Non-locality of Symmetric States

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Motivation

- Non-locality as an interpretation/witness for entanglement 'types'?

  (mutlipartite entanglment is a real mess)

- Different 'types' of non-locality?

  (is non-locality also such a mess?)

Look at symmetric states – same tool
'Majorana Representation' used to study both….

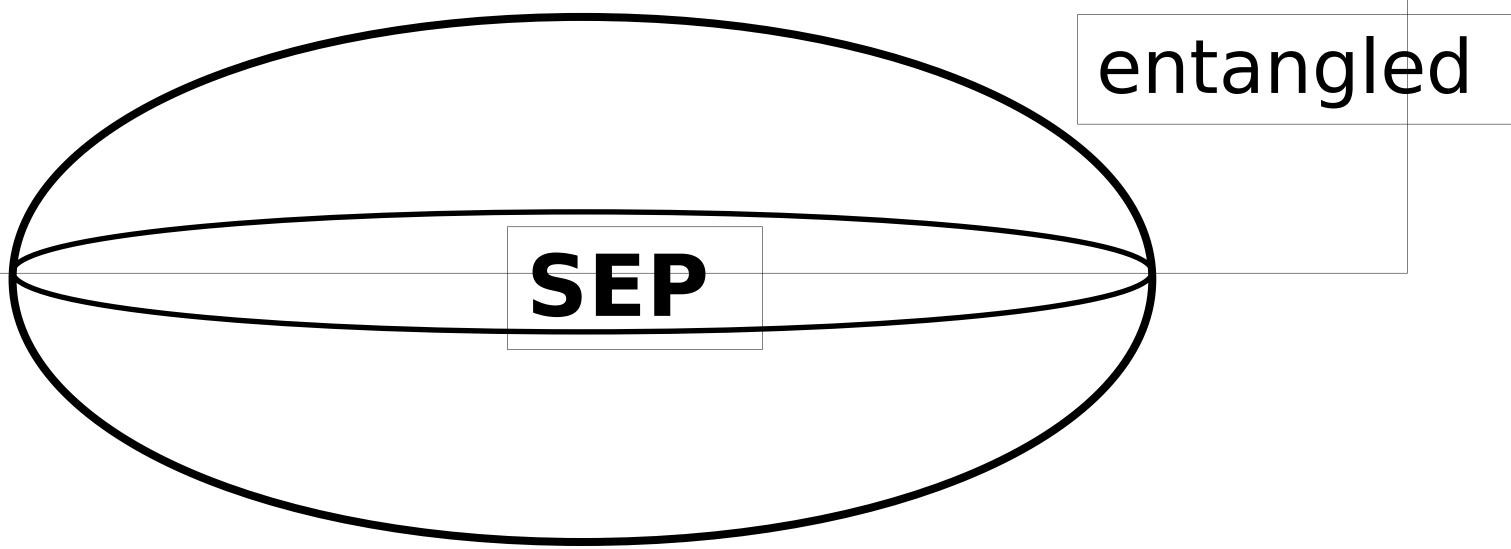
Outline

1. Background (entanglement classes, non-locality, Majorana representation for symmetric states)
1. Hardy’s Paradox for symmetric states of n-qubits
1. Different Hardy tests for different entanglement classes

Entanglement

Definition:

State is entangled iff NOT separapable

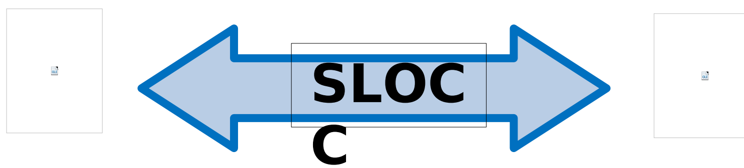


Entanglement

Types of entanglement

Dur, Vidal, Cirac, PRA 62, 062314 (2000)

In multipartite case, some states are incomparable, even under *stochastic* Local Operations and Classical Communications (SLOCC)



Infinitely many different classes!

- Different resources for quantum information processing
- Different entanglement measures may apply for different types

Permutation Symmetric States

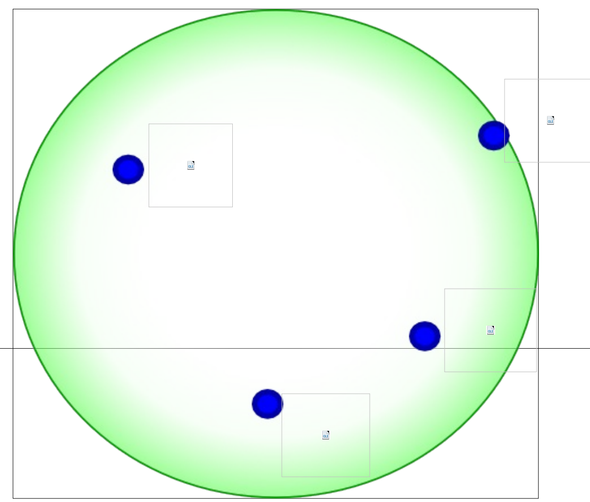
Symmetric under permutation of parties

- Occur as ground states e.g. of some Bose Hubbard models
- Useful in a variety of Quantum Information Processing tasks
- Experimentally accessible in variety of media

Permutation Symmetric States

Majorana representation

E. Majorana, Nuovo Cimento 9, 43 – 50 (1932)



Permutation Symmetric States

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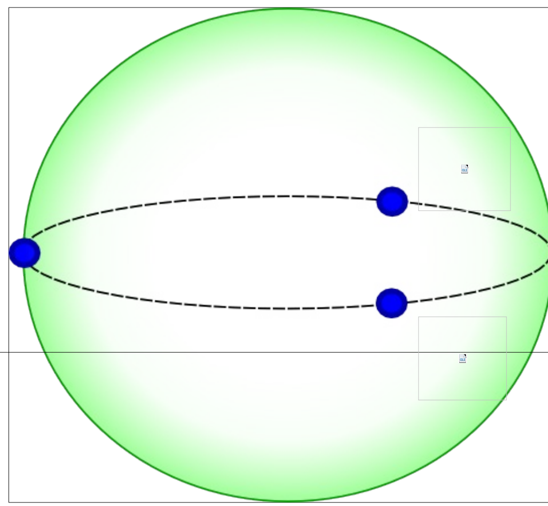

Permutation Symmetric States

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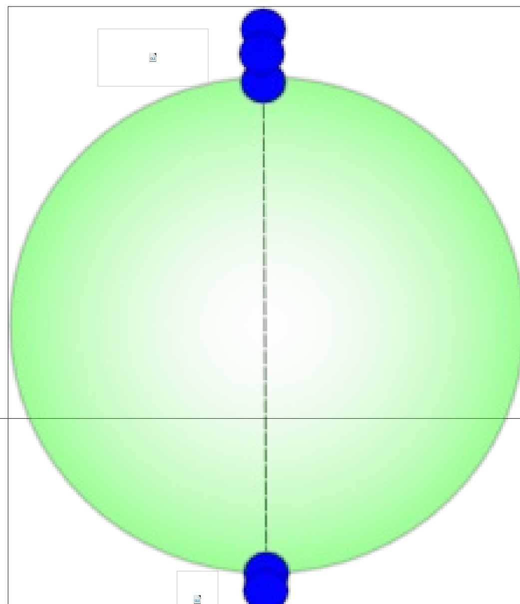

Dicke states

Permutation Symmetric States

Majorana representation

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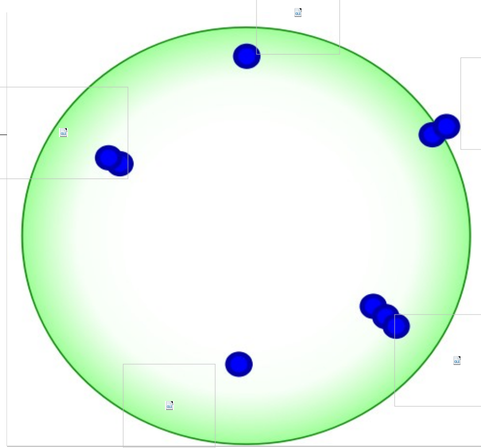
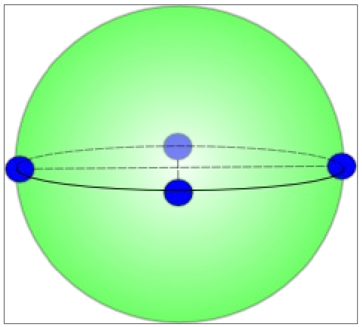


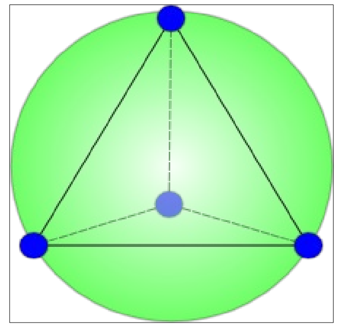
- ***Distribution*** **of points alone determines entanglement features**

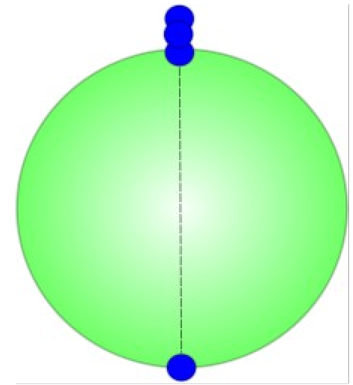
Local unitary $\Leftrightarrow$ rotation of sphere

- **Orthogonality relations**

Permutation Symmetric States







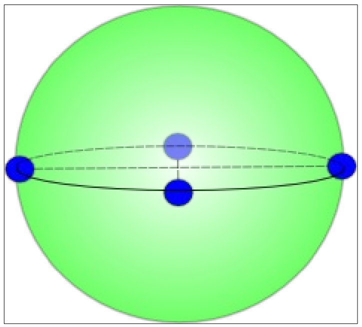
- **Different degeneracy classes***
  (   )

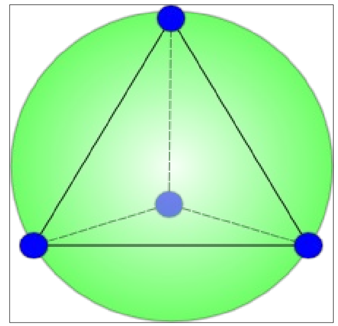
- **Different symmetries^**

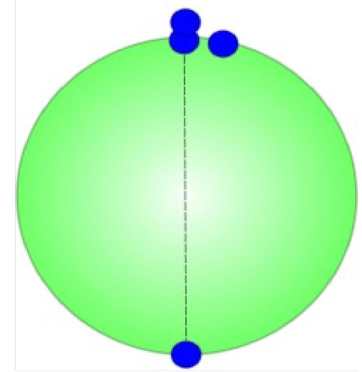
Different entanglement 'types' (w.r.t. SLOCC)

*T. Bastin, S. Krins, P. Mathonet, M. Godefroid, L. Lamata and E. Solano , PRL 103, 070503 (2009)

Permutation Symmetric States







- **Different degeneracy classes ( )**
- **Different symmetries**

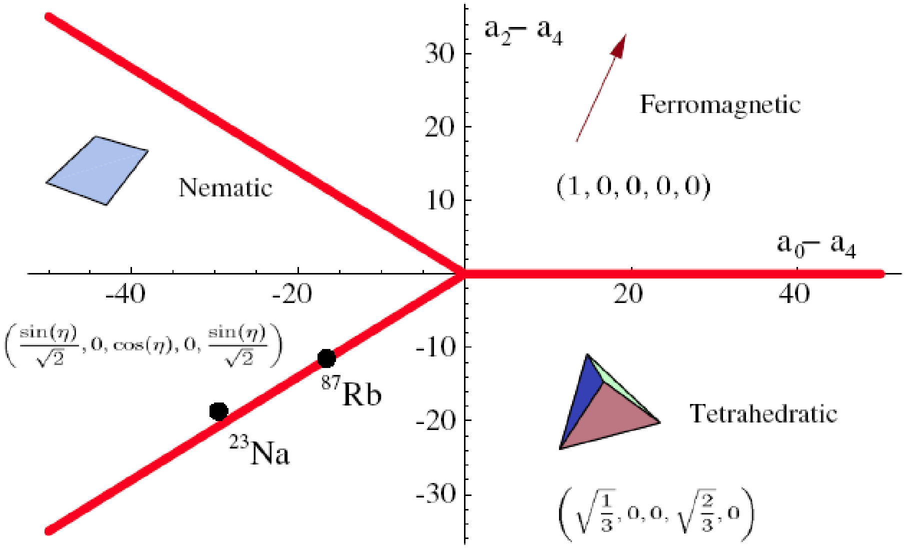
Different entanglement ‘types’ (w.r.t. SLOCC)

- **NOTE: Almost identical states can be in different classes**

Comparison to Spinor BEC

E.g. S=2

R. Barnett, A. Turner and E. Demler, PRL 97, 180412 (2007)



Phase diagram for spin 2 BEC in single optical trap
(Fig taken from PRL 97, 180412)

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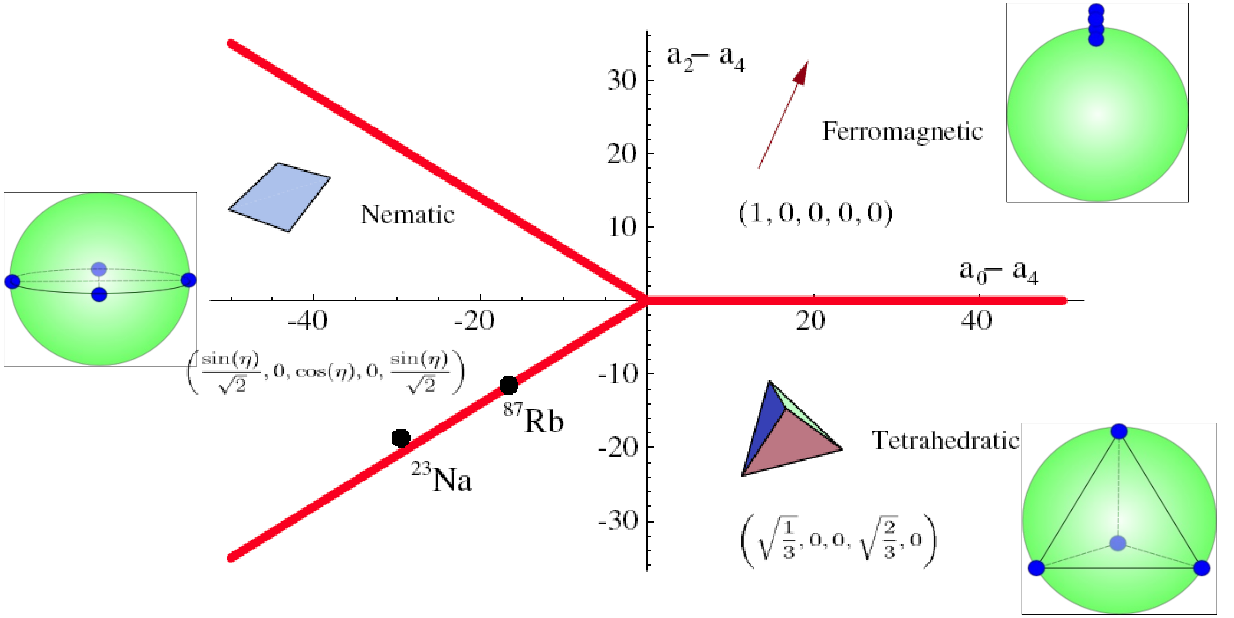
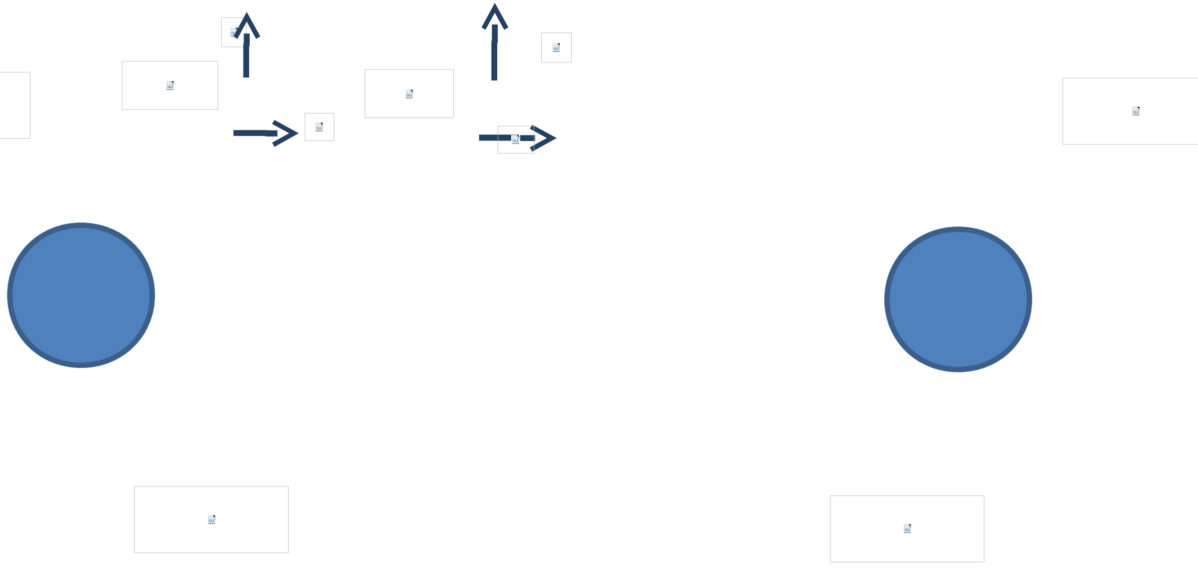


Phase diagram for spin 2 BEC in single optical trap
(Fig taken from PRL 97, 180412)

Non-Locality

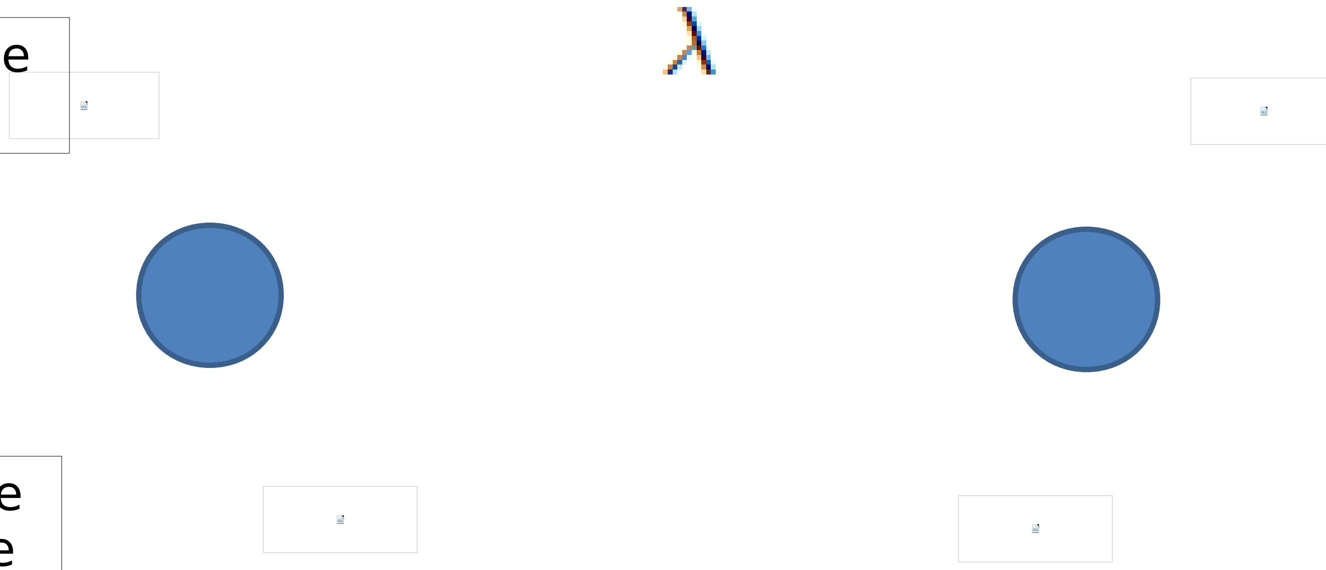
Measurement basis



Measurement outcome

Non-Locality

Measurement basis



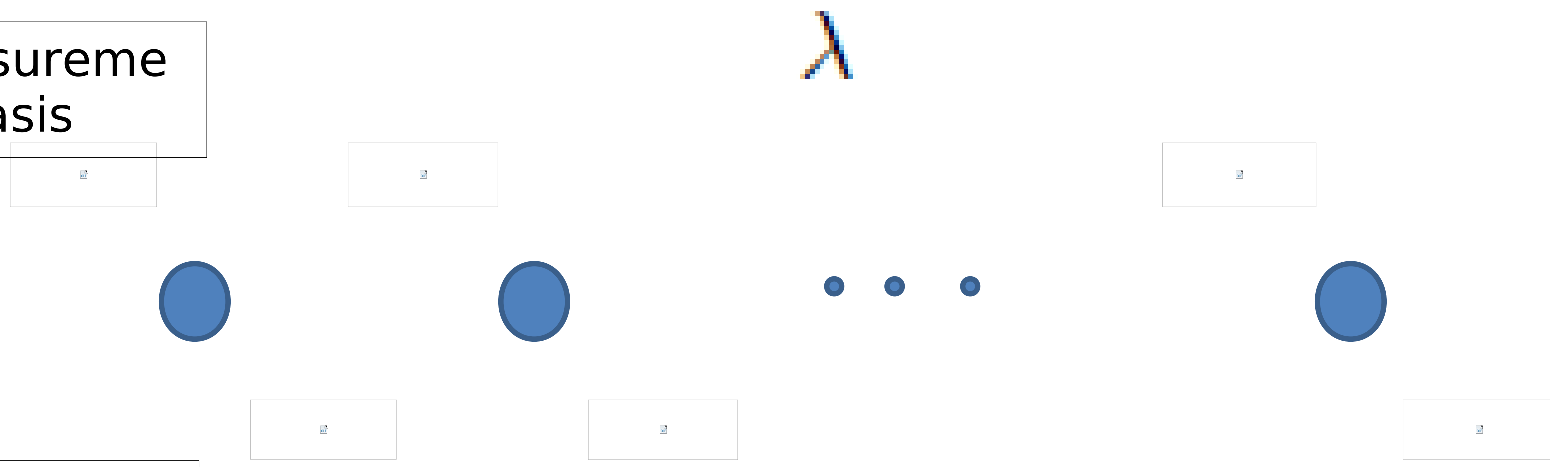
Measurement outcome

- **Local Hidden**

$$P(AB|ab) = \int p(\lambda)P(A|a,\lambda)P(B|b,\lambda)d\lambda$$

Non-Locality

Measurement basis



Measurement outcome

- **Local Hidden Variable model**

$$P(M_1M_2...M_n|m_1m_2...m_n) = \int p(\lambda)P(M_1|m_1,\lambda)P(M_2|m_2,\lambda)...P(M_n|m_n,\lambda)d\lambda$$

n Party case (Hardy's Paradox)

· For all symmetric sta $|\Psi\rangle$ of n qubits (except Dicke states)

- set of probabilities which contradict LHV
- set of measurement which achieve these probabilities

measurement result

$$P(M_1M_2...M_n|m_1m_2...m_n)$$

measurement basis

$$P(00...0|00...0) > 0$$

$$P(00...0|10...0) = 0$$

$$P(00...0|01...0) = 0$$

$$...$$

$$P(00...0|00...1) = 0$$

$$P(11...1|11...1) = 0$$

n Party case (Hardy's Paradox)

· For all symmetric sta $|\Psi\rangle$ of n qubits (except Dicke states)

- **set of probabilities which contradict LHV**
- set of measurement which achieve these probabilities

measurement result

$$P(M_1 M_2 ... M_n | m_1 m_2 ... m_n) = \int p(\lambda) \prod_i P(M_i | m_i, \lambda) d\lambda$$

measurement basis

$P(00...0|00...0) > 0$ ⇒ for some λ , $P(M_i = 0 | m_i = 0, \lambda) > 0$

$P(00...0|10...0) = 0$

$P(00...0|01...0) = 0$

...

$P(00...0|00...1) = 0$

$P(11...1|11...1) = 0$

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$$P(M_1M_2...M_n|m_1m_2...m_n) = \int p(\lambda)\prod_i P(M_i|m_i,\lambda)d\lambda$$

measurement basis

$P(00...0|00...0) > 0$ ⇒ for some λ, $P(M_i = 0|m_i = 0, \lambda) > 0$

$$\left.\begin{aligned} &P(00...0|10...0) = 0\\ &P(00...0|01...0) = 0\\ &...\\ &P(00...0|00...1) = 0 \end{aligned}\right\} \Rightarrow \text{for same } \lambda,\ P(M_i = 0|m_i = 1, \lambda) = 0$$

⇒ $P(M_i = 1|m_i = 1, \lambda) = 1$

$P(11...1|11...1) = 0$

$$\sum_{M_i} P(M_i|m_i\lambda) = 1$$

n Party case (Hardy's Paradox)

- For all symmetric states $|\Psi\rangle$ of n qubits (except Dicke states)
  - **set of probabilities which contradict LHV**
  - set of measurement which achieve these probabilities

measurement result

$$P(M_1M_2...M_n|m_1m_2...m_n) = \int p(\lambda)\prod_i P(M_i|m_i,\lambda)d\lambda$$

measurement basis

$P(00...0|00...0) > 0$ ⇒ for some λ, $P(M_i = 0|m_i = 0, \lambda) > 0$

$P(00...0|10...0) = 0$

$P(00...0|01...0) = 0$

...

$P(00...0|00...1) = 0$

⇒ for same λ, $P(M_i = 0|m_i = 1, \lambda) = 0$

⇒ $P(M_i = 1|m_i = 1, \lambda) = 1$

$\sum_{M_i} P(M_i|m_i\lambda) = 1$

$P(11...1|11...1) = 0$

CONTRADICTION!

n Party case (Hardy's Paradox)

· For all symmetric sta $|\Psi\rangle$ of n qubits (except Dicke states)

- set of probabilities which contradict LHV
- **set of measurement which achieve these probabilities**

measurement result

$$P(M_1 M_2 ... M_n | m_1 m_2 ... m_n)$$

$m_i = 0 \quad \{|0\rangle, |1\rangle\}$

$m_i = 1 \quad \{|\tilde{0}\rangle, |\tilde{1}\rangle\}$

measurement basis

$$\begin{aligned} &P(00...0|00...0) > 0 && \langle 0|\langle 0|...\langle 0||\Psi\rangle \neq 0 \\ &P(00...0|10...0) = 0 && \langle \tilde{0}|\langle 0|...\langle 0||\Psi\rangle = 0 \\ &P(00...0|01...0) = 0 \quad \Leftrightarrow && \langle 0|\langle \tilde{0}|...\langle 0||\Psi\rangle = 0 \\ &... && ... \\ &P(00...0|00...1) = 0 && \langle 0|\langle 0|...\langle \tilde{0}||\Psi\rangle = 0 \\ &P(11...1|11...1) = 0 && \langle \tilde{1}|\langle \tilde{1}|...\langle \tilde{1}||\Psi\rangle = 0 \end{aligned}$$

n Party case (Hardy's Paradox)

- Use Majorana representation

$P(00...0\|00...0) > 0$	$\langle 0\|\langle 0\|...\langle 0\|\|\Psi\rangle \neq 0$
$P(00...0\|10...0) = 0$	$\langle \tilde{0}\|\langle 0\|...\langle 0\|\|\Psi\rangle = 0$
$P(00...0\|01...0) = 0$	$\langle 0\|\langle \tilde{0}\|...\langle 0\|\|\Psi\rangle = 0$
...	...
$P(00...0\|00...1) = 0$	$\langle 0\|\langle 0\|...\langle \tilde{0}\|\|\Psi\rangle = 0$
$P(11...1\|11...1) = 0$	$\langle \tilde{1}\|\langle \tilde{1}\|...\langle \tilde{1}\|\|\Psi\rangle = 0$

n Party case (Hardy's Paradox)

- Use Majorana representation

$$P(00...0|00...0) > 0 \qquad \langle 0|\langle 0|...\langle 0||\Psi\rangle \neq 0$$

$$P(00...0|10...0) = 0 \checkmark \qquad \langle \tilde{0}|\langle 0|...\langle 0||\Psi\rangle = 0$$

$$P(00...0|01...0) = 0 \checkmark \qquad \langle 0|\langle \tilde{0}|...\langle 0||\Psi\rangle = 0$$

$$... \qquad ...$$

$$P(00...0|00...1) = 0 \checkmark \qquad \langle 0|\langle 0|...\langle \tilde{0}||\Psi\rangle = 0$$

$$P(11...1|11...1) = 0 \checkmark \qquad \langle \tilde{1}|\langle \tilde{1}|...\langle \tilde{1}||\Psi\rangle = 0$$

n Party case (Hardy's Paradox)

- Majorana representation always allows satisfaction of lower conditions, what about the top condition?

  - Must find, such that

$$P(00...0|00...0) > 0 \; \textbf{?} \qquad \langle 0|\langle 0|...\langle 0||\Psi\rangle \neq 0$$

$$P(00...0|10...0) = 0 \; \checkmark \qquad \langle \tilde{0}|\langle 0|...\langle 0||\Psi\rangle = 0$$

$$P(00...0|01...0) = 0 \; \checkmark \qquad \langle 0|\langle \tilde{0}|...\langle 0||\Psi\rangle = 0$$

$$... \qquad ...$$

$$P(00...0|00...1) = 0 \; \checkmark \qquad \langle 0|\langle 0|...\langle \tilde{0}||\Psi\rangle = 0$$

$$P(11...1|11...1) = 0 \; \checkmark \qquad \langle \tilde{1}|\langle \tilde{1}|...\langle \tilde{1}||\Psi\rangle = 0$$

n Party case (Hardy's Paradox)

- Majorana representation always allows satisfaction of lower conditions, what about the top condition?
  - Must find, such that
  - It works for *all* cases except product or Dicke states

    Dicke states **too** symmetric!

$$P(00...0|00...0) > 0 \checkmark \qquad \langle 0|\langle 0|...\langle 0||\Psi\rangle \neq 0$$

$$P(00...0|10...0) = 0 \checkmark \qquad \langle\tilde{0}|\langle 0|...\langle 0||\Psi\rangle = 0$$

$$P(00...0|01...0) = 0 \checkmark \qquad \langle 0|\langle\tilde{0}|...\langle 0||\Psi\rangle = 0$$

$$... \qquad ...$$

$$P(00...0|00...1) = 0 \checkmark \qquad \langle 0|\langle 0|...\langle\tilde{0}||\Psi\rangle = 0$$

$$P(11...1|11...1) = 0 \checkmark \qquad \langle\tilde{1}|\langle\tilde{1}|...\langle\tilde{1}||\Psi\rangle = 0$$

(almost)

All permutation symmetric states are non-local!

Testing Entanglement class

Testing Entanglement class

- Use the Majorana representation to add constraints implying degeneracy

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- Use the Majorana representation to add constraints implying degeneracy

$$
\begin{aligned}
&P(00...0|00...0) > 0\\
&P(00...0|10...0) = 0\\
&P(00...0|01...0) = 0\\
&...\\
&P(00...0|00...1) = 0\\
&P(11...1|11...1) = 0 \qquad \langle\tilde{1}|\langle\tilde{1}|...\langle\tilde{1}||\Psi\rangle = 0
\end{aligned}
$$

Testing Entanglement class

- Use the Majorana representation to add constraints implying degeneracy

$$P(00...0|00...0) > 0$$
$$P(00...0|10...0) = 0$$
$$P(00...0|01...0) = 0$$
$$...$$
$$P(00...0|00...1) = 0$$
$$P(11...1|11...1) = 0 \quad \langle\tilde{1}|\langle\tilde{1}|...\langle\tilde{1}||\Psi\rangle = 0$$
$$P(1...1|1...1) = 0$$
$$...$$
$$P(\underbrace{1..1}_{n-d+1} \,|\, \underbrace{1..1}_{n-d+1}) = 0$$

Only satisfied by
With degeneracy

Testing Entanglement class

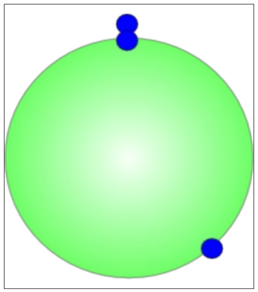
- Use the Majorana representation to add constraints implying degeneracy

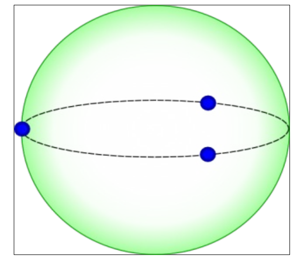
- Correlation persists to fewer sets, *only* for degenerate states.

$$P(00...0|00...0) > 0$$
$$P(00...0|10...0) = 0$$
$$P(00...0|01...0) = 0$$
$$...$$
$$P(00...0|00...1) = 0$$
$$P(11...1|11...1) = 0 \qquad \langle\tilde{1}|\langle\tilde{1}|...\langle\tilde{1}||\Psi\rangle = 0 \quad \Longleftrightarrow$$
$$P(1...1|1...1) = 0$$
$$...$$
$$P(\underbrace{1..1}_{n-d+1} \mid \underbrace{1..1}_{n-d+1}) = 0$$

Only satisfied by
With degeneracy

e.g. Hardy Test for W class...





TW

$$\begin{aligned} P(0000|000) &> 0 \\ P(0000|001) &= 0 \\ P(0000|010) &= 0 \\ P(0000|100) &= 0 \\ P(111|111) &= 0 \\ P(11|11) &= 0 \end{aligned}$$

Get REAL!!!

Get REAL!!!

- We really need to talk about inequalities

$$\mathcal{P}^n := P(0\ldots0|00\ldots00) - \sum_\pi P(00\ldots00|\pi(00\ldots01)) - P(1\ldots1|11\ldots11)$$

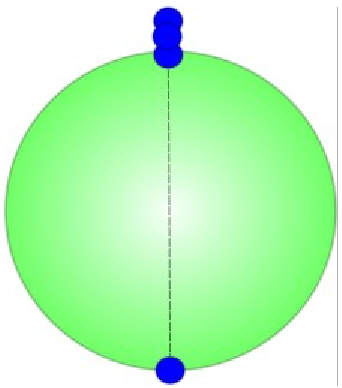
$$\mathcal{Q}_d^n := \mathcal{P}^n - P(\underbrace{11\ldots1}_{n-1}|\underbrace{11\ldots1}_{n-1}) - \ldots - P(\underbrace{11\ldots1}_{n-d+1}|\underbrace{11\ldots1}_{n-d+1})$$

$$\mathcal{P}^n \leq 0$$

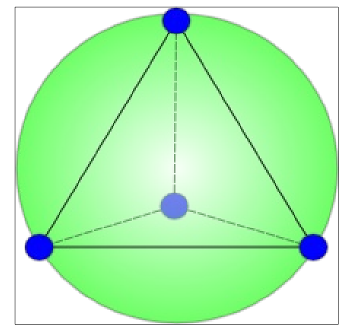
$$\mathcal{Q}_d^n \leq 0$$

- ALL states with one MP of degeneracy d or greater violate for $\mathcal{Q}_d^n$
- CanNOT be any longer true that 'only' states of the correct type violate
  - states arbitrarily close which are in different class

Persistency of degenerate states



$$
\begin{aligned}
Q_3^4 = \quad & P(0000|0000) \\
& -P(0000|0001) \\
& -P(0000|0010) \\
& -P(0000|0100) \\
& -P(0000|1000) \\
& -P(1111|1111) \\
& -P(111|111) \\
& -P(11|11)
\end{aligned}
$$



$Q_3^4 = 0.1703$

$Q_3^4 = -0.609$

Conclusions

- Hardy tests for almost all symmetric states

- Different entanglement types $\Leftrightarrow$ Different Hardy tests

  - Non-locality to 'witness' / interpret entanglement types
    - Different flavour of non-local arguments to Stabiliser states/ GHZ states e.t.c.

- More?

    - 'Types' of non-locality? (from operational perspective)
    - Witness phase transitions by different locality tests?

Thank you!

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