Use of Genetic Algorithm for Quantum Information Processing by NMR

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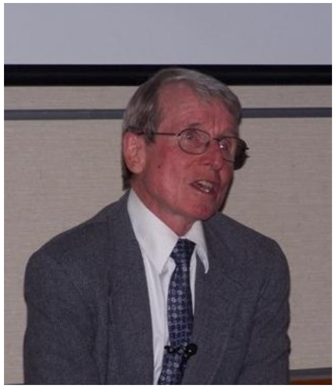
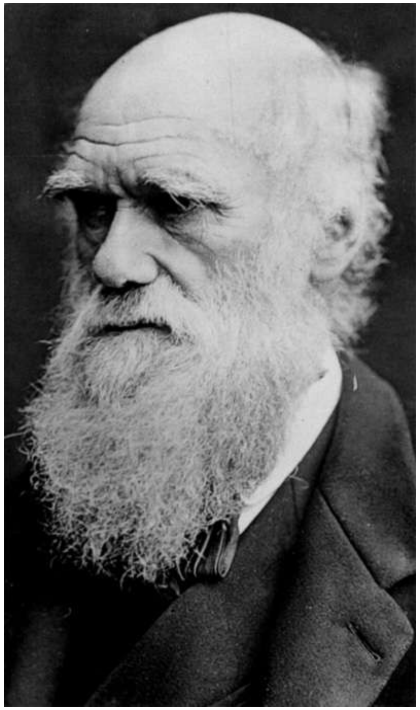
The Genetic Algorithm


John Holland

Directed search algorithms based on the mechanics of biological evolution

Developed by John Holland, University of Michigan (1970's)



Charles Darwin 1866
1809-1882

"Genetic Algorithms are good at taking large, potentially huge, search spaces and navigating them, looking for optimal combinations of things, solutions you might not otherwise find in a lifetime"

Here we apply Genetic Algorithm to

Quantum Computing and Quantum Information Processing

Quantum Algorithms

1. **PRIME FACTORIZATION**

Classically : **400 digit**
exp [2(ln c)$^{1/3}$(ln ln c)$^{2/3}$] **10^{10}years** (Age of the Universe)

Shor's algorithm : **(1994)** **3 years**
(ln c)^3

2. **SEARCHING 'UNSORTED' DATA-BASE**

Classically : ***N/2*** **operations**
Grover's Search Algorithm : (1997) $\sqrt{N}$ **operations**

3. **DISTINGUISH CONSTANT AND BALANCED FUNCTIONS:**

Classically : **(2^{N-1} + 1) steps**
Deutsch-Jozsa(DJ) Algorithm : (1992) . **1 step**

4. Quantum Algorithm for Linear System of Equation:

Harrow, Hassidim and Seth Lloyd; Phys. Rev. Letters, 103, 150502 (2009).
Exponential speed-up

Recent Developments

5. Simulating a Molecule: Using Aspuru-Guzik Algorithm

(i) J.Du, et. al, Phys. Rev. letters 104, 030502 (2010).

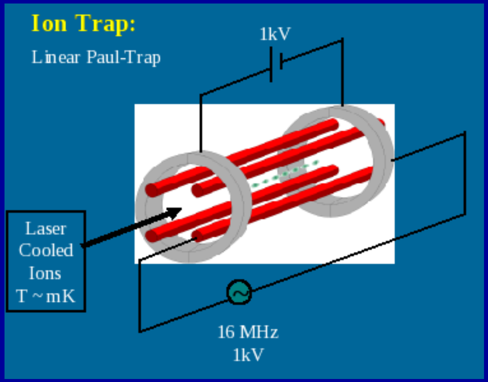
Used a 2-qubit NMR System ($^{13}CHCl_3$) to calculated the ground state energy of Hydrogen Molecule up to 45 bit accuracy.

(ii) Lanyon et. al, Nature Chemistry 2, 106 (2010).

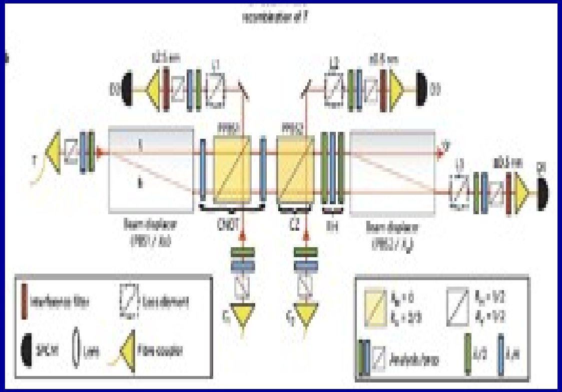
Used Photonic system to calculate the energies of the ground and a few excited states up to 20 bit precision.

Experimental Techniques for Quantum Computation:

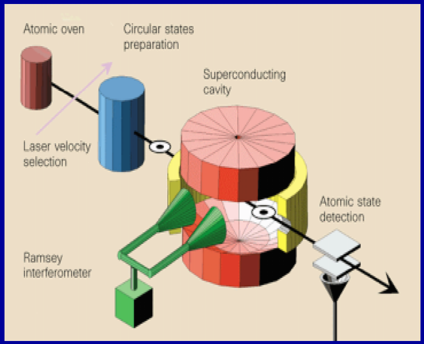
1. Trapped Ions



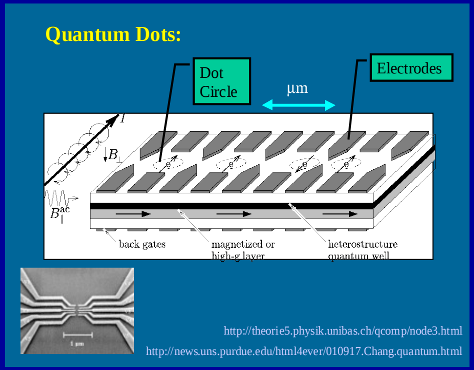
2. Polarized Photons Lasers



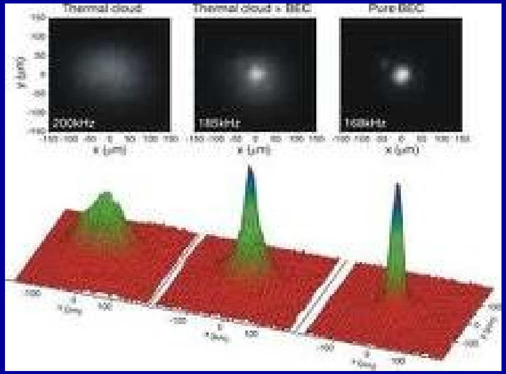
3. Cavity Quantum Electrodynamics (QED)



4. Quantum Dots



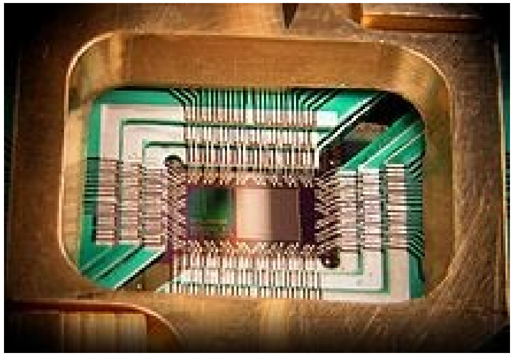
5. Cold Atoms



6. NMR

7. Josephson junction qubits

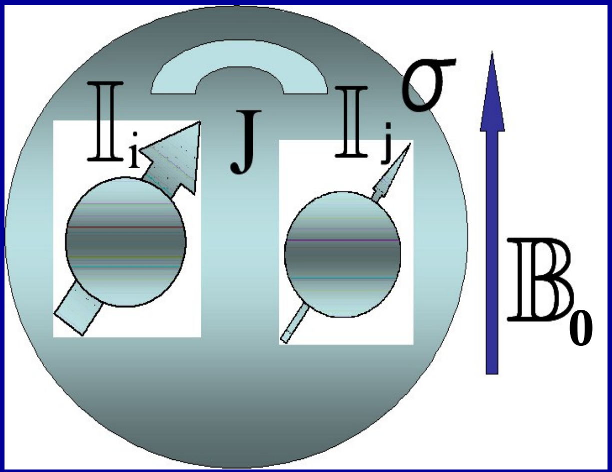
8. Fullerence based ESR quantum computer

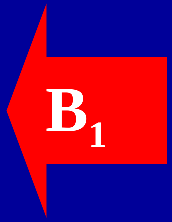


Photograph of a chip constructed by D-Wave Systems Inc., designed to operat
as a 128-qubit superconducting adiabatic quantum optimization processor, mounted in a sample holder.

2011

Nuclear Magnetic Resonance (NMR)





1. **Nuclear spins have small magnetic moments (I) and behave as tiny quantum magnets.**

2. **When placed in a large magnetic field B_0, they oriented either along the field ($|0\rangle$ state) or opposite to the field ($|1\rangle$ state) .**

3. **A transverse radiofrequency field (B_1) tuned at the Larmor frequency of spins can cause transition from $|0\rangle$ to $|1\rangle$ (NOT Gate by a 180^0 pulse).**
Or put them in coherent superposition (Hadamard Gate by a 90^0 pulse).
Single qubit gates.

4. **Spins are coupled to other spins by indirect spin-spin (J) coupling, and controlled (C-NOT) operations can be performed using J-coupling.**
Multi-qubit gates

SPINS ARE QUBITS

NMR Research Centre, IISc

Field/ Frequency stability = 1:10 ^{9}

1 PPB

Why NMR?

> A major requirement of a quantum computer is that the coherence should last long.

> Nuclear spins in liquids retain coherence ~ 100's millisec and their longitudinal state for several seconds.

> A system of N coupled spins (each spin 1/2) form an N qubit Quantum Computer.

> Unitary Transform can be applied using R.F. Pulses and J-evolution and various logical operations and quantum algorithms can be implemented.

Achievements of NMR - QIP

√ 1. **Preparation of Pseudo-Pure States**

√ **2. Quantum Logic Gates**

√ **3. Deutsch-Jozsa Algorithm**

√ **4. Grover's Algorithm**

√ **5. Hogg's algorithm**

√ **6. Berstein-Vazirani parity algorithm**

√ **7. Quantum Games**

√ **8. Creation of EPR and GHZ states**

√ **9. Entanglement transfer**

√ **10. Quantum State Tomography**

√ **11. Geometric Phase in QC**

√ **12. Adiabatic Algorithms**

√ **13. Bell-State discrimination**

14. Error correction

15. Teleportation

16. Quantum Simulation

17. Quantum Cloning

18. Shor's Algorithm

√ **19. No-Hiding Theorem**

√ **Also performed in our Lab.**

Maximum number of qubits achieved in our lab: 8

In other labs.: 12 qubits;

Negrevergne, Mahesh, Cory, Laflamme et al., Phys. Rev. Letters, 96, 170501 (2006).

NMR sample has ~ 10^{18} spins.

↓

Do we have 10^{18} qubits?

No - because, all the spins can’t be individually addressed.

↓ **Progress so far**

Spins having different Larmor frequencies can be individually addressed → as many “qubits”

↓

One needs resolved couplings between the spins in order to encode information as qubits.

NMR Hamiltonian

$$\mathbf{H} = \mathbf{H}_{\text{Zeeman}} + \mathbf{H}_{\text{J-coupling}}$$

$$= \sum_i \omega_i I_{zi} + \sum_{i<j} J_{ij}\, \mathbf{I}_i \cdot \mathbf{I}_j$$

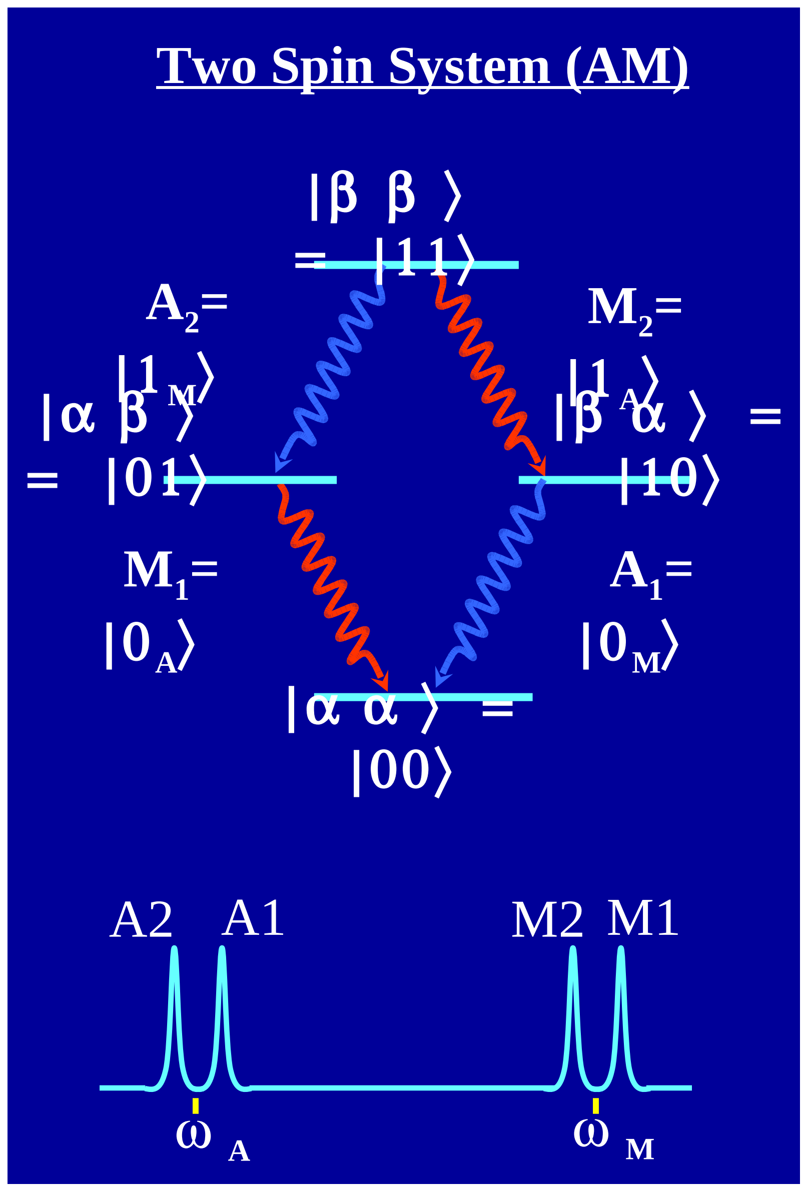
Weak coupling Approximation

$$|\omega_i - \omega_j| >> J_{ij}$$

$$\underbrace{\mathbf{H} = \sum_i \omega_i I_{zi} + \sum_{i<j} J_{ij} I_{zi} I_{zj}}$$

Spin States are eigenstates

Under this approximation all spins having same Larmor Frequency can be treated as one Qubit

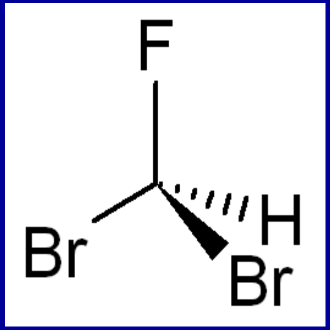


An example of a three qubit system.

A molecule having three different nuclear spins having different Larmor frequencies all coupled to each other

forming a 3-qubit system

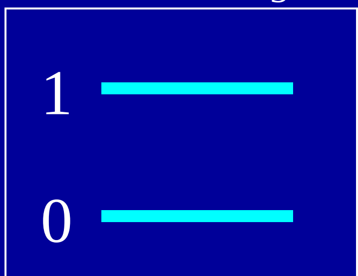
$^{13}CHFBr_2$

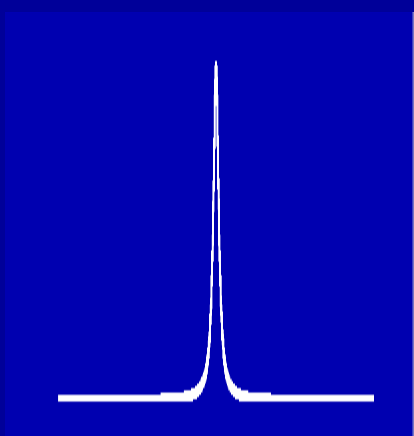


Homo-nuclear spins having different Chemical shifts (Larmor frequencies) also form multi-qubit systems

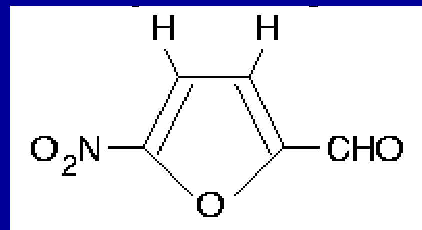
1 Qubit

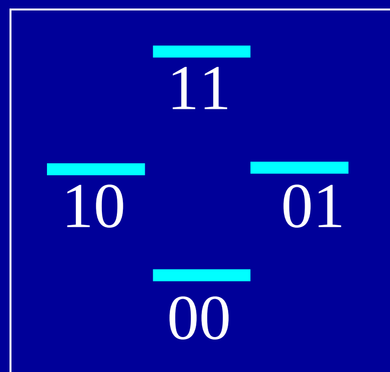
$CHCl_3$

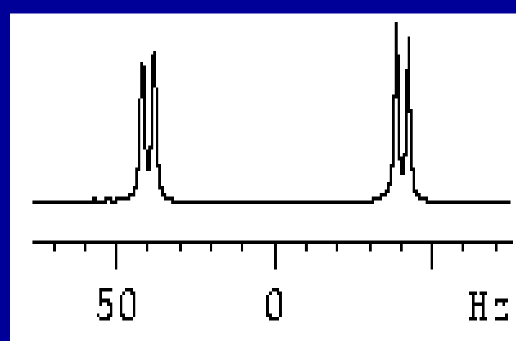




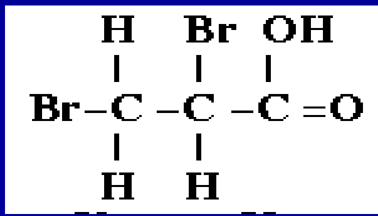
2 Qubits

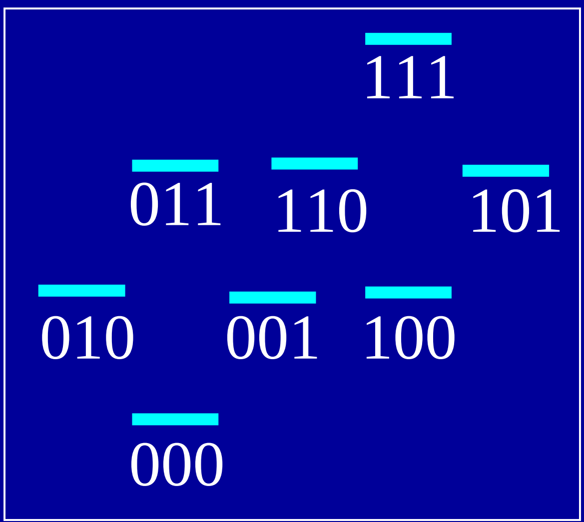


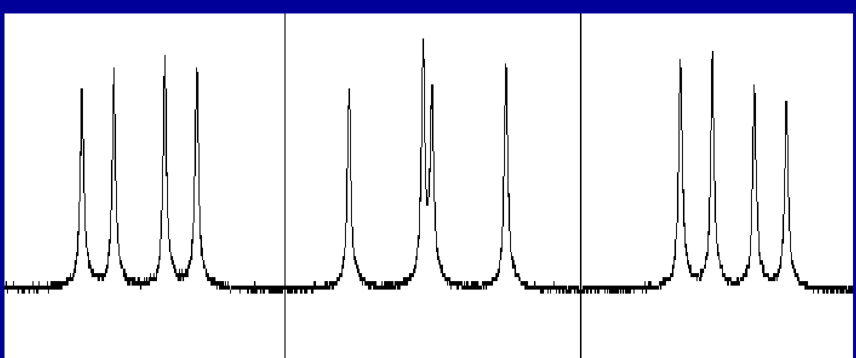




3 Qubits







Unitary Transforms in NMR

1. **Rational Pulse design.**
   (using RF Pulses and coupling (J)-evolution)

2. **Optimization Techniques**

$$\sigma_{initial} \xrightarrow{U_1} \sigma_1 \xrightarrow[C_1 = |\sigma_1 - \sigma_f|]{\text{Goodness criterion}} \text{Iterate to minimize } C_1 \rightarrow \sigma_{final}$$

Various optimization Techniques used in NMR

(a) Strongly Modulated Pulses (SMP) (Cory, Mahesh et. al)
(b) Control Theory (Navin Kheneja et. al (Harvard))
(c) Algorithmic Technique (Ashok Ajoy et. al)
(d) Genetic Algorithm (Manu)

1. Rational Pulse design.

using RF Pulses and coupling (J)-evolution

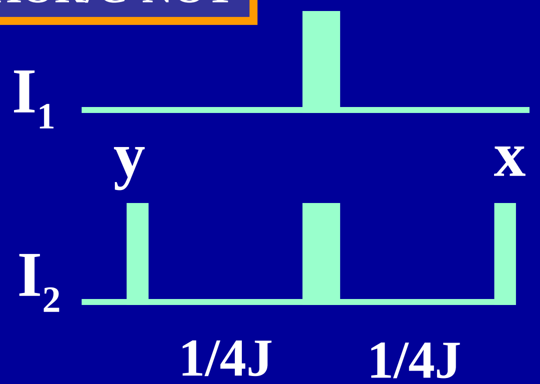
The two methods

Coupling (J) Evolution

Transition-selective Pulses

Examples

XOR/C-NOT



$I_{1z}+I_{2z}$ →

y ↓

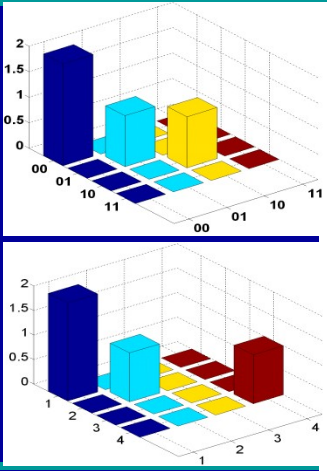
$I_{1z}+I_{2x}$

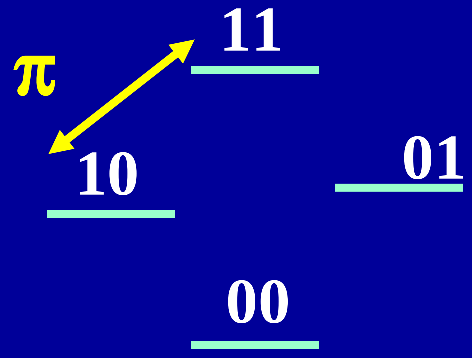
(1/2J) ↓

$I_{1z}+2I_{1z}I_{2y}$

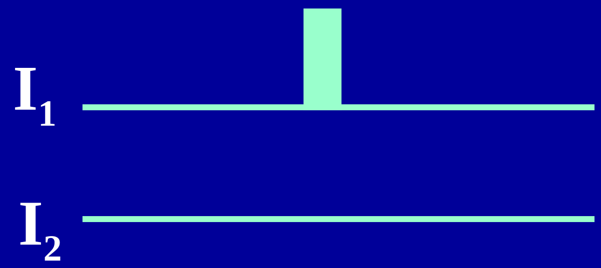
x ↓

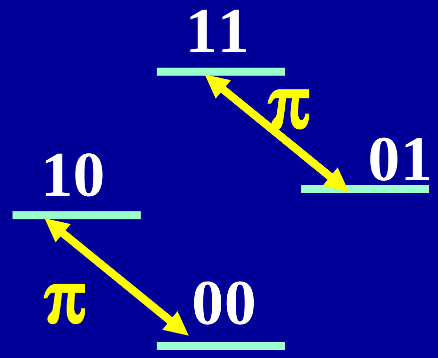
$I_{1z}+2I_{1z}I_{2z}$ →

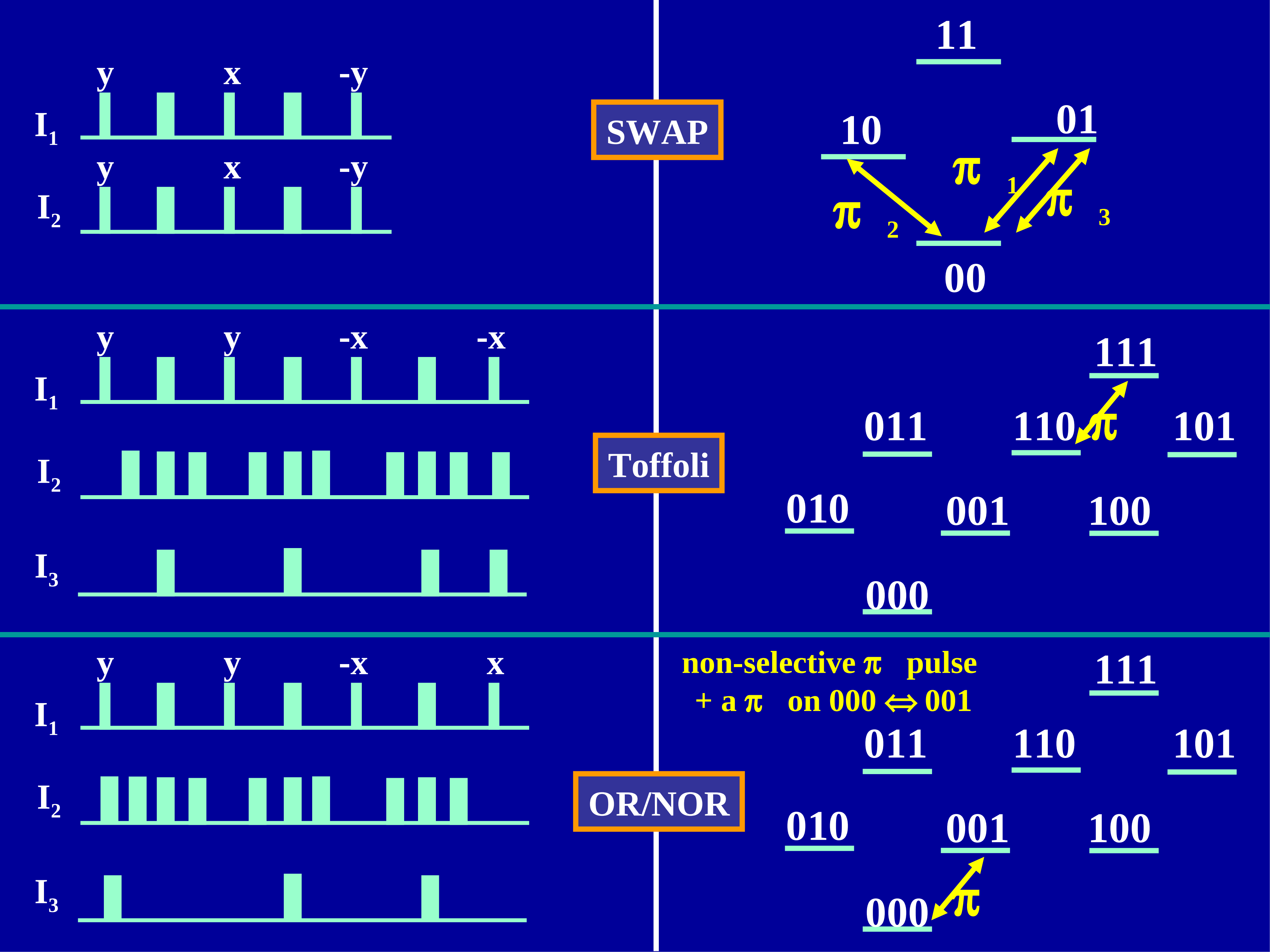




NOT1







y
x
-y
I_1
I_2
SWAP
11
10
01
π
1
π
2
π
3
00
y
y
-x
-x
I_1
I_2
I_3
Toffoli
111
011
110
π
101
010
001
100
000
y
y
-x
x
I_1
I_2
I_3
OR/NOR
non-selective π pulse
+ a π on 000 ⇔ 001
111
011
110
101
010
001
100
000
π

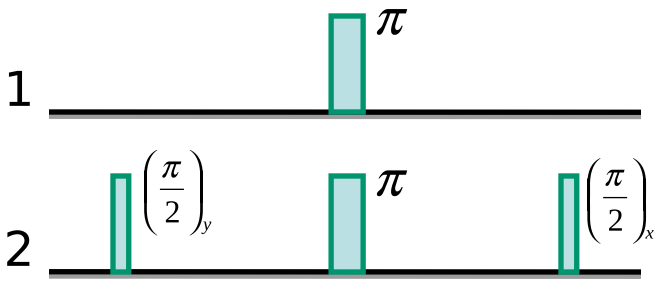
CNOT GATE

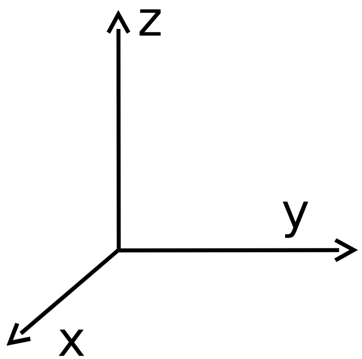
$$\rho_{eq} \propto \gamma_1 I_z^1 + \gamma_2 I_z^2$$

$$\rho_2 = \gamma_1 I_z^1 + \gamma_2 I_x^2$$

$$\rho_3 = \gamma_1 I_z^1 + \gamma_2 \left[-2 I_z^1 I_y^2\right]$$

$$\rho_4 = \gamma_1 I_z^1 + \gamma_2 \left[2 I_z^1 I_z^2\right]$$





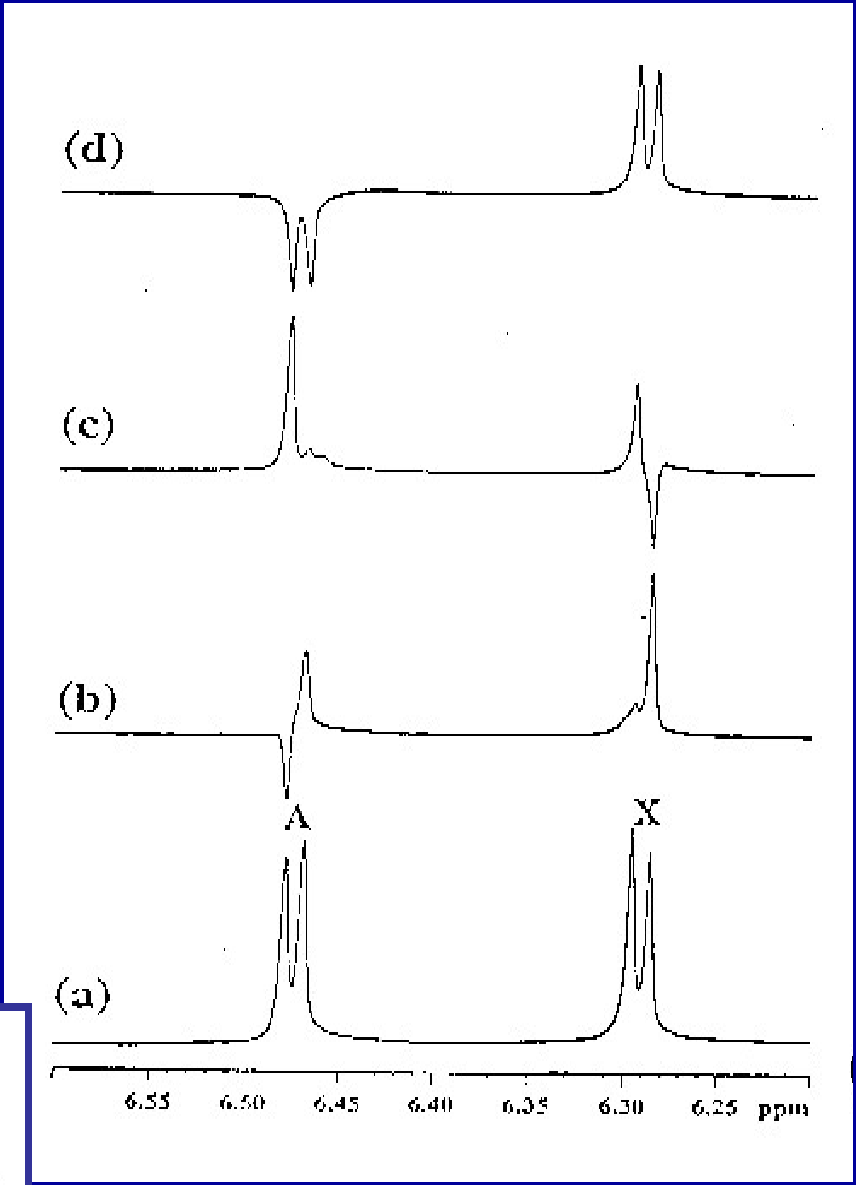
IN	ρ_{eq}	ρ_4	OUT
|00>	$\frac{1}{2}(\gamma_1+\gamma_2)$	$\frac{1}{2}(\gamma_1+\gamma_2)$	|00>
|01>	$\frac{1}{2}(\gamma_1-\gamma_2)$	$\frac{1}{2}(\gamma_1-\gamma_2)$	|01>
|10>	$-\frac{1}{2}(\gamma_1-\gamma_2)$	$-\frac{1}{2}(\gamma_1-\gamma_2)$	|11>
|11>	$-\frac{1}{2}(\gamma_1+\gamma_2)$	$-\frac{1}{2}(\gamma_1+\gamma_2)$	|10>

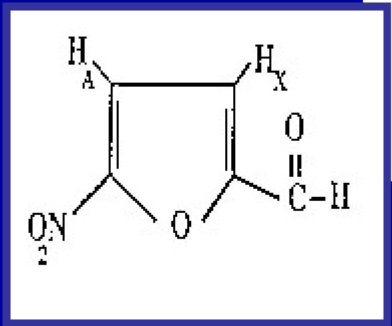
Logic Gates Using 1D NMR

NOT(I_1)

C-NOT-2
XOR2

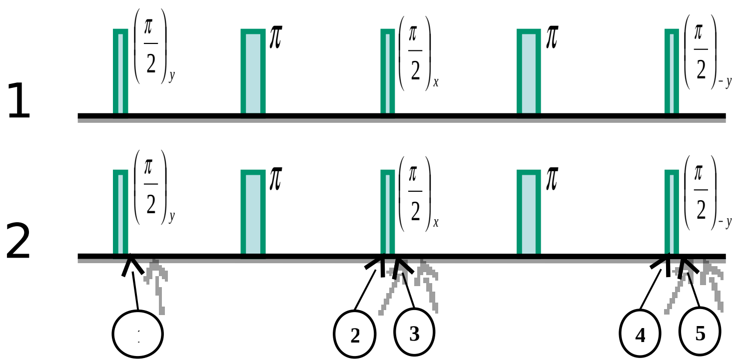
C-NOT-1
XOR1

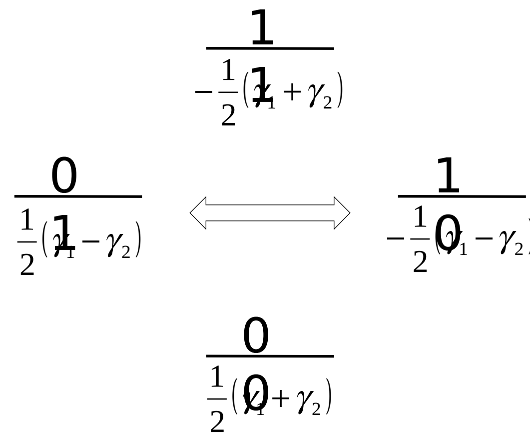




Kavita Dorai, PhD Thesis, IISc, 2000.

SWAP GATE





$$\rho_{eq} \propto \gamma_1 I_z^1 + \gamma_2 I_z^2$$

$$\rho_1 = \gamma_1 I_x^1 + \gamma_2 I_x^2$$

$$\rho_2 = \gamma_1\left[-2I_y^1 I_z^2\right] + \gamma_2\left[-2I_y^1 I_z^2\right]$$

$$\Big\Downarrow \left(\frac{\pi}{2}\right)_x^{1,2}$$

$$\rho_3 = \gamma_1\left[2I_z^1 I_y^2\right] + \gamma_2\left[-2I_z^1 I_y^2\right]$$

$$\Big\Downarrow J$$

$$\rho_4 = \gamma_1\left[I_x^2\right] + \gamma_2\left[I_x^1\right]$$

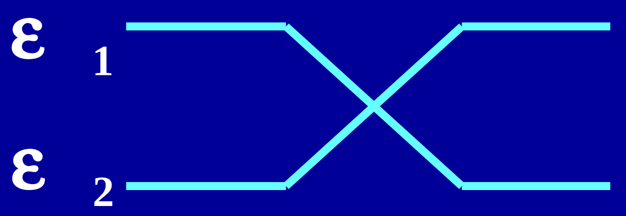
$$\Big\Downarrow \left(\frac{\pi}{2}\right)_{-x}^{1,2}$$

$$\rho_5 = \gamma_1 I_z^2 + \gamma_2 I_z^1$$

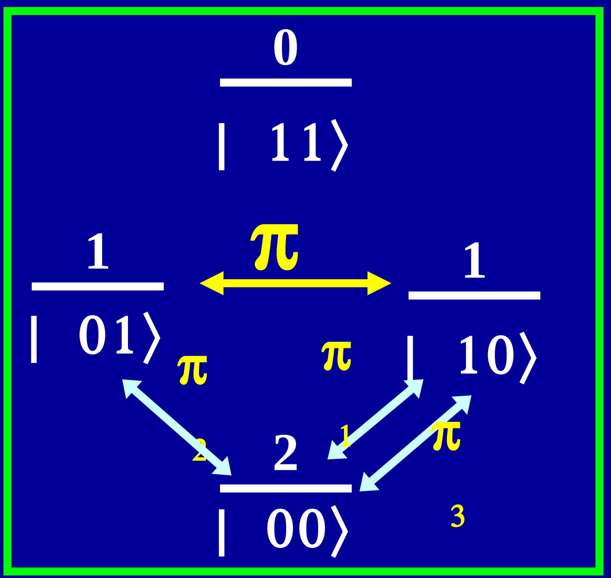
|00>	$\frac{1}{2}(\gamma_1+\gamma_2)$	$\frac{1}{2}(\gamma_1+\gamma_2)$
|01>	$\frac{1}{2}(\gamma_1-\gamma_2)$	$-\frac{1}{2}(\gamma_1-\gamma_2)$
|10>	$-\frac{1}{2}(\gamma_1-\gamma_2)$	$\frac{1}{2}(\gamma_1-\gamma_2)$
|11>	$-\frac{1}{2}(\gamma_1+\gamma_2)$	$-\frac{1}{2}(\gamma_1+\gamma_2)$

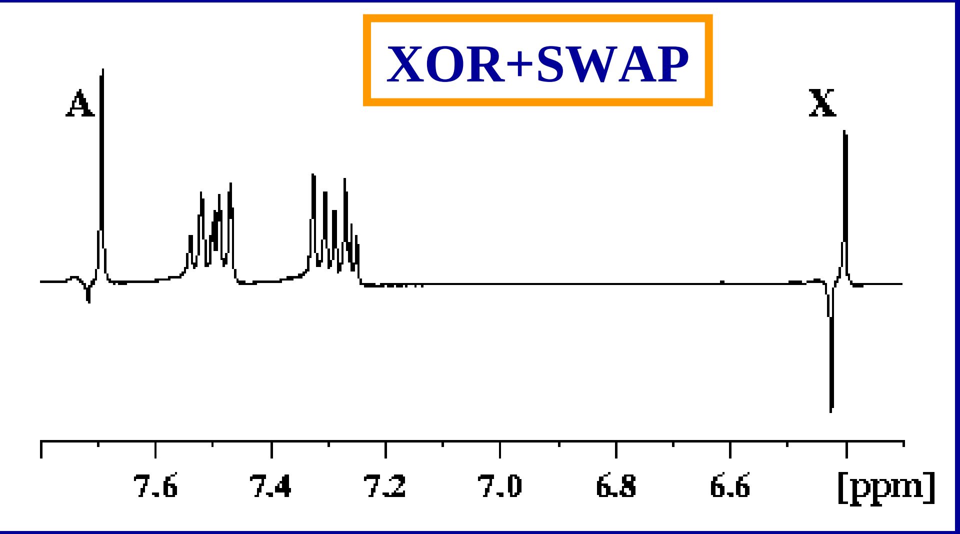
Logical SWAP

$|\varepsilon_1, \varepsilon_2\rangle \rightarrow |\varepsilon_2, \varepsilon_1\rangle$



INPUT		OUTPUT
$\|0\ 0\rangle$		$\|0\ 0\rangle$
$\|0\ 1\rangle$	π	$\|1\ 0\rangle$
$\|1\ 0\rangle$	π	$\|0\ 1\rangle$
$\|1\ 1\rangle$		$\|1\ 1\rangle$





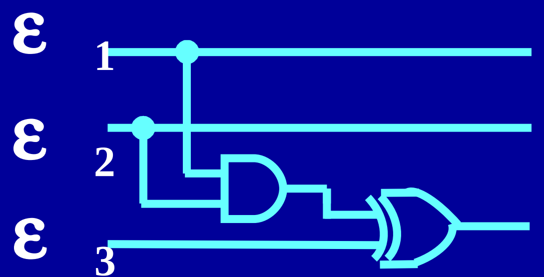
Kavita, Arvind, and Anil Kumar
Phys. Rev. **A 61**, 042306 (2000).

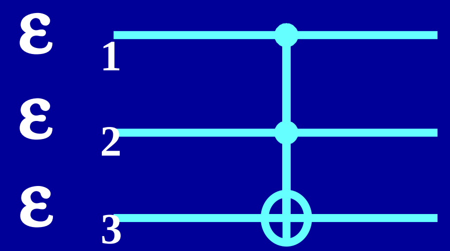
Toffoli Gate = C²-NOT

$$|\varepsilon_1, \varepsilon_2, \varepsilon_3\rangle$$

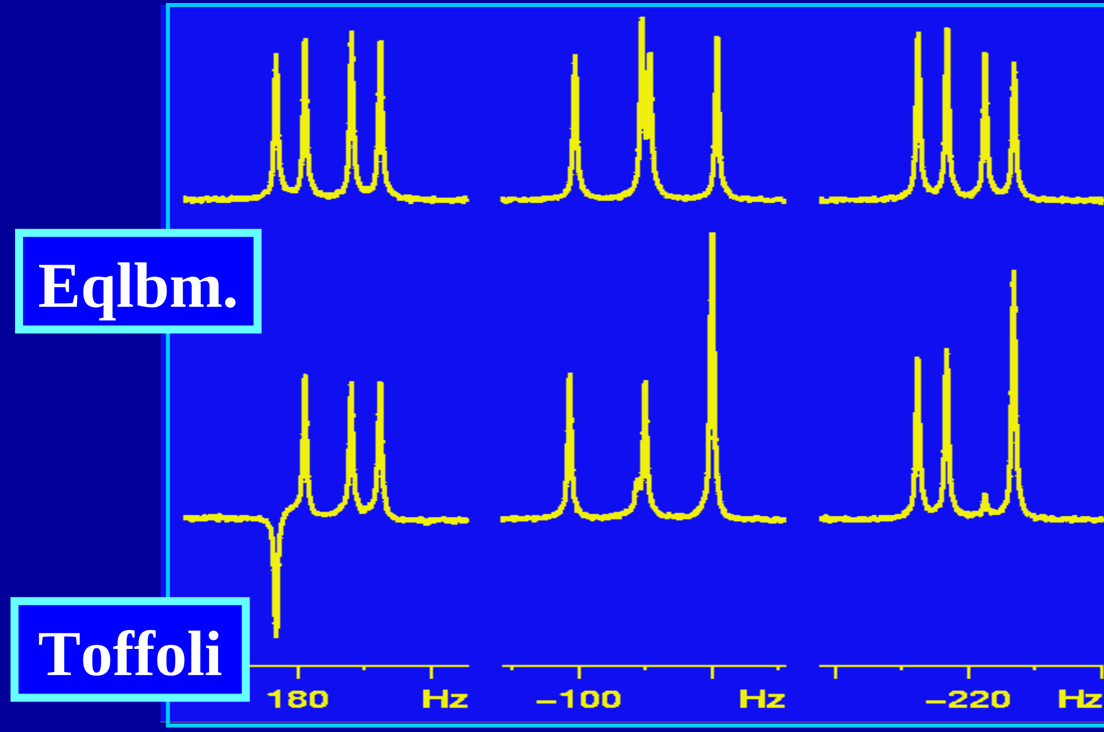
$$\downarrow$$

$$|\varepsilon_1, \varepsilon_2, \varepsilon_3 \oplus (\varepsilon_1 \wedge \varepsilon_2)\rangle$$





	Input	Output
AND	000	000
	001	001
	010	010
	011	111
NAND	100	100
	101	101
	110	110
	111	011



Kavita Dorai, PhD Thesis, IISc, 2000.

Strongly Modulated Pulses (SMP)

(Cory, Mahesh et. al)

Adiabatic Satisfibility problem using Strongly Modulated Pulses

Avik Mitra

In a Homonuclear spin systems

spins are close (~ kHz) in frequency space → Pulses are of longer duration → Decoherence effects cannot be ignored

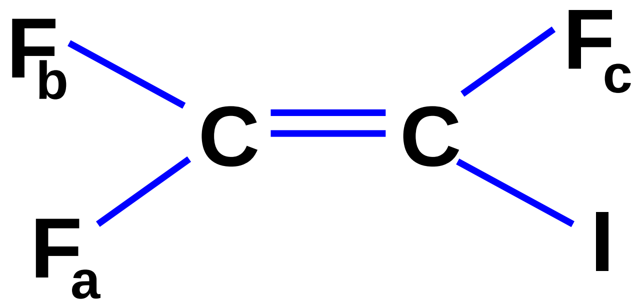
↓

Strongly Modulated Pulses circumvents the above problems

NMR Implementation, using a 3-qubit system.

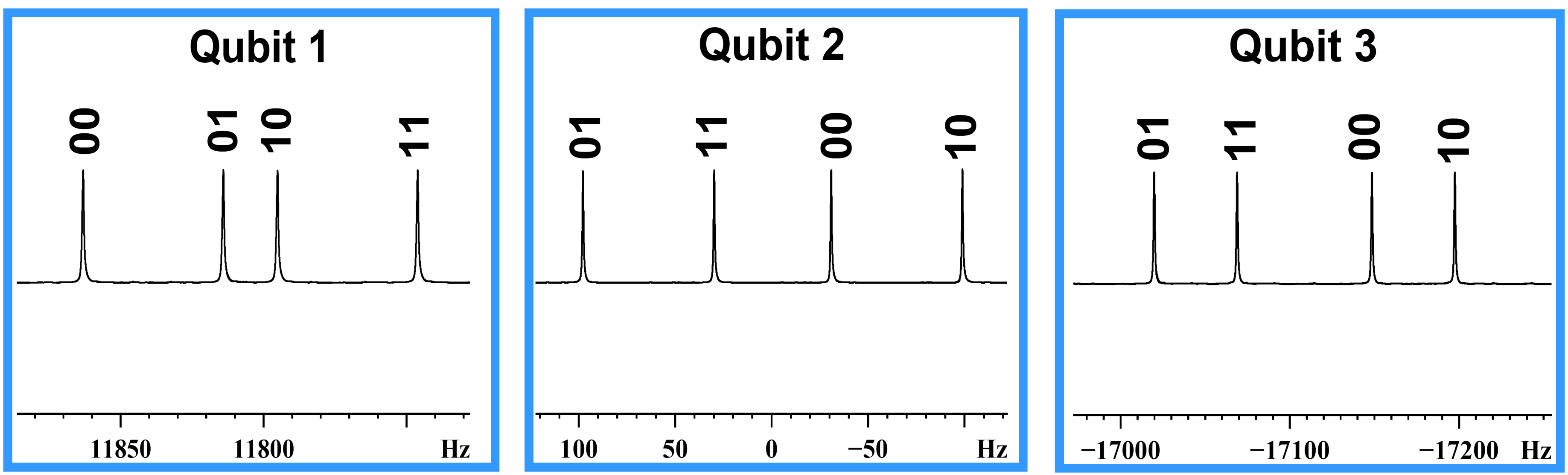
❑ The Sample.

Iodotrifluoroethylene(C_2F_3I)



$$J_{ab} = 68.1 \text{ Hz}$$
$$J_{ac} = 48.9 \text{ Hz}$$
$$J_{bc} = -128.8 \text{ Hz}$$

❑ Equilibrium Specrum.



Implementation of Adiabatic Evolution

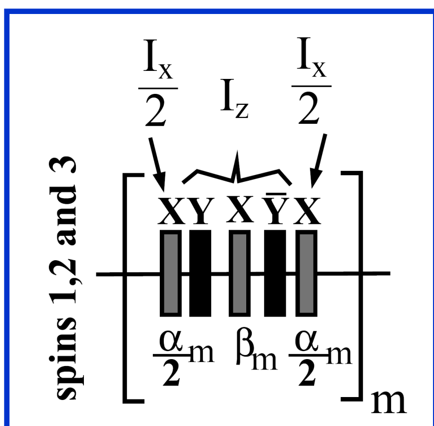
$$H_B = I_x^1 + I_x^2 + I_x^3 = I_x \qquad H_F = I_z^1 + I_z^2 + I_z^3 = I_z$$

$$H(m) = \left(1 - \frac{m}{M}\right) H_B + \frac{m}{M} H_F$$

m^th^ step of the interpolating Hamiltonian .

$$U_m \approx \underbrace{e^{-iI_x\left(1-\frac{m}{M}\right)\frac{180}{2\pi}}}_{\left[\frac{\alpha_m}{2}\right]_x^{1,2,3}\ \textit{pulse}} \cdot \underbrace{e^{-iI_z\left(\frac{m}{M}\right)\frac{180}{\pi}}}_{\left[\beta_m\right]_z^{1,2,3}\ \textit{pulse}} \cdot \underbrace{e^{-iI_x\left(1-\frac{m}{M}\right)\frac{180}{2\pi}}}_{\left[\frac{\alpha_m}{2}\right]_x^{1,2,3}\ \textit{pulse}}$$

m^{th} step of evolution operator

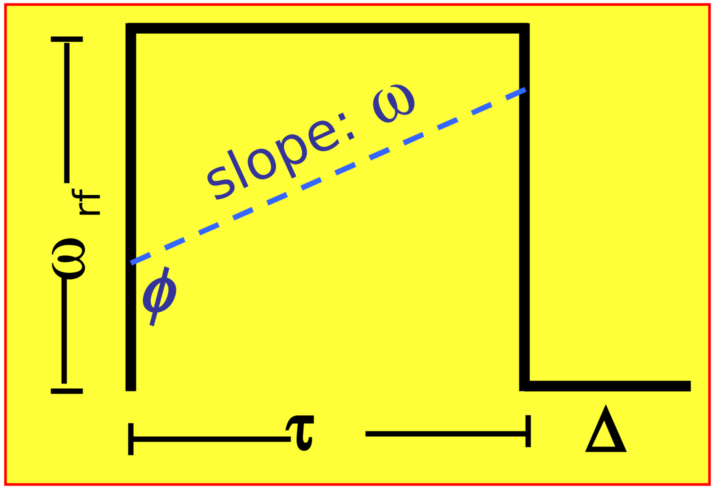


- Pulse sequence for adiabatic evolution
- Total number of iteration is 31
- **time needed = 62 ms**

(400μ s x 5 pulses x 31 repetitions)

Strongly Modulated Pulses.

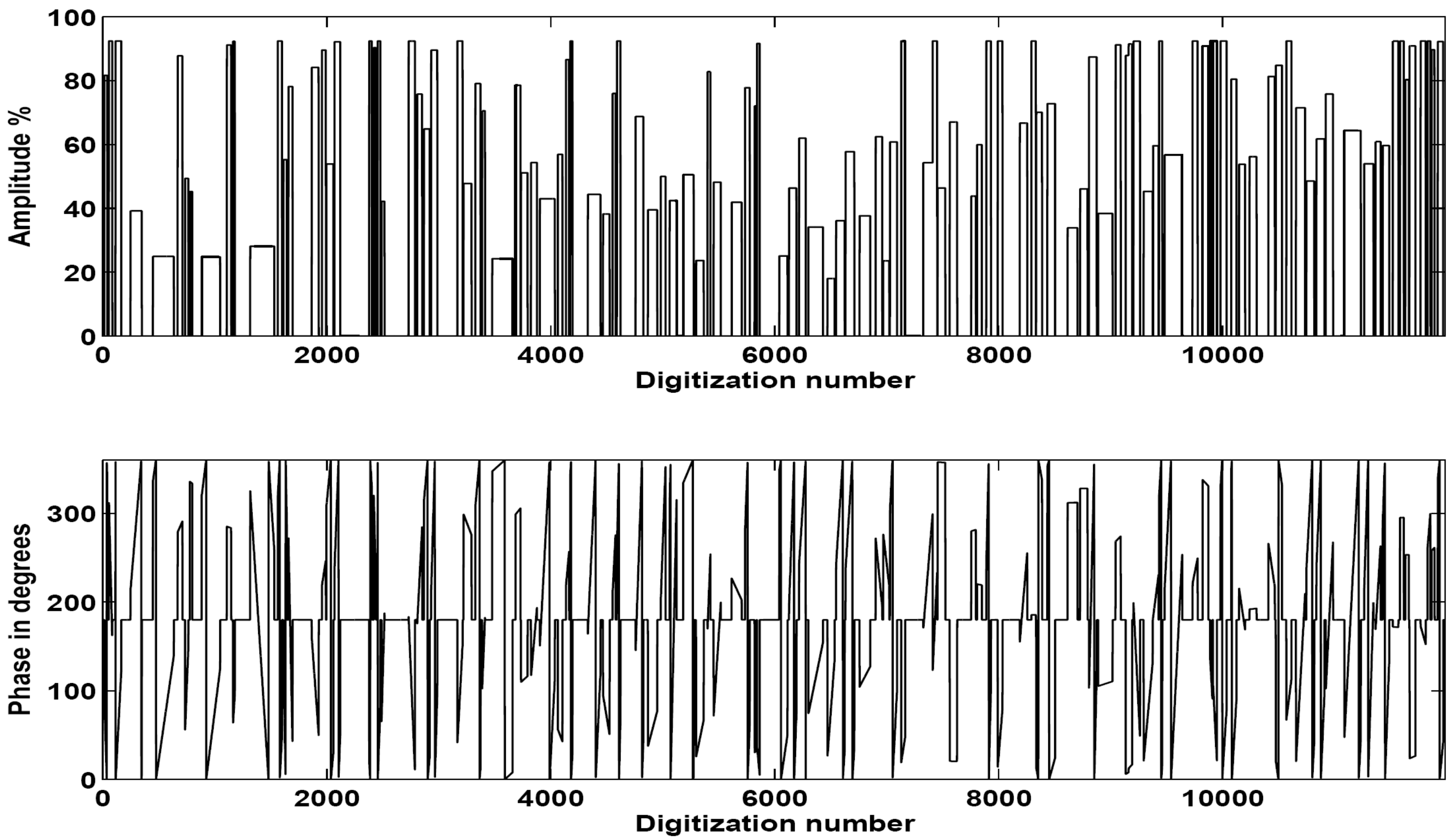
$$U_{SMP} = \prod_{l} \Delta_l(\delta_l) \cdot U_z^{-1}(\tau_l) e^{-iH_{eff}(\omega^l \omega_{rf}^l \phi^l)\tau^l}$$



$$F = \left| \frac{Tr[U_T \cdot U_{SMP}]}{N} \right|^2$$

Nedler-Mead
Simplex Algorithm
(fminsearch)

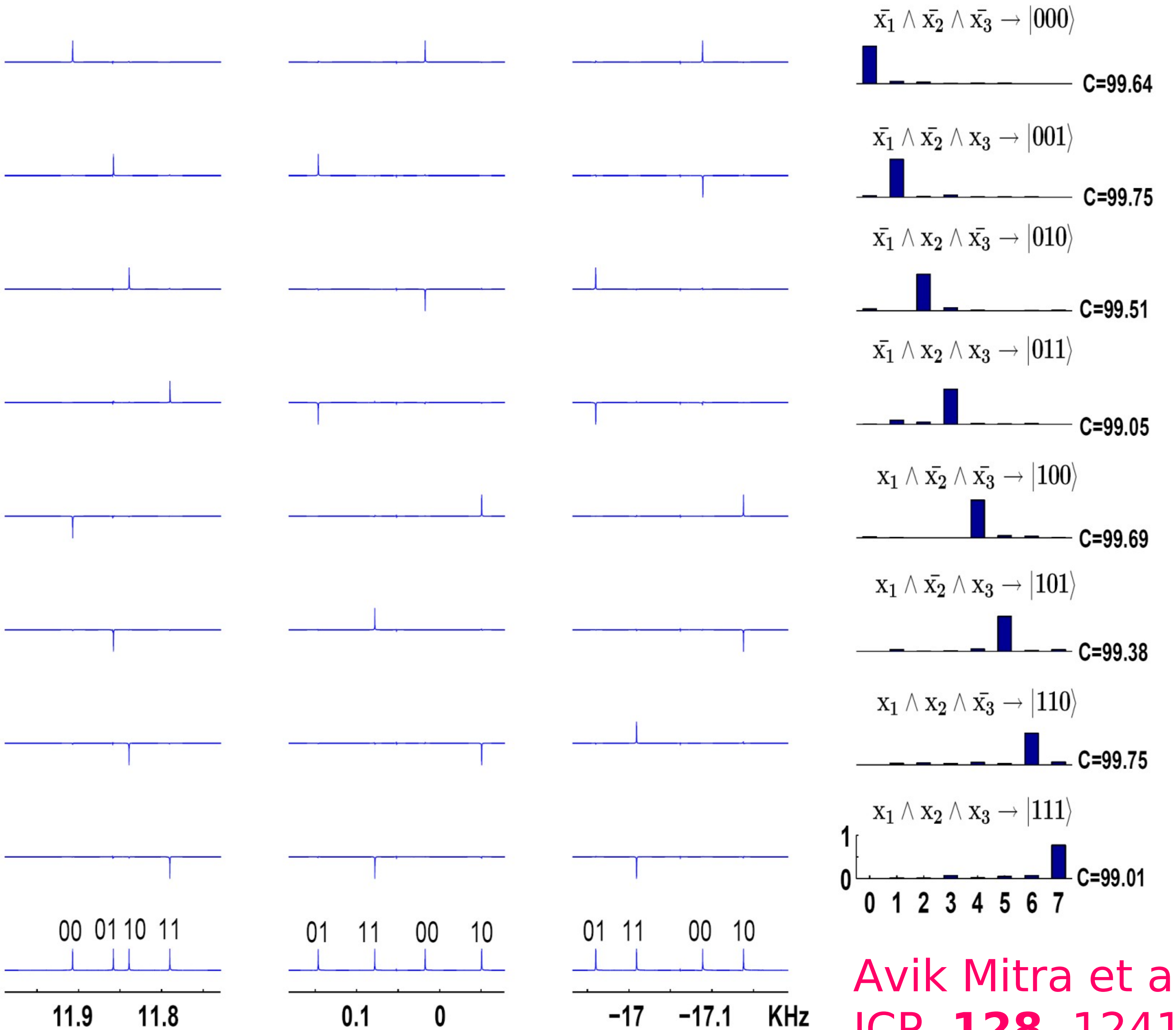
Using Concatenated SMPs



Duration: Max 5.8 ms, Min. 4.7 ms

Avik Mitra et al, JCP, **128**, 124110 (2008)

Results for all Boolean Formulae



Avik Mitra et al,
JCP, **128**, 124110 (2008)

Algorithmic Technique

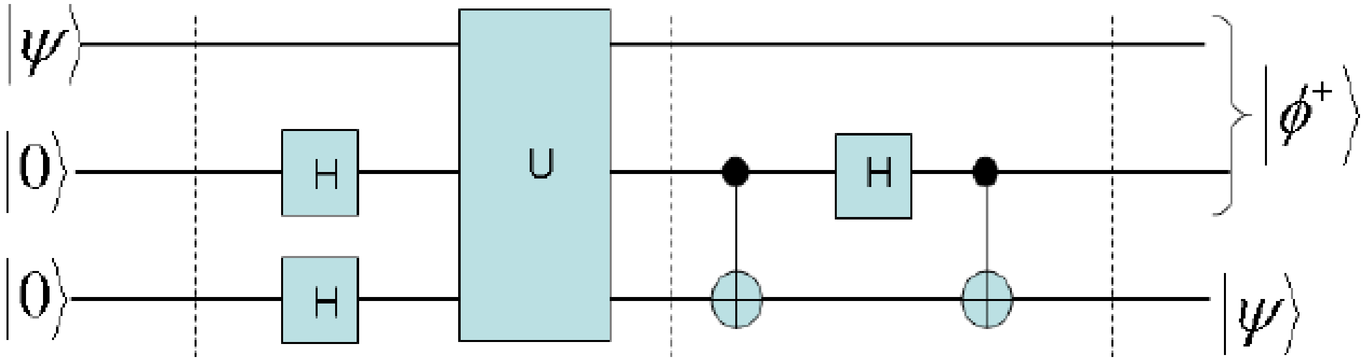
(Ashok Ajoy et. al PRL under review)

Applied for proving Quantum No-Hiding Theorem by NMR

Jharana Rani Samal, Arun K. Pati and Anil Kumar, Phys. Rev. Letters, 106, 080401 (25 Feb., 2011)

Quantum Circuit for Test of No-Hiding Theorem using State Randomization (operator U).

H represents Hadamard Gate and dot and circle represent CNOT gates.



After randomization the state |ψ> is transferred to the second Ancilla qubit proving the No-Hiding Theorem.

(S.L. Braunstein, A.K. Pati, PRL 98, 080502 (2007).

The Randomization Operator is obtained as

$U =$

<table>
<tr><th></th><th>|000></th><th>|001></th><th>|010></th><th>|011></th><th>|100></th><th>|101></th><th>|110></th><th>111></th></tr>
<tr><td>|000></td><td>1</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>|001></td><td></td><td></td><td></td><td></td><td></td><td>1</td><td></td><td></td></tr>
<tr><td>|010></td><td></td><td></td><td></td><td></td><td></td><td></td><td>1</td><td></td></tr>
<tr><td>|011></td><td></td><td></td><td></td><td>1</td><td></td><td></td><td></td><td></td></tr>
<tr><td>|100></td><td></td><td></td><td></td><td></td><td>1</td><td></td><td></td><td></td></tr>
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<tr><td>|110></td><td></td><td></td><td>-1</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>|111></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>-1</td></tr>
</table>

Blanks = 0

Conversion of the U-matrix into an NMR Pulse sequence has been achieved here by a Novel Algorithmic Technique, developed in our laboratory by Ajoy et. al (to be published). This method uses Graphs of a complete set of Basis operators and develops an algorithmic technique for efficient decomposition of a given Unitary into Basis Operators and their equivalent Pulse sequences.

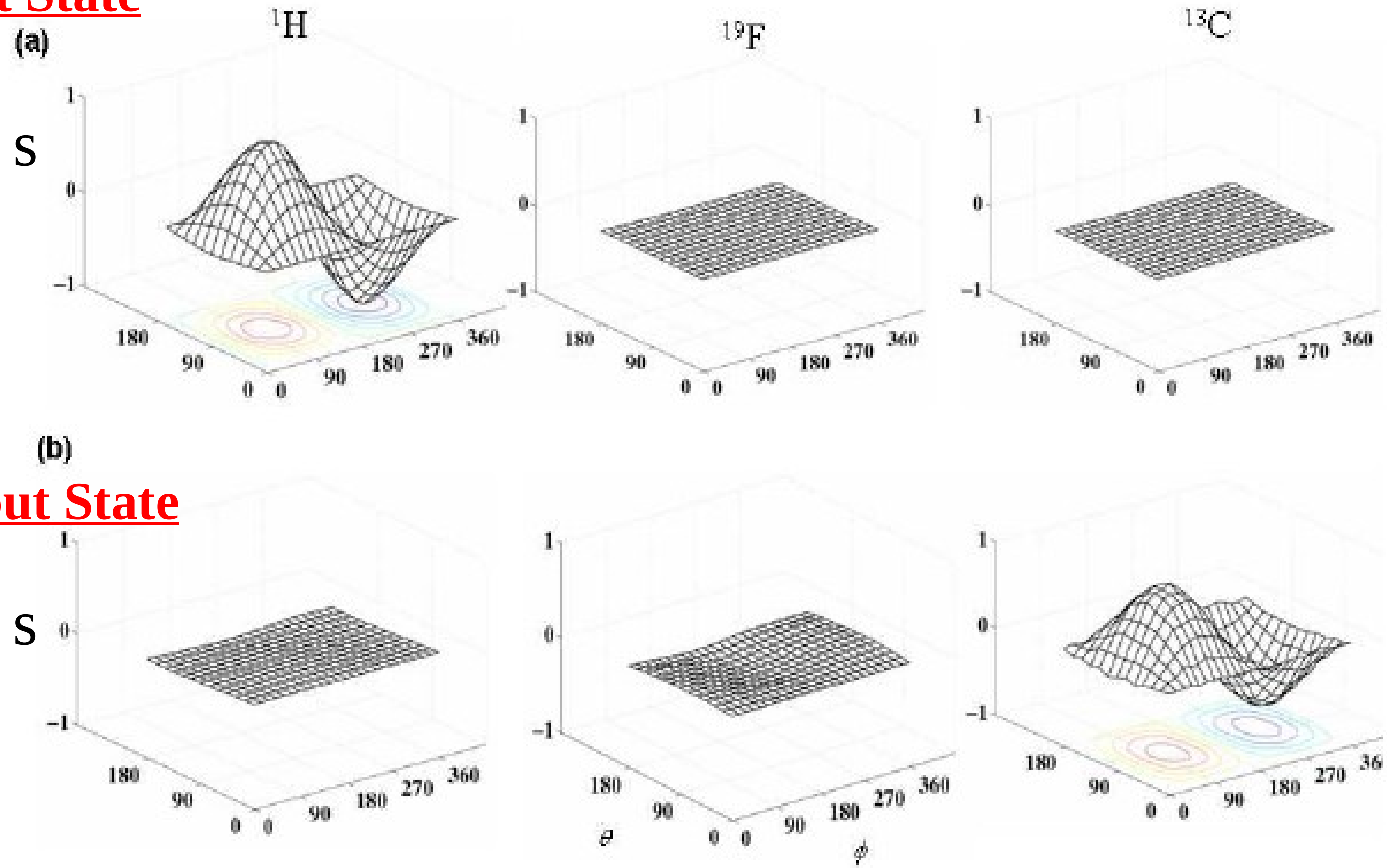
The equivalent pulse sequence for the U-Matrix is obtained as

$$U = exp(-i\frac{\pi}{4}1)exp(i\frac{\pi}{2}I_{3z})exp(-i\pi I_{1y}I_{2z})exp(-i\pi I_{1z}I_{3z})exp(i\frac{\pi}{2}I_{1x})exp(i\frac{\pi}{2}I_{1z}).$$

Experimental Result for the No-Hiding Theorem.

The state ψ is completely transferred from first qubit to the third qubit

Input State



Output State

S = Integral of real part of the signal for each spin

325 experiments have been performed by varying θ and φ in steps of 15º

All Experiments were carried out by Jharana (Dedicated to her memory)

Genetic Algorithm

We present here our latest attempt to use Genetic Algorithm (GA) for direct numerical optimization of rf pulse sequences and devise a probabilistic method for doing universal quantum computing using non-selective (hard) RF Pulses.

We have used GA for

Quantum Logic Gates (Operator optimization)

and

Quantum State preparation (state-to-state optimization)

Representation Scheme

Representation scheme is the method used for encoding the solution of the problem to individual genetic evolution. Designing a good genetic representation is a hard problem in evolutionary computation. Defining proper representation scheme is the first step in GA Optimization.

In our representation scheme we have selected the gene as a combination of

(i) an array of pulses, which are applied to each channel with amplitude (θ) and phase (φ),

(ii) An arbitrary delay (d).

It can be shown that the repeated application of above gene forms the most general pulse sequence in NMR

The Individual, which represents a valid solution can be represented as a matrix of size (n+1)x2m. Here 'm' is the number of genes in each individual and 'n' is the number of channels (or spins/qubits).

$$
\begin{pmatrix}
\theta_{11} & \varphi_{11} & & & \theta_{m1} & \varphi_{m1} \\
\theta_{12} & \varphi_{12} & \cdot & \cdot & \theta_{m1} & \varphi_{m1} \\
\vdots & & \cdot & \cdot & \vdots & \\
& & & & & \\
\theta_{1n} & \varphi_{1n} & \cdot & \cdot & \theta_{mn} & \varphi_{mn} \\
d_1 & 0 & & & d_m & 0
\end{pmatrix}
$$

So the problem is to find an optimized matrix, in which the optimality condition is imposed by a "Fitness Function"

Fitness function

In operator optimization

GA tries to reach a preferred target Unitary Operator (U_{tar}) from an initial random guess pulse sequence operator (U_{pul}).

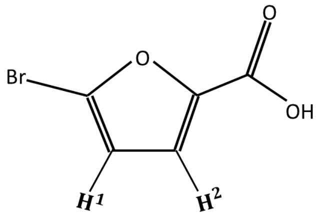
Maximizing the Fitness function

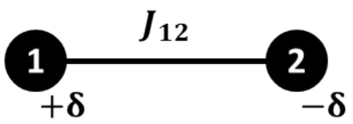
$$F_{pul} = \text{Trace}\,(U_{pul} \times U_{tar})$$

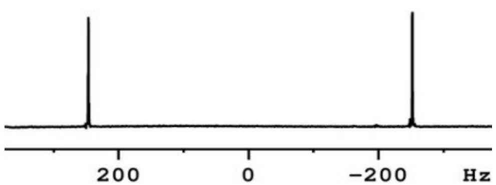
In State-to-State optimization

$$F_{pul} = \text{Trace}\,\{ U_{pul}\,(\rho_{in})\, U_{pul}^{(-1)}\, \rho_{tar}^{\dagger} \}$$

Two-qubit Homonuclear case







δ = 500 Hz, J= 3.56 Hz

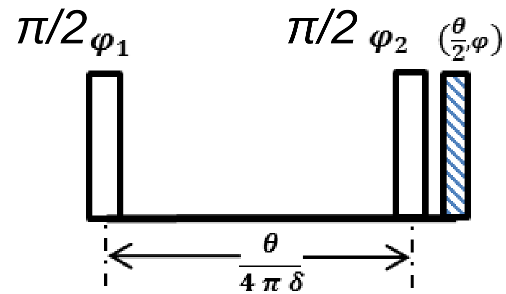
$$H = 2\pi\,\delta\,(I_{1z} - 1_{2z}) + 2\pi\,J_{12}\,(I_{1z}I_{2z})$$

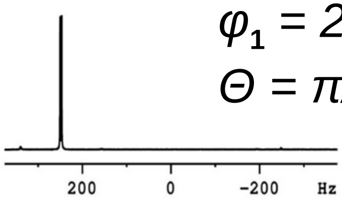
Hamiltonian used

$$H = H_{cs} = 2\,\pi\,\delta\,(I_z^1 - I_z^2)$$

Non-Selective (*Hard*) *Pulses applied in the centre*

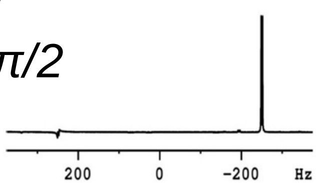
Single qubit rotation





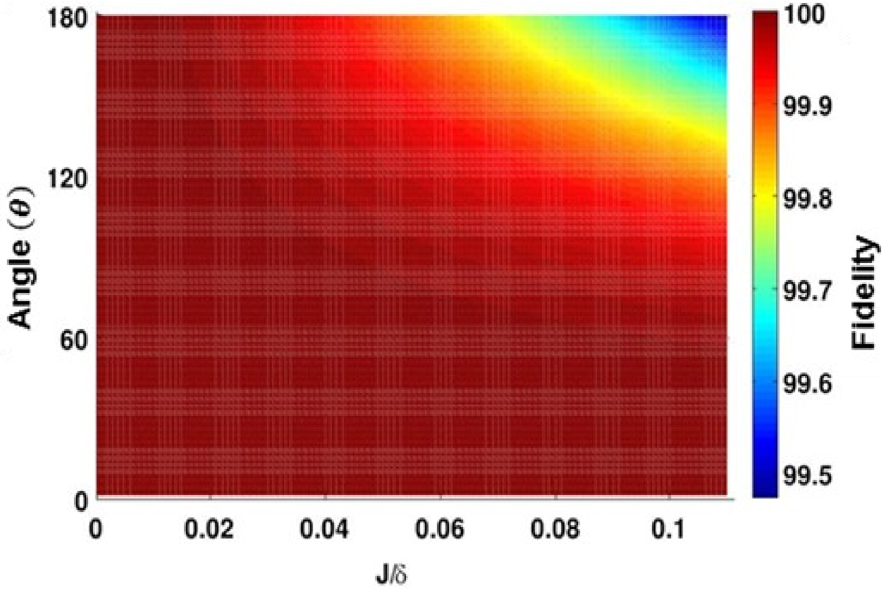
$\varphi_1 = 2\pi$, $\varphi_2 = \pi$,
$\Theta = \pi/2$, $\varphi = \pi/2$

$\varphi_1 = \pi$, $\varphi_2 = 0$
$\Theta = \pi/2$, $\varphi = \pi/2$



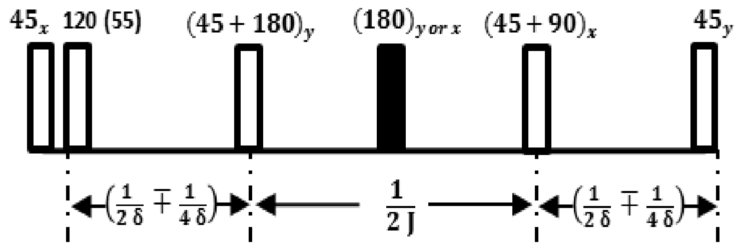
Simulated using J = 0

Fidelity for finite J/δ

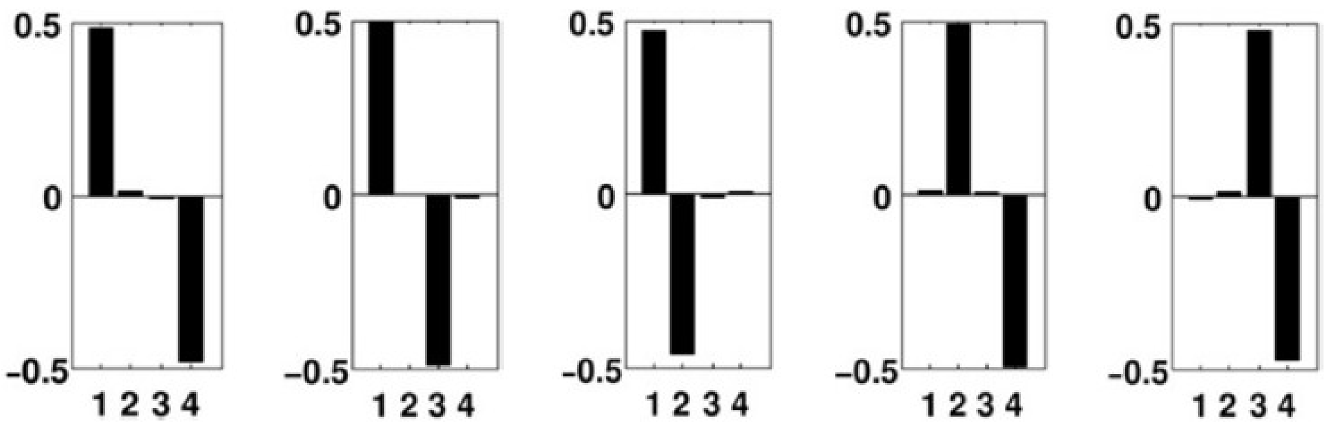


Controlled- NOT:

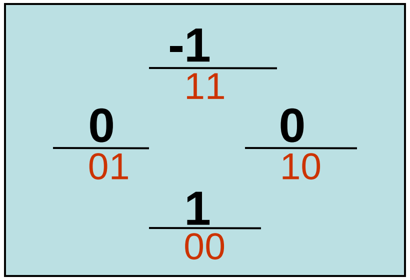
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



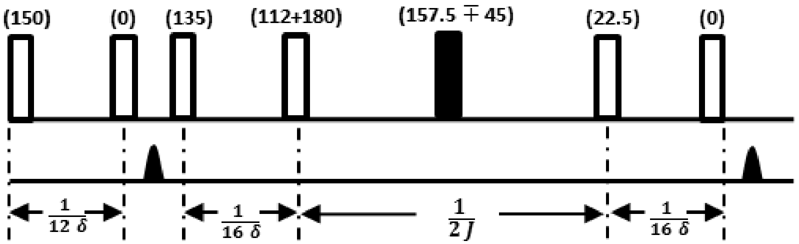
The phase of π pulse (*y or x*) decides the operation type (C-NOT or C-$\overline{NOT}$) and $\mp$ in first and third delays decides the Control Qubit.



Equilibrium　C-NOT(1,2)　C-NOT(2,1)　C-$\overline{\text{NOT}}$(1,2)　C-$\overline{\text{NOT}}$ (2,1)

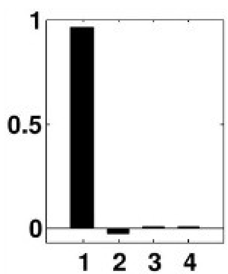


Pseudo Pure State (PPS) creation

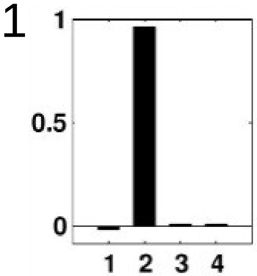


All unfilled rectangles represent 90^0 pulse
The filled rectangle is 180^0 pulse.
Phases are given on the top of each pulse.

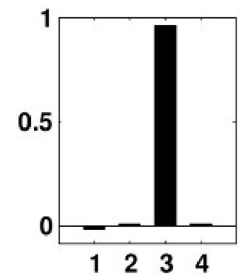
00



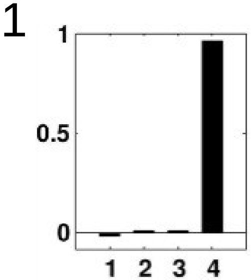
01



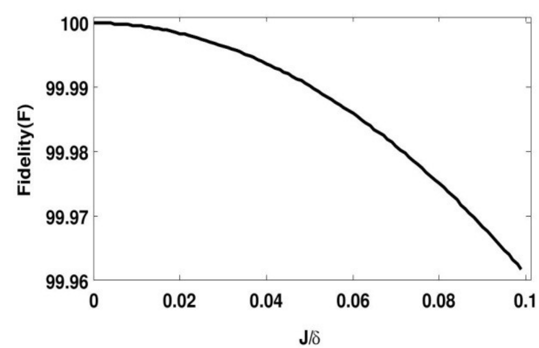
10



11

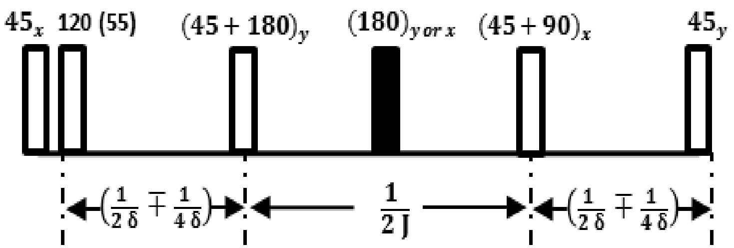


Fidelity w.r.t. to J/δ

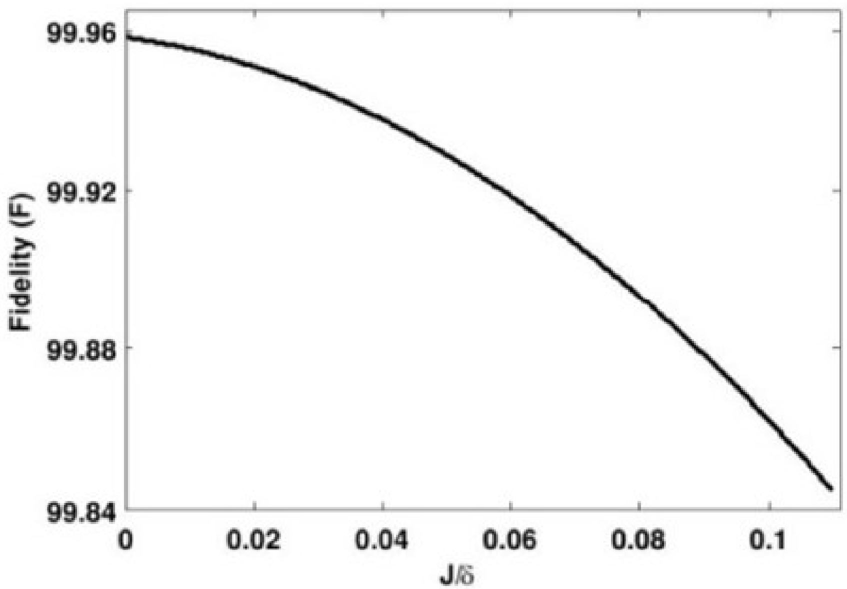


Controlled- Hadamard:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

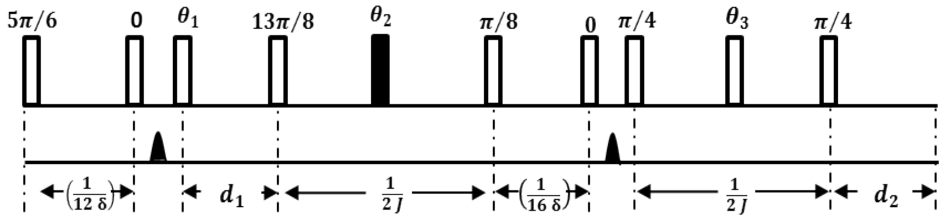


$\mp$ in phase of 180 pulse decides whether the operator is C-H or C-$\overline{H}$ (-for C-H). $\mp$ in delay decides the control qubit(- for first qubit as control).



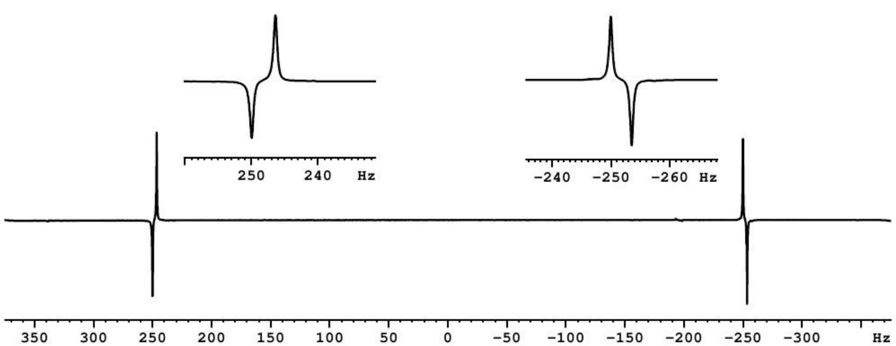
Bell state creation: From Equilibrium (No need of PPS)

Bell states are maximally entangled two qubit states.



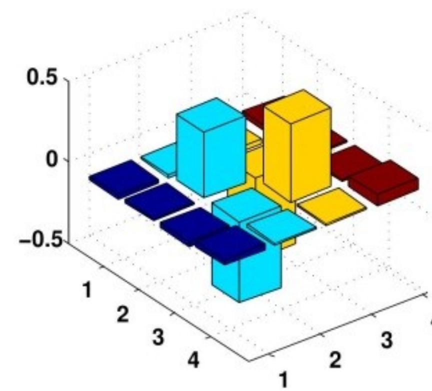
All blank pulses are 90^0 pulses. Filled pulse is a 180^0 pulse. Phases and delays Optimized for best fidelity.

	θ_1	θ_2	θ_3	d_1	d_2
$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$	0	$5\pi/8$	$3\pi/4$	$9/48\delta$	$1/8\delta$
$\frac{1}{\sqrt{2}}(\|01\rangle - \|10\rangle)$	0	$5\pi/8$	$\pi/4$	$9/48\delta$	$1/8\delta$
$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$	$3\pi/4$	$9\pi/8$	$3\pi/4$	$1/16\delta$	0
$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$	$3\pi/4$	$9\pi/8$	$\pi/4$	$1/16\delta$	0



Experimental Fidelity > 99.5 %

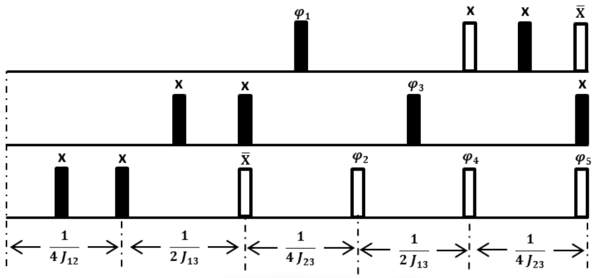
Shortest Pulse Sequence for creation of Bell States directly from Equilibrium



The Singlet Bell State

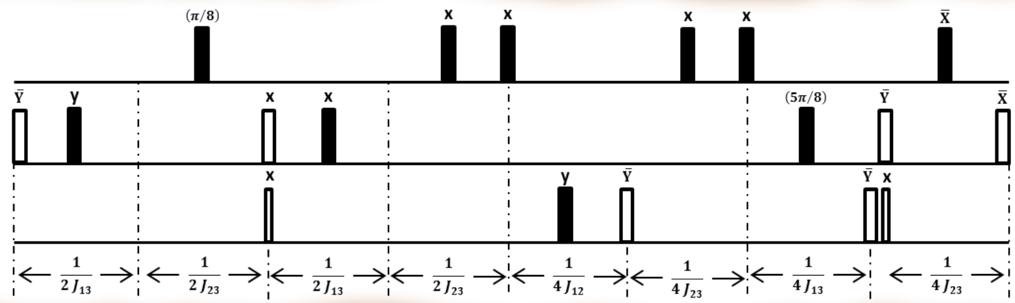
Three qubit system :

CC-NOT

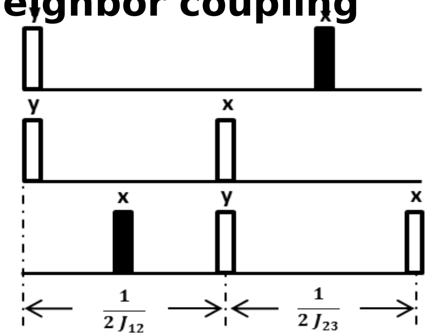


	φ_1	φ_2	φ_3	φ_4	φ_5
$C^2NOT(\bar{1},\bar{2})3$	$3\pi/8$	$3\pi/4$	$7\pi/8$	$\pi/4$	$\pi/2$
$C^2NOT(\bar{1},2)3$	$5\pi/8$	$5\pi/4$	$15\pi/8$	$3\pi/4$	$\pi/2$
$C^2NOT(1,\bar{2})3$	$11\pi/8$	$3\pi/4$	$\pi/8$	$\pi/8$	$3\pi/2$
$C^2NOT(1,2)3$	$\pi/8$	$\pi/4$	$\pi/8$	$3\pi/4$	$3\pi/2$

Controlled SWAP:



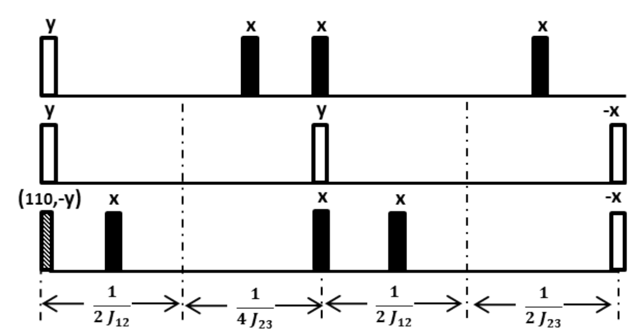
Creating GHZ state using nearest neighbor coupling



Initial state	Final state
$\lvert 000\rangle$	$\frac{1}{\sqrt{2}}(\lvert 010\rangle - \lvert 101\rangle)$
$\lvert 001\rangle$	$\frac{1}{\sqrt{2}}(\lvert 011\rangle + \lvert 100\rangle)$
$\lvert 010\rangle$	$\frac{1}{\sqrt{2}}(\lvert 001\rangle + \lvert 110\rangle)$
$\lvert 011\rangle$	$\frac{1}{\sqrt{2}}(\lvert 000\rangle - \lvert 111\rangle)$
$\lvert 100\rangle$	$\frac{1}{\sqrt{2}}(\lvert 010\rangle + \lvert 101\rangle)$
$\lvert 101\rangle$	$\frac{1}{\sqrt{2}}(\lvert 011\rangle - \lvert 100\rangle)$
$\lvert 110\rangle$	$\frac{1}{\sqrt{2}}(\lvert 001\rangle - \lvert 110\rangle)$
$\lvert 111\rangle$	$\frac{1}{\sqrt{2}}(\lvert 000\rangle + \lvert 111\rangle)$

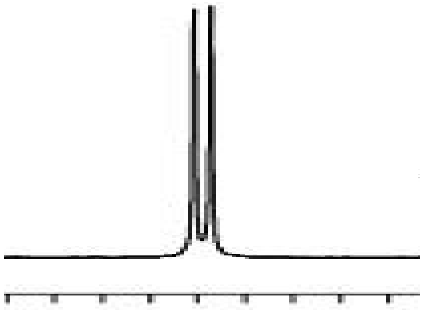
Creating w state using nearest neighbour coupling

$$\left(\lvert 000\rangle \rightarrow \frac{1}{\sqrt{3}}(\lvert 001\rangle + \lvert 010\rangle + \lvert 100\rangle)\right)$$



We plan to use these GA methods for implementation of various Algorithms

Thank You



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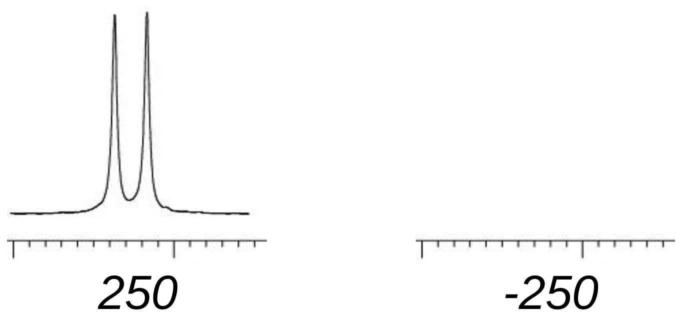
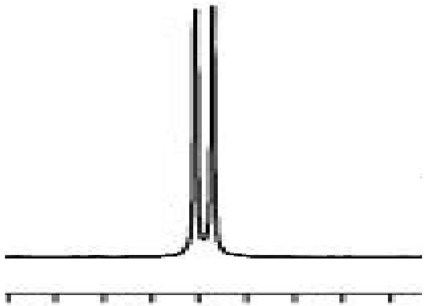

250
-250



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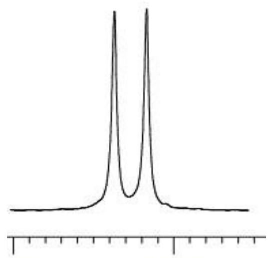

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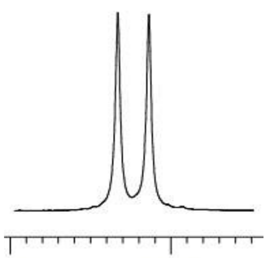

-250