Benford Law detects quantum phase transitions similarly as earthquakes

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Benford Law detects quantum phase transitions similarly as earthquakes

A. Sen(De) & US, EPL'11

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QIC group, Harish-Chandra Research Institute, Allahabad

Outline

- Simon Newcomb (1881) & Frank Benford (1938)
- Benford law
- Violation parameter
- Detecting earthquakes by Benford law
- Detecting QPT by Benford law

First of all ...

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There are at least two Benford laws!

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One states that …

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Passion is inversely proportional to the amount of ***real*** information available.

First of all …

There are at least two Benford laws!

One states that …

Passion is inversely proportional to the amount of ***real*** information available.

We will discuss here the other one!

A simple question

Are the first digits of numbers,
in nature and mathematics,
randomly distributed?

A simple question

significant

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in nature and mathematics,

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A simple question

significant

Are the first digits of numbers,

in nature and mathematics,

randomly distributed?

1.1

0.01

123

0.1

0.000013

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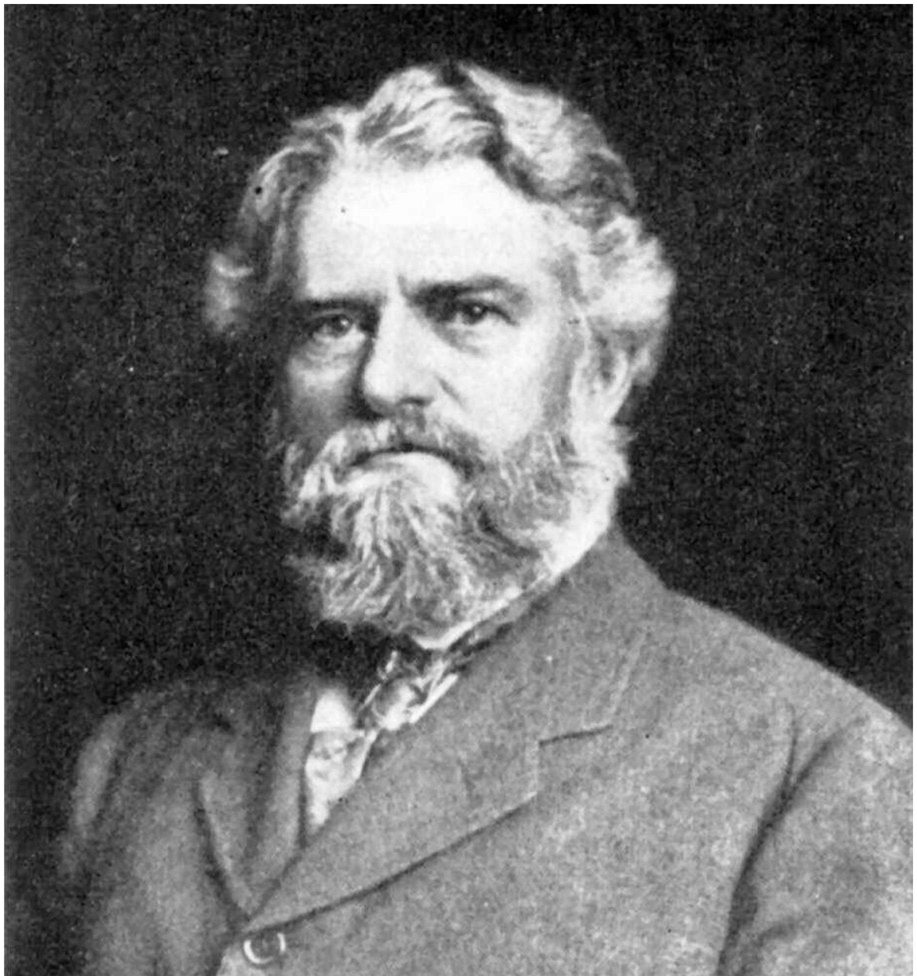
1881



Simon Newcomb

- Observed that the earlier pages of log tables are more worn out.
- Proposed that in any list of data from any source, the first digit will be more often 1.

Newcomb's conjecture

The logarithms of numbers are equally distributed, instead of the numbers themselves.

Newcomb's conjecture

(for single-digit numbers)

A number x starts with digit 1 if $1 <= x < 2$

with digit 2 if $2 <= x < 3$

…

with digit 9 if $9 <= x < 10$

Newcomb's conjecture

A number x starts with digit 1 if $\log 1 <= \log x < \log 2$

with digit 2 if $\log 2 <= \log x < \log 3$

…

with digit 9 if $\log 9 <= \log x < \log 10$

Newcomb's conjecture

A number x starts with digit 1 if $\log 1 \le \log x < \log 2$

with digit 2 if $\log 2 \le \log x < \log 3$

…

with digit 9 if $\log 9 \le \log x < \log 10$

0.301

Newcomb’s conjecture

A number x starts with digit 1 if $\log 1 <= \log x < \log 2$

with digit 2 if $\log 2 <= \log x < \log 3$

…

with digit 9 if $\log 9 <= \log x < \log 10$

0.301

Log is base 10.

Newcomb’s conjecture

A number x starts with digit 1 if log1 <= logx < log2

with digit 2 if log2 <= logx < log3

…

with digit 9 if log9 <= logx < log10

0.301

0.046

Newcomb's conjecture

A number x starts with digit 1 if $\log 1 <= \log x < \log 2$

If logs are equally distributed, it is more likely for a log to fall here than here!

with digit 9 if $\log 9 <= \log x < \log 10$

0.301

0.046

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1st significant digit is

the distance between log(D) and log(D+1)

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1st significant digit is

$$\log(D+1) - \log(D)$$

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1^{st} significant digit is

$$\log(1+1/D)$$

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1st significant digit is

log(1+1/D)

a) The base of the logarithm is 10.

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1^{st} significant digit is

$$\log(1+1/D)$$

b) The base changes if numbers are not written in decimal system.

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1st significant digit is

$$\log(1+1/D)$$

c) Have shown the law holds if logs are equally distributed.
But only for single-digit numbers.
Can be shown for higher-digit numbers also.

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1st significant digit is

$$\log(1+1/D)$$

d) total distance = log10 – log1 = 1 = total probability.
So, normalization is not needed.

Newcomb's conjecture

If logs are equally distributed,
prob that x has D as 1st significant digit is

$$P(D)=\log(1+1/D)$$

Newcomb's conjecture

If logs are equally distribut
prob that x has D as 1st sign

Moral:
Next time you spill tea on a library book,
don't feel guilty –
your act may just inspire
scientists of future generations.

That was in 1881.

There was apparent silence on this front
for the next half a century …

1938

- Frank Benford
- Checked it for a wide variety of data sets.
- The law is known after him, as Newcomb's conjecture had gone unnoticed.

1938

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- Checked it for a wide variety of data sets.
- The law is known after him, as Newcomb's

It has since been checked for a huge variety of data.

1938

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These include …

1938

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- Checked it for a wide variety of data sets.
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rotation frequencies of pulsars

1938

- Frank Benford
- Checked it for a wide variety of data sets.
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national greenhouse gas emission amounts

1938

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- Checked it for a wide variety of data sets.
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depths of earthquakes

1938

- Frank Benford
- Checked it for a wide variety of data sets.
- The law is known after him, as Newcomb's

global infectious disease cases

1938

- Frank Benford
- Checked it for a wide variety of data sets.
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326 fundamental physical constants

1938

- Frank Benford
- Checked it for a wide variety of data sets.
- The law is known after him, as Newcomb's

masses of extrasolar planets

1938

- Frank Benford
- Checked it for a wide variety of data sets.
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global monthly-averaged temp anomalies
from 1880 to 2008

1938

- Frank Benford
- Checked it for a wide variety of data sets.
- The law is known after him, as Newcomb's

and what not …

308, 32, 32, 32, 70, 913, 195, 398, 136, 189, 99, 128, 94, 602, 96, 214, 71, 325, 159, 126, 169, 202, 400, 165, 124, 145, 63, 383, 96, 192, 247, 268, 147, 133, 195, 294, 294, 232, 304, 219, 211, 324, 20*, 303, 144*, 110, 121, 131, 172, 512, 96, 48, 16*, 96.

Total # of pages of the books on the highest shelf of our flat

308, 32, 32, 32, 70, 913, 195, 398, 136, 189, 99, 128, 94, 602, 96, 214, 71, 325, 159, 126, 169, 202, 400, 165, 124, 145, 63, 383, 96, 192, 247, 268, 147, 133, 195, 294, 294, 232, 304, 219, 211, 324, 20*, 303, 144*, 110, 121, 131, 172, 512, 96, 48, 16*, 96.

Frequency chart

First digit	Actual rel frequency
1	.3704
2	.1852
3	.1852
4	.0370
5	.0185
6	.0370
7	.0370
8	0
9	.1296

Frequency chart

First digit	Actual rel frequency	Benford prediction
1	.3704	.3010
2	.1852	.1761
3	.1852	.1249
4	.0370	.0969
5	.0185	.0792
6	.0370	.0669
7	.0370	.0580
8	0	.0512
9	.1296	.0458

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Violation parameter

- O(D) = observed frequency of digit D

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  $= NP(D)$, N is sample size

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Deviation = [O(D) ~ E(D)]

Violation parameter

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Relative deviation = [O(D) ~ E(D)]/E(D)

Violation parameter

- O(D) = observed frequency of digit D
- E(D) = expected frequency of digit D

  = NP(D), N is sample size

Total relative deviation = [O(D) ~ E(D)]/E(D), summed over D

Violation parameter

- O(D) = observed frequency of digit D
- E(D) = expected frequency of digit D

  = NP(D), N is sample size

Violation parameter = [O(D) ~ E(D)]/E(D), summed over D

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Violation parameter = 5.7883

Frequency chart

First digit	Actual rel frequency	Benford prediction
1	.3704	.3010
2	.1852	.1761
3	.1852	.1249
6	.0370	.0669
7		580
8	0	.0512
9	.1296	.0458

Violation parameter = 5.7883

Sample size = 54

1995-1998

- Some mathematical insights into the law have been obtained due to T.P. Hill.

Applications

- Fraud detection. Fraudulent tax return fillings tend to have the first significant digit as random.

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- Malpractices in elections as seen in election results.

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2010

- Sambridge, Tkalcic, & Jackson, in Geophys. Res. Lett.

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- Earthquake detection by Benford law.

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- Detects earthquakes *after* it has occurred.

2010

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- Earthquake detection by Benford law.

- Detects earthquakes *after* it has occurred.

- Potential for use to detect small trembles
  before the bigger one.

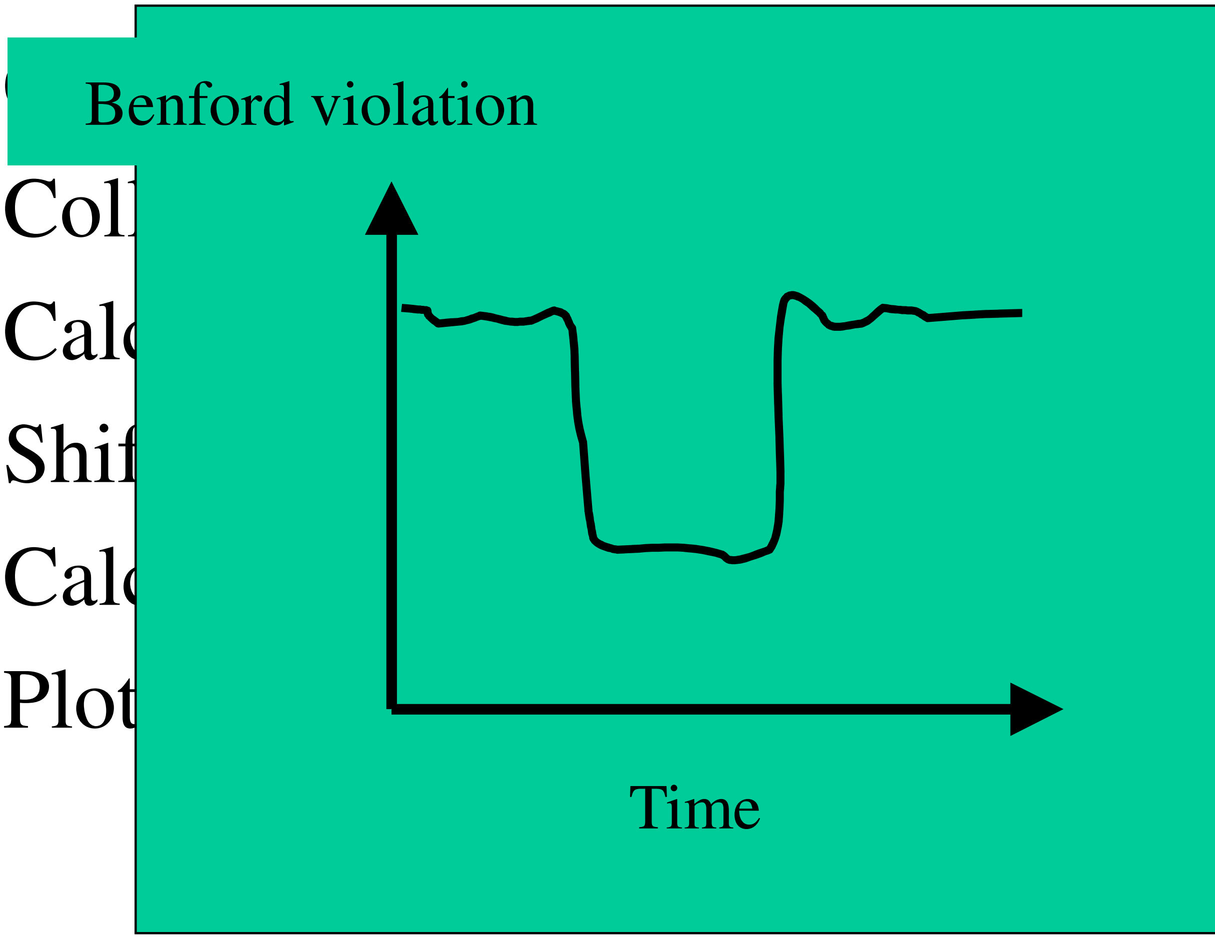
Earthquake detection method

- Characteristic of seismograph pointer.
- Collect data for a certain *time window*.
- Calculate Benford violation parameter.

- Shift time window by a small value.
- Calculate violation parameter again, …
- Plot violation vs. midpoint of window.

A glimpse of the earthquake detection method

- … er.
- Col… ow.
- Calc… ter.
- Shif… .
- Calc… .
- Plot… ow.



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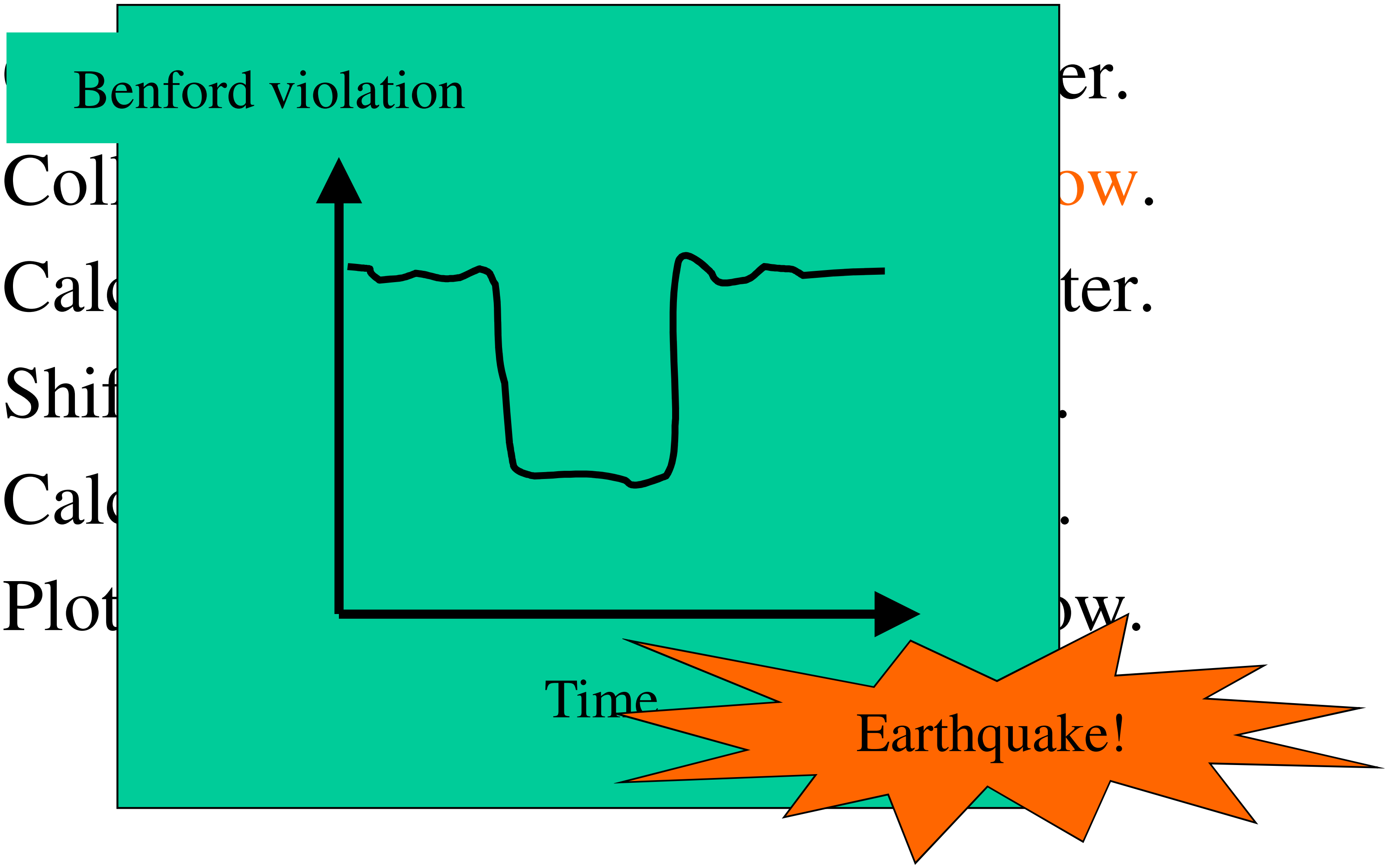

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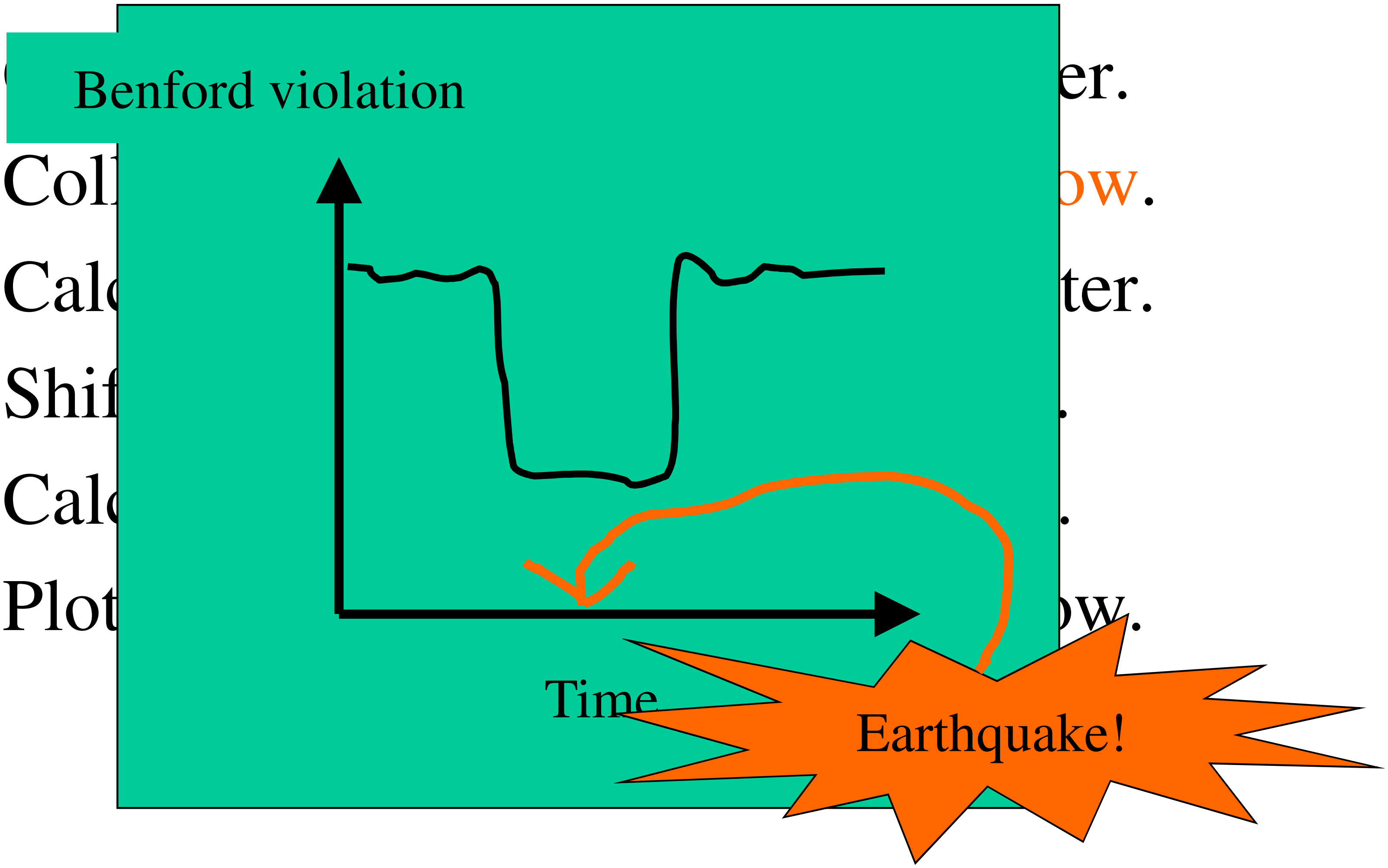
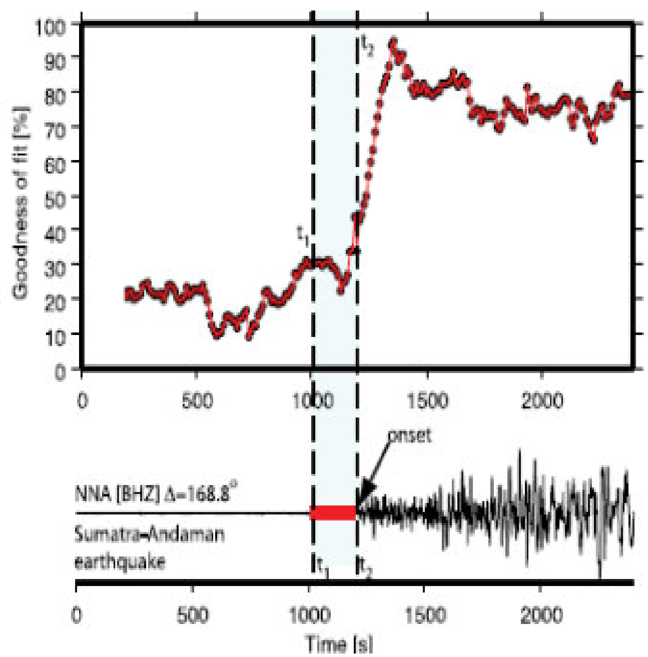
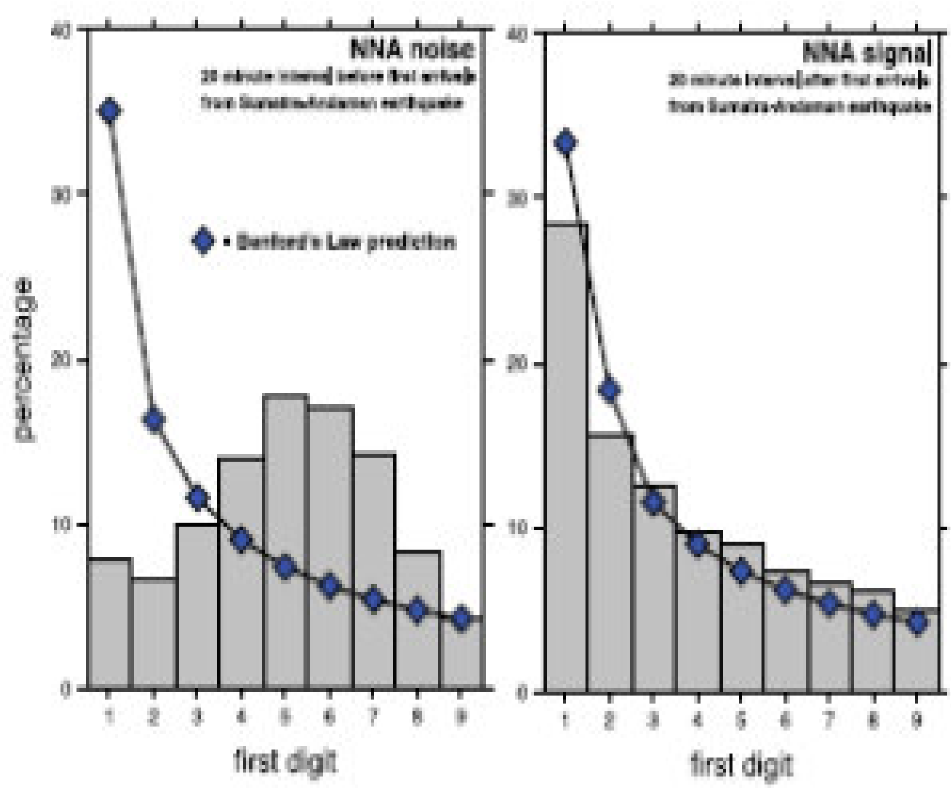


Sumatra-Andaman earthquake December 2004



Sumatra-Andaman earthquake December 2004



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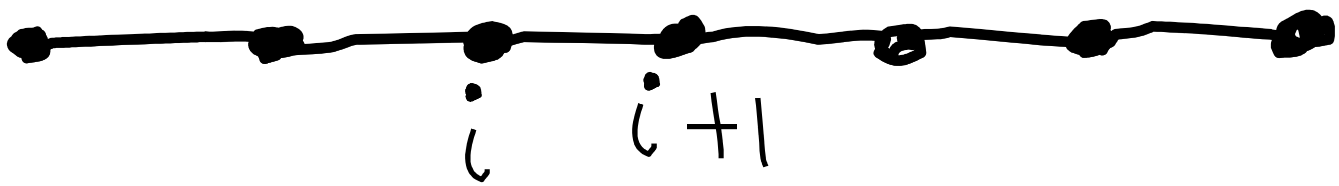
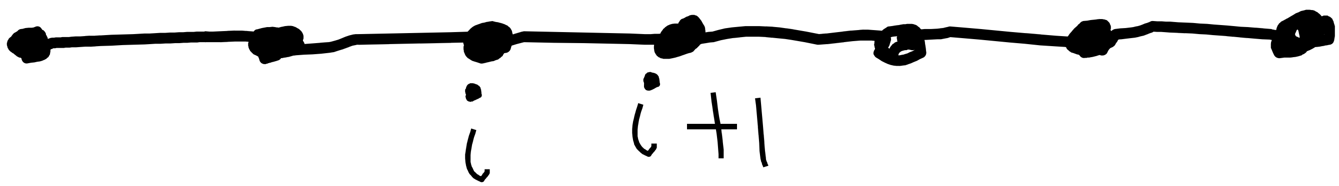
Quantum XY spin model



$$\Sigma\, J\, [(1 + \gamma)\, S_x^{i} S_x^{i+1} + (1 - \gamma)\, S_y^{i} S_y^{i+1}] - a\, S_z^{i}$$

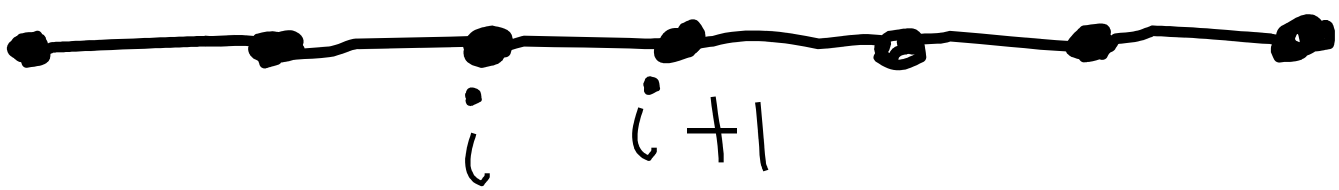
Quantum XY spin model



$$\Sigma J \,[(1 + \gamma)\, S_x^{i} S_x^{i+1} + (1 - \gamma)\, S_y^{i} S_y^{i+1}] - a\, S_z^{i}$$

S are half of Pauli matrices.

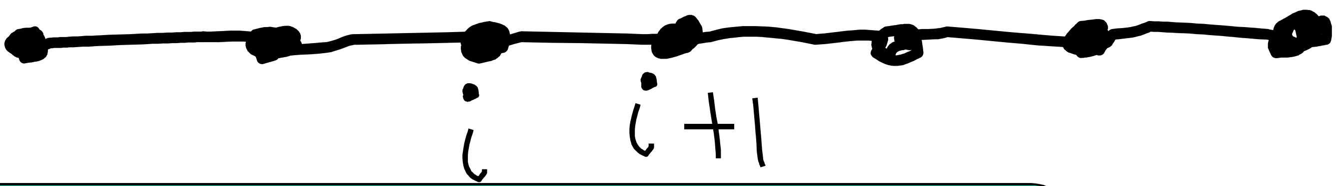
Quantum XY spin model



$$\Sigma J \left[(1 + \gamma)\, S_x^{i} S_x^{i+1} + (1 - \gamma)\, S_y^{i} S_y^{i+1}\right] - a\, S_z^{i}$$

Quantum phase transition at h=1.

Quantum XY spin model



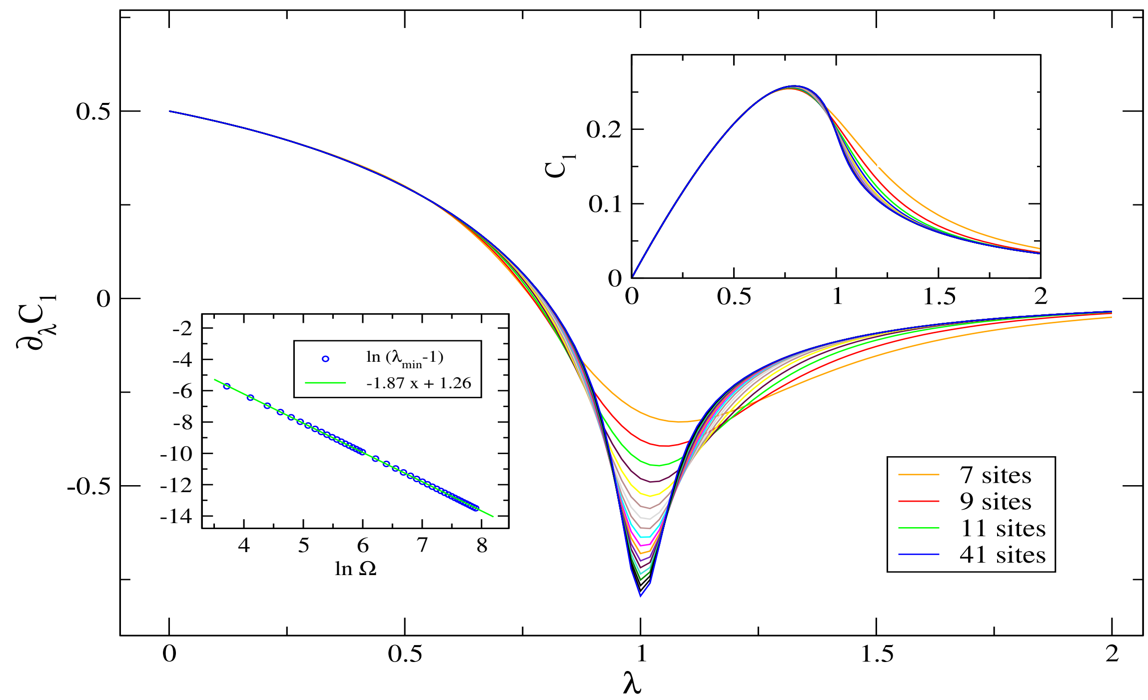
For $\gamma = 1$: Transverse Ising Model.

$$\Sigma\, J\, [(1 + \gamma)\, S_x^i S_x^{i+1} + (1 - \gamma)\, S_y^i S_y^{i+1}] - a\, S_z^i$$

Quantum phase transition at h=1.

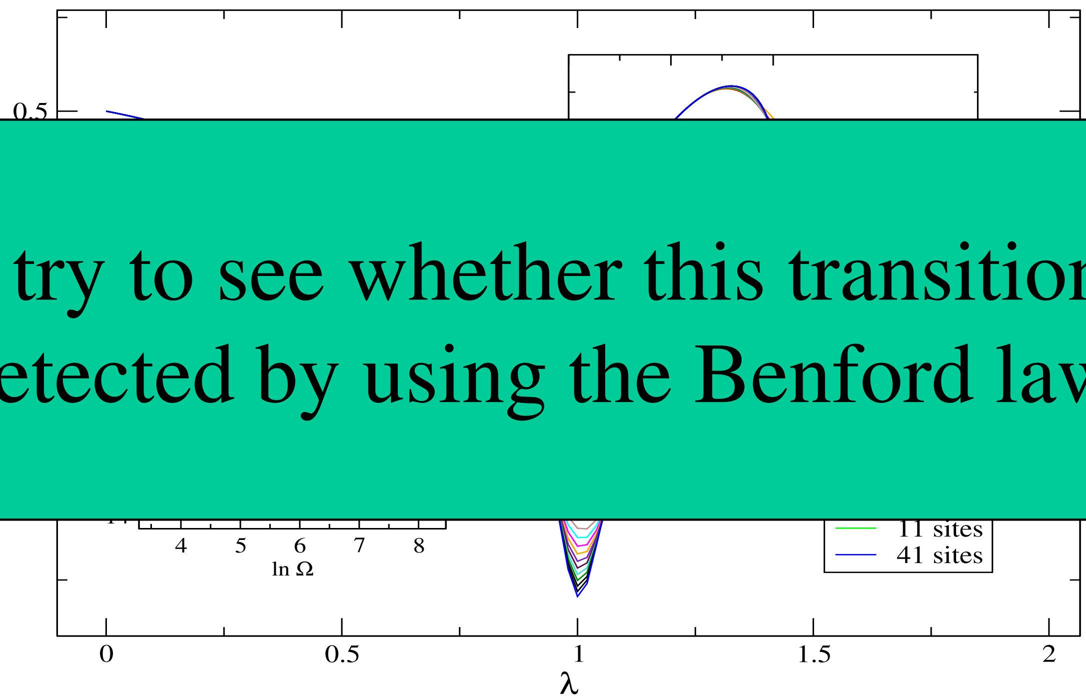
There r many ways to see the QPT …

QPT of transverse Ising



Osterloh, Amico, Falci, & Fazio, Nature 2002;
Osborne & Nielsen, Phys. Rev. A 2002.

QPT of transverse Ising



We now try to see whether this transition can be detected by using the Benford law.

Osterloh, Amico, Falci, & Fazio, Nature 2002;
Osborne & Nielsen, Phys. Rev. A 2002.

Violations and trivial violations

Violations and trivial violations

- Benford law is known to be *not* universally satisfied.

Violations and trivial violations

- Benford law is known to be *not* universally satisfied.
- But, some violations, like the following, are "trivial".

Violations and trivial violations

CESU promises to supply 220V.

Violations and trivial violations

Of course, there are some fluctuations.

Violations and trivial violations

The fluctuation is + or – 10V.

Violations and trivial violations

Any reading will have 2 as the first digit.

Violations and trivial violations

An obvious but trivial violation!

Violations and trivial violations

A way out is to shift and scale the data
to the range (0,1).

Violations and trivial violations

For any random variable X,
we find min and max.

Violations and trivial violations

Shift and scale X to obtain Y.

Y = (X – min)/(max-min).

Ignore the 0 and 1 obtained.

Violations and trivial violations

Thereafter, check status of Benford law
for the random variable Y.

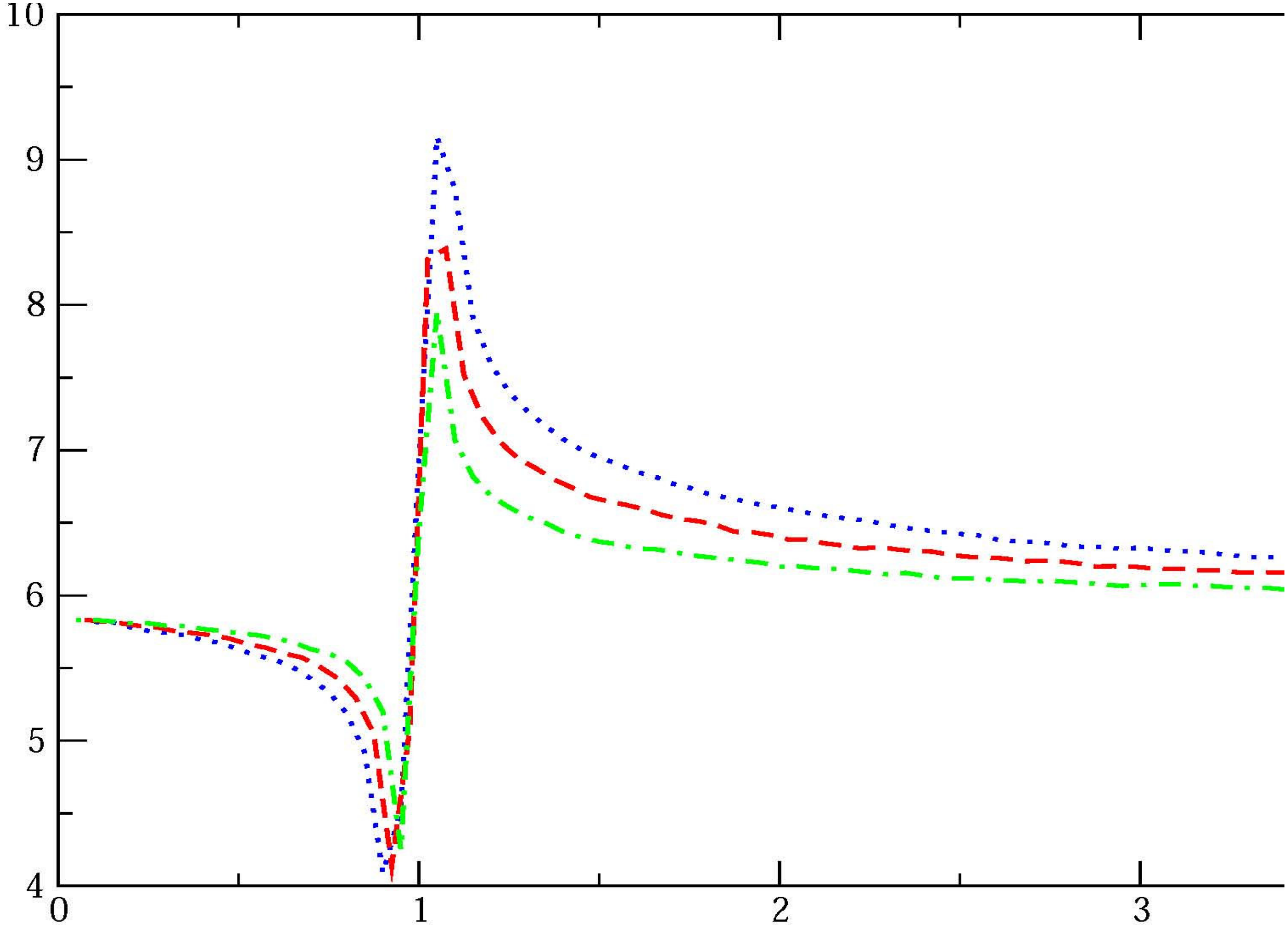
Violations and trivial violations

We call Y as “Benford X”.

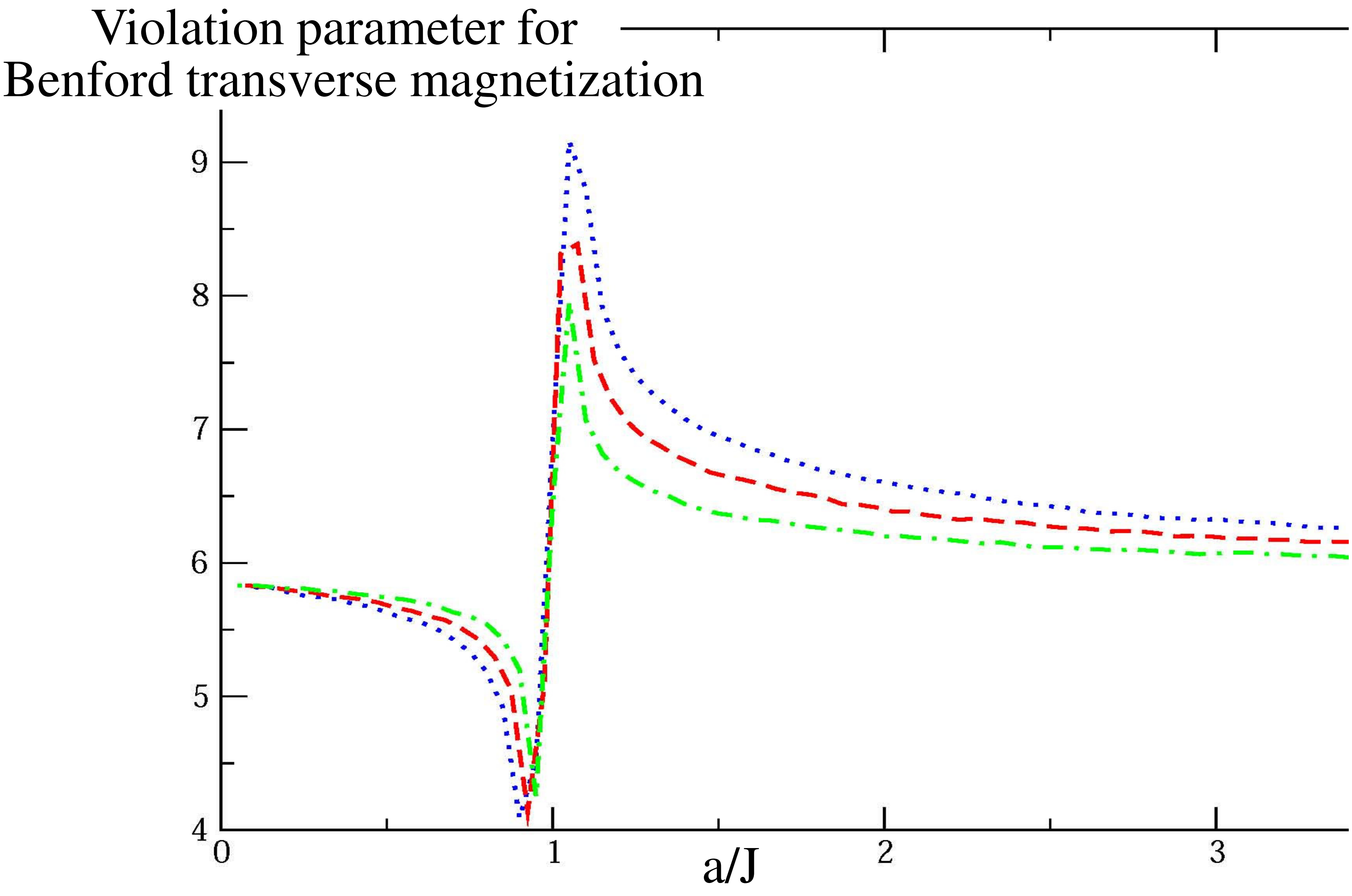
Benford law detects QPT in TIM

We replace the shifting time window by a shifting field window.

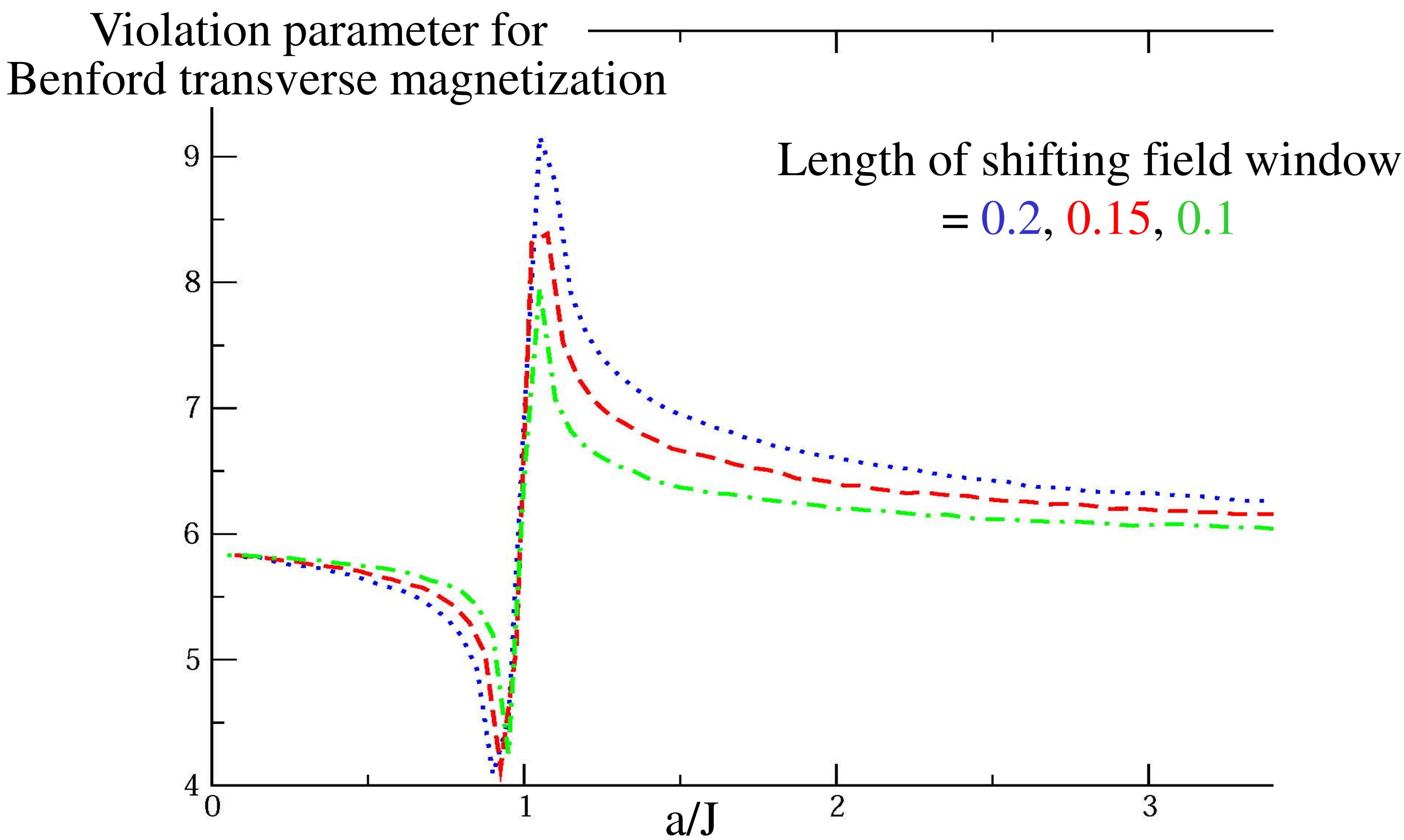
Benford law detects QPT in TIM



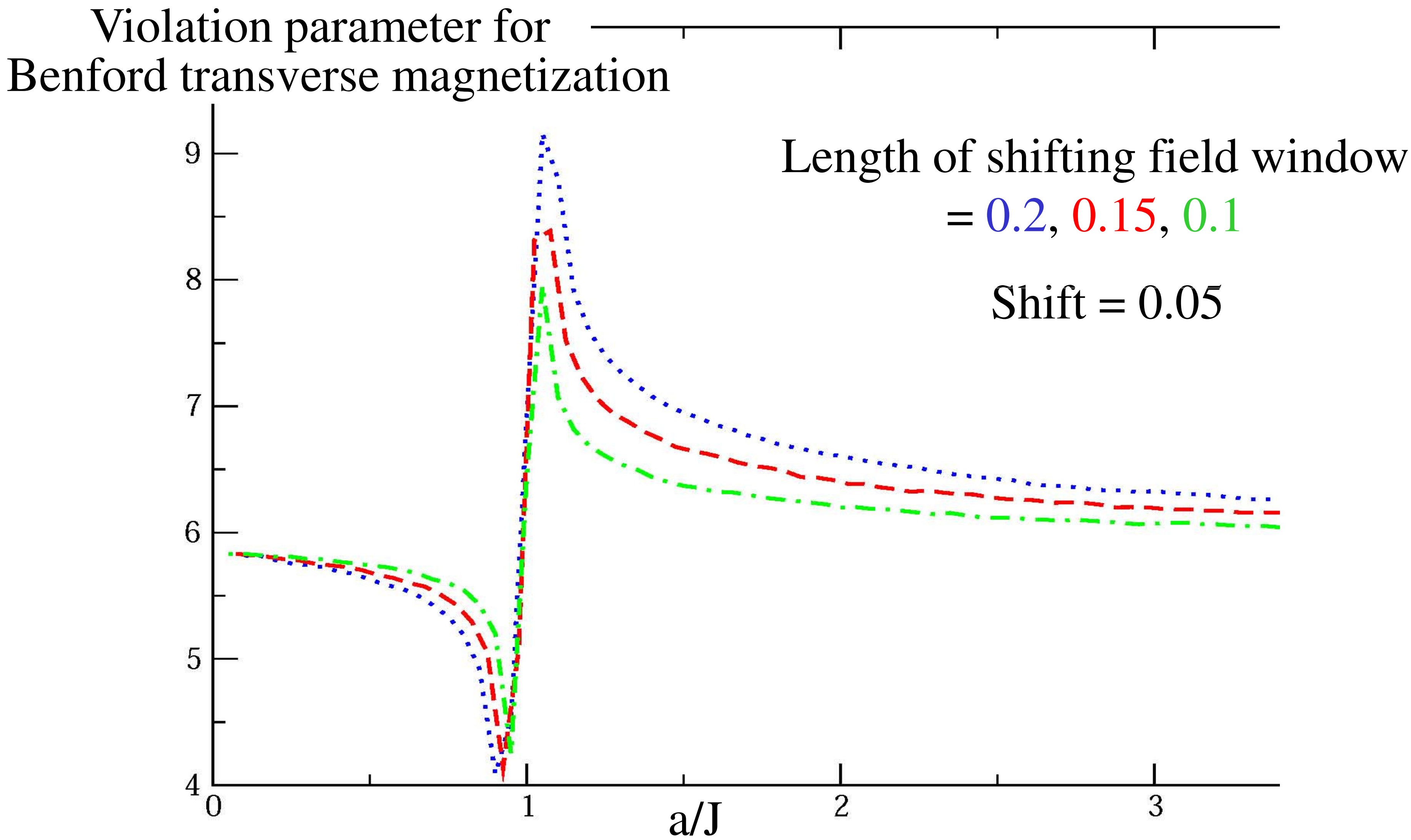
Benford law detects QPT in TIM



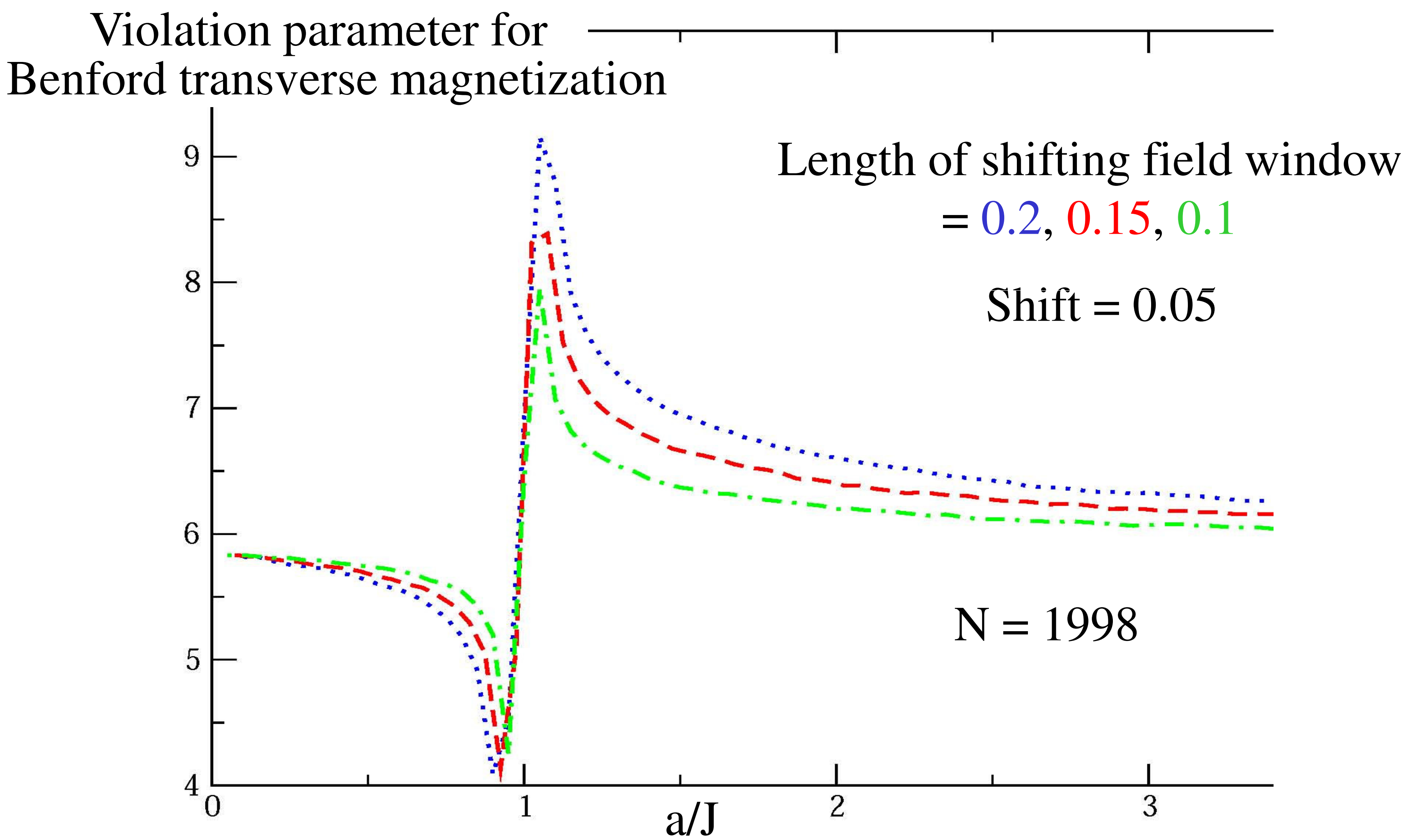
Benford law detects QPT in TIM



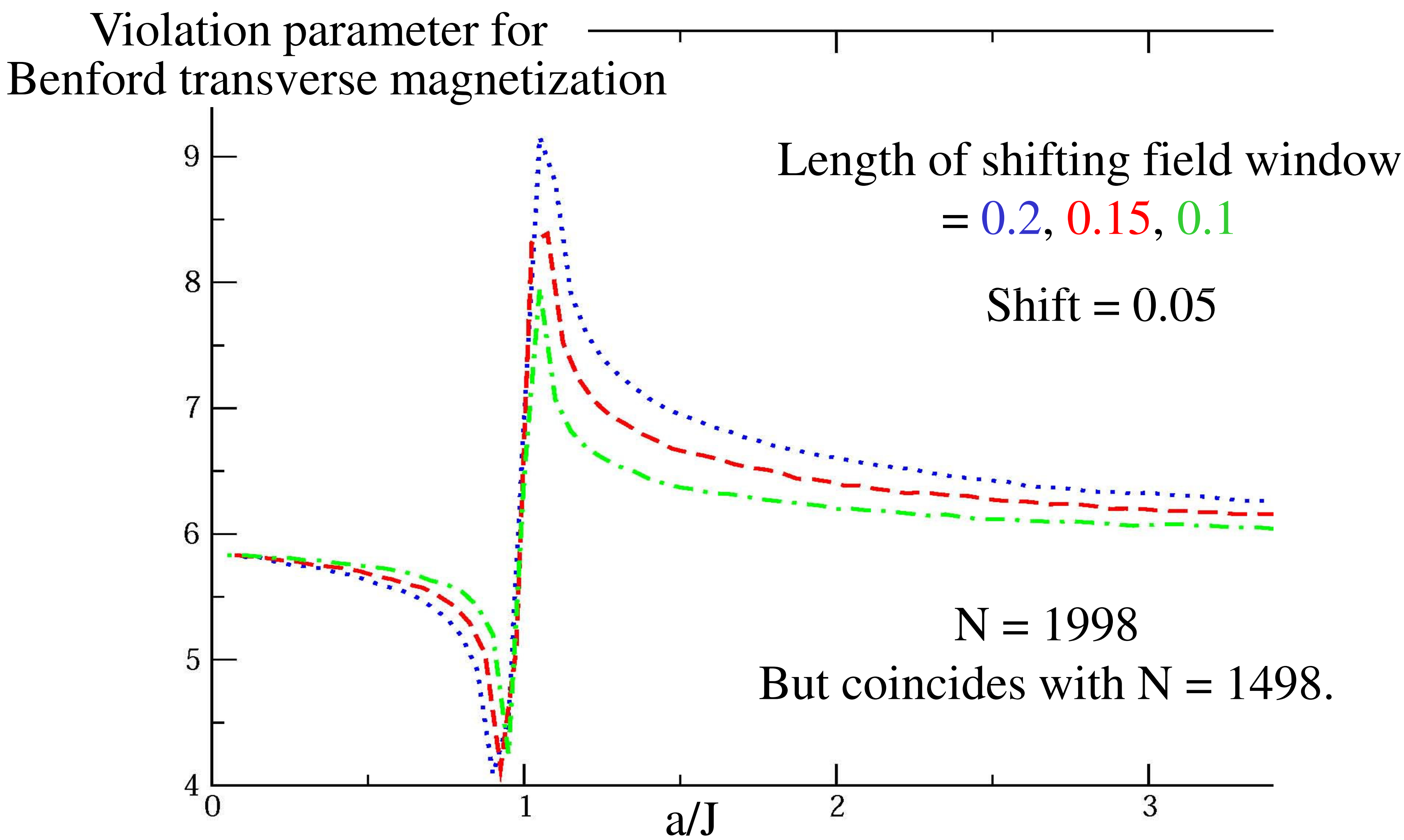
Benford law detects QPT in TIM



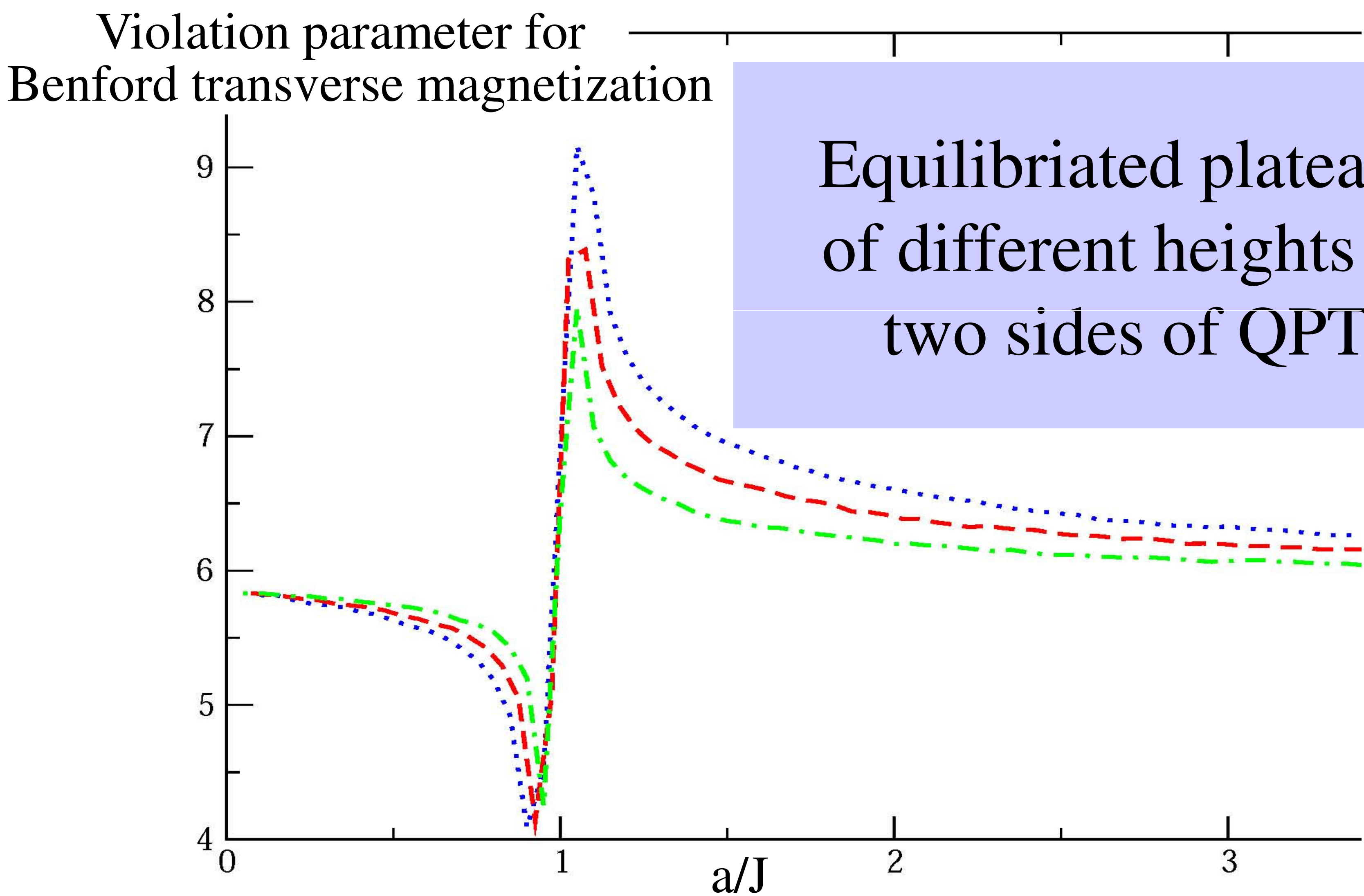
Benford law detects QPT in TIM



Benford law detects QPT in TIM

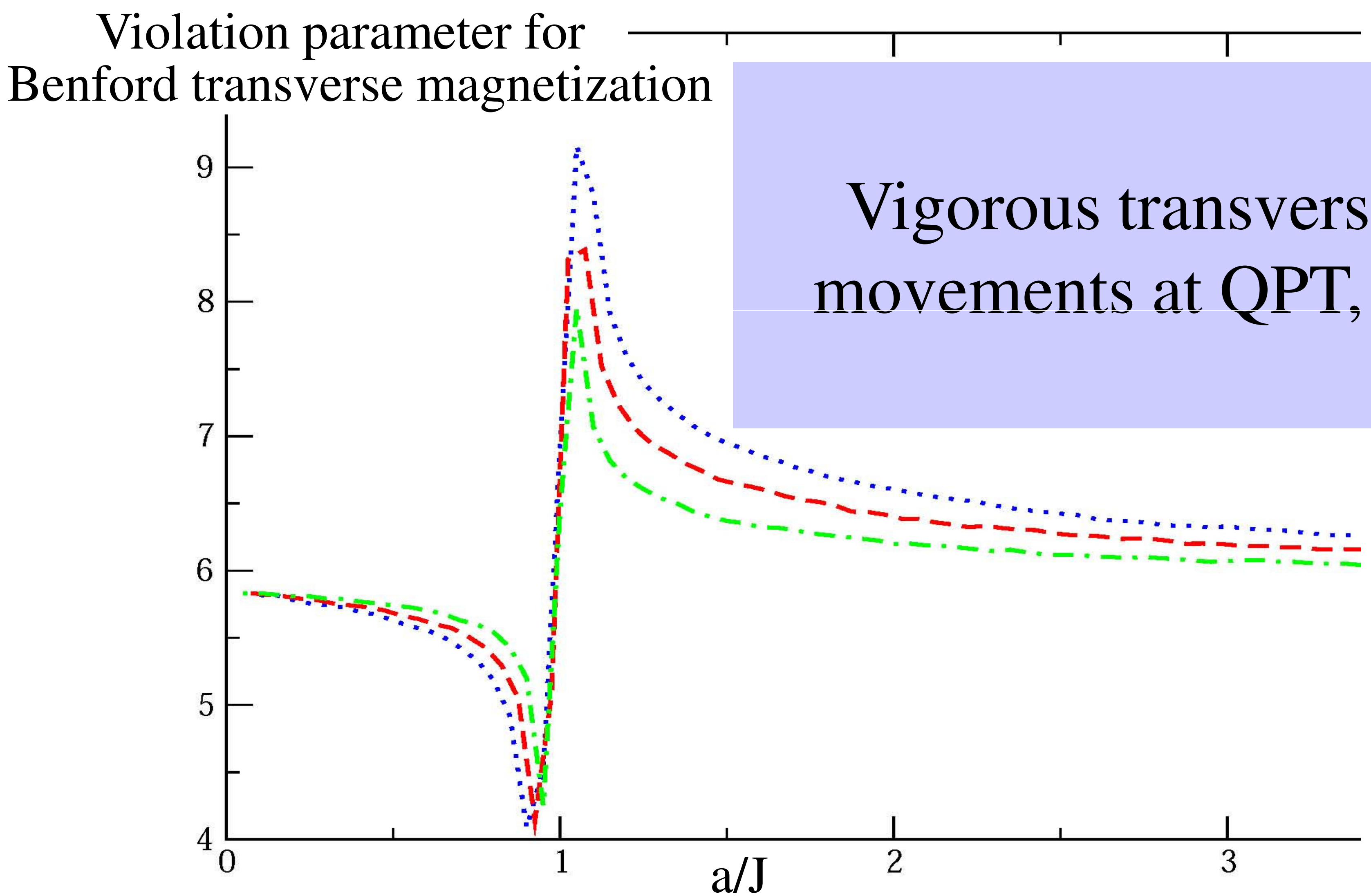


Benford law detects QPT in TIM



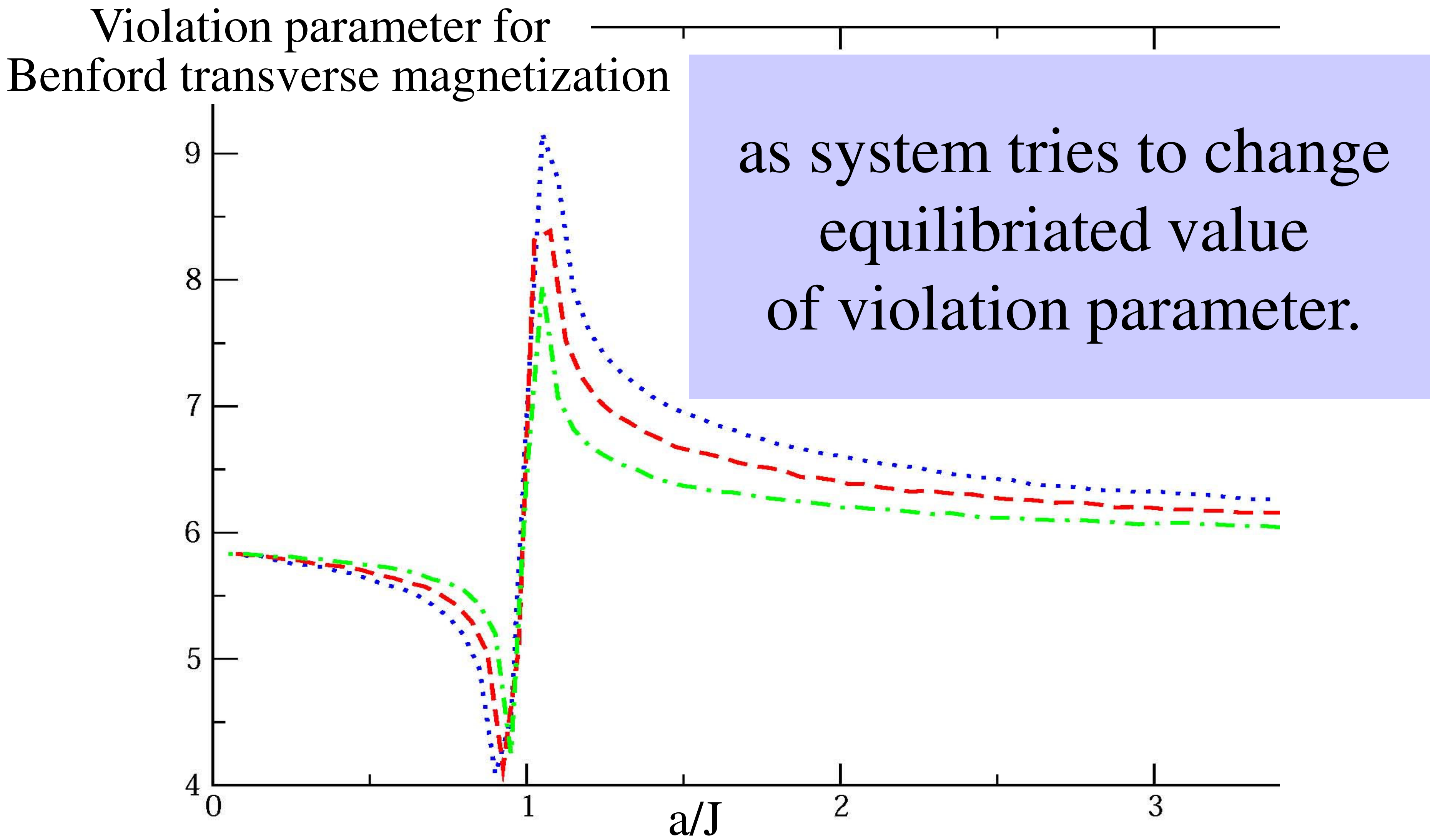
Equilibriated plateaus of different heights on two sides of QPT.

Benford law detects QPT in TIM



Vigorous transverse movements at QPT, …

Benford law detects QPT in TIM



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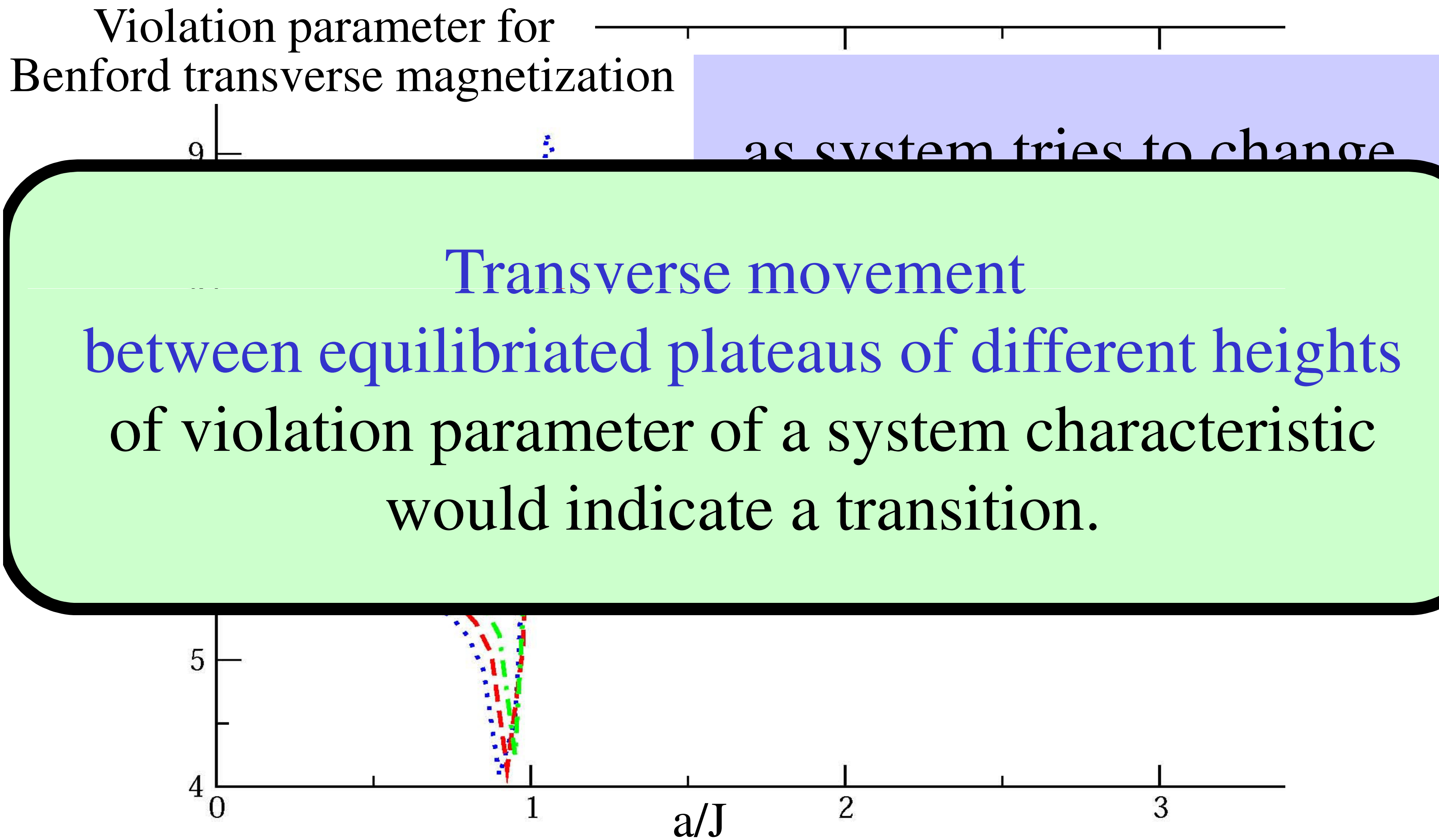


Transverse movement
between equilibriated plateaus of different heights
of violation parameter of a system characteristic
would indicate a transition.

Frequency distribution of 1st digits

Violation parameter is a characteristic of the frequency distribution of the 1st digits.

The distribution itself hides further info.

Frequency distribution of 1st digits

a/J $\in$ (0.82,0.9)

a/J $\in$ (1.1,1.18)

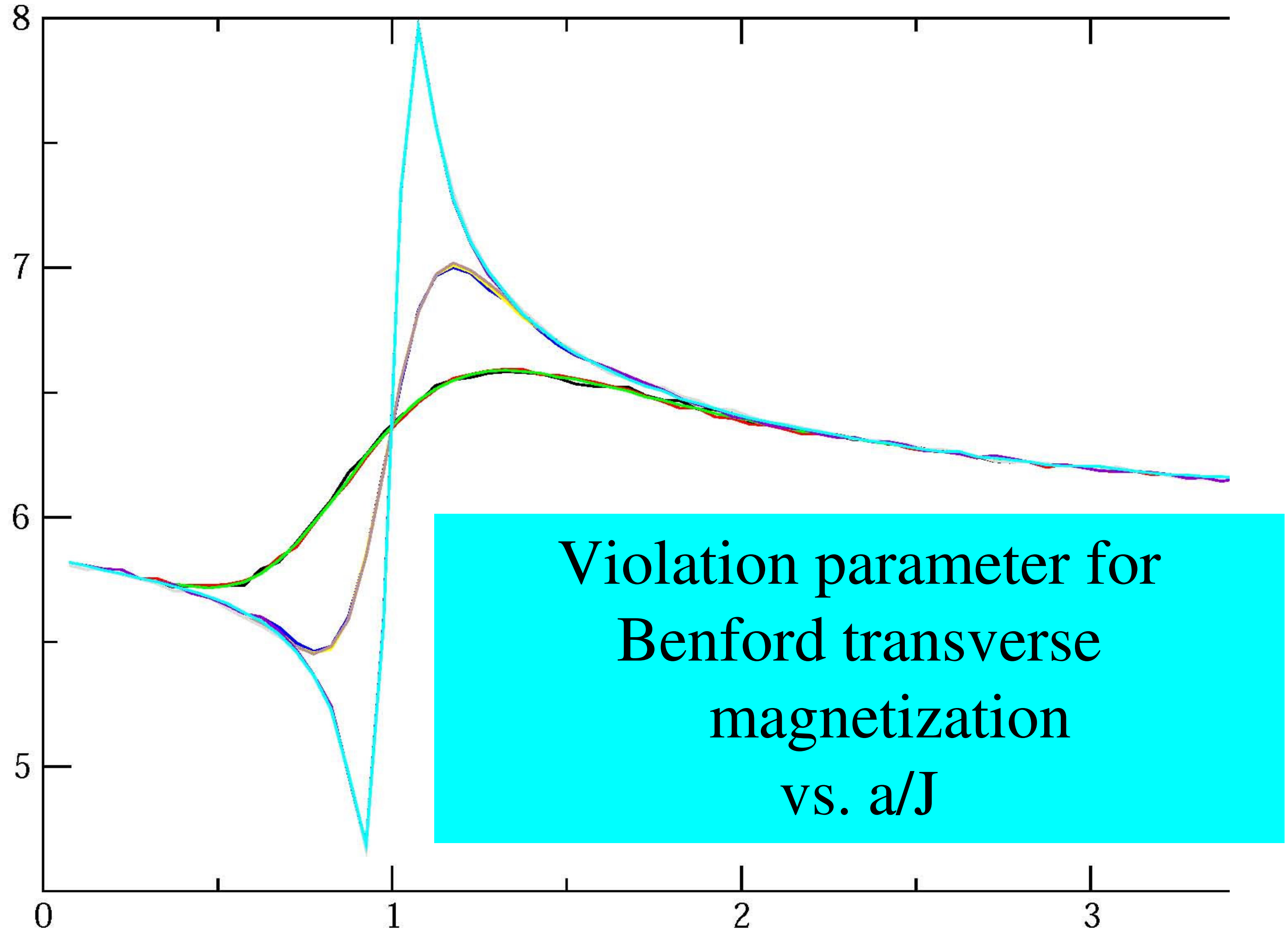
N = 1998

Benford law detects QPT in TIM

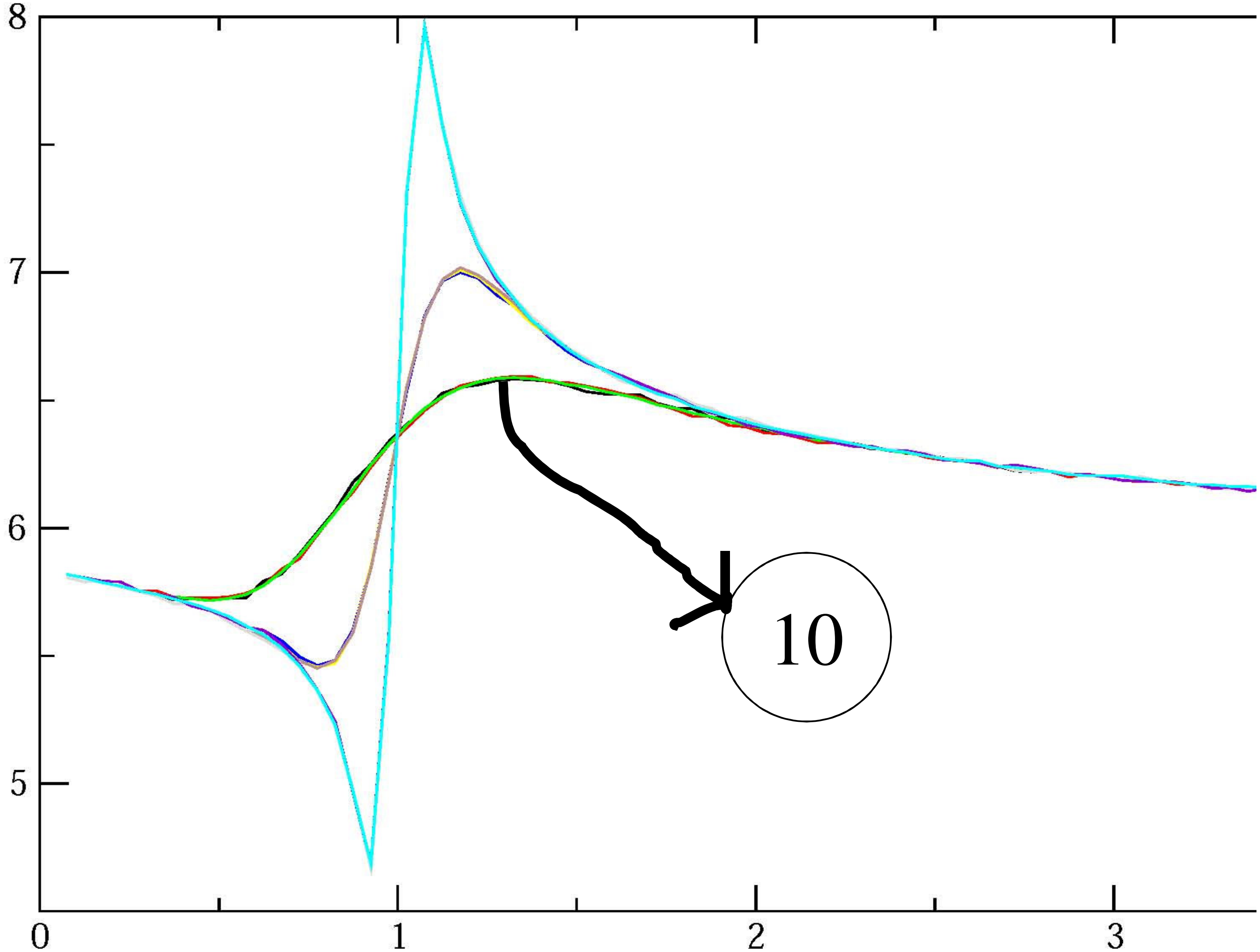
- Have also checked by using

a. nearest-neighbor classical correlations
b. single-site von Neumann entropy
c. nearest-neighbor entanglement

Finite systems



Finite systems



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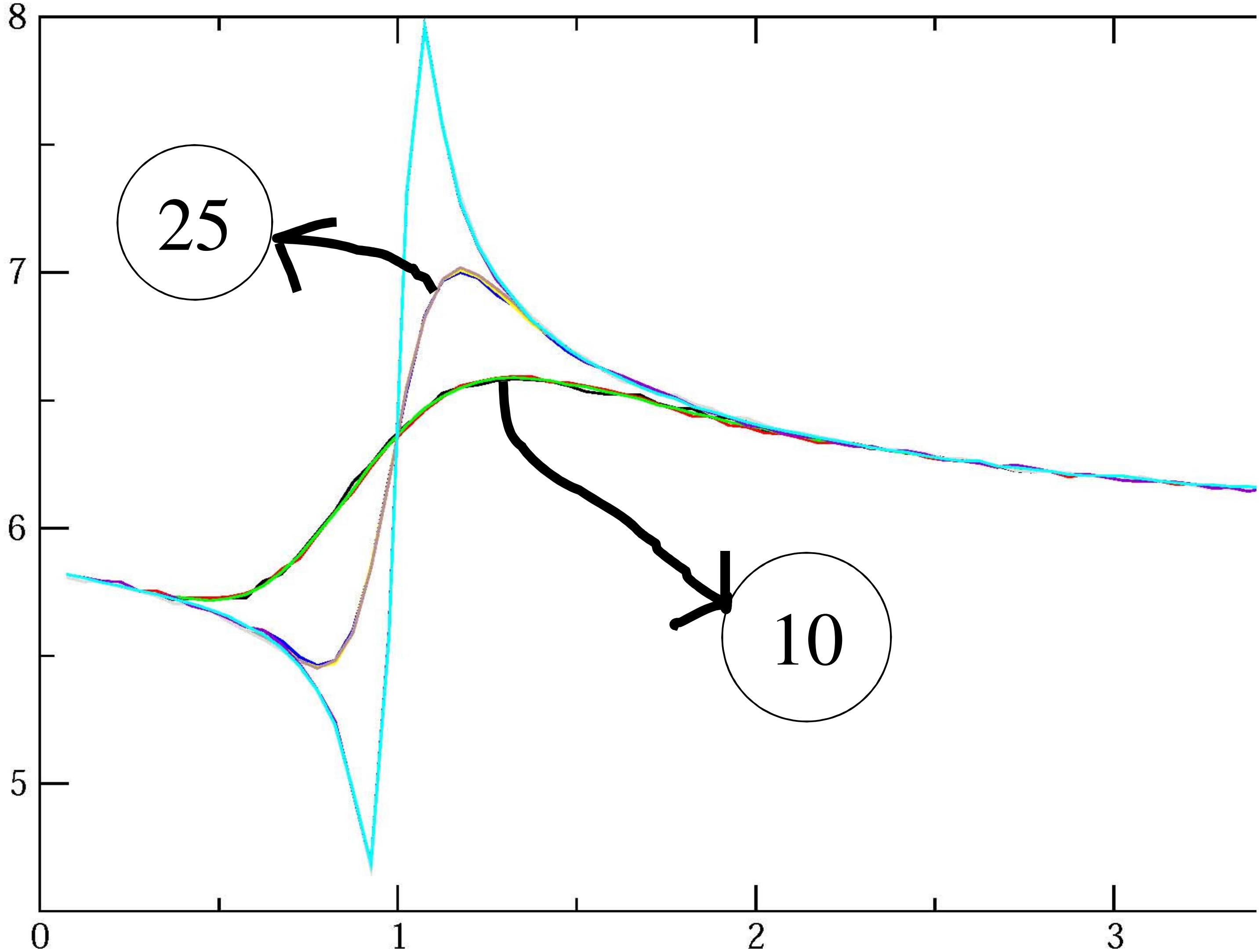
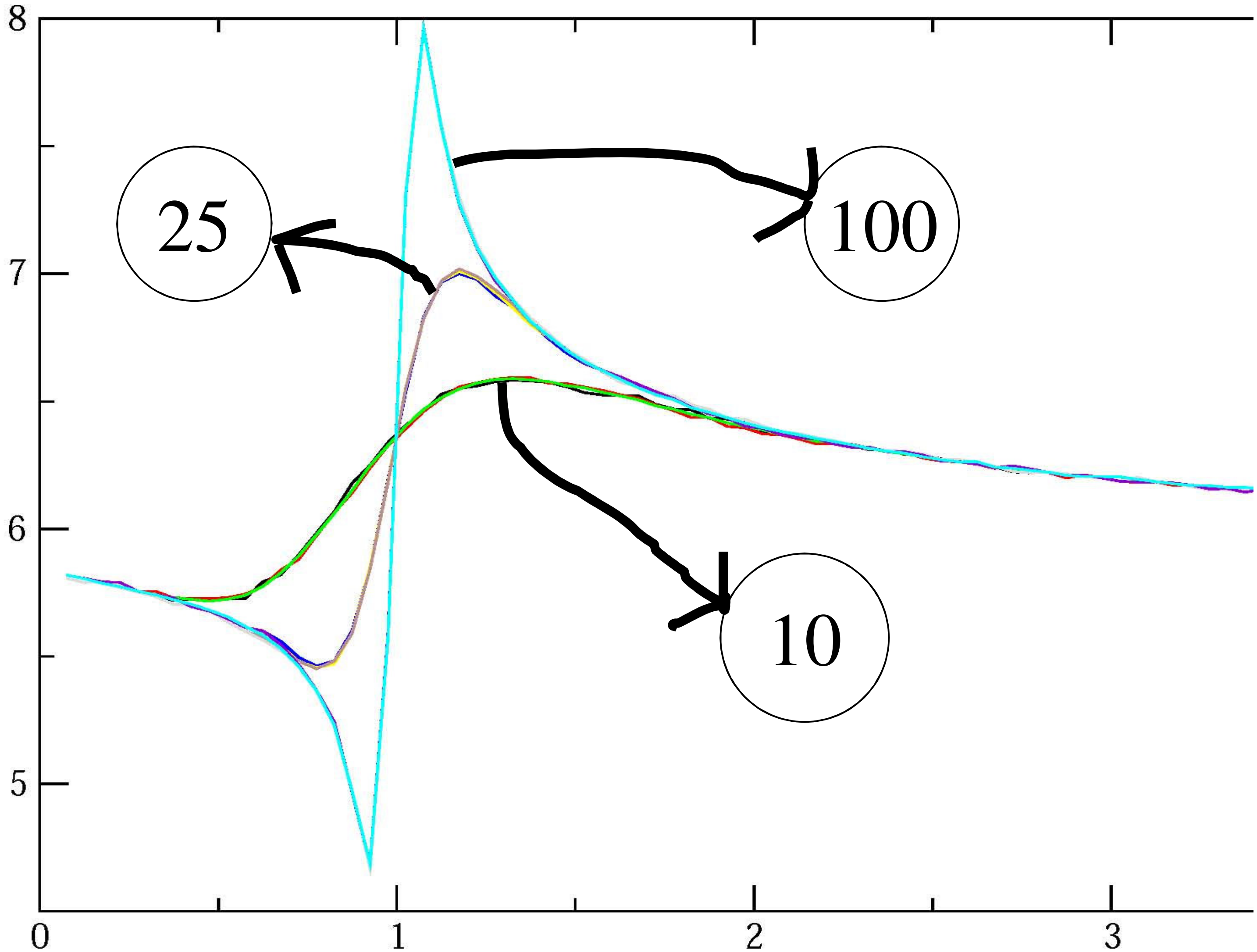
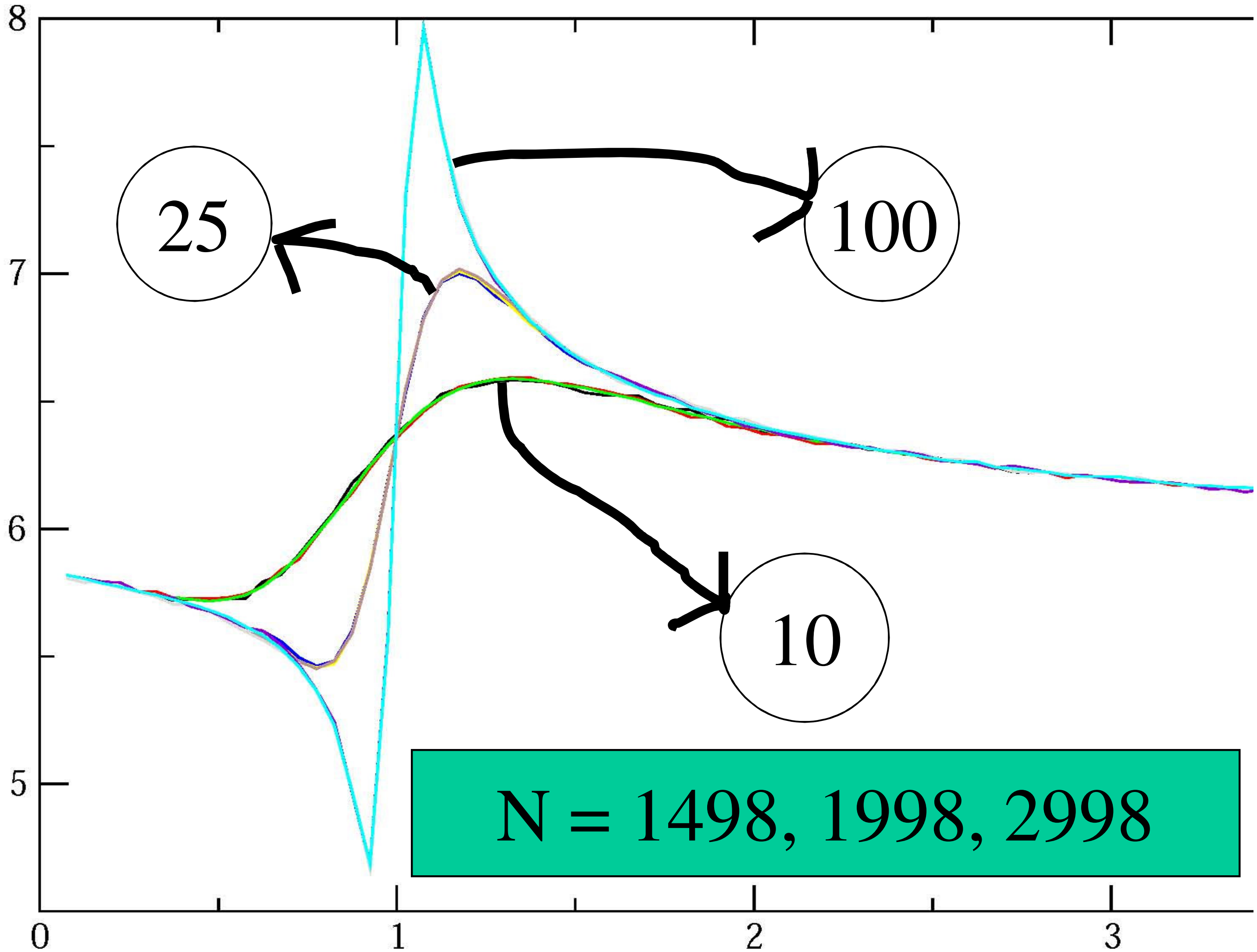


Finite systems



Finite systems



In summary …

- Benford law is interesting.
- Benford law can detect QPT.
- The method of detection is similar to that of detecting earthquakes.

12345678910

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