Quantum interferometric visibility as a witness of general relativistic proper time

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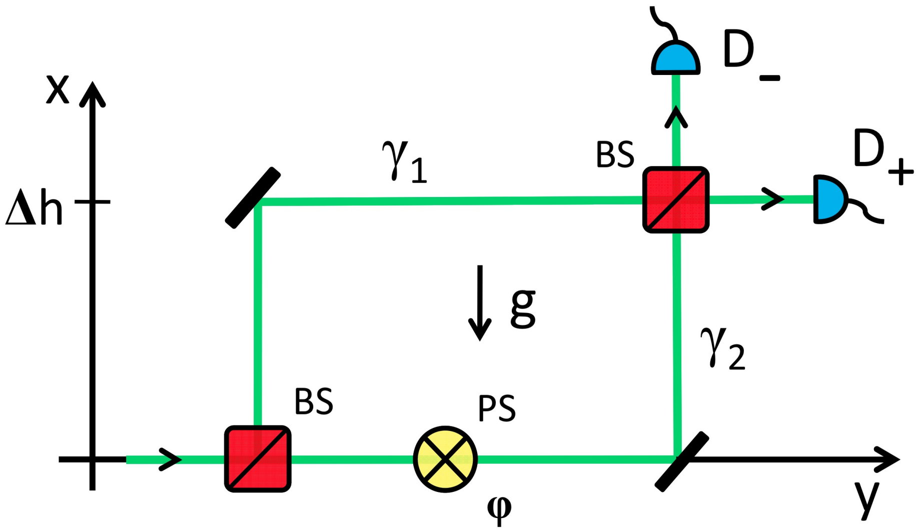
Bhubaneswar, 21st December 2011

Interpretation ambiguity of gravitationally induced phase shifts



Mach-Zehnder interferometer in the gravitational field

- two beam splitters (BS),
- phase shifter (PS),
- two detectors $D_{\pm}$.

$\gamma_{1,2}$ – two possible paths through the setup,
g - homogeneous gravitational field,
Δh - separation between the paths

modes associated with the corresponding paths $\gamma_{1,2}$

The state inside the setup

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(ie^{-i\phi_1}|r_1\rangle + e^{-i\phi_2+i\varphi}|r_2\rangle\right)$$

Probabilities of detection

neutrons, COW, 1975

$$P_{\pm} = \frac{1}{2} \pm \frac{1}{2}\cos\left(\Delta\phi + \varphi\right) \qquad \Delta\phi := \phi_1 - \phi_2$$

Interpretation ambiguity of gravitationally induced phase shifts

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(ie^{-i\phi_1}|r_1\rangle + e^{-i\phi_2+i\varphi}|r_2\rangle\right)$$

non-relativistic quantum mechanics

- gravity: potential force (possibly non-Newtonian)
- there exists a global time paramter,
- flat space-time

$$\phi_{i=1,2} \propto \frac{1}{\hbar}\int_{\gamma_i} dt\, V_{eff}(x)$$

ΔΦ: gravitational analog of an Aharonov-Bohm effect

general relativity

- gravity : metric theory,
- proper time τ may flows at different rates ,
- curved space-time geometry

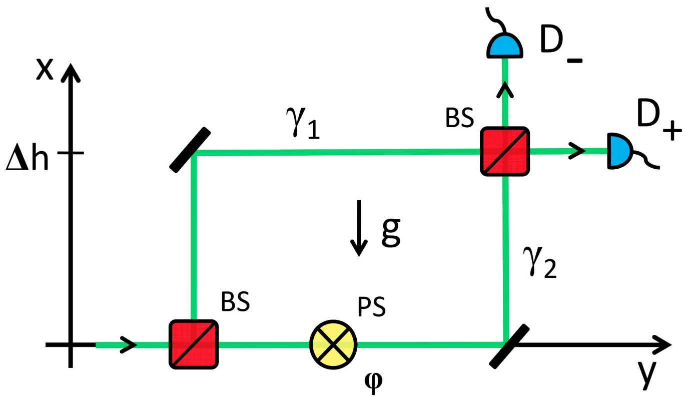
$$\phi_{i=1,2} \propto -\frac{mc^2}{\hbar}\int_{\gamma_i} d\tau$$

ΔΦ: measure of a general relativisitc time dilation

Outline

- General idea: test of general relativistic time dilation in conjunction with the principle of quantum complementarity;
- Derivation of the main result and the experimental proposal - quantitative predictions;
- Feasibility of practical implementations;
- Discussion and extensions: how to test theories in which proper time is supposed to be a quantum degree of freedom;
- Conclusion

Interferometric visibility as a witness of proper time



"clock" - a system with an evolving in time degree of freedom

state of the "clock", which followed the path $\gamma_{1,2}$

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(i|r_1\rangle|\tau_1\rangle e^{-i\phi_1} + |r_2\rangle|\tau_2\rangle e^{-i\phi_2+i\varphi}\right)$$

Probabilities of detection

$$\langle\tau_1|\tau_2\rangle = |\langle\tau_1|\tau_2\rangle|e^{i\alpha}$$

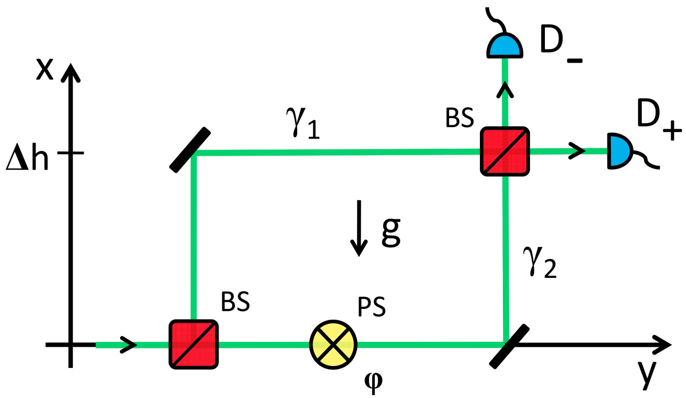
$$P_\pm = \frac{1}{2} \pm \frac{1}{2}|\langle\tau_1|\tau_2\rangle| \cos\left(\Delta\phi + \alpha + \varphi\right)$$

Visibility of the interference pattern: $\mathcal{V} = |\langle\tau_1|\tau_2\rangle|$

Distunguishability of the paths: $\mathcal{D} = \sqrt{1 - |\langle\tau_1|\tau_2\rangle|^2}$

$$\mathcal{V}^2 + \mathcal{D}^2 = 1$$

Interferometric visibility as a witness of proper time



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quantum complementarity + time dilation = drop in the interferometric visibility

Results

"clock" d.o.f. implemented in internal states of a massive particle (neglecting finite size effects)

rest frame:

evolution w.r.t. proper time

$$i\hbar\frac{\partial}{\partial\tau} \equiv H_{\circlearrowleft}$$

laboratory frame:

evolution w.r.t. laboratory time t

$$i\hbar\frac{\partial}{\partial t} = \dot{\tau} H_{\circlearrowleft}$$

$$\dot{\tau} := \frac{d\tau}{dt} = \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}$$

metric, signature mostly plus

energy of a mass m in a space-time with metric $g_{\mu\nu}$

$$T^{\mu\nu} := \frac{\delta S}{\delta g_{\mu\nu}}$$

$$E = mc^2 \frac{-g_{00}}{\sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}}$$

canonical quantization of E → H_0, Hamiltonian of the external d.o.f.

$$H_{Lab} = H_0 + \dot{\tau} H_{\circlearrowleft}$$

Particle in a Schwarzschild metric; up to quadratic terms in the kinetic, potential and internal energy:

$$H_{\text{Lab}} \simeq mc^2 + H_{\circlearrowleft} + E_k^{GR} + \frac{\phi(x)}{c^2}\left(mc^2 + H_{\circlearrowleft} + E_{corr}^{GR}\right)$$

$$\phi(x) = -\frac{GM}{x}$$

$$E_k^{GR} = \frac{p^2}{2m}\left(1 + 3\left(\frac{p}{2mc}\right)^2 - \frac{1}{mc^2}H_{\circlearrowleft}\right)$$

$$E_{corr}^{GR} = \frac{1}{2}m\phi(x) - 3\frac{p^2}{2m}$$

Results

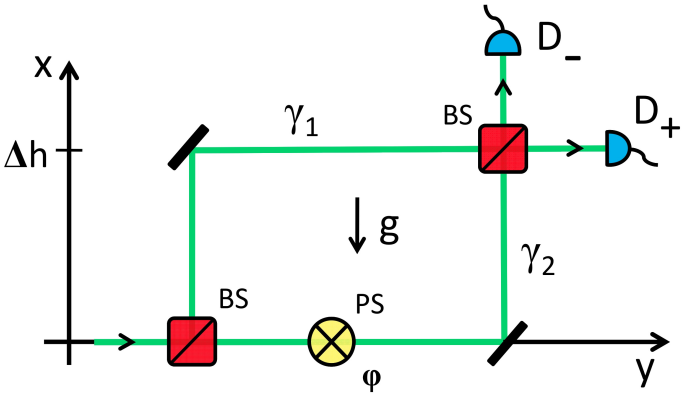
$$H_{\text{Lab}} \simeq mc^2 + H_{\circ} + E_k^{GR} + \frac{\phi(x)}{c^2}\left(mc^2 + H_{\circ} + E_{corr}^{GR}\right)$$

Up to a phase

$$|\Psi_i\rangle = e^{-\frac{i}{\hbar}\int_{\gamma_i} dt\, \frac{\phi(x)}{c^2}\left(mc^2 + H_{\circ} + E_{corr}^{GR}\right)} |x^{in}\rangle |\tau^{in}\rangle$$

$$H_{\circ} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

$$|\tau^{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



- $\Delta E := E_1 - E_0$,
- Δh: distance between the paths
- ΔV:=gΔh, gravitational potential (up to linear terms in Δh)
- ΔT: time for which the particle travels in superposition at constant heights,

$$P_{\pm}(\varphi, m, \Delta E, \Delta V, \Delta T) =$$

$$= \frac{1}{2} \pm \frac{1}{2}\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\cos\left(\left(mc^2 + \langle H_{\circ}\rangle + \bar{E}_{corr}^{GR}\right)\frac{\Delta V \Delta T}{\hbar c^2} + \varphi\right)$$

expectation value taken w.r.t. the $|\tau^{in}\rangle$

E^{GR}_{corr} averaged over the two paths

Results

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relative phase from the Newtonian potential

GR corrections to the relative phase from the path d.o.f.

new effects appearing with the "clock":

change in the interferometric visibility

$$\mathcal{V} = \left|\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\right|$$

phase shift proportional to the internal average energy

Results

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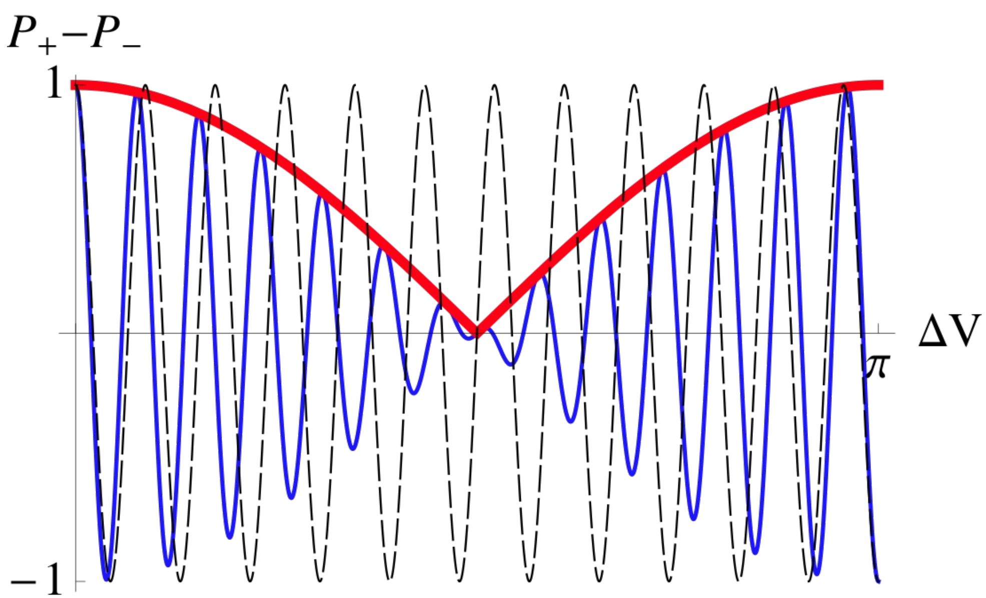


- dashed, black line - interference without the "clock"
- blue line - interference with the "clock"
- thick, red line - modulation in the visibility

Generalization

$$\mathcal{V} = \left|\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\right|$$

$$H = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1| \qquad |\tau^{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- orthogonalization time (of a quantum system):

$$t_\perp := \min\{t > 0 \mid \ \langle\Psi(0)|\Psi(t)\rangle = 0\}\,, \quad |\Psi(t)\rangle = e^{-\frac{i}{\hbar}\int_0^t H ds}|\Psi(0)\rangle$$

here: $\quad t_\perp = \frac{\pi\hbar}{\Delta E}$

- total time dilation between the trajectories: $\Delta\tau = \frac{\Delta V \Delta T}{c^2}$

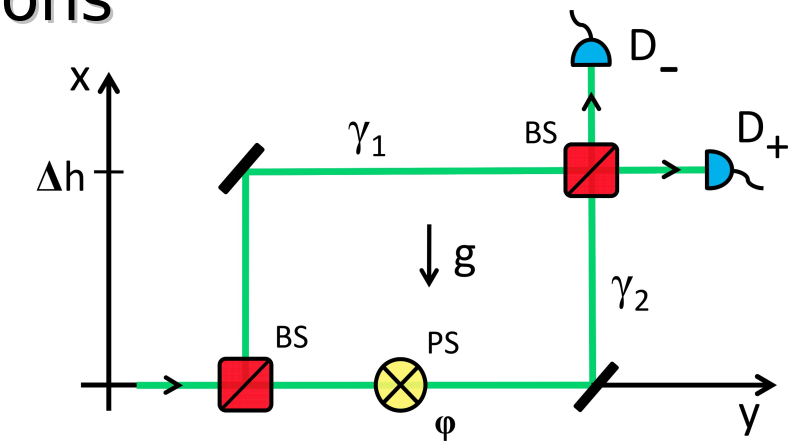
$$\boxed{\mathcal{V} = \left|\cos\left(\frac{\Delta\tau}{t_\perp}\frac{\pi}{2}\right)\right|}$$

time dilation between the interferometric paths = orthogonalization time of the “clock” => maximal which-way information & no interference

Implementations

$$\mathcal{V} = \left|\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\right|$$

- $\Delta E := E_1 - E_0$,
- Δh: distance between the paths
- $\Delta V := g\Delta h$, gravitational potential,
- $g = 10\ \mathrm{m/s^2}$
- ΔT: time for which the particle travels in superposition at constant heights,



"clock" frequency $\omega = \Delta E/\hbar$

previously used interferometers testing e.g. gravitational phase shift

for the full loss of the interferometric visibility

system	clock	ω [Hz]	$\Delta h \Delta T$ [ms] achieved	$\Delta h \Delta T$ [ms] required
atoms	hyperfine states	10^{15}	10^{-5}	10
electrons	spin precession	10^{13}	10^{-6}	10^3
molecules	vibrational modes	10^{12}	10^{-8}	10^4
neutrons	spin precession	10^{10}	10^{-7}	10^6

Discussion and final remarks

- phase shift occurs independently of the implementation of the „clock“

- interferometric visibility drops ⇔ proper time has operational meaning

$$\mathcal{V} = \left| \cos\left(\frac{\Delta\tau}{t_\perp} \frac{\pi}{2} \right) \right|$$

take an eigenstate of the internal energy Hamiltonian

⇒ only the phase of the state changes...

the „clock“ does not „tick“

⇒ the concept of proper time has no operational meaning

⇒ visibility is maximal!

Discussion and final remarks

- phase shift occurs independently of the implementation of the „clock“

- interferometric visibility drops ⇔ proper time has operational meaning

- ~~each massive particle = Compton clock measuring proper time along ist path~~

$$\Delta\phi = \frac{mc^2}{\hbar}\Delta\tau$$

redshift of a clock ticking at the Compton frequency

interference should always be lost! (since the which-path information is stored „somewhere“ in the particle)

In quantum mechanics it makes no sense to speak about quantities without specifying how they are measured!

Discussion and final remarks

- phase shift occurs independently of the implementation of the „clock“

  tests the corrections to the gravitational potential, analogous to the A-B effect in the electromagnetism

- interferometric visibility drops ⇔ proper time has operational meaning

  tests quantum complementarity principle
  in the conjunction with
  the general relativistic time dilation

toward testing new theories

Theories which assume that proper time is a new quantum degree of freedom can be tested with our proposal.

- V_m - measured visibility, with estimated error $\Delta\nu$;
- V_{QM} - visibility predicted by quantum mechanics

experimental visibility	**possible explanation**	**current experimental status**
$\mathcal{V}_m = 0$	proper time: quantum d.o.f., sharply defined	disproved in e.g. Ref. [1,2]
$0 < \mathcal{V}_m < \mathcal{V}_{QM}$	proper time: quantum d.o.f with uncertainty σ_τ	consistent with current data for $\sigma_\tau > \frac{\lvert\Delta\tau\rvert}{\sqrt{-8\ln(1-\Delta\mathcal{V})}}$
$\mathcal{V}_m = \mathcal{V}_{QM}$	proper time: not a quantum d.o.f. or has a very broad uncertainty	consistent with current data
$\mathcal{V}_m > \mathcal{V}_{QM}$	quantum interferometric complementarity does not hold when general relativistic effects become relevant	not tested

Conclusion

Drop in the visibility of quantum interference due to gravitational time dilation

- new paradigm for tests of genuine general relativistic effects in quantum mechanics

- clarification of the notion of proper time in the quantum context - only operationally well defined physical quantities have meaning in quantum mechanics!

- Test of theories in which proper time is assumed to be a quantum degree of freedom;

- previously not considered mechanism of decoherence (important for quantum-to-classical transition)

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Nat. Commun. 2:505 doi: 10.1038/ncomms1498 (2011)

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Thank you for your attention!