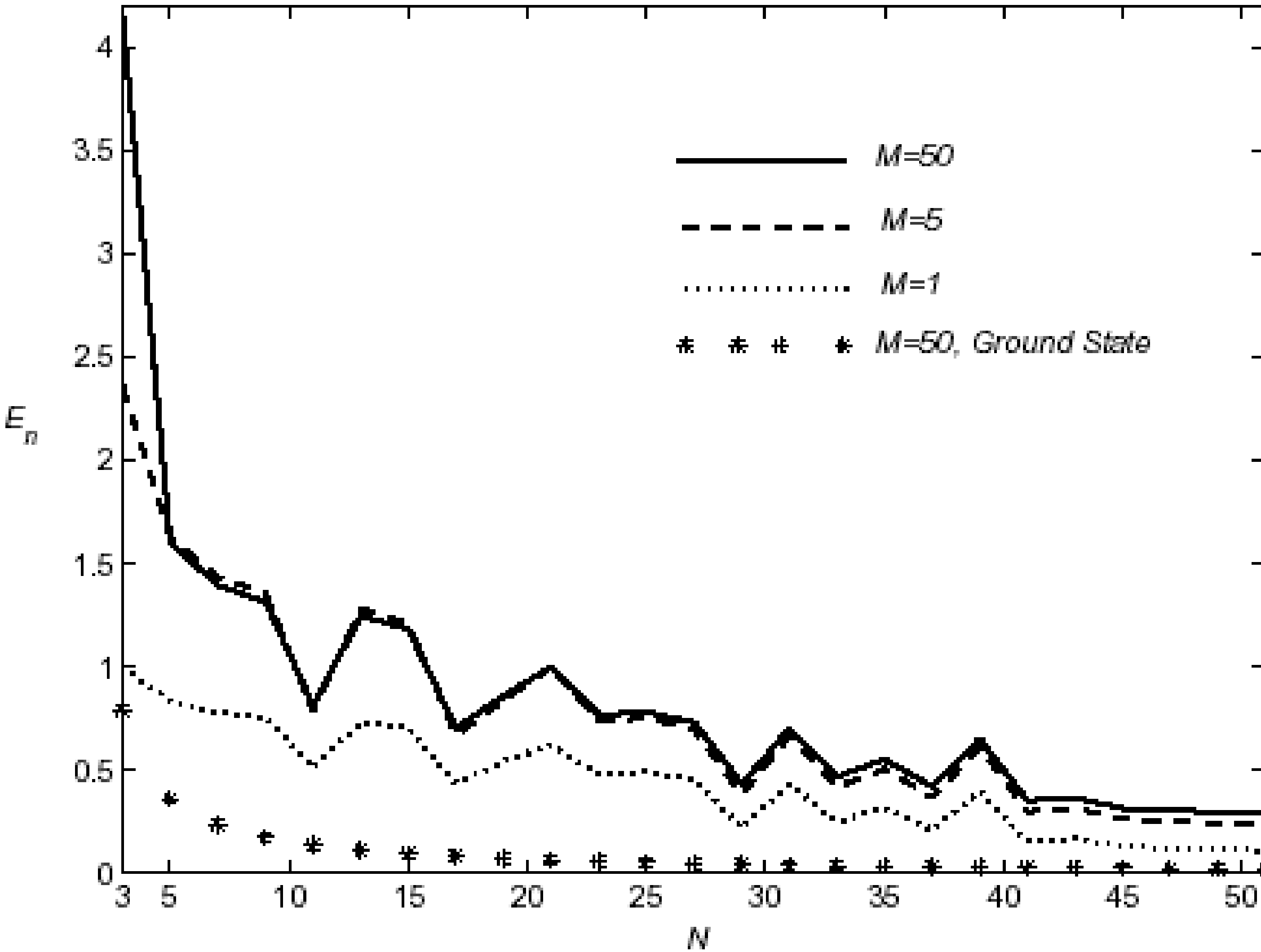
$$\rho(t)_{1N} = \sum_{r=0}^{M} P_r(t) |\psi_r\rangle\langle\psi_r|_{1N}, \quad (1)$$

where $P_r(t) = {}^M C_r (2|f_1(t)|^2)^r (1 - 2|f_1(t)|^2)^{M-r}$.

$$|\psi_r\rangle_{1N} = \frac{1}{2^{r/2}} \sum_{k=0}^{r} \sqrt{{}^r C_k} |k\rangle_1 |r-k\rangle_N. \quad (2)$$

- Time dependence of the state $\rho(t)_{1N}$ stems from $f_1(t)$
- For $N = 3$ (smallest non-trivial lattice), sites 1 and 3 go to the pure state $|\psi_M\rangle_{13}$ at $t = \pi/2J\sqrt{2}$.

Entanglement from dynamics only



Entanglement from dynamics plus measurements

- At an optimal time, we measure the populations of sites 2 to $N-1$
- Average von Neumann entropy of entanglement: $\langle E_v \rangle = \sum_r P_r E_v(|\psi_r\rangle_{1N})$
- Analytic approximation: At $t \sim (N + 0.81 N^{1/3})/4J$, $\langle E_v \rangle \sim \log_2\{1.7\sqrt{M\pi e}/N^{1/3}\}$
- An order of magnitude increase of M gives $\log_2 \sqrt{10} = 1.66$ more singlets
- For 10 singlets across a distance of ~ 1000 lattice sites we require $M \sim 10^7$

Physical Impl. of Spin Chain Comm.

- **Arrays of superconducting qubits:** Romito, Fazio, Bruder, PRB 71, 100501(R) (2005)— Charge qubits; Lyakhov, Bruder, NJP 7 (2005) 181 —- flux qubits.
- **Quantum Dot Arrays:** Irene D'Amico, cond-mat/0511470, Microelec. J. **37**, 1440 (2006). — with excitons. Spiller, D'Amico, Lovett, NJP 9, 20 (2007)— entanglement gene+dis in forked chains.
- **Coupled Cavities in PBGs:** Bose, Angelakis, Burgarth, recent quant-ph (April 2007) – To appear in JMO.
- **NMR** J. Fitzsimons, *et. al.*, quant-ph/0606188 (Phys. Rev. Lett, to appear) — Ising chains; J. Zhang, *et.al.*, quant-ph/07060352 –End gates scheme; P. Cappellaro, *et.al.*, quant-ph/07060483.
- **Electrons in Penning Traps:** G. Ciaramicoli, *et.al.*, Phys. Rev. A **75**, 032348 (2007).

Work Covered in this talk:

- S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).
- D. Burgarth & S. Bose, Phys. Rev. A **71**, 052315 (2005).
- K. Shizume, K. Jacobs, D. Burgarth and S. Bose, Phys. Rev. A **75**, 062328 (2007).
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- A. Bayat and S. Bose, arXiv:0706.4176v2 [quant-ph] (2007).
- S. Bose, arXiv:cond-mat/0610024v1 [cond-mat.other] (2006).
- S. Bose, Contemporary Physics **48**, Issue 1, pages 13 - 30 (2007).

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