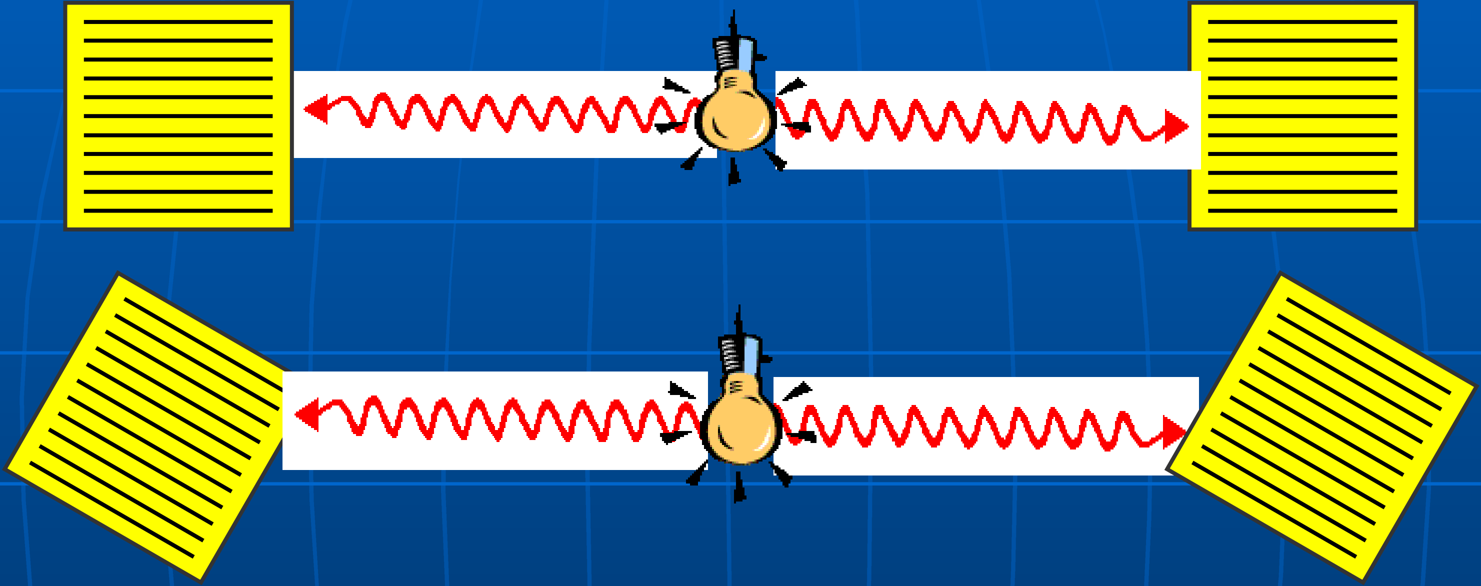
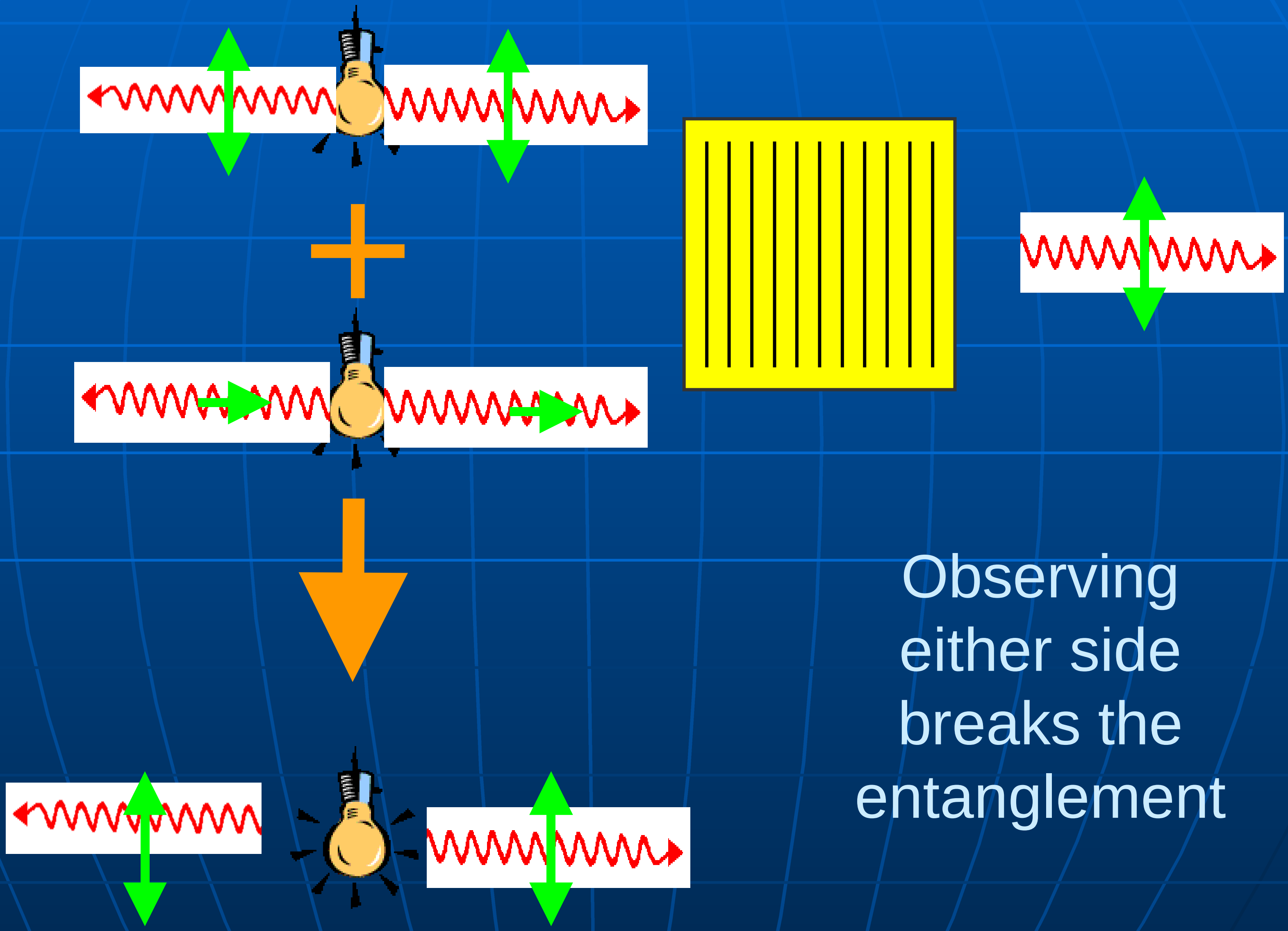
Einstein-Podolsky-Rosen argument



If one photon passes through the polaroid, so does the other one.

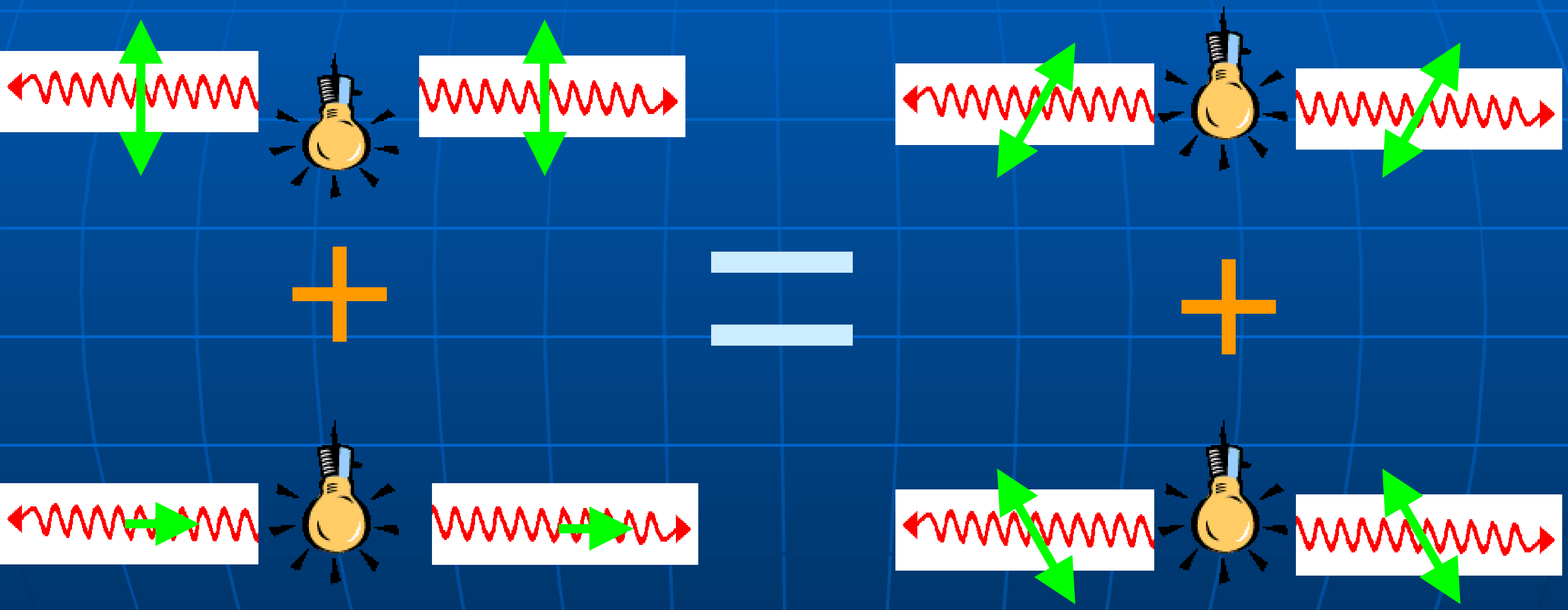
Therefore each photon must already have its instructions
about what to do when it meets the polaroid.

Entanglement



Observing either side breaks the entanglement

Entangled every which way



EPR AND BELL

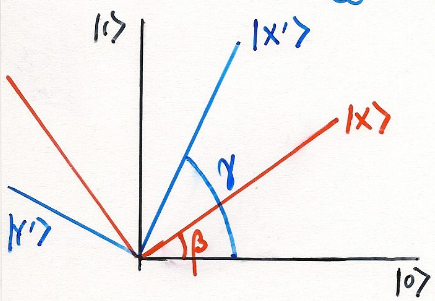
EPR: Look!

$$|\Psi_+\rangle = |0\rangle|0\rangle + |1\rangle|1\rangle$$
$$= |X\rangle|X\rangle + |Y\rangle|Y\rangle$$

where $|X\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$
$|Y\rangle = -\sin\theta|0\rangle + \cos\theta|1\rangle$

If Alice measures "0 or 1?" her result tells her what result Bob would get; so Bob's qubit must already have a definite answer. Likewise for "X or Y" (for any θ).

Bell: OK, suppose you're right. Then Alice's qubit



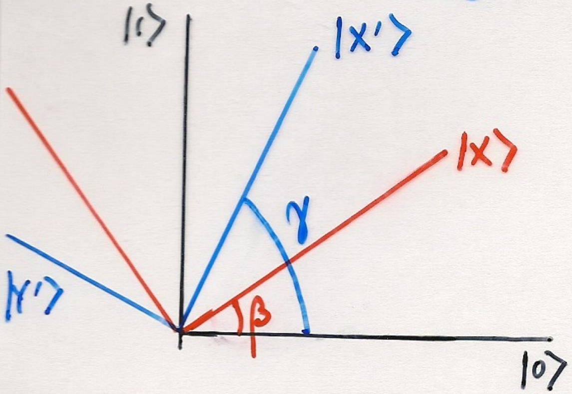
has bits a, b, c (= 0 or 1) answering "$|0\rangle$ or $|1\rangle$?", "$|X\rangle$ or $|Y\rangle$?" & "$|X'\rangle$ or $|Y'\rangle$?"

and she can measure two of them.

But $P(a=0, b=0) \leq P(a=0, c=0) + P(b=0, c=1)$

$$\tfrac{1}{2}\cos^2\beta \leq \tfrac{1}{2}\cos^2\gamma + \tfrac{1}{2}\sin^2(\gamma-\beta)$$

Bell: OK, suppose you're right. Then Alice's qubit



has bits a, b, c $(= 0 \text{ or } 1)$

answering "$|0\rangle$ or $|1\rangle$?",

"$|X\rangle$ or $|Y\rangle$?" & "$|X'\rangle$ or $|Y'\rangle$?"

and she can measure two of them.

But $P(a=0, b=0) \leq P(a=0, c=0) + P(b=0, c=1)$

$$\tfrac{1}{2}\cos^2\beta \leq \tfrac{1}{2}\cos^2\gamma + \tfrac{1}{2}\sin^2(\gamma-\beta)$$

Not true if $\beta = \frac{\pi}{3}$, $\gamma = \frac{\pi}{6}$.

Experiments support quantum mechanics.

If qubits do have definite answers to every question, they can be changed instantaneously by an action at a distance.

ELSEVIER

30 October 2000

Physics Letters A 276 (2000) 1–7

PHYSICS LETTERS A

www.elsevier.nl/locate/pla

The speed of quantum information and the preferred frame: analysis of experimental data

Valerio Scarani *, Wolfgang Tittel, Hugo Zbinden, Nicolas Gisin

Group of Applied Physics, University of Geneva, 20, rue de l'Ecole-de-Médecine, CH-1211 Geneva, Switzerland

Received 10 August 2000; accepted 18 September 2000
Communicated by P.R. Holland

Abstract

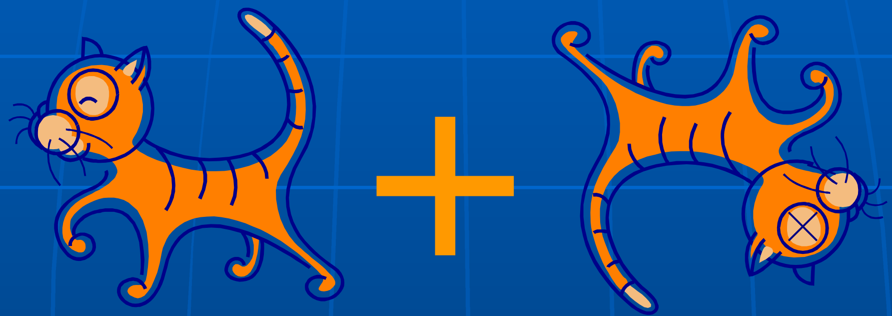
The results of EPR experiments performed in Geneva are analyzed in the frame of the cosmic microwave background radiation, generally considered as a good candidate for playing the role of preferred frame. We set a lower bound for the speed of quantum information in this frame at $2 \times 10^4\ c$.

Keywords: Optical EPR experiments; Preferred frame; Cosmic background radiation

Schrödinger's Cat

$$|CAT\rangle = |ALIVE\rangle + |DEAD\rangle$$

Entanglement killed the cat



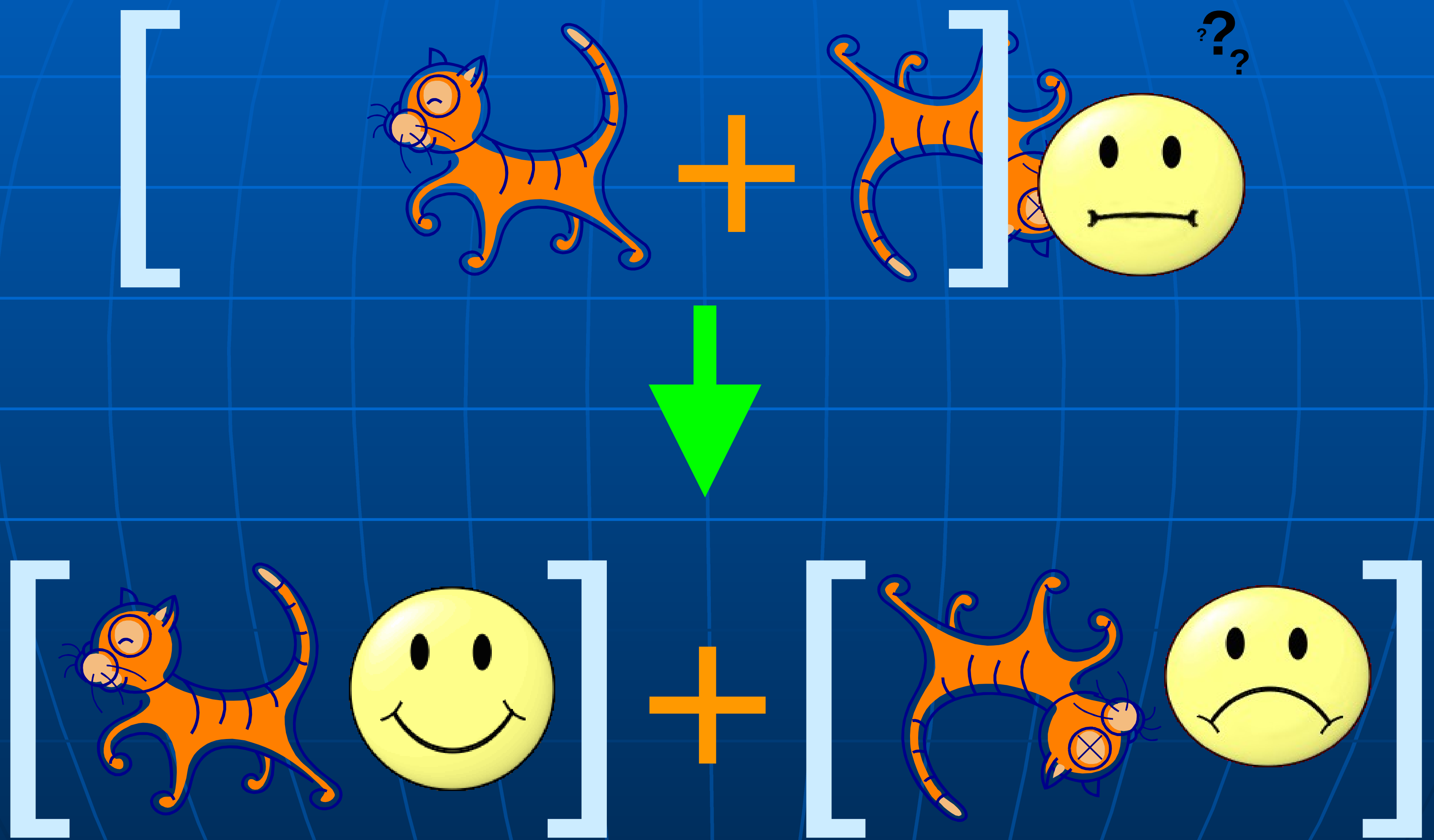
According to quantum theory, if a cat can be in a state $|\mathrm{ALIVE}\rangle$ and a state $|\mathrm{DEAD}\rangle$, it can also be in a state

$$|\mathrm{ALIVE}\rangle + |\mathrm{DEAD}\rangle.$$

Why don't we see cats in such superposition states?

Entanglement killed the cat

ANSWER: because the theory actually predicts…..



SUPERDENSE CODING

How to transmit two bits by sending one qubit.

Alice  Bob

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$

ENCODING

Alice encodes 2 bits by choosing U_+, U_-, V_+ or V_-

$$U_\pm = \begin{pmatrix} 1 & 0 \\ 0 & \pm 1 \end{pmatrix} \qquad V_\pm = \begin{pmatrix} 0 & 1 \\ \pm 1 & 0 \end{pmatrix}$$

$$(U_\pm \otimes 1)|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle \pm |1\rangle|1\rangle\right) = |\Psi_\pm\rangle$$

$$(V_\pm \otimes 1)|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle \pm |1\rangle|0\rangle\right) = |\Phi_\pm\rangle$$

TRANSMISSION

Alice sends her 1 qubit to Bob

DECODING

Bob receives a 2-bit message $|\Psi_+\rangle$, $|\Psi_-\rangle$, $|\Phi_+\rangle$ or $|\Phi_-\rangle$

QUANTUM TELEPORTATION

Alice  Bob

Message O

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$

$$|\varphi_M\rangle = a|0\rangle + b|1\rangle$$

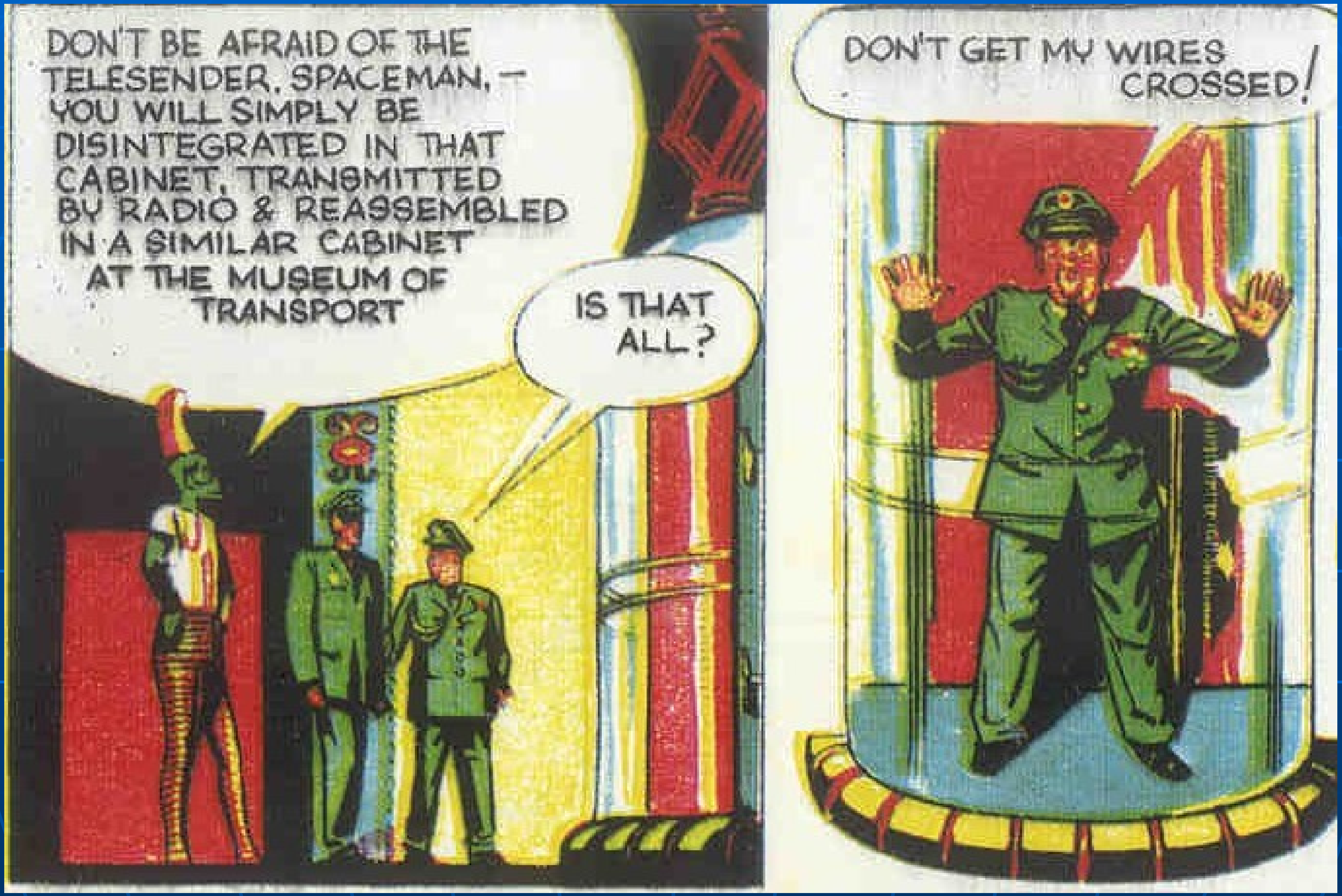
$$|\text{initial}\rangle = \frac{1}{\sqrt{2}}\left(a|0\rangle + b|1\rangle\right)\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)$$

$$\begin{aligned} &= \tfrac{1}{2}|\Psi_+\rangle\left(a|0\rangle + b|1\rangle\right) && |\varphi_M\rangle \\ &+ \tfrac{1}{2}|\Psi_-\rangle\left(a|0\rangle - b|1\rangle\right) && Z|\varphi_M\rangle \\ &+ \tfrac{1}{2}|\Phi_+\rangle\left(a|1\rangle + b|0\rangle\right) && X|\varphi_M\rangle \\ &+ \tfrac{1}{2}|\Phi_-\rangle\left(a|1\rangle - b|0\rangle\right) && Y|\varphi_M\rangle \end{aligned}$$

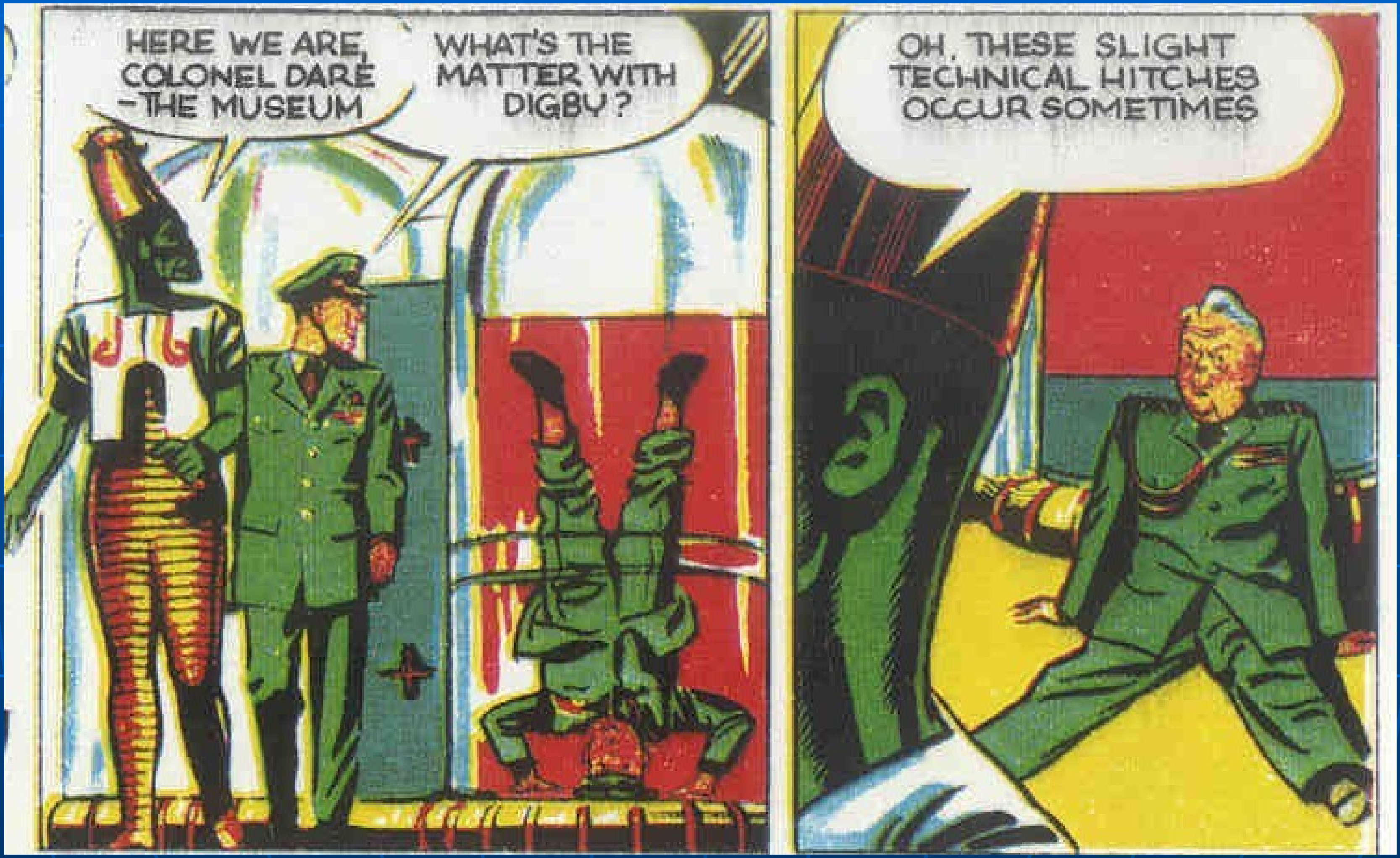
Alice measures in the basis $|\Phi_\pm\rangle, |\Psi_\pm\rangle$

& tells Bob the result by a classical channel.

Bob then performs I, Z^{-1}, X^{-1} or Y^{-1} to obtain $|\varphi_M\rangle$



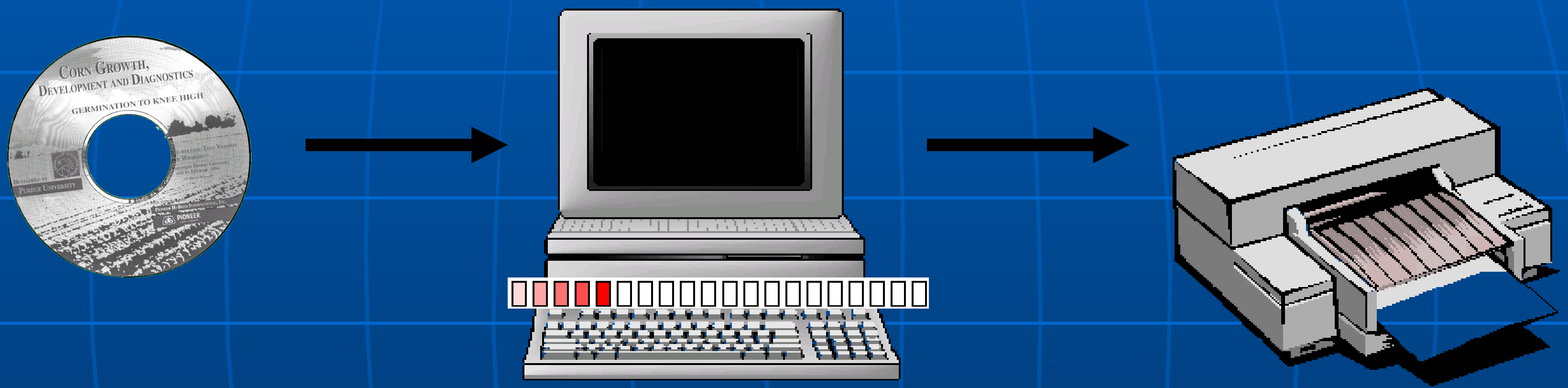
Dan Dare, Pilot of the Future. Frank Hampson, Eagle (19



Dan Dare, Pilot of the Future. Frank Hampson, Eagle (19

Nature 362, 586-587 (15 Apr 1993)

Computing



INPUT
N digits

COMPUTATION
Running time T

OUTPUT

How fast does T grow as you increase N?

Quantum Computing

In 1 unit of time, many calculations can be done but only one answer can be seen

But you can choose your question

E.g. Are all the answers the same?

Two Easy Sums

7873 x 6761 = 53 229 353

? x ? = 26 292 671

Not so easy

N	T for multiplying two N-digits	T for factorising a 2N-digit number
1	1	2
2	4	4
3	9	8
4	16	16
5	25	32
10	100	1,024
20	400	1,048,576
30	900	1,073,741,824
40	1600	1,099,511,627,776
50	2500	1,125,899,906,842,620
	$T \approx N^2$	$T \approx 2^N$

But on a quantum computer, factorisation can be done in roughly the same time as multiplication

$T \approx N^2$

(Peter Shor, 1994)

TWO QUBITS : PURE STATES

General state $\alpha|\psi_1\rangle|\varphi_1\rangle + \beta|\psi_2\rangle|\varphi_2\rangle$

$\alpha, \beta \in \mathbb{R}^+$; $\langle\psi_1|\psi_2\rangle = \langle\varphi_1|\varphi_2\rangle = 0$

Canonical state $\alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle$

$\alpha^2 + \beta^2 = 1$ 1 parameter

Amount of entanglement:

Reduced density matrix $\rho_1 = \alpha^2|0\rangle\langle 0| + \beta^2|1\rangle\langle 1|$

Entanglement $E = S(\rho_1) = -\alpha^2 \log \alpha^2 - \beta^2 \log \beta^2$

$$= \lim_{n\to\infty} \left(\frac{k}{n}\right)$$

k = max. no. of Bell states that can be produced from n copies of $|\Psi\rangle$ by local operations & classical communication

DIRECTIONS OF GROWTH

$$a|0\rangle|0\rangle + b|1\rangle|1\rangle$$

Pure 2 - qubit state

mixed

multipartite

(n parties)

d-state parties

(qudits)

MANIPULATION OF ENTANGLEMENT

Principle

The total amount of entanglement cannot be increased by LOCAL OPERATIONS (including measurement) together with CLASSICAL COMMUNICATION

Theorem (Nielsen)

Let $|\Psi\rangle$ & $|\Phi\rangle$ be two pure states of a 2-party system (d states each) with Schmidt decompositions

$$|\Psi\rangle = \sum \alpha_i |\psi_i\rangle|\psi_i'\rangle, \qquad |\Phi\rangle = \sum \beta_i |\varphi_i\rangle|\varphi_i'\rangle$$

Then $|\Phi\rangle$ can be converted to $|\Psi\rangle$ by LOCC

$\Longleftrightarrow$ $(\alpha_1, \dots, \alpha_d)$ majorises $(\beta_1, \dots, \beta_d)$

$$\sum_{i=1}^{l}{}^{\downarrow} \alpha_i \geqslant \sum_{i=1}^{l}{}^{\downarrow} \beta_i \qquad l = 1, \dots, d$$

$\Longrightarrow$ entropy of $|\Psi\rangle \leq$ entropy of $|\Phi\rangle$

AN ENTANGLED FORMULA

Theorem (Wootters)

The entanglement of formation of a mixed 2-qubit state ρ is

$$\mathcal{E}_F(\rho) = -\frac{1+\sqrt{1-C^2}}{2}\log\frac{1+\sqrt{1-C^2}}{2} - \frac{1-\sqrt{1-C^2}}{2}\log\frac{1-\sqrt{1-C^2}}{2}$$

where $$C = \max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4,\ 0)$$

& $\lambda_1, \ldots, \lambda_4$ are, in descending order, the square roots of the eigenvalues of

$$\rho\tilde{\rho} = \rho(\rho - \rho_1 - \rho_2 + 1).$$

C is called the <u>concurrence</u> of ρ.

PPT & BOUND ENTANGLEMENT

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|_{AB} \qquad |\psi_i\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$$

ρ separable $\Longleftrightarrow$ $\rho = \sum_i p_i \rho_{iA} \otimes \rho_{iB}$

$$\Longrightarrow \rho^{T_B} = \sum_i p_i \rho_{iA} \otimes \rho_{iB}^{T} \geq 0 \qquad \boxed{\text{PPT}}$$

Th$^{\underline{m}}$ 1 (Horodeckis 1996) If $(d_A, d_B) = (2,2)$ or $(2,3)$,

ρ is separable $\Longleftrightarrow$ ρ has PPT

In other dimensions, $\exists$ PPT entangled states.

DISTILLATION

ρ is distillable $\Longleftrightarrow$ for some n, $\rho^{\otimes n}$ can be converted by LOCC to $\rho_{max}^{\otimes m}$ $(0 < m \leq n)$

$$\rho_{max} = \frac{1}{d} |\psi_+\rangle\langle\psi_+| \qquad |\psi_+\rangle = \sum_{i=1}^{d} |i\rangle_A |i\rangle_B$$

$$d = \min(d_A, d_B)$$

Th$^{\underline{m}}$ 2 (HHH 1997) Any ρ is distillable if $d_A = d_B = 2$.

Th$^{\underline{m}}$ 3 (HHH 1998) A distillable ρ does not have PPT.