Dynamical Decoupling Applied to Polarization Qubits

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Outline

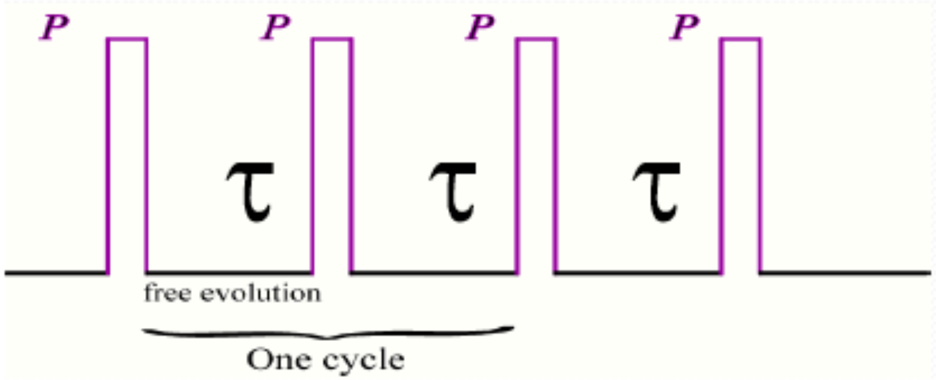
- Dynamical Decoupling (DD)-Introduction.
- Group theoretic interpretation of DD.
- Different DD Schemes-PDD, CDD, UDD, QDD.
- DD applied to polarization qubits in optical fiber.
- Results and Discussion.
- Summary.

DD-General Idea

- Motivation: To protect the qubit in presence of decoherence.
- Applying pulses to time-reverse the system-bath interaction.
- A pulse producing a unitary evolution P, such that

  $PH_{SB} P^{\dagger}=-H_{SB}$, i.e. $\{P, H_{SB}\}=0$

Suppression of Decoherence

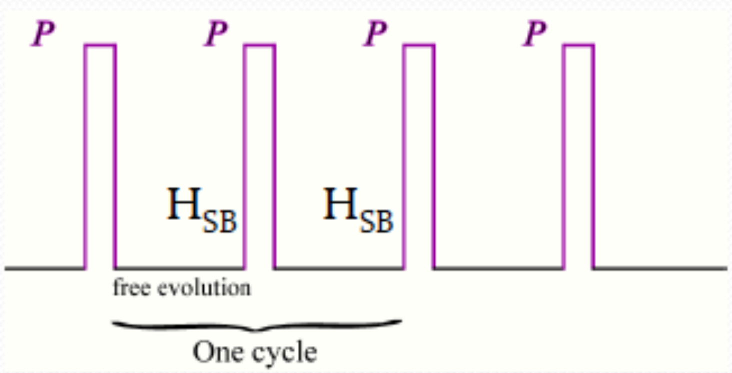


For ideal (zero-width) pulses

$P\exp(-i\tau H_{SB})P^{\dagger}\exp(-i\tau H_{SB})=$
$\exp(-i\tau P H_{SB} P^{\dagger})\ \exp(-i\tau H_{SB})$
$=\exp(i\tau H_{SB})\ \exp(-i\tau H_{SB})$
$=I$

Quick Example: Pure dephasing->$H_{SB}=\lambda Z\otimes B$

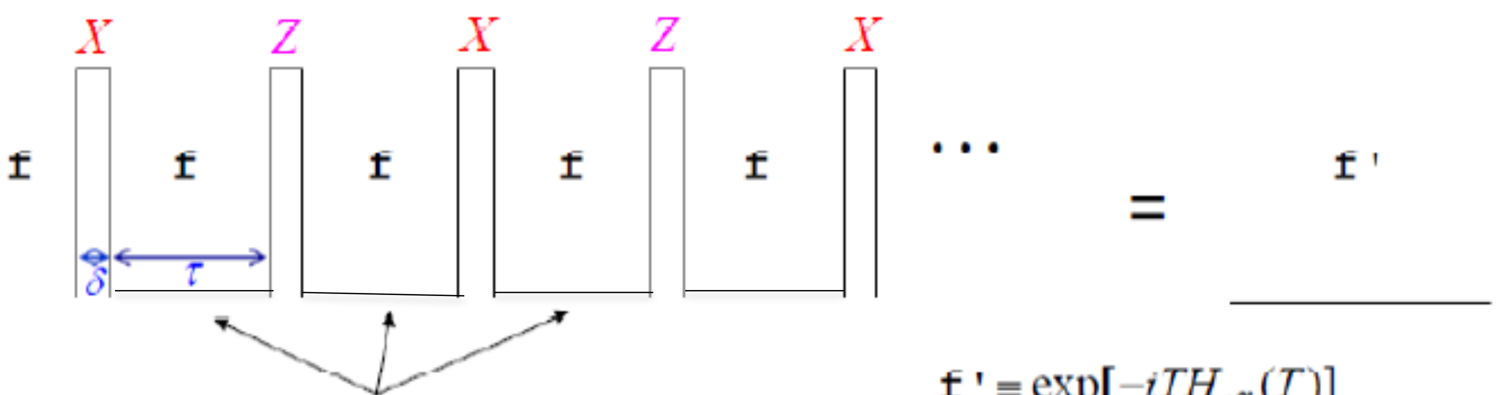
$XZX=-Z$=>
"time-reversed",
H_{SB} averaged to zero (in 1st order Magnus expansion)



H_{SB} = System-Bath Interaction
P = Pulse(typically π pulses)

Effective Hamiltonian

Another view of the universal decoupling sequence:



$\mathtt{f} \equiv \exp[-i\tau H_{SB}]$

$\mathtt{f}' \equiv \exp[-iTH_{eff}(T)]$

$H_{eff}(T)$=0, ideally

But, errors accumulate…: $H_{eff}(T) \neq 0$

DD AS SYMMETRIZATION

"Symmetrizing group" of pulses $\{g_i\}$ and their inverses are applied in series:

$$\boxed{(g_N^\dagger \mathtt{f} g_N)\cdots(g_2^\dagger \mathtt{f} g_2)(g_1^\dagger \mathtt{f} g_1) \approx \exp(-i\tau \sum_i g_i^\dagger H_{SB} g_i)}$$

$$\mathtt{f} \equiv \exp(-iH_{SB}\tau)$$

first order Magnus expansion

Choose the pulses so that:

$$\boxed{H_{SB} \mapsto H_{\text{eff}}^{(1)} \equiv \sum_i g_i^\dagger H_{SB} g_i = 0 \qquad \textbf{Dynamical Decoupling Condition}}$$

For a qubit the Pauli group $G=\{X,Y,Z,I\}$ (π pulses around all three axes) removes an *arbitrary* H_{SB}:

$$\boxed{(\mathtt{XfX})\,(\mathtt{YfY})\,(\mathtt{ZfZ})\,(\mathtt{IfI}) = \underline{\mathtt{XfZfXfZf}}}$$

Periodic DD: periodic repetition of the universal DD pulse sequence

DD as a Rescaling Transformation

$$J = \|H_{SB}\|_\infty$$
$$\beta = \|H_B\|_\infty$$

- Interaction terms are rescaled after the DD cycle

$$J = J^{(0)} \mapsto J^{(1)} \propto \max[\tau(J^{(0)})^2, \tau\beta J^{(0)}]$$

$$\beta \mapsto \beta + O((J^{(0)})^3\tau^2)$$

- We need a mechanism to continue this

Periodic Dynamical Decoupling

PDD Strategy: repeat the basic XfZfZfXfZ cycle with total of N pulses.

The total duration is fixed at T. N can be changed.

Pulse interval: $\tau = T/N$

Recall noise strength $\eta \equiv ||H_{\text{eff}}(T)||T$

norm of final effective system-bath Hamiltonian times the total duration.

PDD leading order result for error:

$$\eta \propto N^{-1}$$

Can we do better?

Concatenated Universal Dynamical Decoupling

Nest the universal DD pulse sequence into its own free evolution periods **f** :

```
p(1)= X f   Z f   X f   Z f
p(2)= X p(1)Z p(1)X p(1)Z p(1)
      p(n+1)= X p(n)Z p(n)X p(n)Z p(n)
```

Level	Concatenated DD Series after multiplying Pauli matrices
1	XfZfXfZf
2	fZfXfZfYfZfXfZffZfXfZfYfZfXfZf
3	XfZfXfZfYfZfXfZffZfXfZfYfZfXfZfZfZfXfZfYfZfXfZffZfXfZfYfZfXfZfXfZf XfZfYfZfXfZffZfXfZfYfZfXfZfZfZfXfZfYfZfXfZffZfXfZfYfZfXfZf

Length grows exponentially; how about error reduction?

Performance of Concatenated Sequences

$$\text{error} \mapsto (\text{error})^2 \mapsto ((\text{error})^2)^2 \mapsto (((\text{error})^2)^2)^2 \mapsto \underset{k}{\cdots} \mapsto (\text{error})^{2^k}$$

For fixed total time $T=N\tau$ and N zero-width (ideal) pulses:

$$\boxed{\eta \propto N^b N^{-c \log N}}$$

Compare to periodic DD: $\eta \propto N^{1}$

PDD vs. CDD

- CDD outperforms PDD for the bounded bath with ***super-polynomial*** advantage.
- In PDD, errors accumulate if not removed in the basic cycle.
- While in CDD, the next-layer-up removes the errors left from the last layer.
- Particularly important up to the 2^{nd} order errors in the Magnus expansion

BETTER THAN CONCATENATED DD

Question

Does there exist an ***optimal*** pulse sequence?

Optimal-> removes maximum decoherence with least possible number of pulses.

Uhrig Dynamical Decoupling

- Optimization of the switching instants.
- Good for *pure dephasing* as originally proposed by Uhrig.
- Requires $O(\mathrm{n})$ number of pulses.
- Cycles with n pulses are used with the pulses applied at

$$\delta_j = T\sin^2[\, j\pi/(2n + 2)]$$

Spin- Boson Model with Pure Dephasing

$$H = \sum_i \omega_i b_i^\dagger b_i + \frac{1}{2}\sigma_z \sum_i \lambda_i (b_i^\dagger + b_i) + E$$

- $\{\lambda_i, \omega_i\}$ – Properties of the bath
- Spectral density $J(\omega) = \sum_i \lambda_i^2 \delta(\omega - \omega_i).$
- Free induction decay $e^{-2\chi(t)}$ where the decoherence function

$$\chi(t) := \int_0^\infty \frac{S(\omega)}{\omega^2} |y_n(\omega t)|^2 d\omega.$$

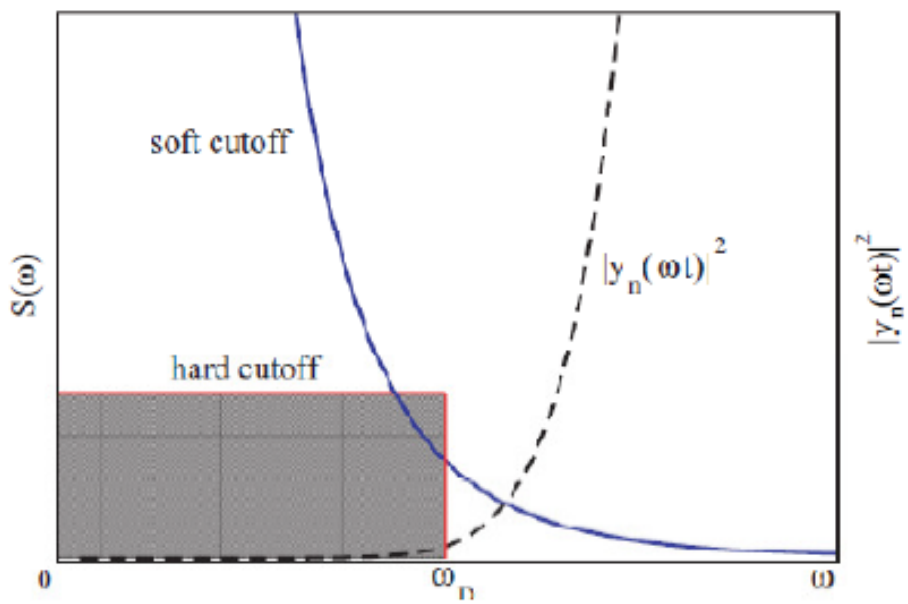
$$S(\omega) := \tfrac{1}{4} J(\omega) \coth(\beta\omega/2)$$

y_n- > Filter function for n pulses.

UDD Continued:

- **Aim:** We have to minimize χ.

$e^{-2\chi(t)} \sim 1$ when χ (t) is written in powers of t.



- Analytically shown first n derivatives of $y_n(\omega t) = 0$ at $\omega t = 0$ for n pulses.

Power Spectra

$$\frac{S(\omega)}{\omega^2} = \frac{S_0}{\omega^{\alpha+1}}.$$ where α=power law exponent

- Convergence of $$\chi(t) = S_0 \int_0^\infty \frac{1}{\omega^{\alpha+1}} |y_n(\omega t)|^2 d\omega$$
- For $y_n(\omega t) \propto (\omega t)^{m+1}$, we must have $\alpha < 2m + 2.$
- UDD applies at all temperatures as long as the cut-off is hard
- Experimentally verified case: $S(\omega) \propto 1/\omega^4$ $(\alpha = 5)$

Quadratic Dynamical Decoupling (QDD)

[West, Fong and Lidar 2010]

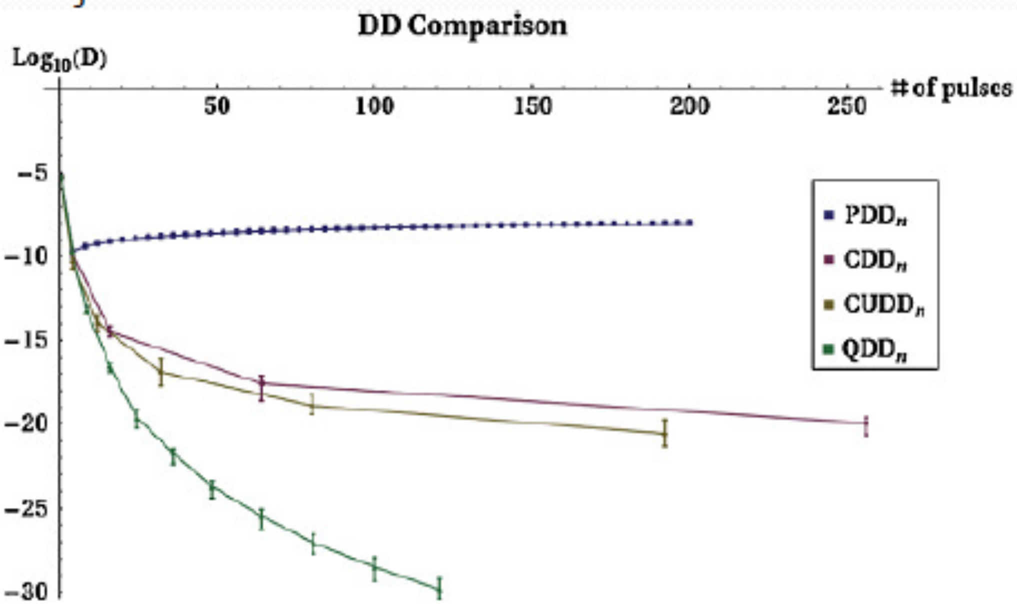
- One UDD sequence in one direction nested within another UDD sequence in an orthogonal direction.
- *X*-type UDD sequence (for pure dephasing) with a *Z*-type UDD sequence (for longitudinal relaxation).
- Relative sizes of the pulse intervals are considered

$$s_j \equiv \frac{t_j - t_{j-1}}{t_1 - t_0} = \sin\left(\frac{(2j-1)\pi}{2n+2}\right)\csc\left(\frac{\pi}{2n+2}\right),$$

- Pulse intervals $\sim(n+1)^2$ are required.

QDD (Contd.)

- Total normalized time $S_n \equiv \sum_{j=1}^{n+1} s_j = \frac{t_{n+1}}{t_1} = \csc^2\left(\frac{\pi}{2n+2}\right)$,
- Z-type UDD$_n$ $Z_n(\tau) \equiv Z^n U(s_{n+1}\tau) Z U(s_n\tau) \cdots Z U(s_2\tau) Z U(s_1\tau)$
- $U(s_j\tau)$ is replaced by time-scaled sequence $X_n(s_j\tau)$.



West *et al*, PRL 104 130501 (2010)

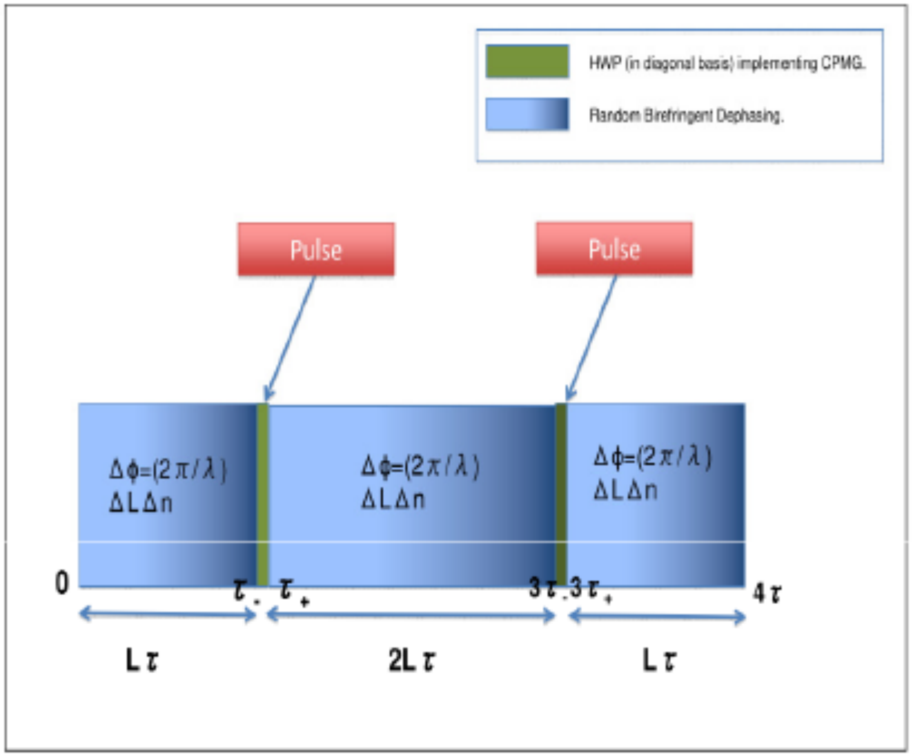
DD with Polarization Qubits

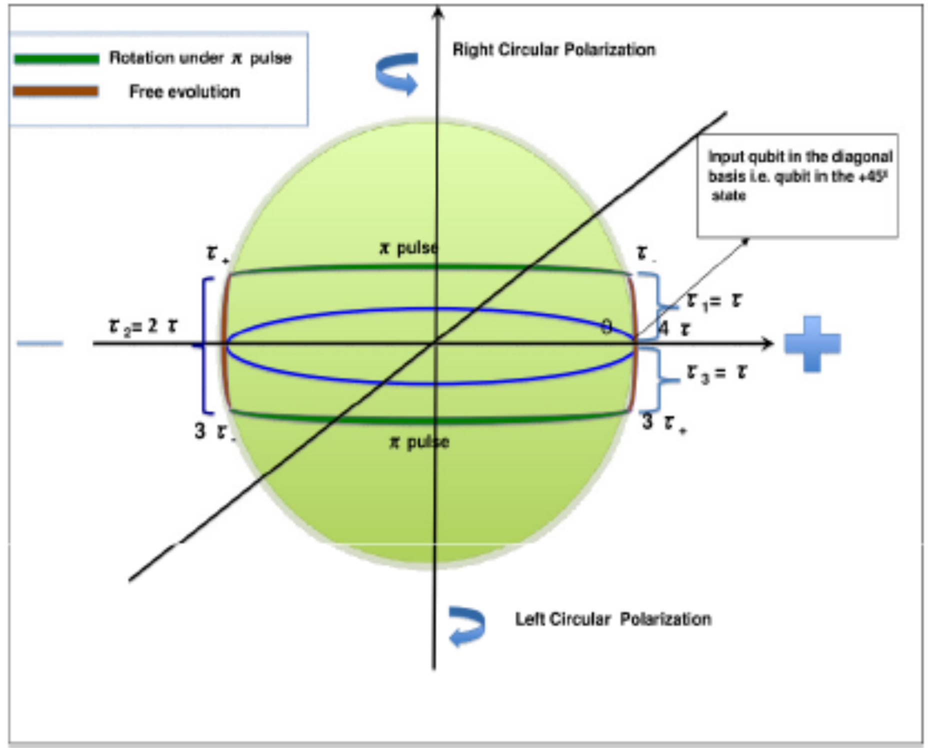
Motivation

- Wu and Lidar [2004] analytically showed `bang-bang' decoupling could be used to suppress dephasing in optical fibers.
- However, CPMG sequence has been shown to be very robust against a variety of dephasing and rotation errors.
- In the BB84 protocol, it is crucial to preserve the input polarized signals against decoherence effects through the fiber.

- L. A. Wu and D. A. Lidar, Phys. Rev. A 70, 062310 (2004).
- S. Meiboom and D. Gill, Rev. Sci. Instrum. 29, 688 (1958).

Dynamical Decoupling with Polarization Qubits



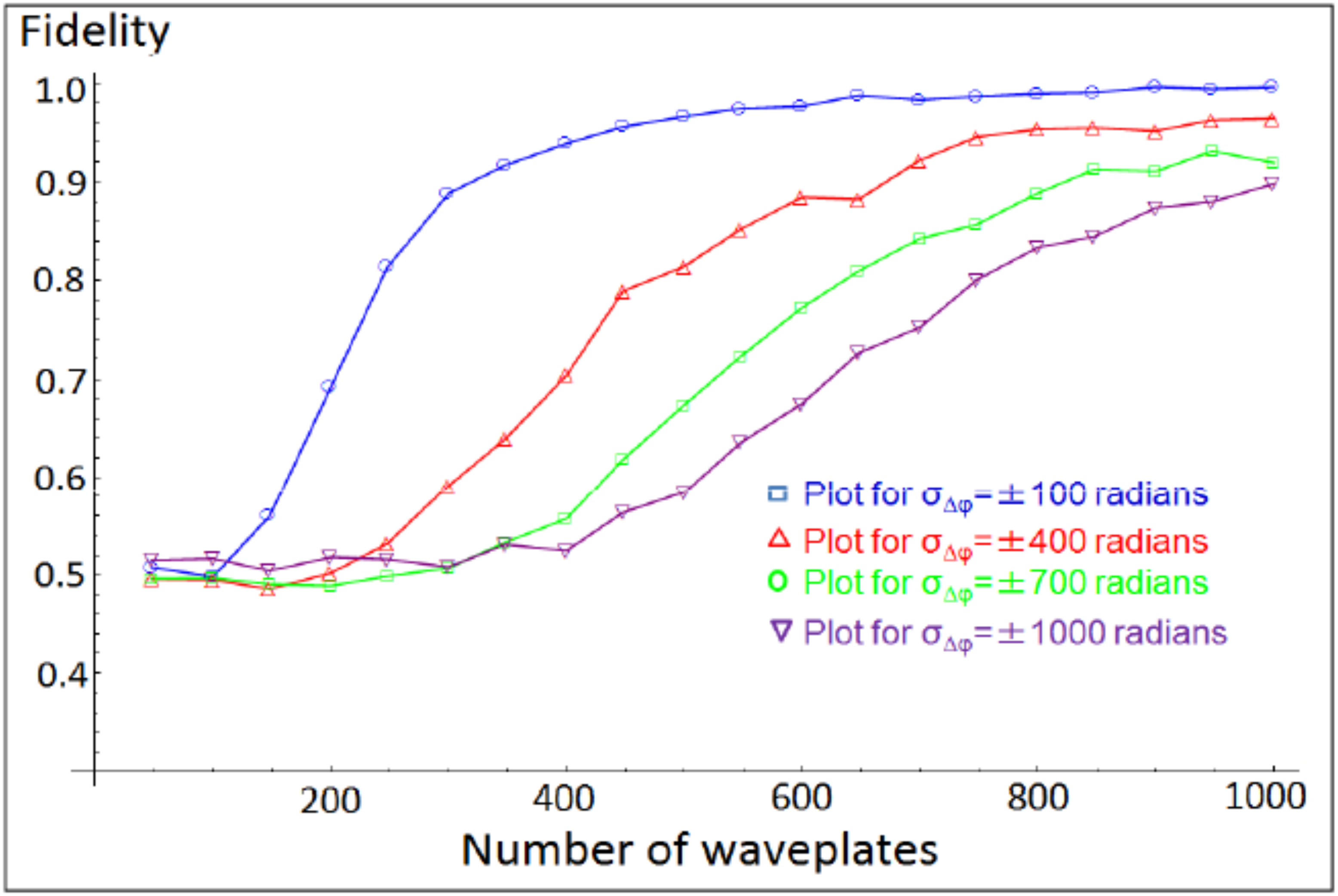


- Dephasing after a propagation length ΔL is $\Delta\phi=(2\pi/\lambda)\Delta L\ \Delta n$;

  Δn = Refractive index difference, λ=Wavelength in vacuum.

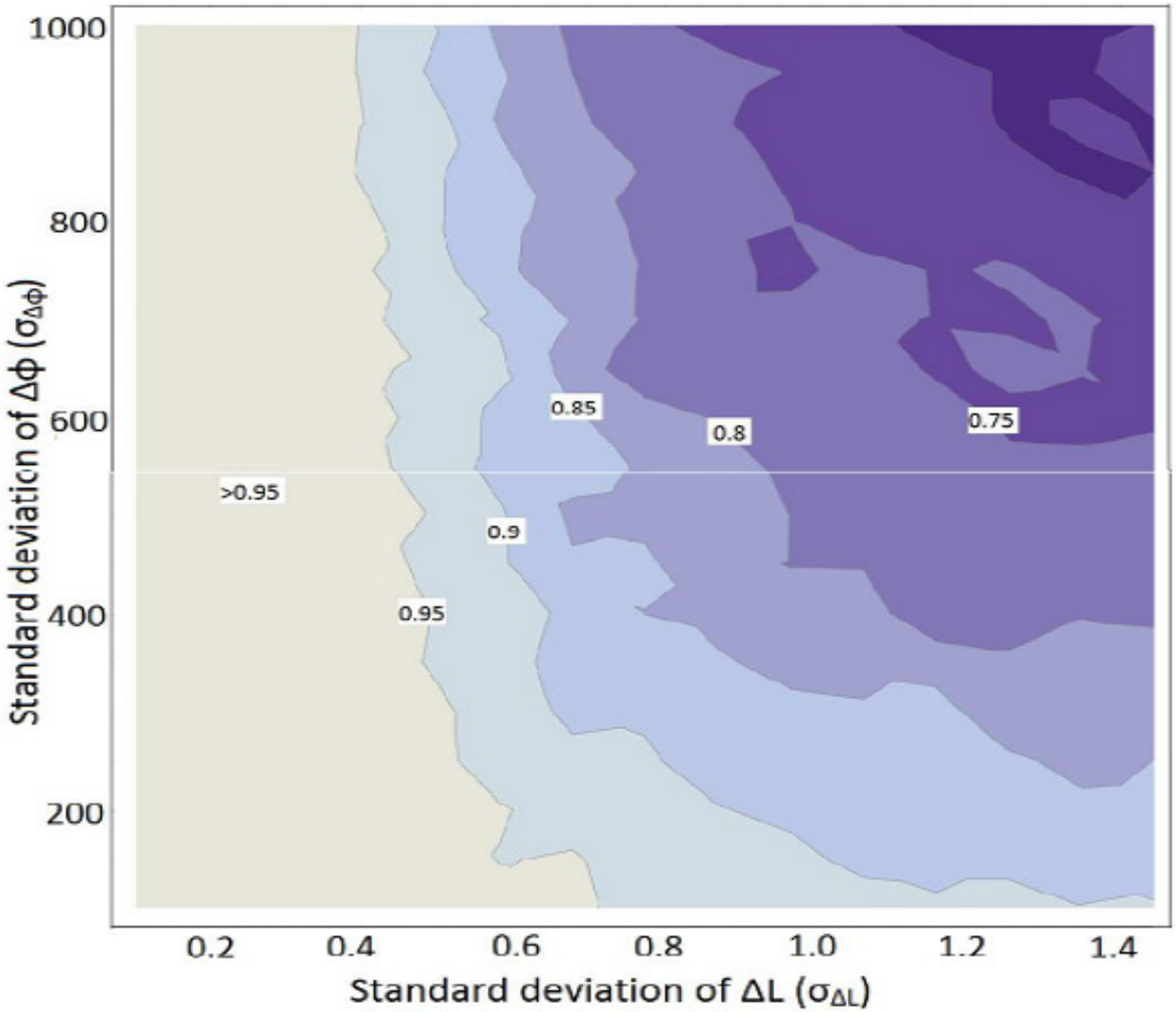
- **CPMG** sequence implemented by half-wave plates along the fiber.
- Both ΔL and Δn were generated randomly to simulate the random dephasing.

DD Preserves the Polarization State

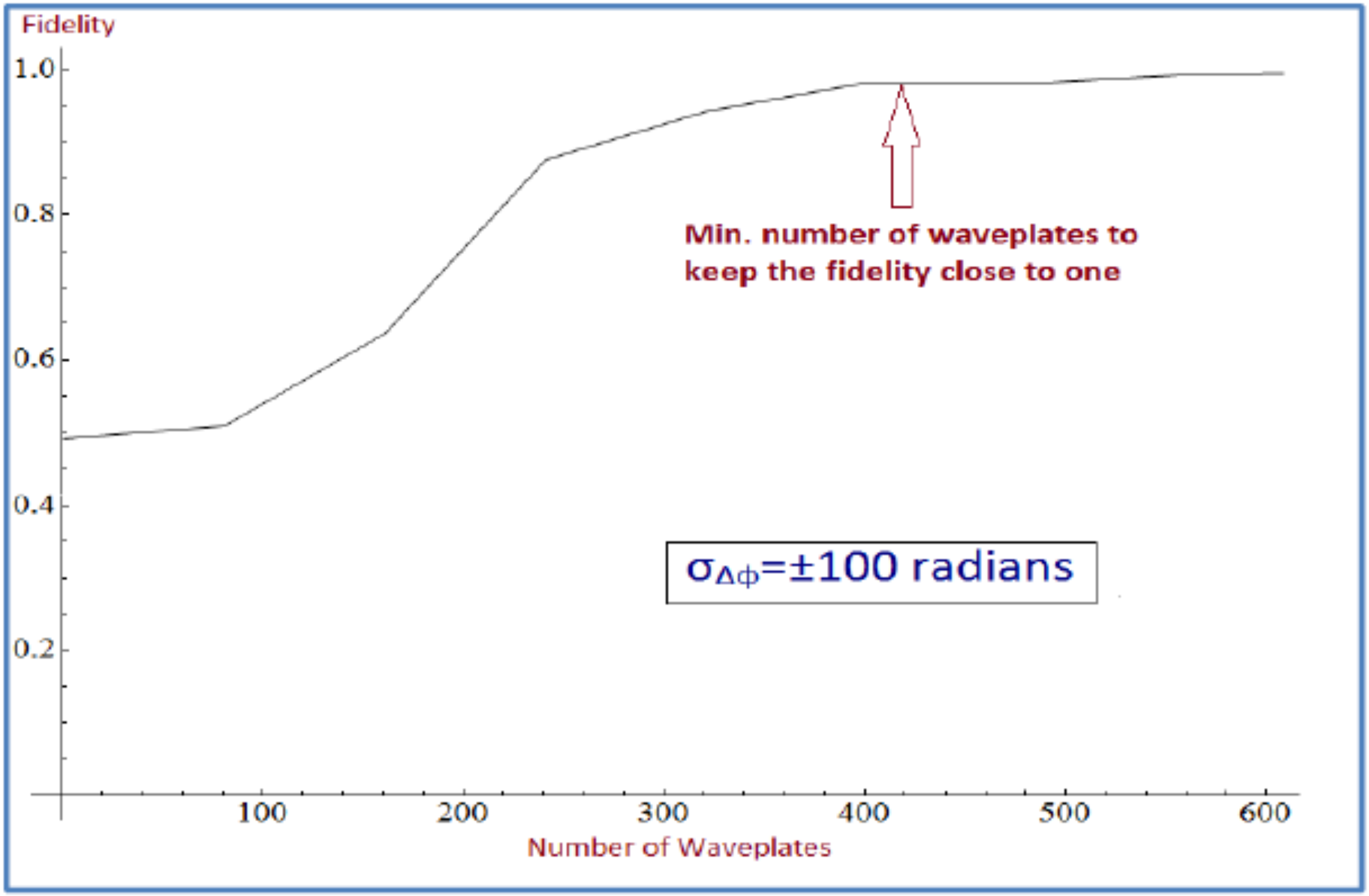


Fidelity increases significantly when we apply DD in the fiber, even when the fluctuations in $\Delta\phi$ are high.

Contour plot of fidelity

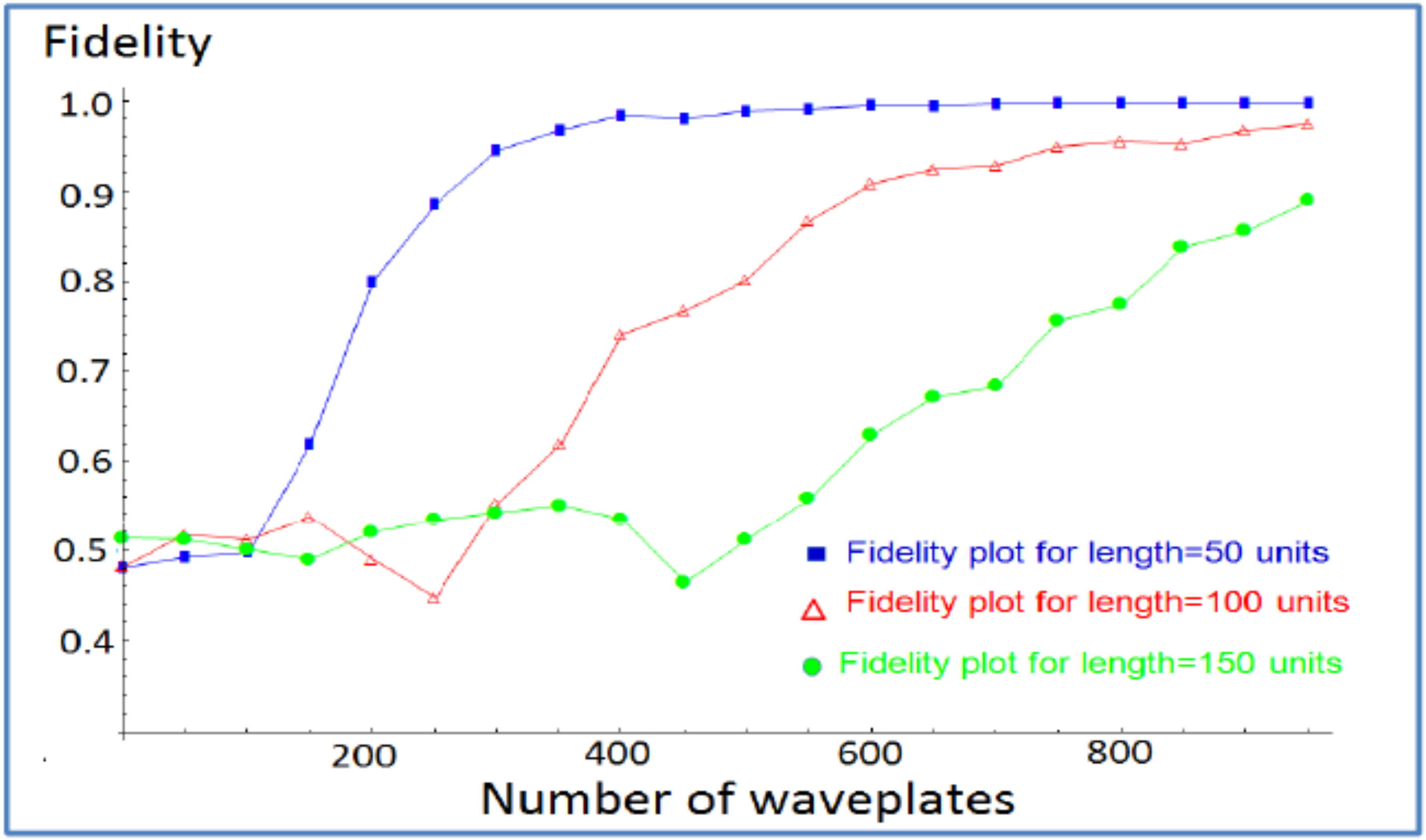


Estimating Minimum Number of Waveplates



- Equivalently, the **optimal distance between the waveplates** can be estimated for a given length of the fiber.

Fidelity for Different Fiber Lengths



- For a fixed range of $\Delta\phi$ variation (i.e. fixed standard deviation σ of $\Delta\phi$), the fidelity varies when we change the total length of the fiber.

Mean value of ΔL

- **Dimensionless** : $\Delta\phi = (2\pi/\lambda)\, \Delta L\, \Delta n = [(2\pi/\lambda)\, \Delta n < \Delta L>][\Delta L/< \Delta L>]$

=> **any** fiber length can be used.

Summary

- We could successfully apply the CPMG sequence to the optical fiber with flying polarization qubits.
- Waveplate seperation for achieving fidelity close to 1 could be estimated precisely for any arbitrary and practical fiber length.
- This is valid for any general polarization qubit.
- In future, we wish to incorporate finite width of the waveplate as well as use DD within gates for polarization qubits.

Thank You