

Interplay of Classical and Quantum Correlations in Many-Body Systems

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Collaborators: R. Prabhu, Ujjwal Sen









• What is entanglement?







- What is entanglement?
- Quantum many-body systems and entanglement ---brief overview





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 overview
 Lectures in School





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- Opposite statistical behavior between entanglementseparability and information-theoretic paradigms





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mechanical

• Opposite statistical behavior between entanglementseparability and information-theoretic paradigms





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Why Entanglement?

- Entanglement is useful in
- Quantum Communication ---
 - sending classical info via quantum states -- Dense coding (Bennett, Wiesner, PRL 1992)



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- Quantum Cryptography-- secure info transfer (*Ekert*, *PRL*, 1991)



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Quantum Computation----

• One-way quantum computer (*Briegel, Raussendorf, PRL2003*)



• Ion trap





R. Blatt's group in Innsbruck









Anil Kumar's group in Bangalore





• Photon



Zeilinger in Vienna;



Gisin in Geneva



• Optical Lattice





I. Bloch's group in Munich



And so on



- Photon
- Ion
- Neutral atoms in optical lattice
- NMR
- Cavity QED

So on.....





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Why this connection interesting?

• Study fundamental properties by using quantum info or vice-versa.



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• Study fundamental properties by using quantum info or vice-versa.

• Realization of quantum computer by using such systems.





VOLUME 91, NUMBER 9

PHYSICAL REVIEW LETTERS

29 AUGUST 2003

Controlling Spin Exchange Interactions of Ultracold Atoms in Optical Lattices

L.-M. Duan,¹ E. Demler,² and M. D. Lukin²

¹Institute for Quantum Information, California Institute of Technology, mc 107-81, Pasadena, California 91125, USA ²Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA (Received 25 October 2002; published 26 August 2003)

We describe a general technique that allows one to induce and control strong interaction between spin states of neighboring atoms in an optical lattice. We show that the properties of spin exchange interactions, such as magnitude, sign, and anisotropy, can be designed by adjusting the optical potentials. We illustrate how this technique can be used to efficiently "engineer" quantum spin systems with desired properties, for specific examples ranging from scalable quantum computation to probing a model with complex topological order that supports exotic anyonic excitations.

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PACS numbers: 03.75.Nt, 03.67.-a, 42.50.-p, 73.43.-f





PRL 93, 250405 (2004)

PHYSICAL REVIEW LETTERS

week ending 17 DECEMBER 2004

Implementation of Spin Hamiltonians in Optical Lattices

J. J. García-Ripoll,¹ M. A. Martin-Delgado,^{1,2} and J. I. Cirac¹ ¹Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, Garching, D-85748, Germany ²Universidad Complutense de Madrid, Fac. de CC. Físicas, Ciudad Universitaria, Madrid, E-28040, Spain (Received 27 April 2004; published 15 December 2004)

We propose an optical lattice setup to investigate spin chains and ladders. Electric and magnetic fields allow us to vary at will the coupling constants, producing a variety of quantum phases including the Haldane phase, critical phases, quantum dimers, etc. Numerical simulations are presented showing how ground states can be prepared adiabatically. We also propose ways to measure a number of observables, like energy gap, staggered magnetization, end-chain spins effects, spin correlations, and the string-order parameter.





PHYSICAL REVIEW LETTERS VOLUME 91, NUMBER 9 29 AUGUST 2003 week ending PHYSICAL REVIEW LETTERS PRL 93, 250405 (2004) 17 DECEMBER 2004 Controlling Spin Exchange Interactions of Ultracold Atoms in Optical Lattices L.-M. Duan,¹ E. Demler,² and M. D. Lukin² Implementation of Spin Hamiltonians in Optical Lattices ¹Institute for Quantum Information, California Institute of Technology, mc 107-81, Pasadena, California 91125, USA ²Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA J. J. García-Ripoll,¹ M. A. Martin-Delgado,^{1,2} and J. I. Cirac¹ (Received 25 October 2002; published 26 August 2003) ¹Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, Garching, D-85748, Germany We describe a general technique that allows one to induce and control strong interaction between spin ²Universidad Complutense de Madrid, Fac. de CC. Físicas, Ciudad Universitaria, Madrid, E-28040, Spain states of neighboring atoms in an optical lattice. We show that the properties of spin exchange (Received 27 April 2004; published 15 December 2004) interactions, such as magnitude, sign, and anisotropy, can be designed by adjusting the optical potentials. We illustrate how this technique can be used to efficiently "engineer" quantum spin systems We propose an optical lattice setup to investigate spin chains and ladders. Electric and magnetic fields with desired properties, for specific examples ranging from scalable quantum computation to probing a allow us to vary at will the coupling constants, producing a variety of quantum phases including the model with complex topological order that supports exotic anyonic excitations. Haldane phase critical phases quantum dimers etc. Numerical simulations are presented showing how Week ending PHYSICAL REVIEW LETTERS VOLUME 91, NUMBER 7 15 AUGUST 2003

Entangling Strings of Neutral Atoms in 1D Atomic Pipeline Structures

U. Dorner,¹ P. Fedichev,¹ D. Jaksch,¹ M. Lewenstein,² and P. Zoller^{1,2} ¹Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria ²Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany (Received 6 December 2002; published 14 August 2003)

We study a string of neutral atoms with nearest neighbor interaction in a 1D beam splitter configuration, where the longitudinal motion is controlled by a moving optical lattice potential. The dynamics of the atoms crossing the beam splitter maps to a 1D spin model with controllable time dependent parameters, which allows the creation of maximally entangled states of atoms by crossing a quantum phase transition. Furthermore, we show that this system realizes protected quantum memory, and we discuss the implementation of one- and two-qubit gates in this setup.





atomic gases or ion traps

VOLUME 92, NUMBER 20

PHYSICAL REVIEW LETTERS

week ending 21 MAY 2004

Effective Quantum Spin Systems with Trapped Ions

D. Porras* and J. I. Cirac[†]

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, Garching, D-85748, Germany (Received 16 January 2004; published 20 May 2004)

We show that the physical system consisting of trapped ions interacting with lasers may undergo a rich variety of quantum phase transitions. By changing the laser intensities and polarizations the dynamics of the internal states of the ions can be controlled, in such a way that an Ising or Heisenberg-like interaction is induced between effective spins. Our scheme allows us to build an analogue quantum simulator of spin systems with trapped ions, and observe and analyze quantum phase transitions with unprecedented opportunities for the measurement and manipulation of spins.

PRL 98, 023003 (2007)

PHYSICAL REVIEW LETTERS

week ending 12 JANUARY 2007

Trapped Ion Chain as a Neural Network: Error Resistant Quantum Computation

Marisa Pons,¹ Veronica Ahufinger,² Christof Wunderlich,³ Anna Sanpera,⁴ Sibylle Braungardt,⁵ Aditi Sen(De),⁵ Ujjwal Sen,⁵ and Maciej Lewenstein⁶ ¹Departamento de Física Aplicada I, Universidad del País Vasco, 20600 Eibar, Spain

⁵ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain ⁶ICREA and ICFO-Institut de Ciències Fotòniques, 08860 Castelldefels (Barcelona), Spain

We demonstrate the possibility of realizing a neural network in a chain of trapped ions with induced long range interactions. Such models permit one to store information distributed over the whole system. The storage capacity of such a network, which depends on the phonon spectrum of the system, can be controlled by changing the external trapping potential. We analyze the implementation of error resistant universal quantum information processing in such systems.



Two Main Directions

- Entanglement of 2-party state
- Entanglement Area Law

- Osterloh, Amico, Falci, & Fazio, Nature '02;
- Vidal, et al., PRL'03;
- Korepin, PRL'04
- Osborne & Nielsen, PRA '02.

Lewenstein, Sanpera, Ahufinger, Damski, ASD, Sen, Adv Phys. 56, 243 ('07);

Amico, Fazio, Osterloh, Vedral, RMP 80, 517 '08











Nearest Neighbor (NN) entanglement: $E(\rho_{23})$







Nearest Neighbor (NN) entanglement: $E(\rho_{23})$

1D transverse Ising:

Ground state two-site entanglement remains short ranged while correlation length diverges. Entanglement, however, does show signs of criticality.

Osterloh, Amico, Falci, & Fazio, Nature '02; Osborne & Nielsen, PRA '02.







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Entanglement wt Classical Correlations

Such multipartite states exist

D. Kaszlikowski, ASD, U. Sen, V. Vedral, A. Winter, PRL '08
Entanglement wt Classical Correlations

Such multipartite states exist

Such bipartite states do not exist

Bipartite entanglement always has a "background" of bipartite classical correlations. Bipartite entanglement always has a "background" of bipartite classical correlations.

Is it true therefore that properties of bipartite ent are always inherited from those of bipartite classical correlations?

Prop. Entanglement wt prop. of correlations









Entanglement in Dynamics R. Prabhu, ASD, U. Sen, arXiv:1103.3836







XY spin model

This model is exactly solvable. Jordan-Wigner transformation

$$H = \sum \left[(1 + \gamma) \sigma_i^x \sigma_{i+1}^x + (1 - \gamma) \sigma_i^y \sigma_{i+1}^y \right] + h(t) \sum \sigma_i^z$$



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$$+ h(t) \sum \sigma_i^z$$
External magnetic field
$$h(t) = a, t = 0$$

$$= 0, t > 0$$



Entanglement in dynamics of many body systems

Magnetization -- Barouch, McCoy, & Dresden, 1970s.

Bipartite entanglement--ASD, Sen, & Lewenstein, Phys. Rev. A (Rapid Com.) 2004.







P is nonergodic

for a given T,

$P^{\infty}(T, h(t), \gamma) \neq P^{can}(T', h(t = \infty), \gamma), \forall T'$

if



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External magnetic field
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Infinite XY Spin Chain







Infinite XY Spin Chain







Infinite XY Spin Chain E_N 0.20 0.15 Nonergodic Magnetization 0.10 Ergodic Entanglement. Entanglement 0.05 Equilibrium 🛧 βJ 0.00 a = 0.2 a = 0.6 a = 1.2Evolved 8 Mz 0.6 0.5 0.4 Magnetization 0.3 0.2 0.1 βJ 0.0



 $\overrightarrow{} Equilibrium$ $\overrightarrow{} a = 0.2$ $\overrightarrow{} a = 0.6$ $\overrightarrow{} a = 1.2$ Evolved











Nonergodic correlations and magnetization can lead to ergodic entanglement



Nonergodic correlations and magnetization can lead to ergodic entanglement



Finite XY Spin Chain





Finite XY Spin Chain



$$\overrightarrow{} Equilibrium$$

$$\overrightarrow{} a = 0.2$$

$$\overrightarrow{} a = 0.6$$

$$\overrightarrow{} a = 1.2$$
Evolved



Finite XY Spin Chain



Possible to tune magnetic field, entanglement is ergodic while its constituent quantities are all nonergodic

$$\overrightarrow{} Equilibrium$$

$$\overrightarrow{} a = 0.2$$

$$\overrightarrow{} a = 0.6$$

$$\overrightarrow{} a = 1.2$$
Evolved



Finite XY Spin Chain



Possible to tune magnetic field, entanglement is ergodic while its constituent quantities are all nonergodic

$$\overrightarrow{} Equilibrium$$

$$\overrightarrow{} a = 0.2$$

$$\overrightarrow{} a = 0.6$$

$$\overrightarrow{} a = 1.2$$
Evolved



Finite system mimics infinite one!!







Entanglement remains ergodic for high magnetic field







Entanglement remains ergodic for high magnetic field

For the same field, some correlations, magnetization remain nonergodic







Entanglement remains ergodic for high magnetic field

For the same field, some correlations, magnetization remain nonergodic

Mimicking 1D

2D Finite System





2D Finite System

∃ magnetic field for which entanglement still remains ergodic





2D Finite System

∃ magnetic field for which entanglement still remains ergodic





For the same field, magnetization remains nonergodic;

2D Finite System

 \exists magnetic field for which entanglement still remains ergodic





For the same field, magnetization remains nonergodic; $T^{zz}(a,\beta)$ is now nonergodic.



Ø-Ø-Q-Q-Q-Q-Q-

Ergodic entanglement with nonergodic correlations

Low dimensional systems,



Ergodic entanglement with nonergodic correlations



Low dimensional systems,





Ergodic entanglement with nonergodic correlations

Low dimensional systems,



0 ---- **0** --- **0** --- **0** --- **0** --- **0**



Ergodic entanglement with nonergodic correlations

Low dimensional systems,



quantum correlations can have statistical mechanical properties, which are not inherited from the same in classical correlations and magnetizations




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Quantum Correlation Measure: Discord

•Total Correlations: Mutual Information

$$I(\rho_{AB})=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$$

• $S(\sigma) = -tr(\sigma \log_2 \sigma) \equiv von Neumann entropy$



$$I(\rho_{AB})=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$$

Classical Correlations:

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$



$$I(\rho_{AB})=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$$

Classical Correlations:

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

• $S(\sigma_{A|B}) = \min_{\{B_i\}} \sum_{i} p_i S(\sigma_{A|i})$



$$I(\rho_{AB})=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$$

Classical Correlations:

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

Discord = Total - Classical = $I(\rho_{AB}) - J(\rho_{AB})$



$$I(\rho_{AB})=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$$

Classical Correlations:

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

Henderson-Vedral, J. Phys. A'01; Olliver,-Zurek, PRL'02

Discord = Total - Classical = $I(\rho_{AB}) - J(\rho_{AB})$





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Opposite Statistical Mechanical Behavior of Quantum Correlation Measures

R. Prabhu , ASD, U. Sen arXiv: 1112.1856



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$$+ h(t) \sum \sigma_i^z$$
External magnetic field
$$h(t) = a, t = 0$$

$$= 0, t > 0$$



For low field, all quantum correlation measures are ergodic in any dimension



For low field, all quantum correlation measures are ergodic in any dimension

provided
$$H = H_{int} - h(t) H_{mag}$$
 and $h(t) \begin{cases} = a, t=0 \\ = 0, t>0 \end{cases}$
 $[H_{int}, H_{mag}] \neq 0$







For high fields, all quantum correlation measures are again ergodic for infinite XY spin chain

provided
$$H = \Sigma [(1 + \gamma) \sigma_i^x \sigma_{i+1}^x + (1 - \gamma) \sigma_i^y \sigma_{i+1}^y] + h(t) \Sigma \sigma_i^z$$

and $h(t) = a, t=0$
= 0, t>0





Entanglement Measures VS.



Information - Theoretic Measures

For moderate fields,











Conclusion

Conclusion

Opposite statistical mechanical behavior of two paradigms

Conclusion

Opposite statistical mechanical behavior of two paradigms: Entanglement-separability paradigm & Information theoretic paradigm

Conclusion

Opposite statistical mechanical behavior of two paradigms: Entanglement-separability paradigm & Information theoretic paradigm

Ergodic entanglement while nonergodic discord/workdeficit

QIC Group @ HRI

























