Quantum correlations with no causal order

International Workshop
Relativistic **Q**uantum **I**nformation

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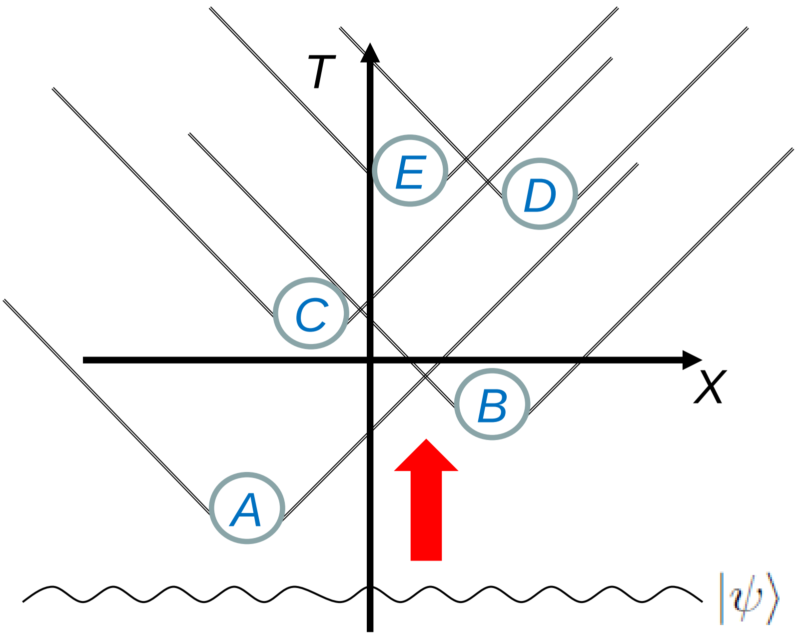
arXiv:1105.4464

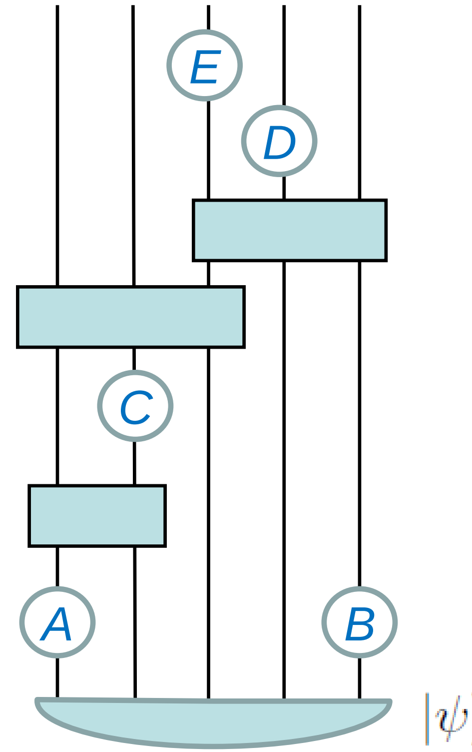
6-8 September 2011 Madrid

Measurements in space-time



- Fix positions wrt coordinates.
- Define initial state.
- Follow Eqs of motion.
- Include causal influences.
- Find joint probabilities

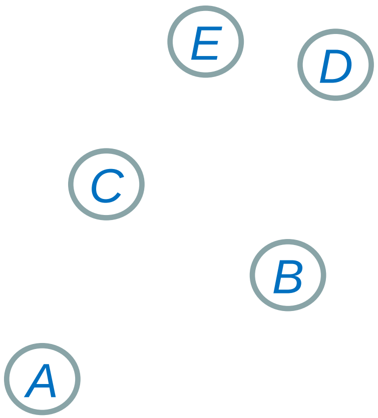
  P(A, B, C, D, E)
- Formalization as Circuit model possible



Space-<u>time</u> is a pre-existing entity

Is (quantum) physics possible without space-<u>time</u>?

Measurements in space-time



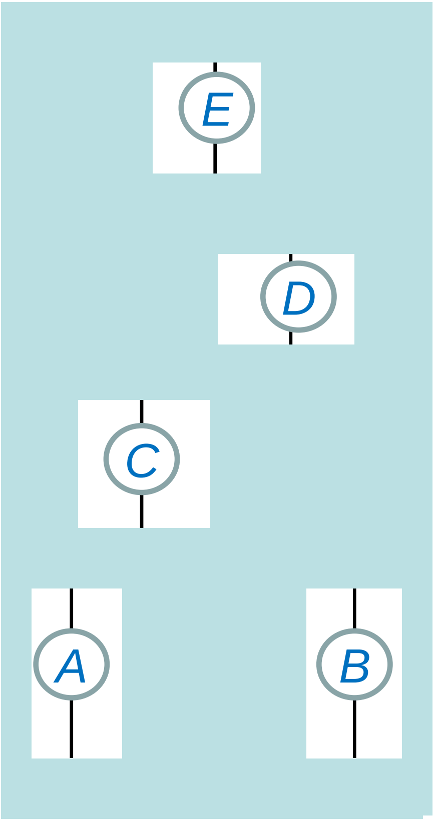
Find joint probabilities

P(A, B, C, D, E)

Formalization as

Circuit model

not possible (?)



Is (quantum) physics possible without space-<u>time</u>?

Questions

- Is a definite causal structure a necessary pre-assumption or does it follow from more primitive concepts?

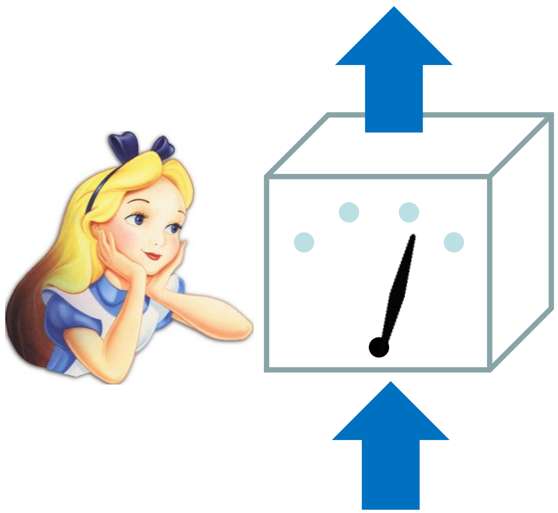
- Is it possible to define operationally well-defined theories with no time or causal structure?

- What happens if one removes time and causal structure from quantum mechanics? What new phenomenology is implied?

Outline

- “Locality” without space-time
- Most general bipartite correlations – with causal structure
- Most general bipartite correlations – with no causal structure
- Causal game → “non-causal” correlations allow to score higher than in any causal scenario
- Conclusions

“Local laboratory”



Output — The system exits the lab

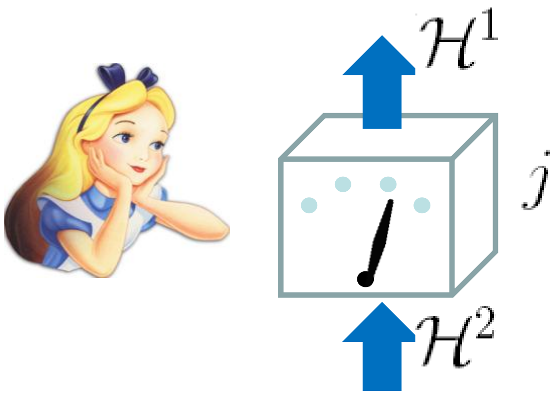
$j = 1, \ldots, n$ — An operation is performed - one out of a set of possible events is recorded

Input — A system enters the lab

This is the **only** way how the lab interacts with the “outside world”.

Local quantum laboratory

Local operations are described by quantum mechanics

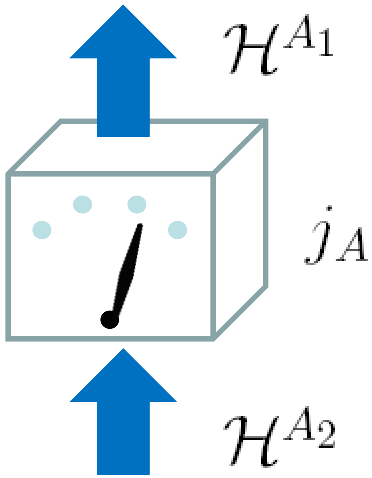


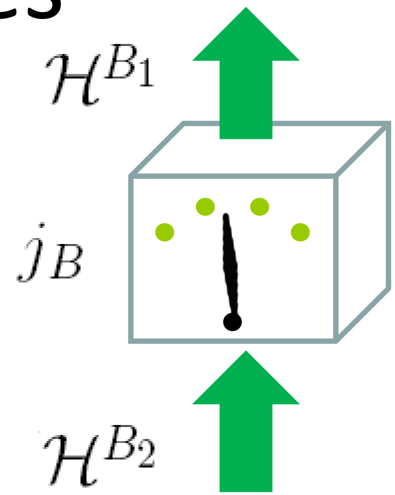
$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2)$$

Selective Measurement (non-deterministic operations)
= completely positive (CP) trace non increasing maps

Non-selective measurement (deterministic operation)= set of CP maps $\{\mathcal{M}_j\}_{j\in J}$ such that $\sum_{j\in J}\mathcal{M}_j$ is CPTP (trace preserving)

Two parties





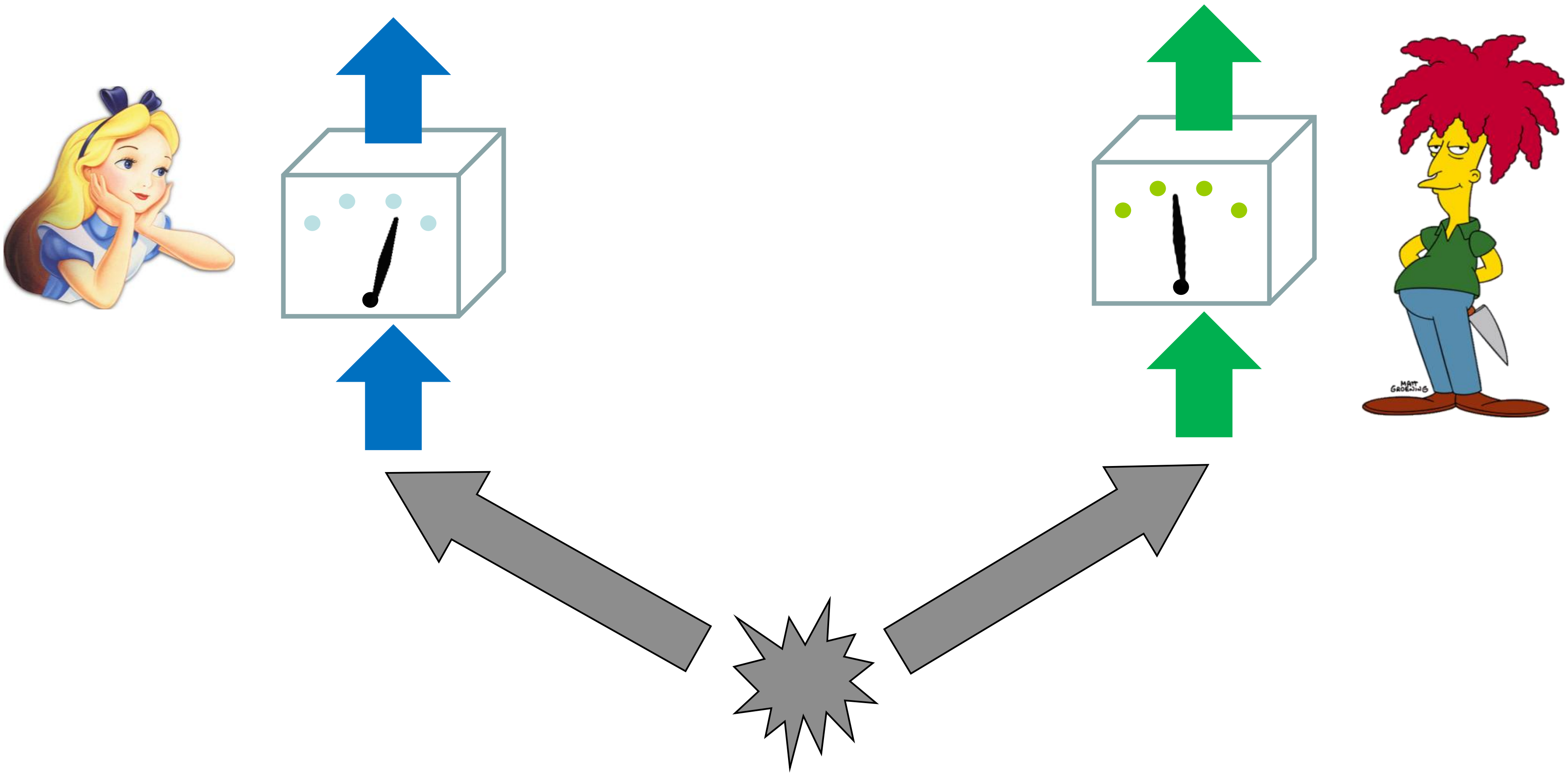
$\mathcal{M}^A_{j_A} : \mathcal{L}(\mathcal{H}^{A_2}) \longrightarrow \mathcal{L}(\mathcal{H}^{A_1})$

$\mathcal{M}^B_{j_B} : \mathcal{L}(\mathcal{H}^{B_2}) \longrightarrow \mathcal{L}(\mathcal{H}^{B_1})$

Question: what is the most general bipartite probability distribution?

$$P(\mathcal{M}^A_{j_A}, \mathcal{N}^B_{j_B})$$

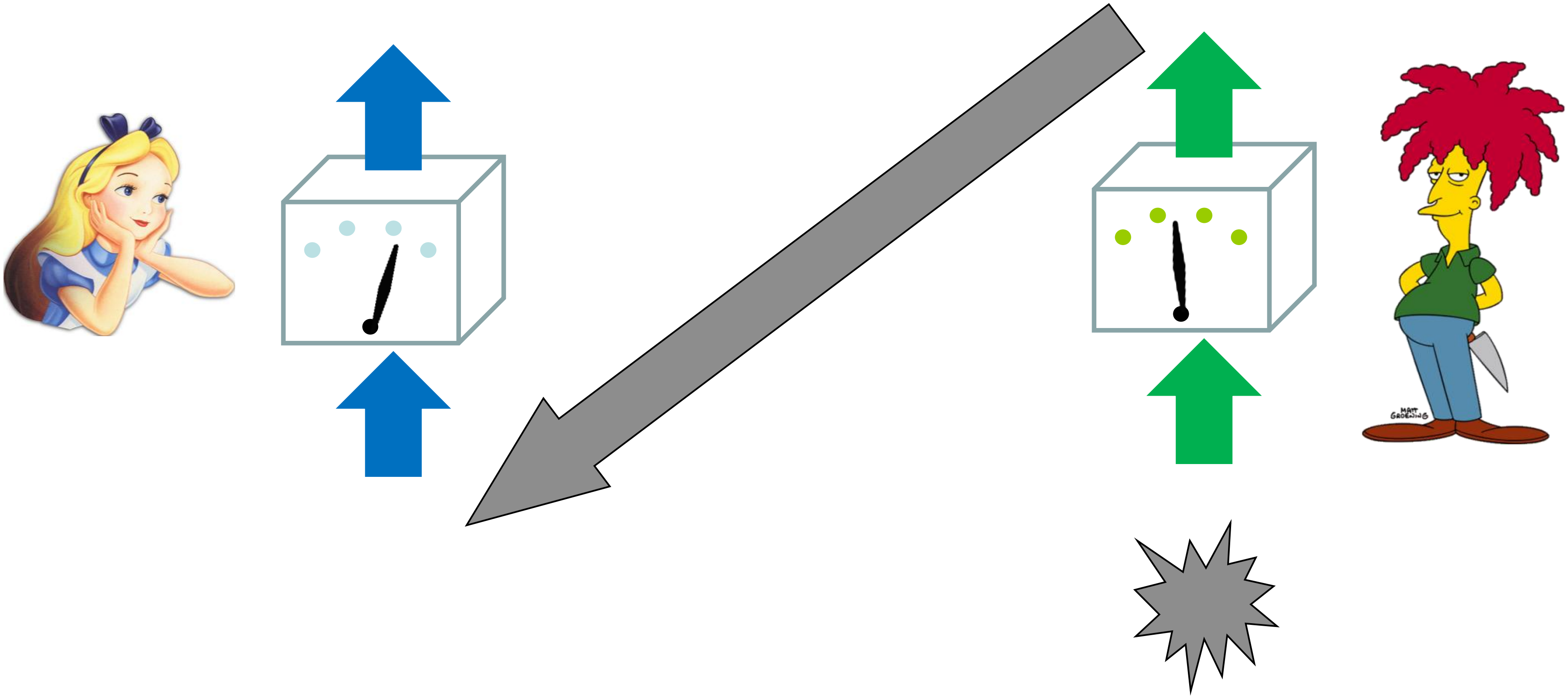
Bipartite state



Sharing a joint state, no signalling

$$P(\mathcal{M}^A, \mathcal{M}^B) \; = \mathrm{Tr}\left[\mathcal{M}^A \otimes \mathcal{M}^B(\rho^{A_2B_2})\right]$$

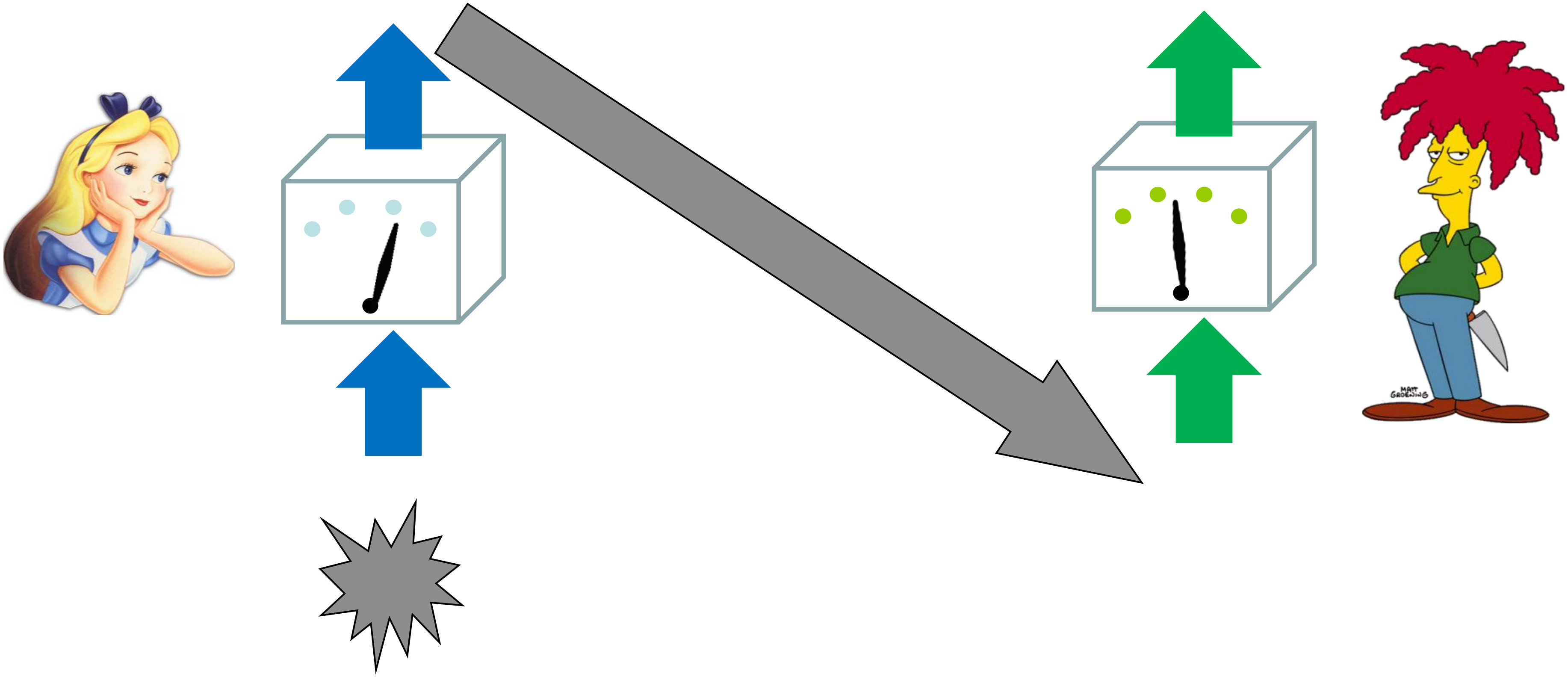
Channel B→A



Sending a state from B to A, possibility of signalling

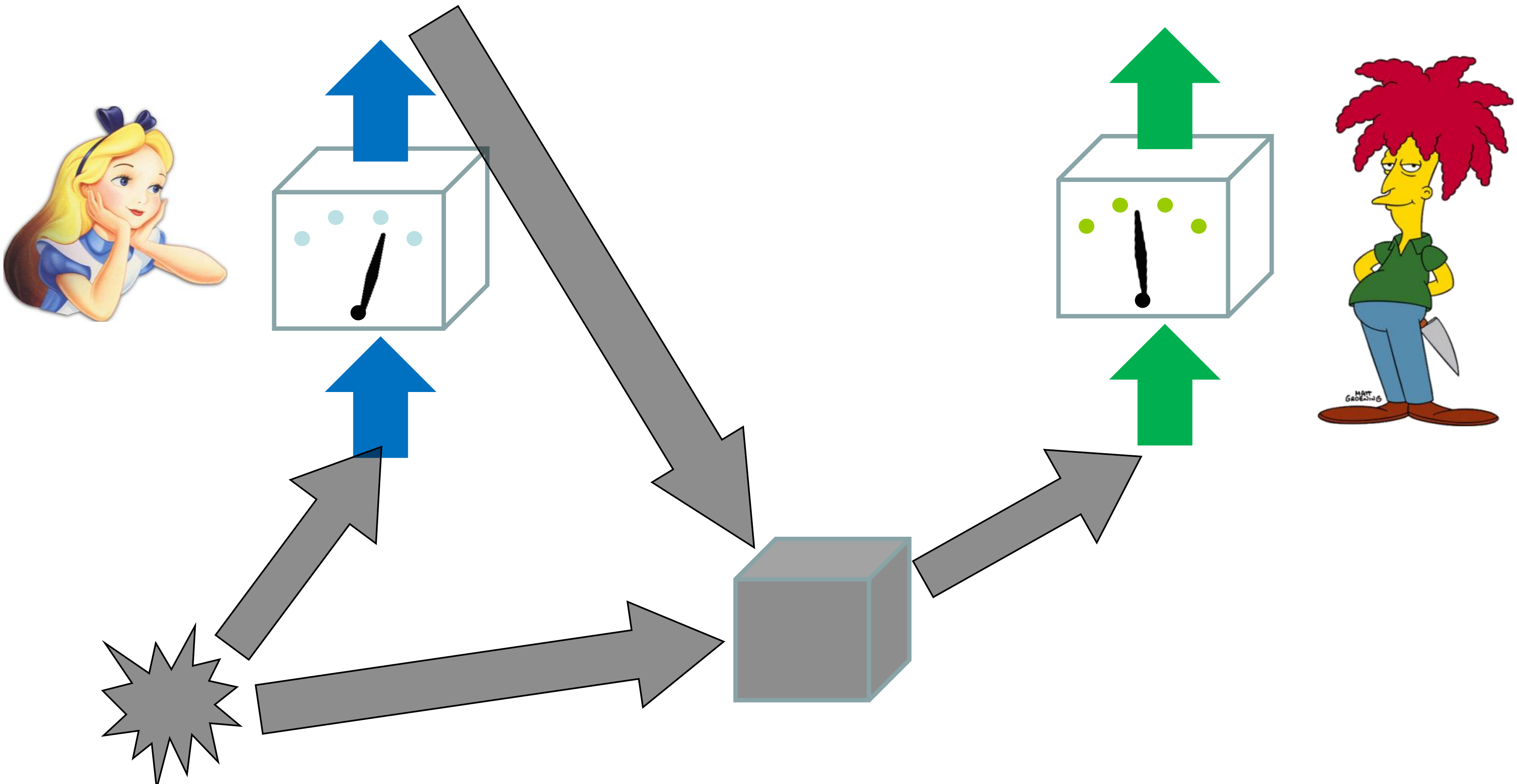
$$P(\mathcal{M}^A, \mathcal{M}^B) \;=\; \mathrm{Tr}\left[\mathcal{M}^A \circ \mathcal{E} \circ \mathcal{M}^B \left(\rho_0^{B_2}\right)\right]$$

Channel A→B



Channel with memory –

most general causally ordered situation



Signalling from A to B, no signalling from B to A

$$P^{A \to B}$$

More generally: allow classical ignorance of the causal order

$$qP^{A\to B} + (1-q)P^{B\to A}$$

$$0 \leq q \leq 1$$

Classical mixture of channels with memory

If no causal order is assumed, are more general situations possible?

Assumption

Probabilities are **bilinear** functions of the CP maps

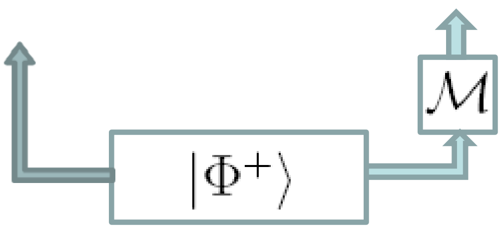
$$P(\mathcal{M}^A, \mathcal{M}^B) = \omega(\mathcal{M}^A, \mathcal{M}^B)$$

Necessary if algebra of quantum operations holds in each laboratory

Choi-Jamiołkowski isomorphism

CP maps

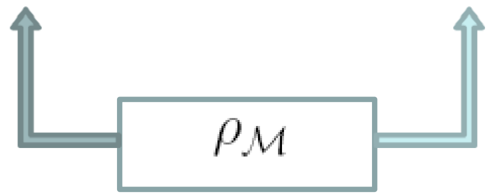
$$\mathcal{M} : \mathcal{L}(\mathcal{H}^2) \rightarrow \mathcal{L}(\mathcal{H}^1)$$



Maximally entangled state

Bipartite positive operators

$$\rho^{12}_{\mathcal{M}} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$$



$$\rho^{12}_{\mathcal{M}} = \mathcal{M} \otimes \mathcal{I}(|\Phi^+\rangle\langle\Phi^+|)$$

$$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle$$

$$|i\rangle \in \mathcal{H}^1$$

Examples

Projection on a pure state $|\psi\rangle$

$$\rho^{12} = |\psi\rangle\langle\psi|^1 \otimes |\psi\rangle\langle\psi|^2$$

Preparation of a new state σ

$$\rho^{12} = \sigma^1 \otimes \mathbb{1}^2$$

Bipartite probabilities

Bilinear functions of CP maps → Bilinear functions of positive operators

$$P(\mathcal{M}^A, \mathcal{M}^B) = \omega(\mathcal{M}^A, \mathcal{M}^B)$$

$$P(\mathcal{M}^A, \mathcal{N}^B) = \omega(\rho^{A_1 A_2}_{\mathcal{M}_\mathcal{A}}, \rho^{B1,B2}_{\mathcal{M}_\mathcal{B}})$$

Representation

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(\rho^{A_1 A_2}_{\mathcal{M}^A} \otimes \rho^{B_1 B_2}_{\mathcal{M}^B}\right)\right]$$

$$W^{A_1 A_2 B_1 B_2} \in \mathcal{L}(\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2} \otimes \mathcal{H}^{B_1} \otimes \mathcal{H}^{B_2})$$

"Process Matrix"

CP maps
(CJ representation)

Cf. Born rule

$$\mathrm{Tr}\,(E\rho)$$

POVM element

Density matrix

Bipartite probabilities

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(\rho^{A_1 A_2}_{\mathcal{M}^A} \otimes \rho^{B_1 B_2}_{\mathcal{M}^B}\right)\right]$$

$$W^{A_1 A_2 B_1 B_2} \in \mathcal{L}(\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2} \otimes \mathcal{H}^{B_1} \otimes \mathcal{H}^{B_2})$$

Conditions on process matrices

1. Probability positive: $W^{A_1 A_2 B_1 B_2} \geq 0$

Assume that parties can share ancillary entangled states

2. Probability 1 on all CPTP maps:

$$\mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(\rho^{A_1 A_2}_{\mathcal{E}^A} \rho^{B_1 B_2}_{\mathcal{E}^B}\right)\right] = 1$$

$$\forall \rho^{A_1 A_2}, \rho^{B_1 B_2} > 0,\ \mathrm{Tr}_1 \rho^{A_1 A_2} = \mathbb{1}^{A_2}, \mathrm{Tr}_1 \rho^{B_1 B_2} = \mathbb{1}^{B_2}$$

G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. A **81**, 062348 (2010).

Additional constraints imply causal order.

Formalism contains all causally ordered situations

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(\rho_{\mathcal{M}^A}^{A_1 A_2} \otimes \rho_{\mathcal{M}^B}^{B_1 B_2}\right)\right]$$

Bipartite state	$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_1 B_1} \otimes \left(\rho^{A_2 B_2}\right)^T$
Channel	$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_1} \otimes \left(\rho_{\mathcal{E}}^{A_2 B_1}\right)^T \rho_0^{B_2^T}$
Channel with memory	$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_1} \otimes W^{A_2 B_1 B_2}$

Most general causally separable situation: probabilistic mixture of ordered ones.

Probabilistic mixture of channels with memory in different orders

$$W^{A_1 A_2 B_1 B_2} = qW^{A \to B} + (1-q)W^{B \to A}$$

Signalling only from A to B

Signalling only from B to A

Are all possible processes always causality separable?

A causal game

a

x

(best estimate of b)

Each part **first** estimates the bit given to the other and **then** receives a bit that the other has to read

Depending on the value of an additional bit b‘, Bob tries either to **read** a or to **send** b

b

y

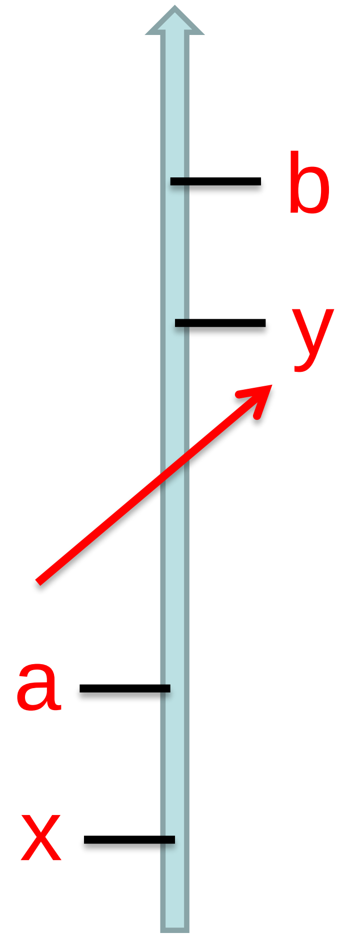
b‘

(best estimate of a)

They try to maximize the quantity

$$p_{succ} := \frac{1}{2}\left[P(x = b|b' = 0) + P(y = a|b' = 1)\right]$$

Causally ordered situation



$$P(x = b) \leq 1/2$$

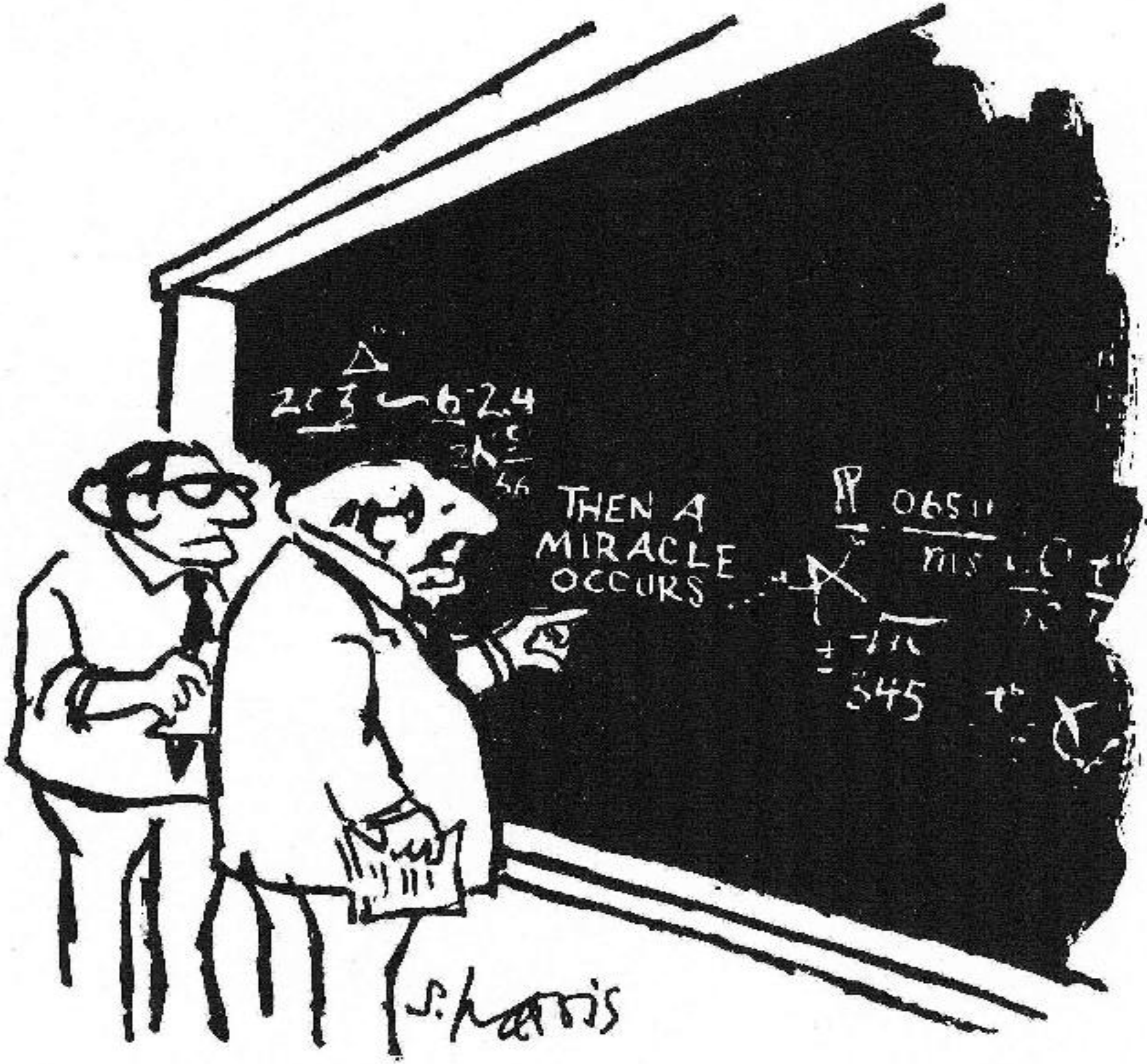
$$P(y = a) \leq 1$$

$$\frac{1}{2}\left[P(x = b) + P(y = a)\right] \leq \frac{3}{4}$$

A causally non-separable example

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

Is a valid process matrix



"I think you should be more explicit here in step two."

A causally non-separable example

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

The probability of success is

$$\tfrac{1}{2}\left[P(y=a|b'=1) + P(x=b|b'=0)\right] = \tfrac{2+\sqrt{2}}{4} > \tfrac{3}{4}$$

This example cannot be realized as a probabilistic mixture of causally ordered situations!

Sketch of the strategy

Alice always encodes a in the z basis

Bob always receives $\frac{1}{2}\left[\mathbb{1} + (-1)^a \frac{1}{\sqrt{2}}\sigma_z\right]$

If Bob wants to read (b'=1) he measures in the z basis

If Bob wants to send (b'=0)

He measures in the x basis, encodes b in the z basis

Alice receives $\frac{1}{2}\left[\mathbb{1} + (-1)^b \frac{1}{\sqrt{2}}\sigma_z\right]$

By measuring in the z basis, Alice can make a good guess of b

Conclusions

- [Not shown]: Classical correlations are always causally separable
- Unified framework for both signalling ("time-like") and non-signalling ("space-like") quantum correlations with no prior assumption of time or causal structure
- Situations where a causal ordering between laboratory operations is not definite → Suggests that causal ordering might not be a necessary element of quantum theory
- What one needs to do in the lab to realize the "processes"? New resource for quantum information processing?

Thank you for your attention!

Questions?

CoQuS ComplexQuantumSystems

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FQXi Foundational Questions Institute

A causally non-separable example

Note: when Alice detects in the basis $\{|\psi_j\rangle\}$ and reprepares $|\psi'\rangle$ Bob effectively „sees“

$$\widetilde{W}^{B_1B_2} = \sum_j \mathrm{Tr}_{A_1A_2}\left[W^{A_1A_2B_1B_2}\left(|\psi'\rangle\langle\psi'|^{A_1}\otimes|\psi_j\rangle\langle\psi_j|^{A_2}\right)\right]$$

Alice always measures in the *z* basis
and reprepares in the *z* basis

$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

$$\langle z_\pm|\sigma_z|z_\pm\rangle = \pm 1 \qquad \langle z_\pm|\sigma_z|z_\pm\rangle = \pm 1$$

Choosen by Alice

Not seen by Bob

⇒ Bob receives the state $\widetilde{W}^{B_1B_2} = \frac{1}{2}\left(\mathbb{1} + (-1)^a\frac{1}{\sqrt{2}}\sigma_z^{B_2}\right)$

If Bob wants to read (b‘ = 1) he measures in the *z* basis and achieves

$$P(y = a|b' = 1) = \frac{2+\sqrt{2}}{4}$$

A causally non-separable example

If Bob wants to send (b‘ = 0) → Measures in the *x* basis

Reprepares in the *z* basis

$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

$$\langle x_\pm|\sigma_z|x_\pm\rangle = 0 \qquad \langle z_\pm|\sigma_z|z_\pm\rangle = \pm 1 \qquad \langle x_\pm|\sigma_x|x_\pm\rangle = \pm 1$$

The encoding can be correlated with the detection result

Alice receives the state $\widetilde{W}^{A_1A_2} = \frac{1}{2}\left(\mathbb{1} + (-1)^b \frac{1}{\sqrt{2}}\sigma_z^{A_2}\right)$

She can read Bob‘s sent bit with probability

$$P(x = b|b' = 0) = \frac{2+\sqrt{2}}{4}$$

Terms appearing in process matrix

$$W^{A_1 A_2 B_1 B_2} = \sum_{\mu_1,\dots,\mu_4} a_{\mu_1 \dots \mu_4} \sigma_{\mu_1}^{A_1} \otimes \dots \otimes \sigma_{\mu_4}^{B_2}$$

$$\sigma_i^{A_1} \otimes \mathbb{1}^{rest} \qquad \text{type } A_1$$

$$\sigma_i^{A_1} \otimes \sigma_j^{A_2} \otimes \mathbb{1}^{rest} \qquad \text{type } A_1 A_2$$

...

1. Probability positive & 2. Probability 1on all CPTP maps

	$A_2,\ B_2\ , A_2 B_2$	$A_1 B_2$	$A_1 A_2 B_2$
		$A_2 B_1$	$A_2 B_1 B_2$
Causal order	States	Channels	Channels with memory