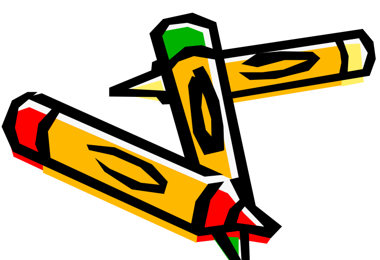
Introduction to Quantum Game Theory

Multiparticipant decision problems

strategic interdependence

Classical game theory

- began in 1944 with '*The Theory of Games and Economic Behavior*', by John von Neumann and Oscar Morgenstern
- Originally based in classical physics
- generalized to include laws of quantum mechanics by Meyer in 1999.



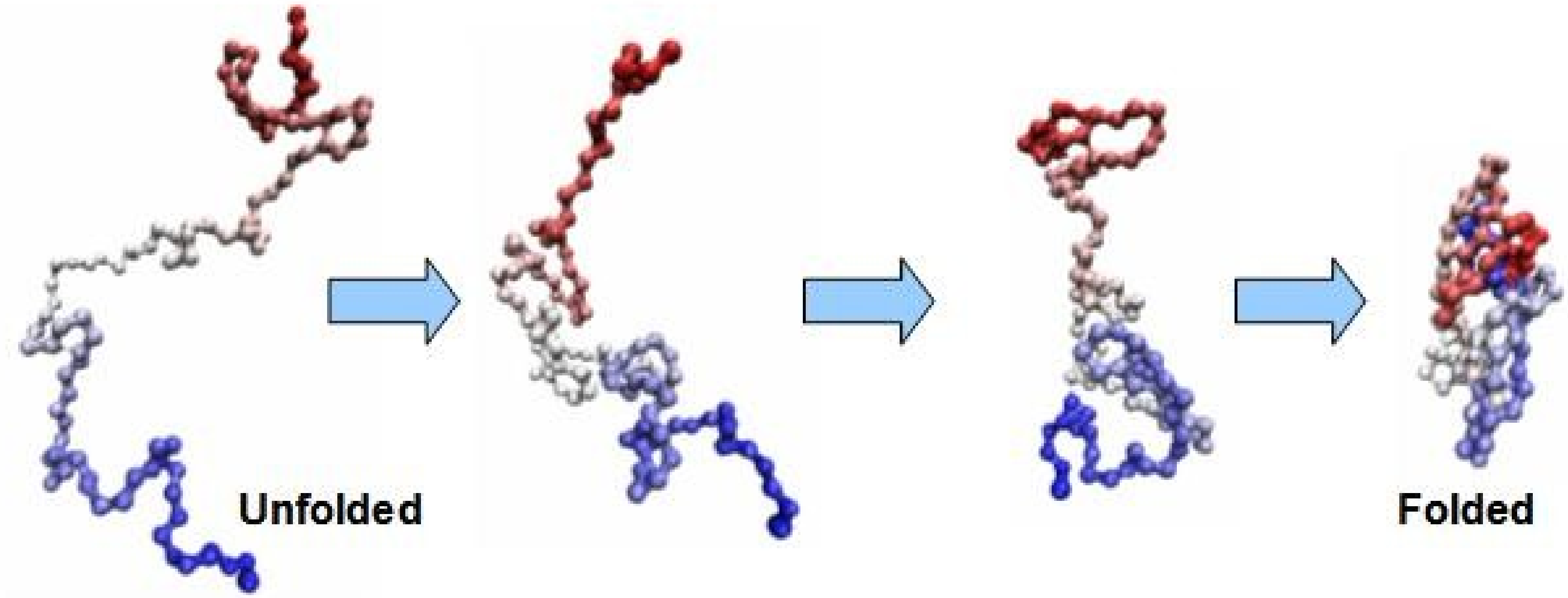
What range of application does game theory have?

Applications

- Economic theory (game of maximizing monetary rewards, stock market, business, ie supply and demand)
- Diplomacy(2 to N players, N>2 coalitions tend to form)
- Secure communications (the quantum-mechanical protocols for eavesdropping [Gisin & Huttner 1997, Ekert 1991] and optimal cloning can be formulated as games
- Quantum algorithms can be formulated as a game between classical and quantum players
- Fundamental questions of Quantum mechanics (e.g. protein folding, and electrons can be viewed as playing a quantum game competing for atomic orbitals)
- Dawkins' dictum of the ‘Selfish Gene’ can be thought of in terms of games of survival. Colonies of bacteria appear to play the game of prisoner dilemma

Hence a `**game**’ is quite a general construct.

Protein folding



E.g. Prisoner Dilemma game

Bob Alice	C	D
C	(3,3)	(0,5)
D	(5,0)	(1,1)

Payoff(Alice,Bob)

Regardless of Bob's choice, Alice always maximizes her payoff by playing D and similarly for Bob thus forming the **pure** Nash equilibrium of D,D with a payoff of 1 unit for each player.

NB: Both players would prefer the outcome C,C!

Bacterium Prisoner's dilemma

	φ6	φH2
φ6	1	0.65
φH2	1.99	0.83

- Realized pay-off matrix for the evolved high MOI phage ØH2 relative to its ancestor Ø6 reveals evolution of an evolutionarily stable strategy conforming to the prisoner's dilemma. Turner & Chao, Letters to Nature, 1999.

Nash equilibrium

- A NE is found if any unilateral deviation of this strategy results in a lower payoff
- A NE, X’,Y’ Can be defined as

$$\Pi_A(X',Y) \geq \Pi_A(X,Y)$$

$$\Pi_B(X,Y') \geq \Pi_B(X,Y)$$

Elements of a Game

- **Information** about the game situation can be full or partial
- **Strategy sets**, e.g. C or D as in PD game, the strategic choice made depends on the information available to the player
- Game equilibrium such as **Nash equilibrium(NE)** arise

Nash Equilibria(NE)

-Player responses to maximize the payoff function. Any variation of this strategy will produce a lower payoff

- Repeated games
- A mixed NE always exist(J. Nash)
- A subset of NE are evolutionary stable strategies
- Bayesian games
- Decoherence can also be included
- Parrondo games can also arise

The Quantum extension…

- The game state can now become entangled
- strategy sets can be expanded to general unitary transformations
- Strategic choices can be quantum superpositions of two separate strategies

Definition of a quantum game

We define a game
$\Gamma = \Gamma(H,\Lambda,\{S_i\}_j,\Pi)$

- H is a Hilbert space
- Λ is the initial state of the game
- $\{S_i\}_j$ are the set of allowed choices for each player j, usually unitary transformations or classical choices
- Π the payoff function determined after measurement
- A strategy is determined by the players after analyzing game setup and the payoff matrix

Penny Flip game

Bob prepares coin Alice can flip coin Bob can flip coin

Heads: Bob wins
Tails: Alice wins

Once placed in box coin is hidden from the players

Penny flip game **classical** solution

- Each player flips with a 50% probability, that is a **mixed** strategy
- A mixed strategy is used so that their choices are unpredictable
- Payoff expectation is zero for each player, and so a fair game

Quantum penny flip game

Let $|0\rangle$ represent heads and $|1\rangle$ represent tails.

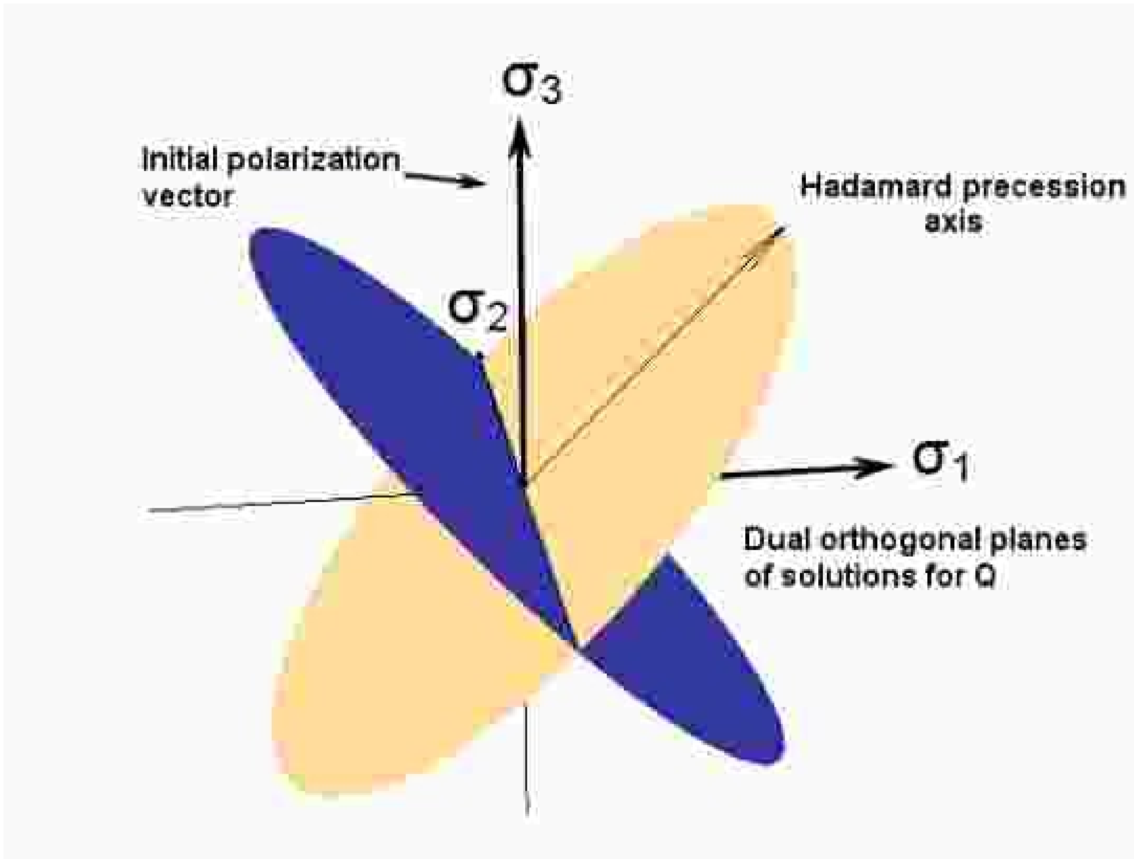
Bob plays $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Whether Alice flips or not the game state is unchanged.

Bob plays H again, and as $H^2 = I$, he returns the state $|0\rangle$.

Hence Bob always wins!

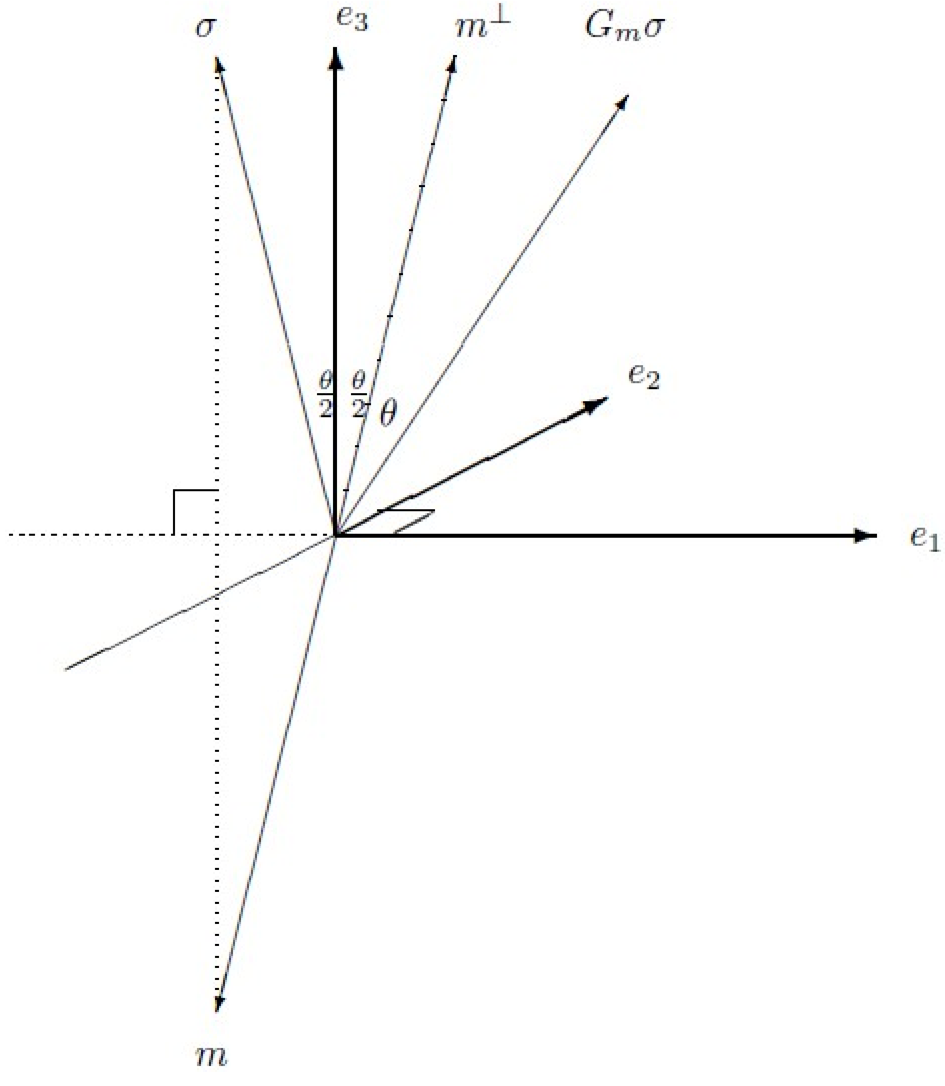
Meyer's Penny flip general solution



Chappell et al. JPSJ 2009

Grover search

Classical observer extracts informatio n from quantum system



Clifford's Geometric Algebra

- Modeling of a qubit in a real space thus avoiding complex numbers and allowing a visual picture(Bloch sphere, density matrix)
- Elegant expressions for probabilistic outcomes for multiple qubits

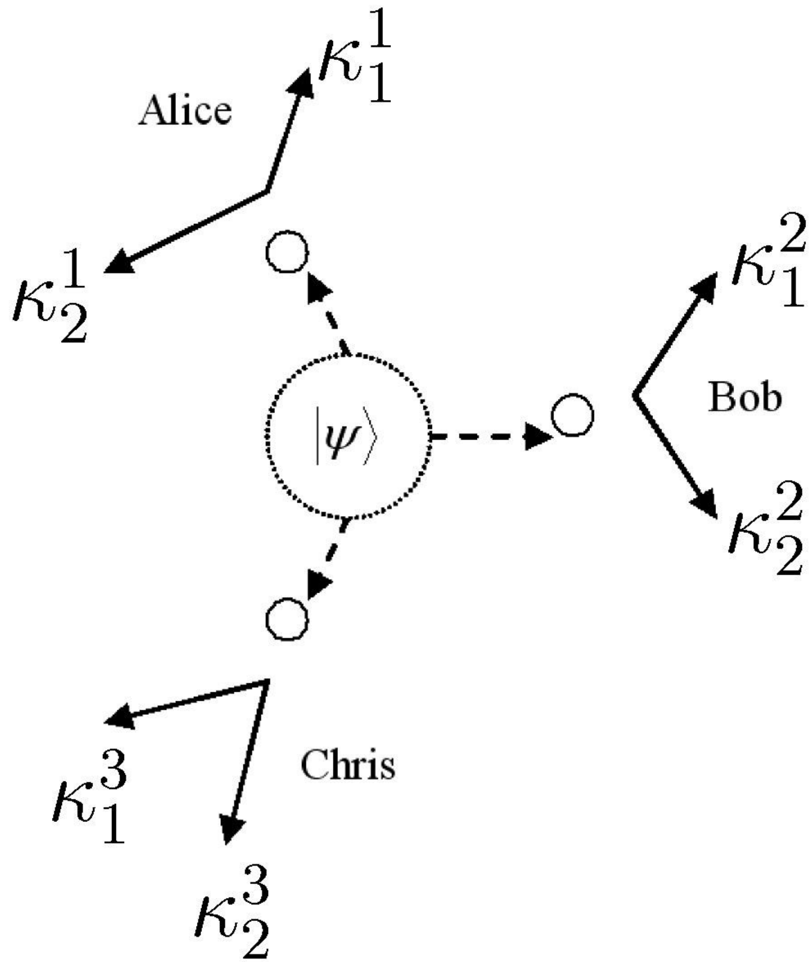
$$P(\psi, \phi) = 2^{N-2}[\langle \psi E \psi^\dagger \phi E \phi^\dagger \rangle_0 - \langle \psi J \psi^\dagger \phi J \phi^\dagger \rangle_0]$$

Quantum games vs gaming the quantum

- We seek a proper extension of a classical game, so that at zero entanglement we recover the classical game.
- For example, we can use an EPR experiment setting, which retains classical strategies
- Avoids arguments by Ent & Pike, that unitary transformations fundamentally change the corresponding classical game.

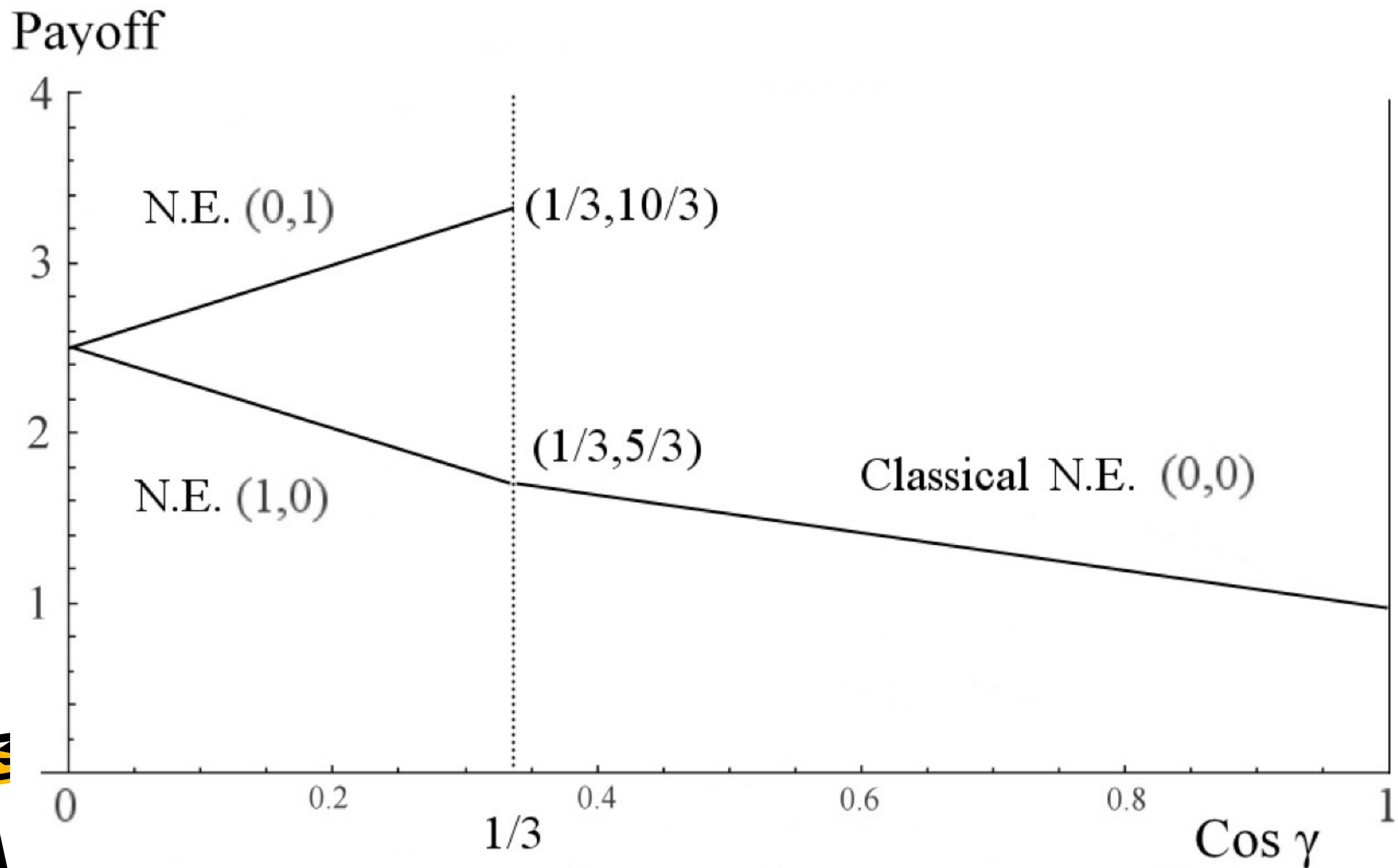
EPR setting

- An entangled qubit is distributed to each player who select one of 2 possible measurement directions.



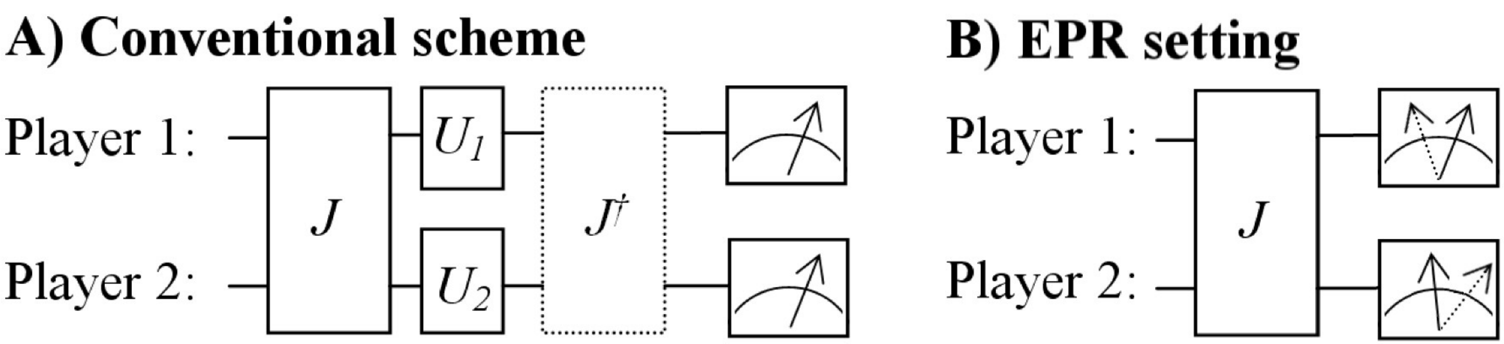
Quantum games-EPR setting

- Prisoner dilemma



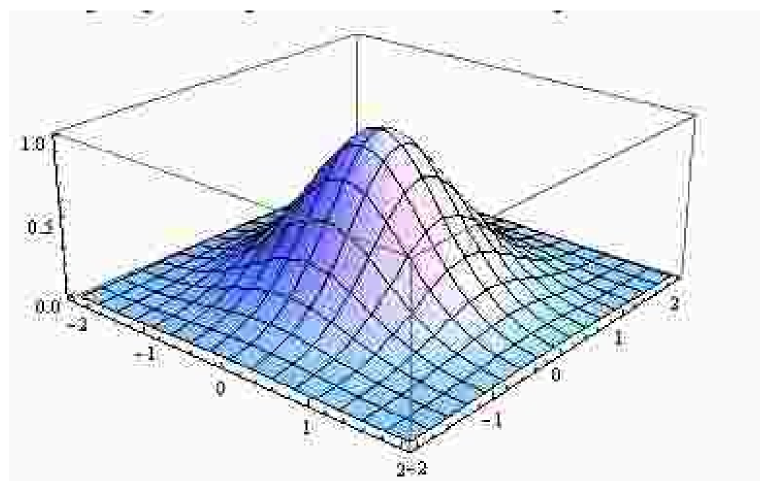
Gaming the quantum

- Utilize the full range of quantum mechanical properties, entanglement, unitary transformations and superposition of states.



Non-factorizable joint probabilities

- Quantum mechanical measurements result in probabilistic outcomes, and hence a alternate framework is non-factorizable joint probabilities. Gives a superset of quantum mechanical correlations
- When the joint probability distribution becomes factorizable(equivalent to being unentangled) then we recover the classical game. Fines theorem.

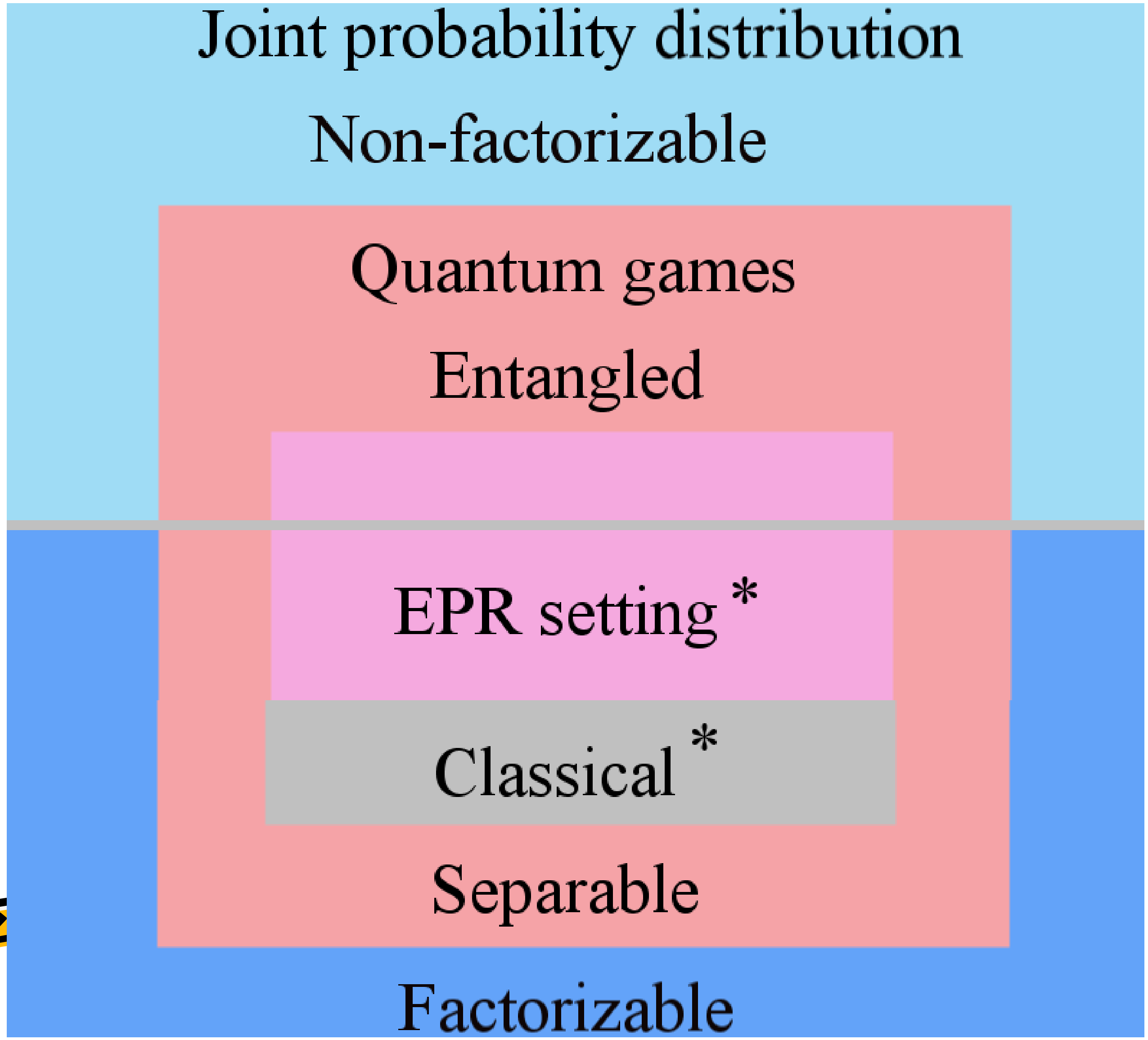


Heirarchy of Games

Framework	Strategy space	Cirel'son bound	Comment
NFJP*		4	
Quantum	Unitary transform	$2\sqrt{2}$	Gaming the quantum
EPR	Probabilistic choice	$2\sqrt{2}$	Quantum gaming
Mixed classical	Probabilistic choice	2	NE always exists,eg PF
Classical	Classical choice	2	eg PD

*NFJP=Non-factorizable joint probability

Structure of game settings



* Classical strategies

Summary

- We highlighted the distinction between *quantum games* and *gaming the quantum*.
- EPR setting provides a proper quantum extension to a classical game as it retains classical strategies
- Geometric algebra a useful tool
- Non-factorizable joint probability provides a general framework for classical and quantum games.
- Wide applicability of quantum game theory to many areas of science