On Measures of Multipartite Correlation

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Motivation:

To search for multipartite measures of correlation.

To begin with, we want to explain some of the background materials.

Bipartite Entanglement

- As far as bipartite entanglement is concerned we have at least some knowledge how to deal with entangled states.
- For pure bipartite states there exists a unique measure of entanglement calculated by Von- Neumann entropy of reduced density matrices.

- However for mixed entangled states there is no unique measure of entanglement. One has to look on different ways to quantify entanglement.
- Some of the measures of entanglement are distillable entanglement, entanglement of formation, relative entropy of entanglement, logarithmic negativity, squashed entanglement, etc.

Difficulty

- In most of the cases it is really hard to calculate exactly the measures of entanglement. Only for some few classes of states, actual values are available.
- It is also hard to find whether a mixed bipartite state is entangled or not.

Multipartite Entanglement

- But the situation in multipartite case is really different from that of bipartite case. E.g., how could we define a measure of entanglement for multipartite states at least for pure states are concerned. It is also very difficult to define maximally entangled states in multipartite systems.

- If we consider that a mixed entangled state in a multipartite system has the property that it has maximal entanglement w.r.t. any bipartite cut (i.e., reduced density matrices corresponding to the cut is proportional to the identity operator), then we observe that for n-qubit ($n \geq 3$) system, there does not exist any maximally entangled states for $n=4$ and $n \geq 8$.

Therefore one has to think how to define maximally entangled states for such situations. Recently, Gour and others have defined maximally entangled states in 4-qubit system considering some operational interpretation. One way: the average bipartite entanglement w.r.t. all possible bipartite cuts the state is maximal.

Depending upon different entanglement measures, such as, tangle, Taslis and Renyi α-entropies one could find different states which are maximally entangled w.r.t. the entangled measures considered. Another attempt is to quantify entanglement of a multipartite state, through the distance measures. E.g., geometric measure.

Correlation measures beyond entanglement:

Consider two newly introduced measures of correlation:

1. Quantum Discord

1. Measurement Induced Non-locality

Quantum Discord

- Consider the following state:
- $\rho = \frac{1}{4}\left[\,|+\rangle\langle+| \otimes |0\rangle\langle 0| + |-\rangle\langle-| \otimes |1\rangle\langle 1| + |0\rangle\langle 0| \otimes |-\rangle\langle-| + |1\rangle\langle 1| \otimes |+\rangle\langle+|\,\right]$
- The above state is separable. However, it has non-zero quantum discord which is defined by difference of measuring mutual information in two different ways, viz., $D(A,B)= I(A:B)-J(A:B)$ where,
- $I(A:B)= S(A)-S(A|B)$ and
- $J(A:B)=S(A)-\min_{\{\Pi_j\}} \sum_j p_j S(A|j)$

The above quantity is a measure of non-classical correlation. It has zero value if and only if there exists a von Neumann-measurement $\Pi_k = |\psi_k\rangle\langle\psi_k|$ such that the bipartite state

$$\rho = \sum_k \Pi_k \otimes I \, \rho \, \Pi_k \otimes I$$

States of the above kind are known as classical-quantum state.

Some Comments

- One could interpret Discord in terms of consumption of entanglement in an extended quantum state merging protocol thus enabling it to be a measure of genuine quantum correlation.
- Physically, discord quantifies the loss of information due to the measurement.
- This correlation measure is invariant under LU but may change under other local operation. It is asymmetric w.r.t the parties.

- The set of Classical-Quantum states is non convex.
- Due to the optimization problem, it is in general very hard to find analytic expression for discord. Exact analytical result is available only for a few classes of states.
- It was found that Quantum discord is always non-negative and it reduces to Von Neumann Entropy of the reduced density matrix for pure bipartite states.

- Recently, different measures of quantum discord and their extensions to multipartite systems have been proposed.E.g.,
- Geometric discord:

$D(\rho)= \min \|\rho-\chi\|$ where the minimum is taken over all zero discord state χ .

- Exact analytical formula for geometric discord is also available for only a few class of states.
- Similarly, for discord in terms of relative entropy: $D(\rho) = \min S(\rho \| \chi\ )$

Measurement Induced Nonlocality

- Consider the state, $\rho = \frac{1}{2} [\, |00\rangle\langle 00| + |11\rangle\langle 11| \,]$
- The state has non-zero value of a new measure of correlation is the Measurement Induced Non-Locality(MIN) .
- It is defined as,
- $$N(\rho) = \max \|\rho - \Pi(\rho)\|$$
- where the maximum is taken over all Von-Neumann measurements that preserves density matrix of the first party.

- Physically, MIN quantifies the global effect caused by locally invariant measurement.
- MIN vanishes for product state and remains positive for entangled states. For pure bipartite state MIN reduces to linear entropy like geometric discord.
- It has explicit formula for $2\otimes N$ system, $m\otimes n$ system(if reduced density matrix of first party is non-

- MIN is invariant under local unitary.
- The set of states with zero MIN is a proper subset of the set of states with zero Discord. Thus, it signifies the existence of non-locality without Discord. The set of all zero MIN states is also non-convex.

Our Attempt

- Consider a new measure of correlation for pure multipartite states.
- Suppose, |ψ> be a multipartite pure state *shared between n number of parties. We define the quantity*
- E= max E(|ψ>)k:n-k, maximum is taken over all bipartite cut.
- For mixed states it is convex roof

- Clearly this measure can detect entanglement even for the states with no genuine multipartite entanglement.
- For example, consider three partite bi-separable pure states. In this case *E = 0 iff states are* bi-separable w.r.t. all three bi-partitions, i.e., fully separable.
- Therefore, this type of correlation can be useful in detecting the presence of global entanglement as well as local entanglement (shared between different subsystems) of multipartite state.
- We now explicitly mention results for some class of states of 3 qubit and 4 qubit

Three Qubit System

- For three qubit fully separable class of states, $E = 0$.
- *In case of bi-separable* classes, there are three types of bi-separations, i.e., $A-BC; B-CA; C-AB$. *In all these cases the maximum* value reaches 1 if any two of the three parties share a two-qubit maximally entangled state.

- *Consider the state,* $|\Phi\rangle = \beta 1|000\rangle + \beta 2 \exp(i\theta)|100\rangle + \beta 3|110\rangle + \beta 4|101\rangle + \beta 5|111\rangle$, and calculate,
- $E = \max\{E(|\Phi\rangle)_{A:BC}, E(|\Phi\rangle)_{B:CA}, E(|\Phi\rangle)_{C:AB}\}$
- We discuss some particular cases:
- Maximum value of E, i.e., E = 1 occurs in the three qubit generic

- Also, E=1 occur for W class state, $|\Phi\rangle = \beta1|010\rangle + \beta2\,|100\rangle + \beta3|001\rangle$ with $|\beta1| = 1/\sqrt{2}$, $|\beta2|^2 + |\beta3|^2 = ½$ in B:CA cut. For every cut E=1 occurs.
- Now we consider the four qubit system:
- Firstly, consider the 4-qubit generic class,
- $|\Phi\rangle = \beta1|B_1B_1\rangle + \beta2\,|B2B2\rangle + \beta3|B3B3\rangle + \beta4|B4B4\rangle$, Bi are Bell

- In 2:2 cut, Maximum value of E for this class of states occurs for 'Cluster States'
- $1/2\,[|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle]$.
- Next we consider, another measure of correlation:
- I= max I(|ψ>)k:n-k, maximum is taken over all bipartite cut, where I(.) implies mutual

- Therefore E has also information theoretic interpretation as far as pure states are concerned.

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