Quantum Correlations in Many-Body Systems

ISCQI – 2011, IOP Bhubaneswar

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Outline

1. Why ***quantum*** information?

2. Separable and Entangled states.

3. Setting the stage: Which "entanglements" can we ***compute***?

4. Two main streams:
   a. Two-site densities of spin-1/2 ground states
   b. Area Law: Scaling of ground state local entropy

5. End remarks

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Why *quantum* computation?

No efficient classical algorithm that factorizes an integer into its prime factors.

If we believe that there are none, then quantum computation promises qualitatively better efficiencies than its classical counterpart:

"Shor's algorithm".

Moreover, quantum computers can be used to simulate complex quantum systems.

Also: With the rapidly decreasing size of computer chips, sooner or later quantum effects will begin to show up.

Feynman 1982, Shor 1994, Grover 1995, ...

Why *quantum* cryptography?

- All practical classical cryptographic schemes rely, for its security, on the unproven premise that integers cannot be efficiently factored.

- They will be insecure if the eavesdropper implements Shor's algorithm.

- Security of **quantum** cryptographic schemes rely on quantum mechanics.

Wiesner 1970s; Bennett & Brassard 1984; Ekert 1991; Bennett, Brassard, & Mermin 1992; Bennett 1992; Bruss 1998.

Why ***quantum*** communication?

- A two-state classical system (e.g. a ball that may be either blue or green) can be used to send at most one bit of classical information.

If the sender and receiver are allowed to share a quantum state, a two-dimensional quantum state can be used to send up to two bits of classical information:

"Quantum dense coding".

Bennett & Wiesner 1992.

Why ***quantum*** communication?

- An infinite amount of classical communication is needed for sending a two-dimensional quantum system.

  Only two bits of classical communication may be needed, if a quantum state is shared between the sender and receiver:

  “Quantum teleportation”.

Bennett, Brassard, Crepeau, Jozsa, Peres, Wootters 1993.

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LOCC paradigm in quantum info

- If the state is shared between two or more parties, the parties would only be able to act locally. Allowed operations: LOCC.

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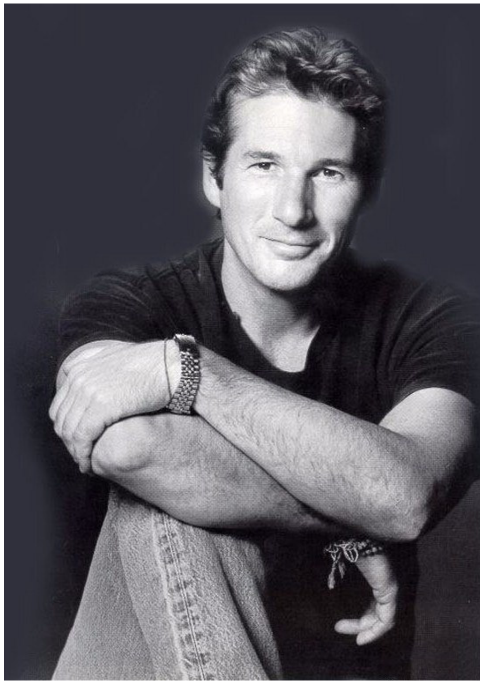
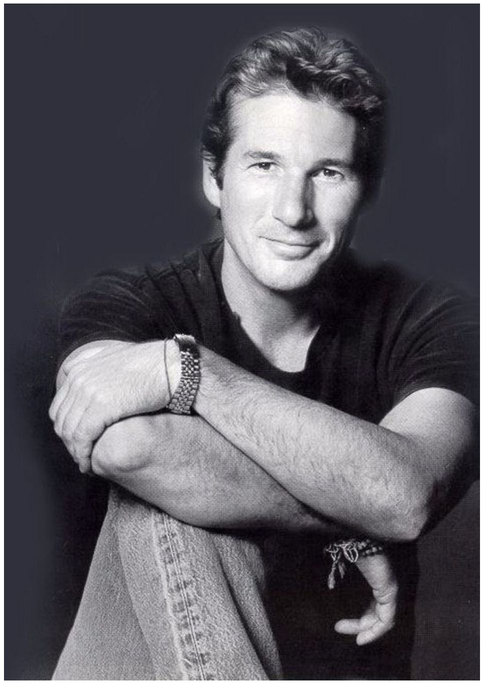
Not this!!

What do we mean by LOCC?



- Alice makes a measurement and communicates her result to Bob (say, by a phone call).

What do we mean by LOCC?



- Alice makes a measurement and communicates her result to Bob (say, by a phone call).
- Then depending on her result, Bob will make his measurement and communicate his result to Alice.
- And so on.

Separable and Entangled states

- Quantum states that can be prepared by LOCC $\rightarrow$ Separable states.
- Otherwise $\rightarrow$ Entangled states.

Separable and Entangled states

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- How do they look like?

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- Separable *pure* states: product over pure states of individual systems.

Separable and Entangled states

- Quantum states that can be prepared by LOCC $\rightarrow$ Separable states.
- How do they look like? Mathematically?
- Separable states: mixture of products over pure states of individual systems.

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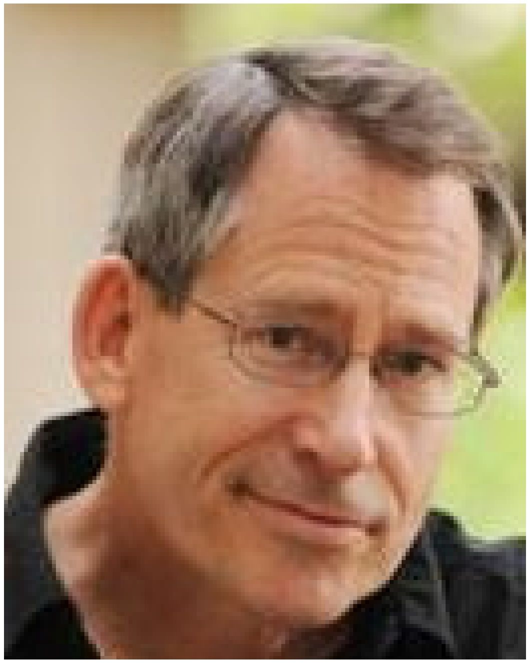
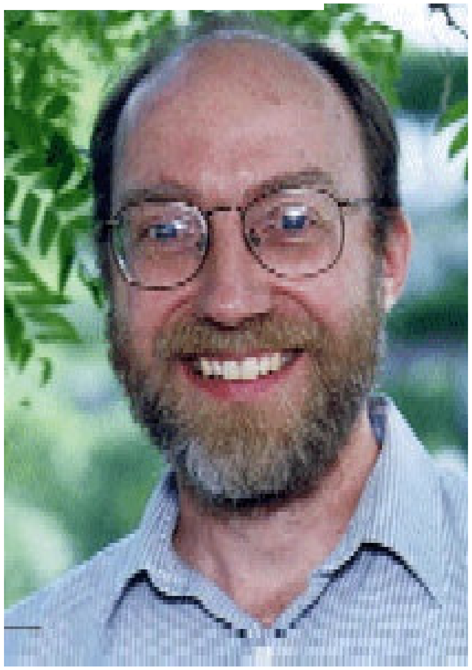
Circa 2000

- Nielsen, Preskill, Wootters *et al.*





Circa 2000

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Idea of using entanglement-like concepts
in quantum many-body phenomena was put forward.

Circa 2000

- Nielsen, Preskill, Wootters *et al.*
- Osborne and Nielsen, QIP'02, PRA'02
- Osterloh, Amico, Falci, Fazio, Nature'02

Which entanglement can we *compute*?

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to understand an entanglement measure conceptually.
We must also be able to ***compute*** it
for the states of the real systems.

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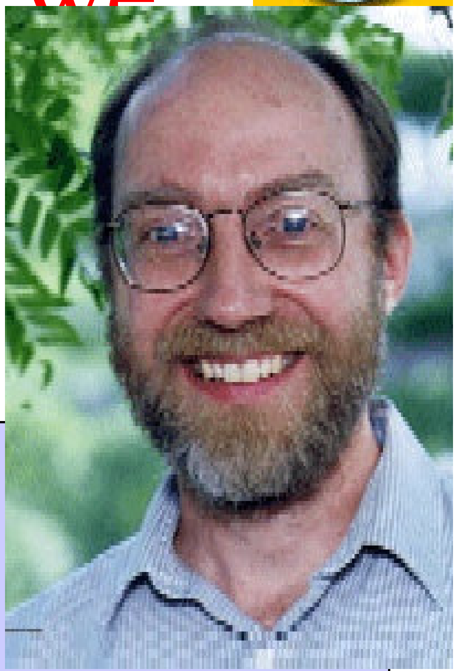

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- In higher dimensions, logarithmic negativity can be calculated. But it cannot detect bound entanglement.

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N = sum of mod of negative eigenvalues
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Which entanglement can we

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The opposite is not true!

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Which entanglement can we

If there [illegible] eigenvalues

Peres, Horodeccy Family, Vidal, Werner

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Possible in arbitrary dimensions.

Which entanglement can we *compute*?

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This sets the stage for the

QI – many-body interface.

Possible in arbitrary dimensions.

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Indeed, two of the main directions of study are

1. EoF of reduced densities of spin-1/2 ground states
2. Scaling of local entropy in ground state partitions

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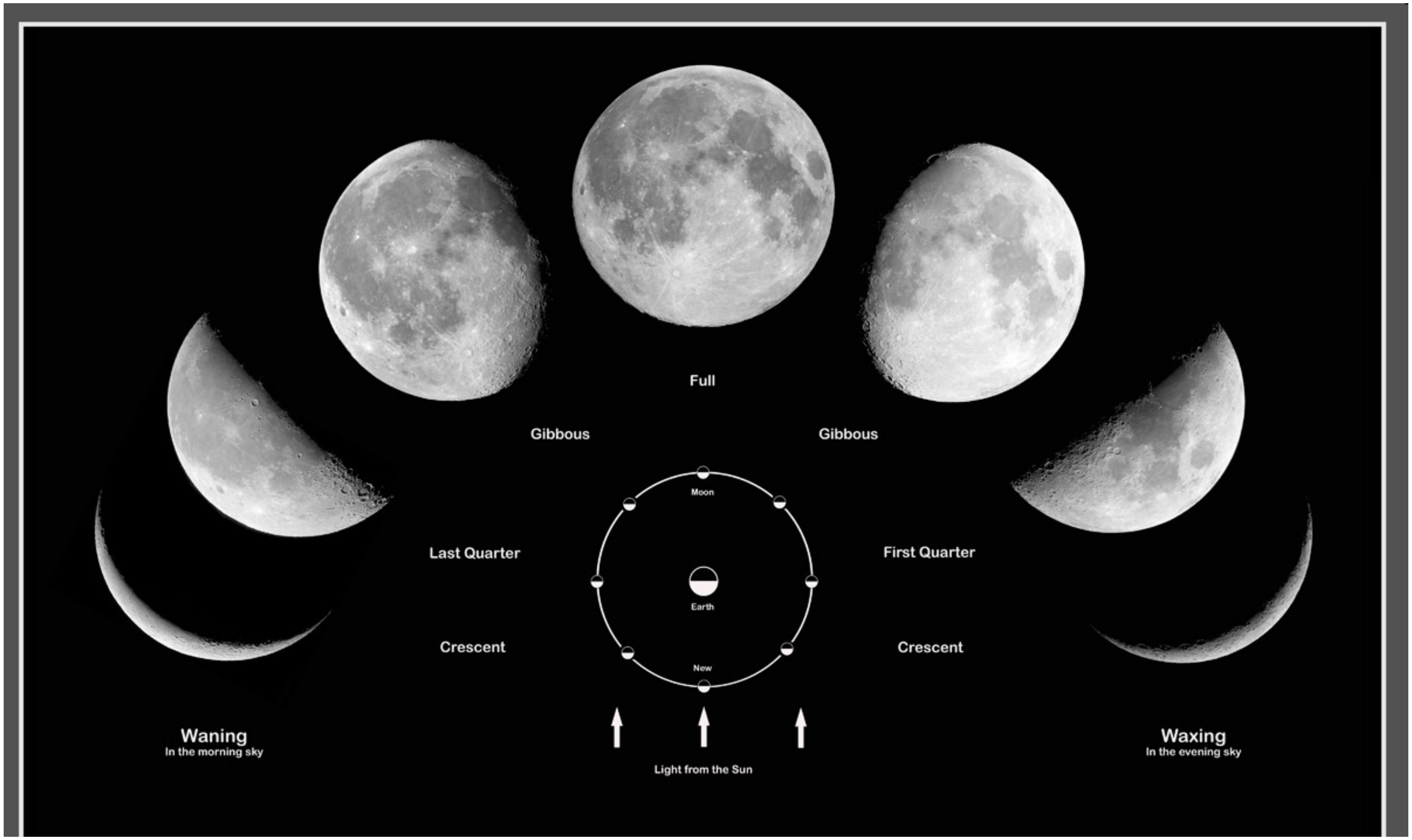
"Area Law"

Possible in arbitrary dimensions.

Phase Transitions

Phase Transitions



Phase Transitions

Phase Transitions

Quantum Phase Transitions

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- Transitions at zero temperature.

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- Implying, transition not temp. driven.

Quantum Phase Transitions

- Transitions at zero temperature.
- Implying, transition not temp. driven.
- Driven by system parameter, like a magnetic field.

S Sachdev, QPT

Quantum Phase Transitions

Typical situation:

- $H = H(\mathrm{int}) + a\, H(\mathrm{field})$

Quantum Phase Transitions

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- $H = H(\mathrm{int}) + a\, H(\mathrm{field})$

"a" is usually denoted as ……

Quantum Phase Transitions

Typical situation:

- H = H(int) + a H(field)
- Ground state of H

Quantum Phase Transitions

Typical situation:

- H = H(int) + a H(field)
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Quantum Phase Transitions

Typical situation:

- H = H(int) + a H(field)
- Ground state of H ← guarantees T=0
- GS depends on "a".
- "a" can be changed.
- Nonanalyticity appears in some physical quantity as "a" is changed.

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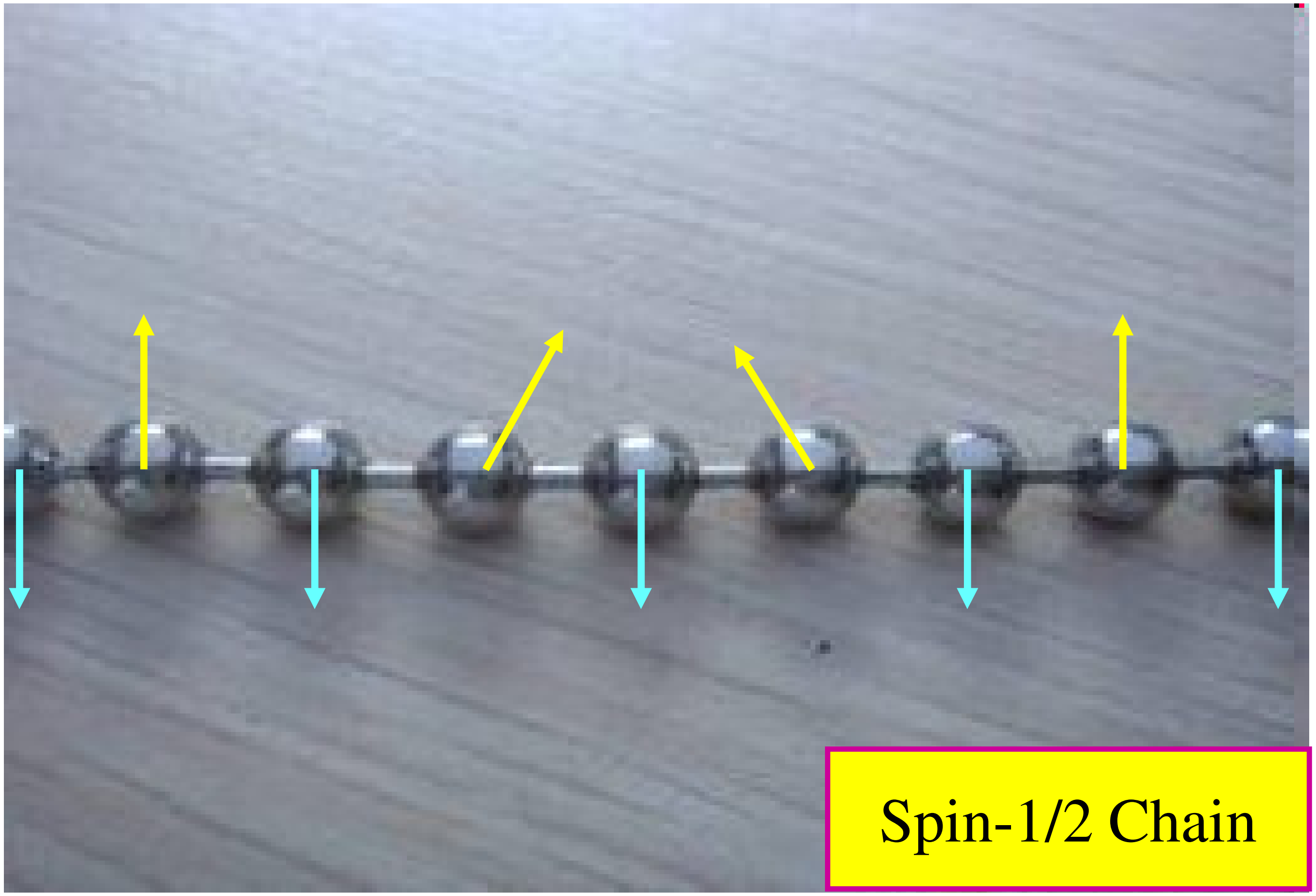
Two-site densities



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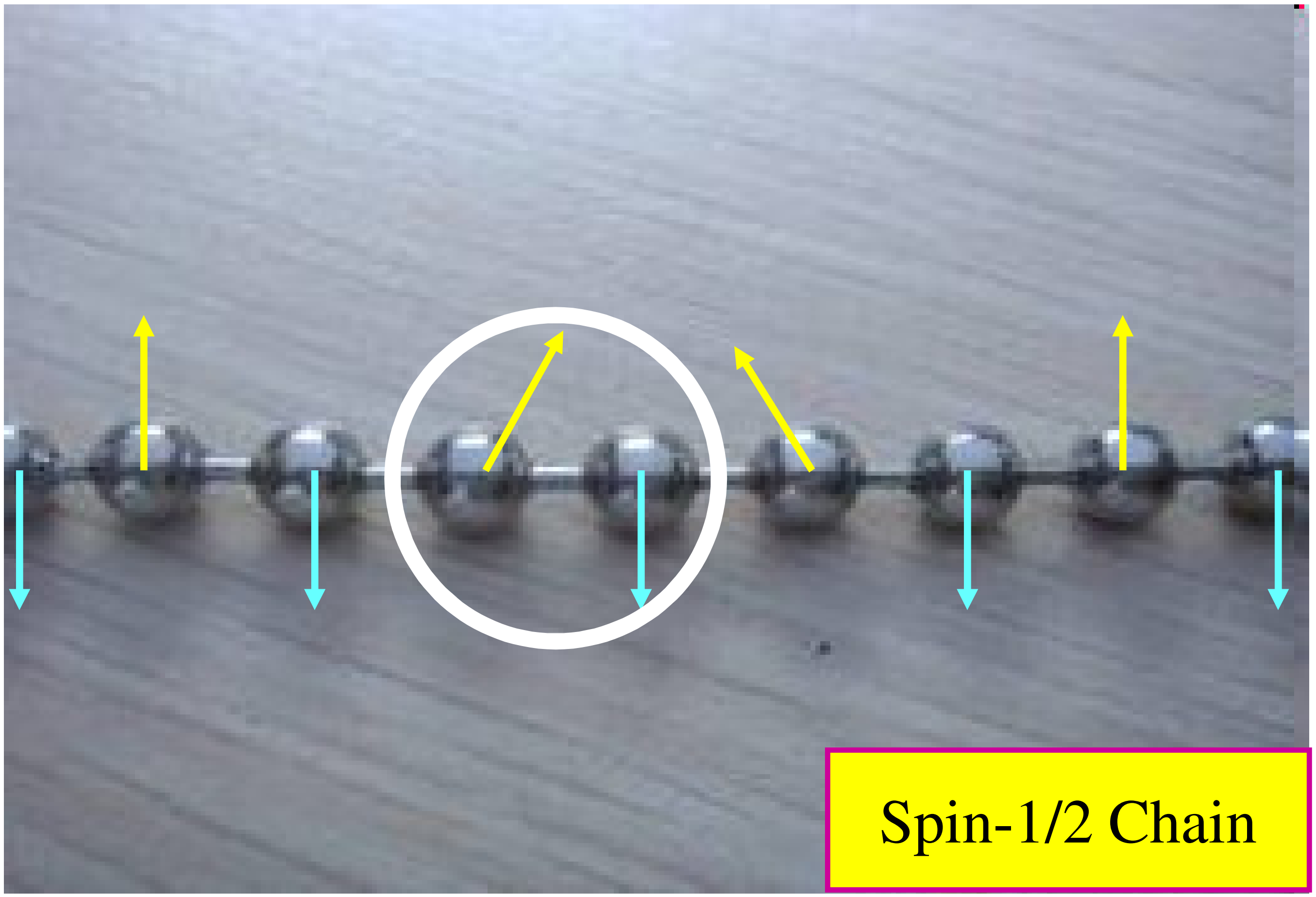
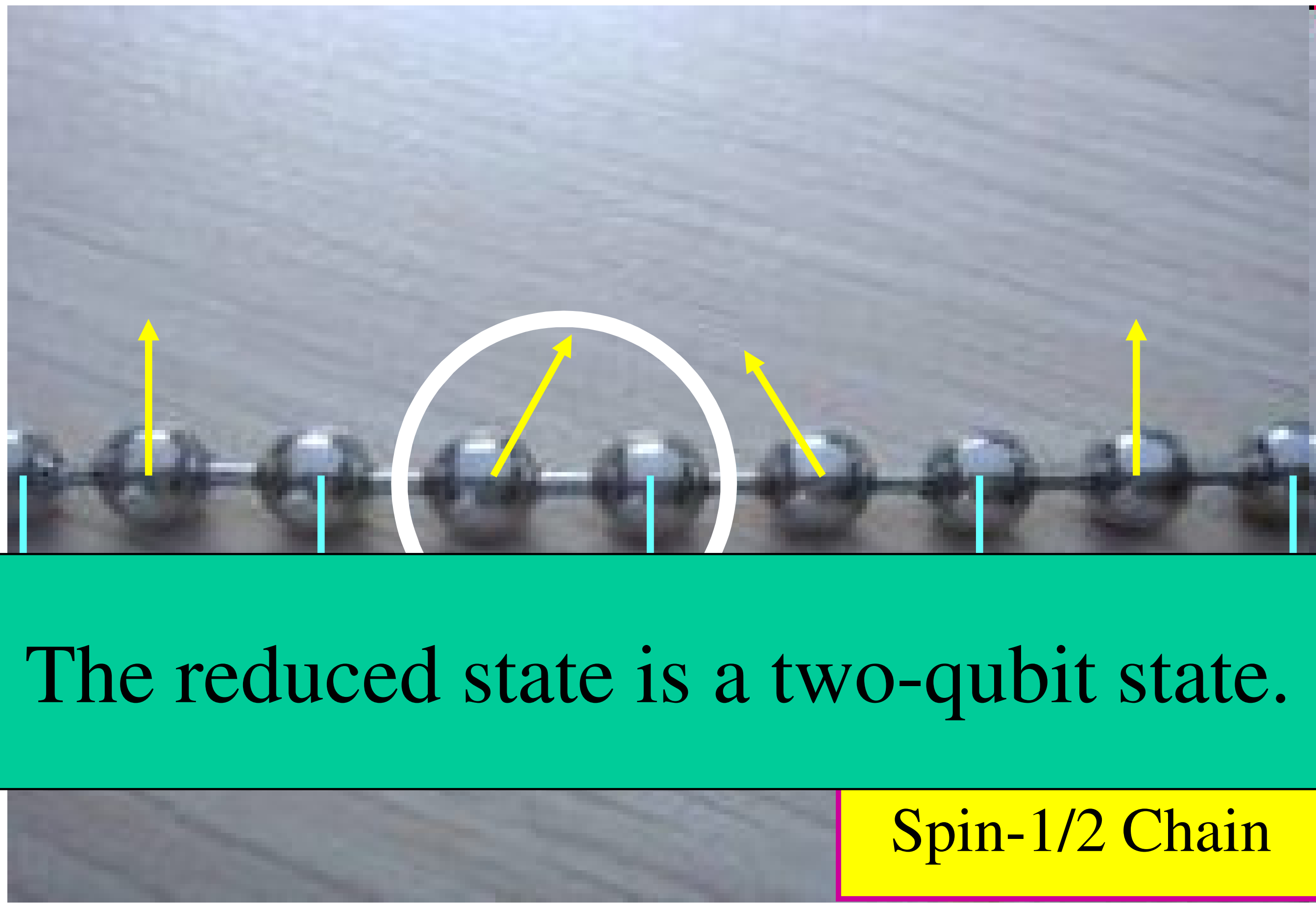


Two-site densities



The reduced state is a two-qubit state.

Two-site densities

The prescription:

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1. Find ground state of spin-1/2 system

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4. Investigate it wrt the relevant system parameter

Quantum XY spin model

$$\Sigma\, J\,[(1+\gamma)\, S_x^{i} S_x^{i+1} + (1-\gamma)\, S_y^{i} S_y^{i+1}] - a\, S_z^{i}$$

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$$\Sigma J \, [(1+\gamma) S_x^i S_x^{i+1} + (1-\gamma) S_y^i S_y^{i+1}] - a\, S_z^i$$

S are half of Pauli matrices.

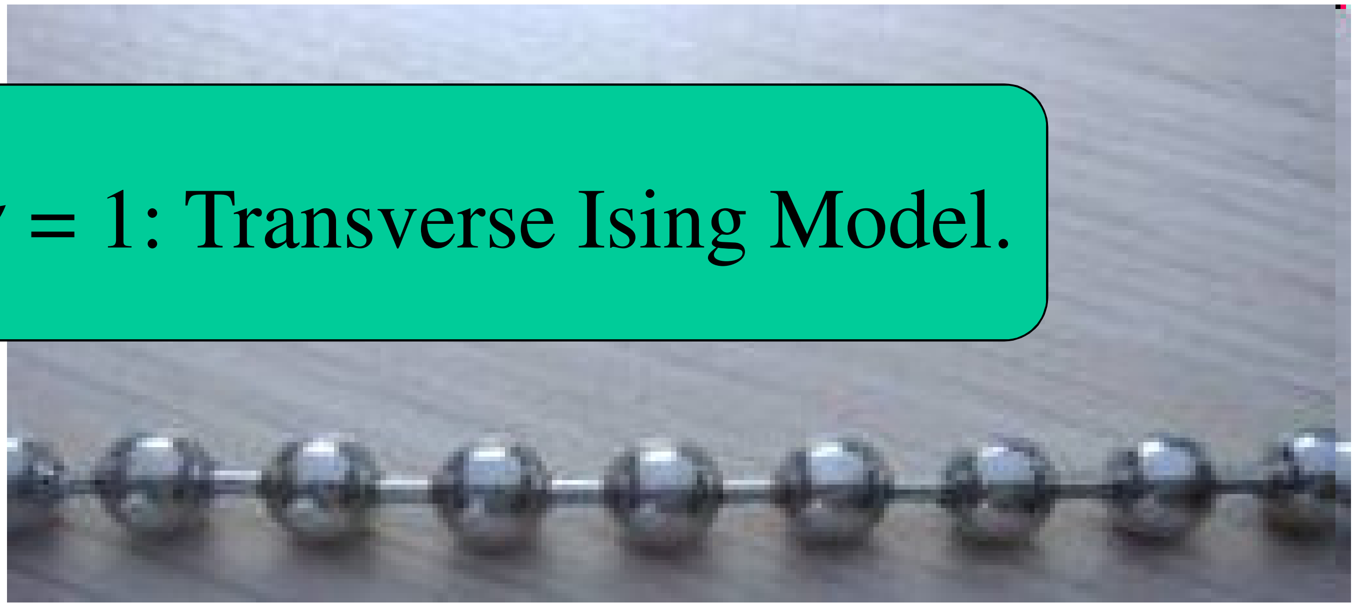
Quantum XY spin model

$$\Sigma J\, [(1+\gamma)\, S_x^i S_x^{i+1} + (1-\gamma)\, S_y^i S_y^{i+1}] - a\, S_z^i$$

Quantum phase transition at h=1.

Quantum XY spin model

For $\gamma = 1$: Transverse Ising Model.



$$\Sigma J \left[(1 + \gamma)\, S_x^i S_x^{i+1} + (1 - \gamma)\, S_y^i S_y^{i+1}\right] - a\, S_z^i$$

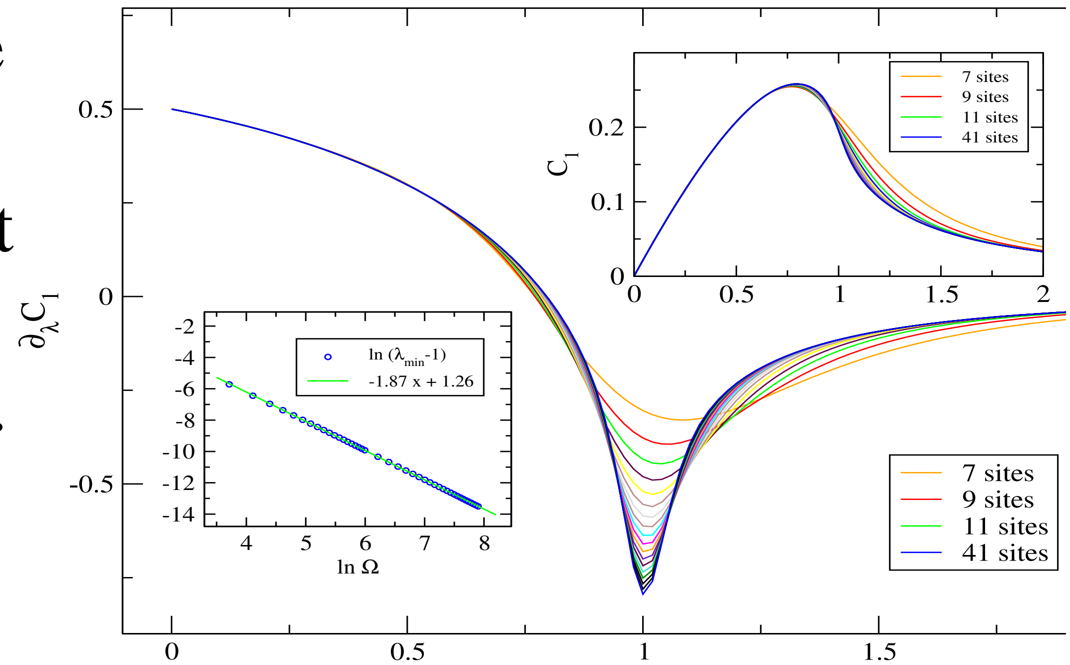
Quantum phase transition at h=1.

Entanglement in states of many body systems

Linking QI with concepts in quantum statistical mechanics and quantum phase transitions.

Near QPT in 1D transverse Ising model, 2-site entanglement remains short ranged, while 2-site correlation length diverges.

Entanglement, however, does show signs of criticality.



Osterloh, Amico, Falci, & Fazio, Nature 2002; Osborne & Nielsen, Phys. Rev. A 2002.

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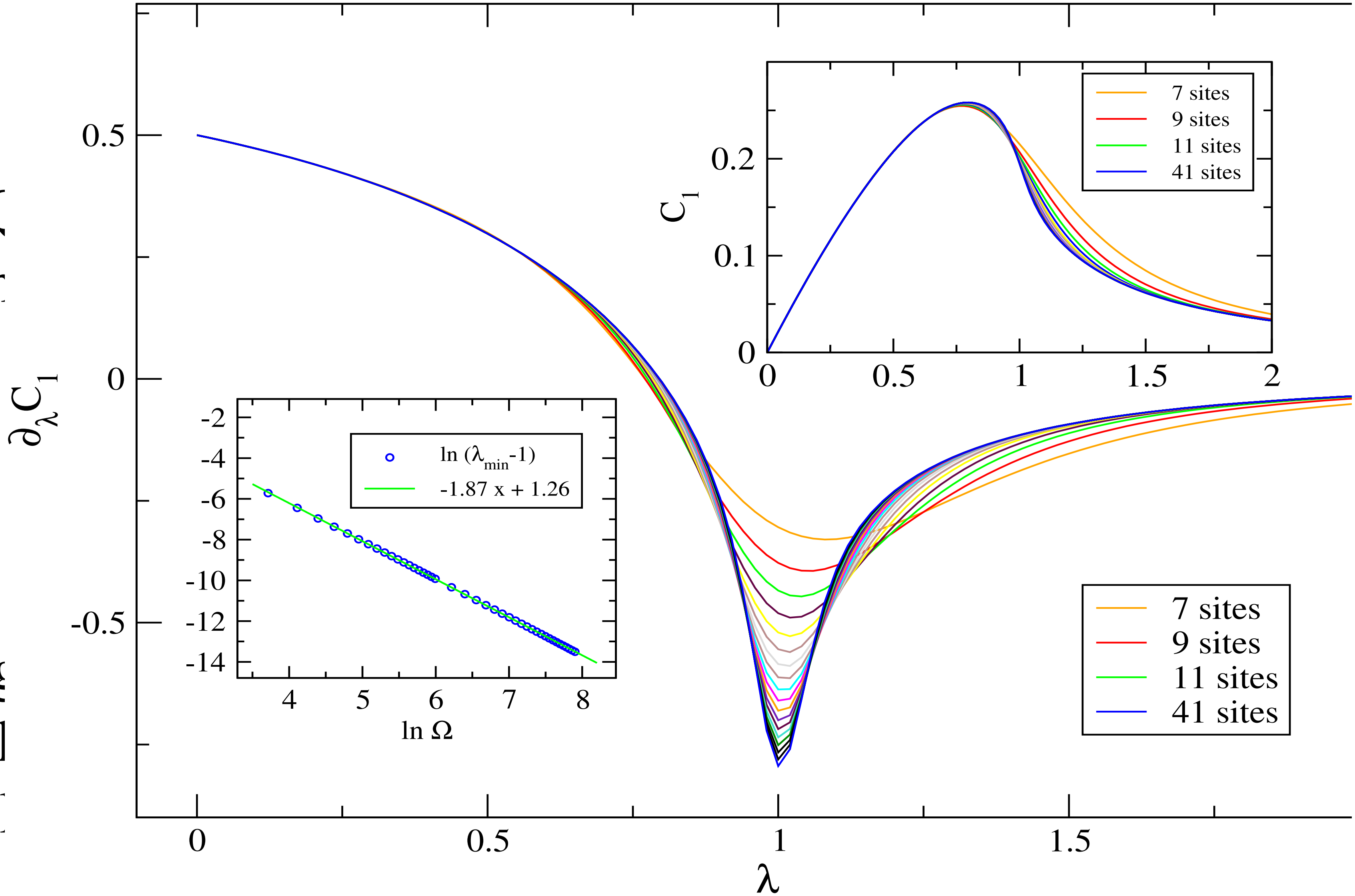

Two-site densities

The prescription:

1. Find ground state of spin-1/2 system
2. Remove all spins except two NNs
3. Find EoF of resulting two-site density
4. Investigate it wrt the relevant system parameter

Two-site densities

Why ***ground*** state?

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Two-site densities

The prescription:

1. Find ground s
2. Remove all sp
3. Find EoF of r
4. Investigate it   er

Why ***ground*** state?

Guarantees that there are no thermal effects.

Two-site densities

Why ***ground*** state?

The prescription:

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Thermal states,
time-evolved states
also considered.

Two-site densities

Why ***NN***?

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Two-site densities

Why *NN*?

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In many instances,
but NOT all,
NNN and so on
have little to no entanglement.

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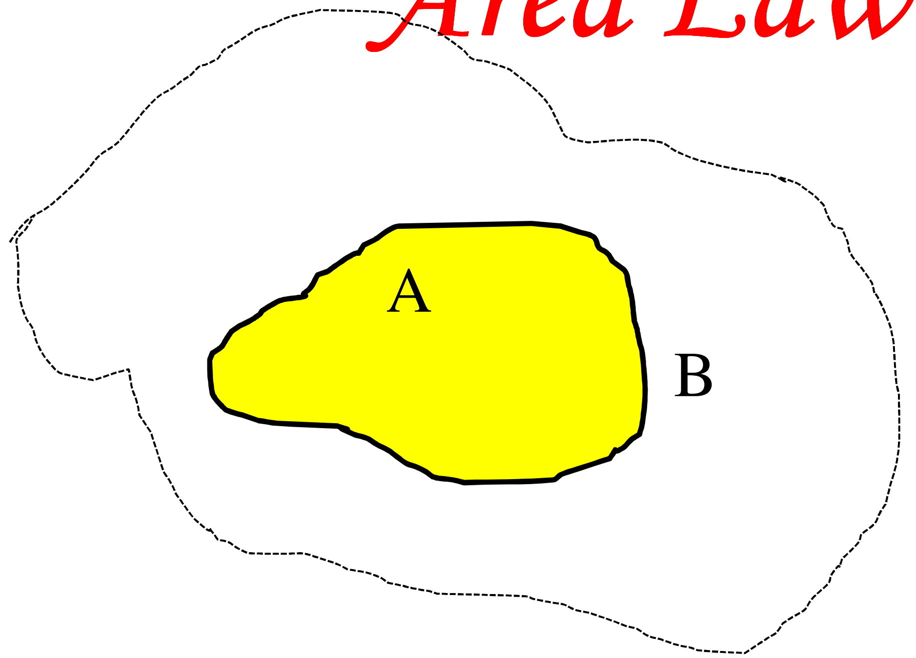
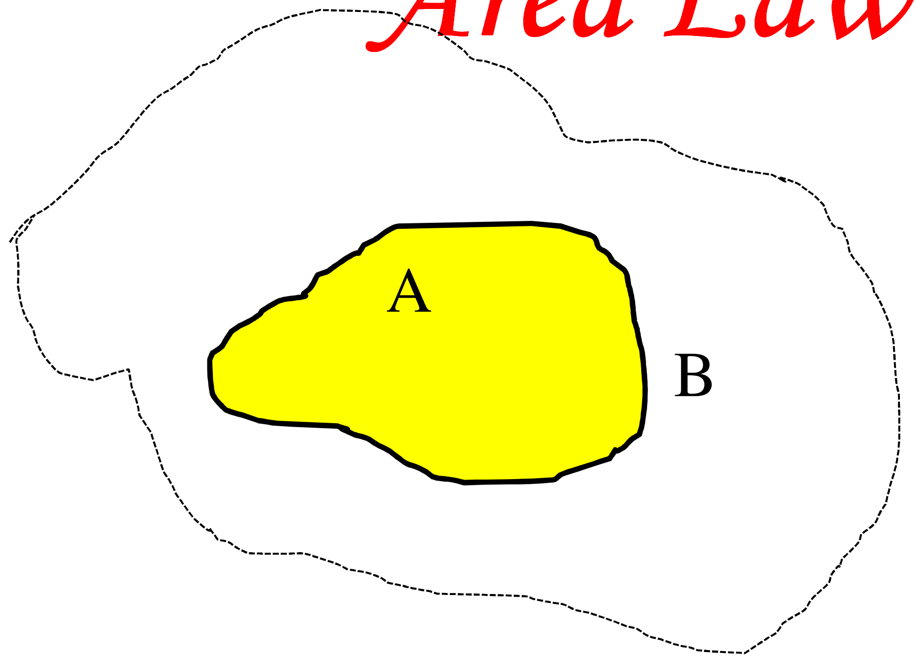
Area Law



Reduced entropy S would depend on the surface of separation between A and B.

Area Law



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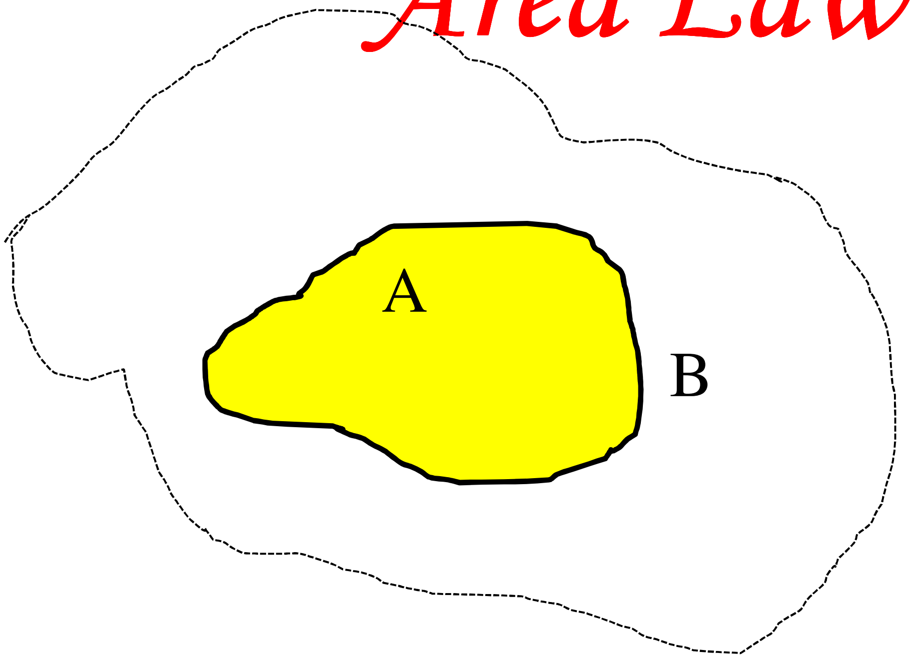
We r talking abt interacting systems.

Area Law



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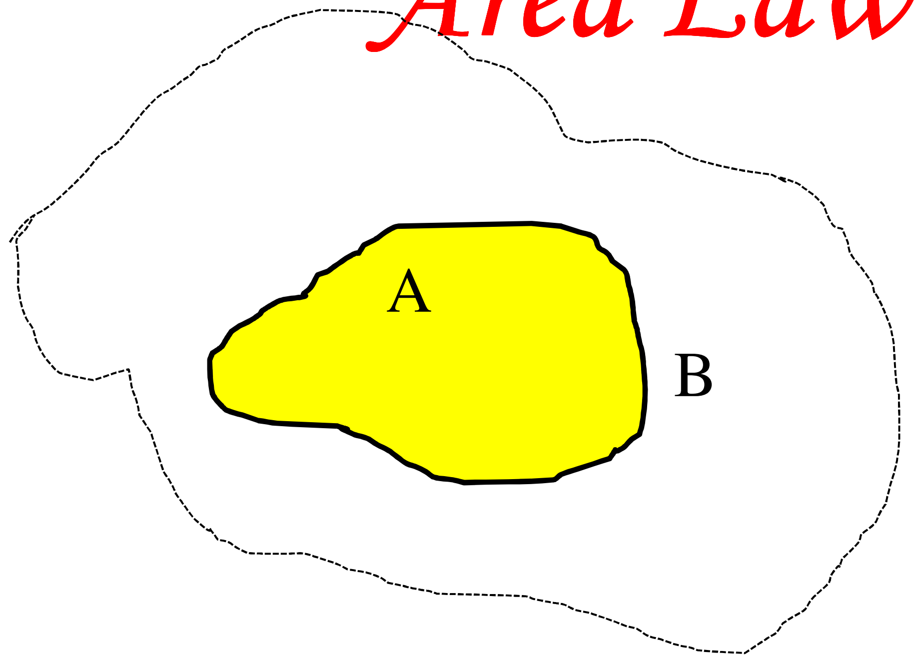
Would be true if …

Area Law



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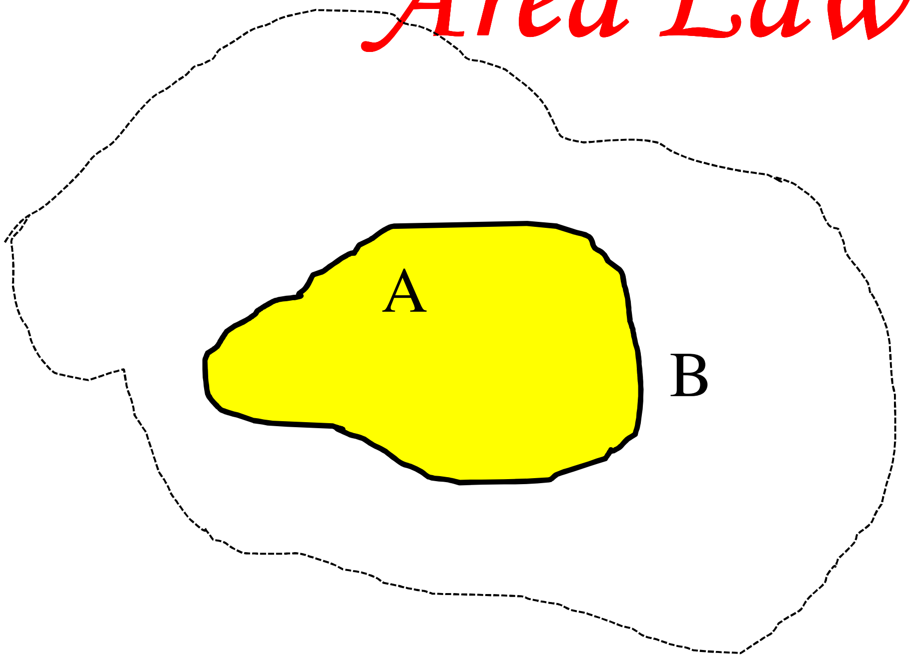
Boundary particles are *pure* entangled states.

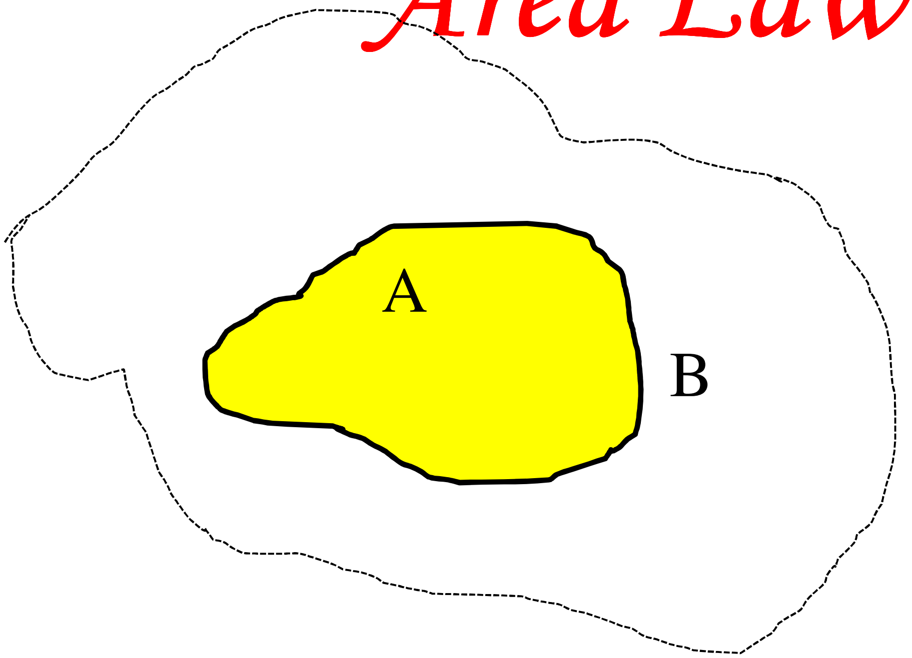
Area Law



Reduced entropy S would depend on the surface of separation between A and B.

Boundary particles are *pure* entangled states.
Plus no long-range entangled pairs.

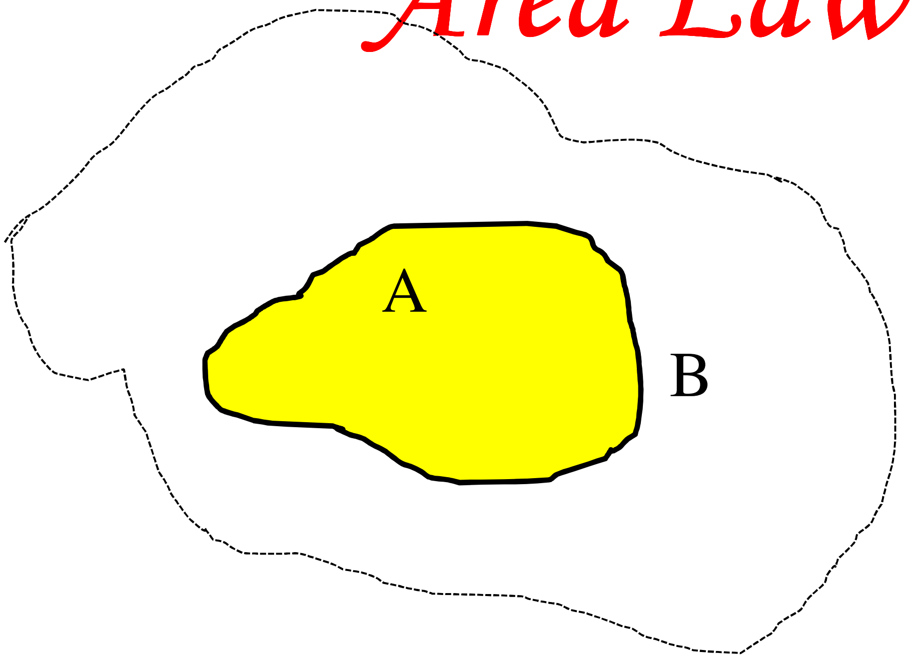
Area Law



Reduced entropy S would depend on the surface of separation between A and B.

Typical situation is far from being such.

Area Law



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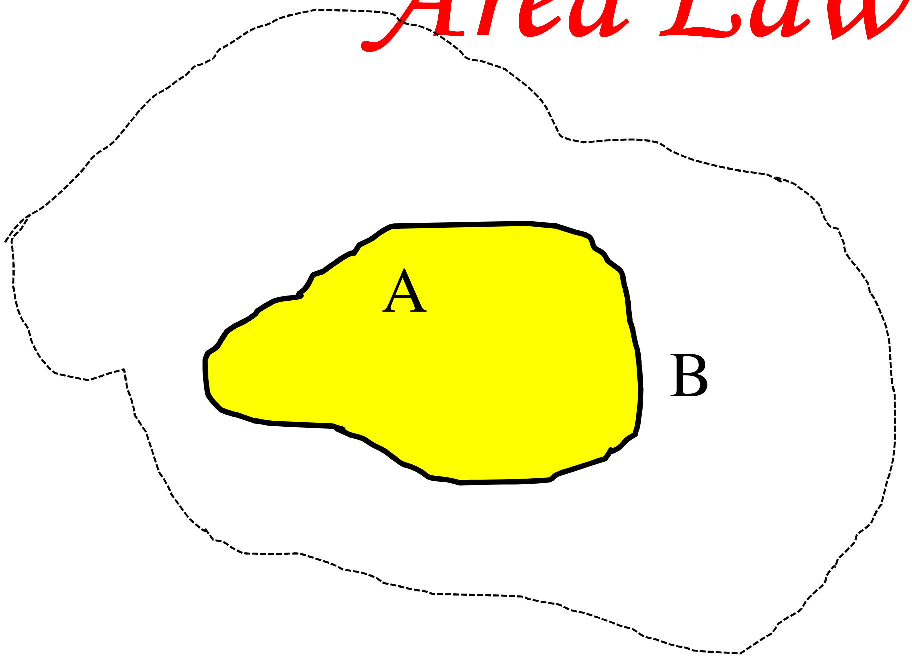
Typical situation is far from being such.
Usually intricately multiparty quantum correlated.

Area Law

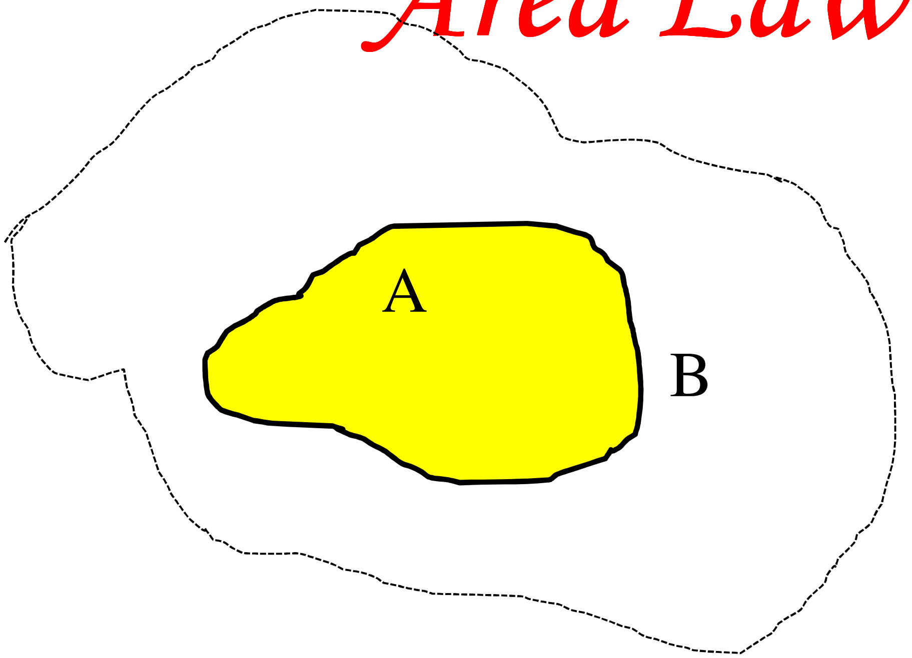


Reduced entropy S would depend on the surface of separation between A and B.

$$S(\rho_L) \sim L^{d-1}$$

L: characteristic length of A

Area Law



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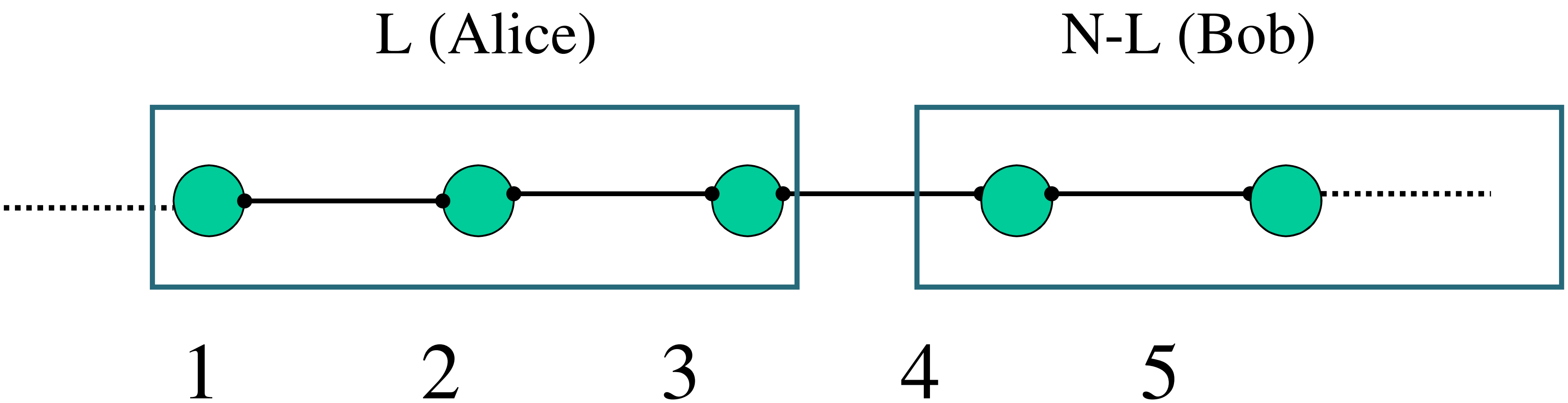
L: characteristic length of A

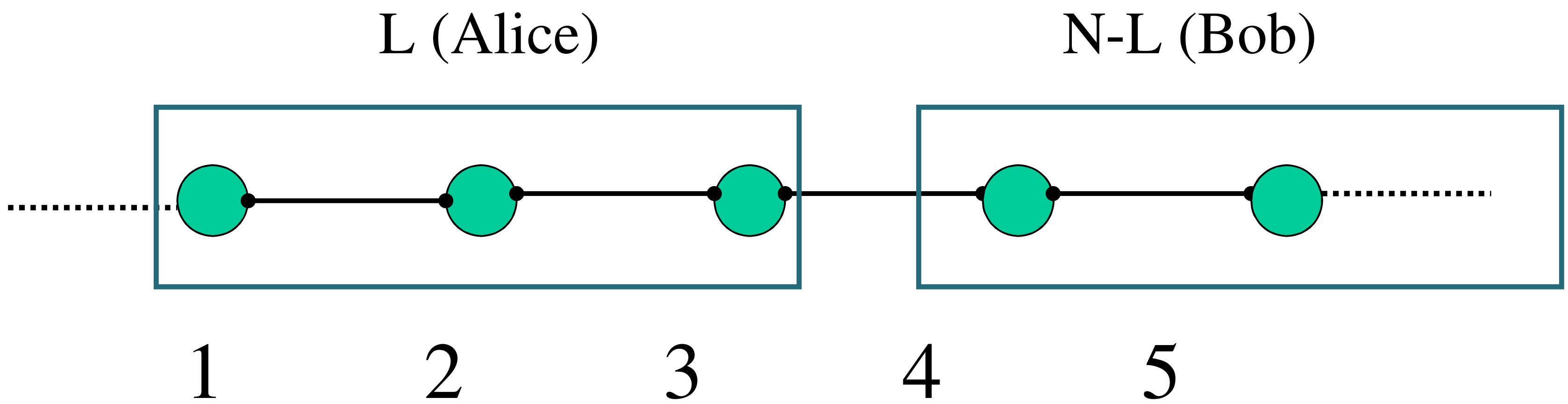
Area Law:1D



Block entanglement: $E(|\Psi\rangle_{L:N-L})$

$$E(|\Psi\rangle_{L:N-L}) = S(\rho_L)$$

Area Law:1D

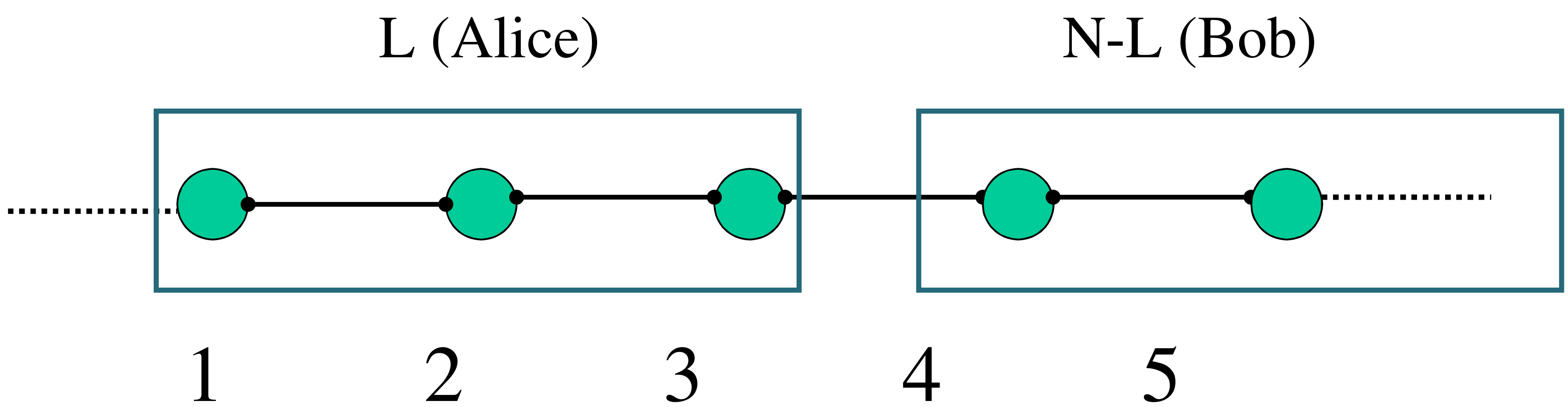


Block entanglement: $E(|\Psi\rangle_{L:N-L})$

$$E(|\Psi\rangle_{L:N-L}) = S(\rho_L) \sim L^{d-1} \equiv \text{constant}$$

away from criticality

Area Law: 1D



Block entanglement: $E(|\Psi\rangle_{L:N-L})$

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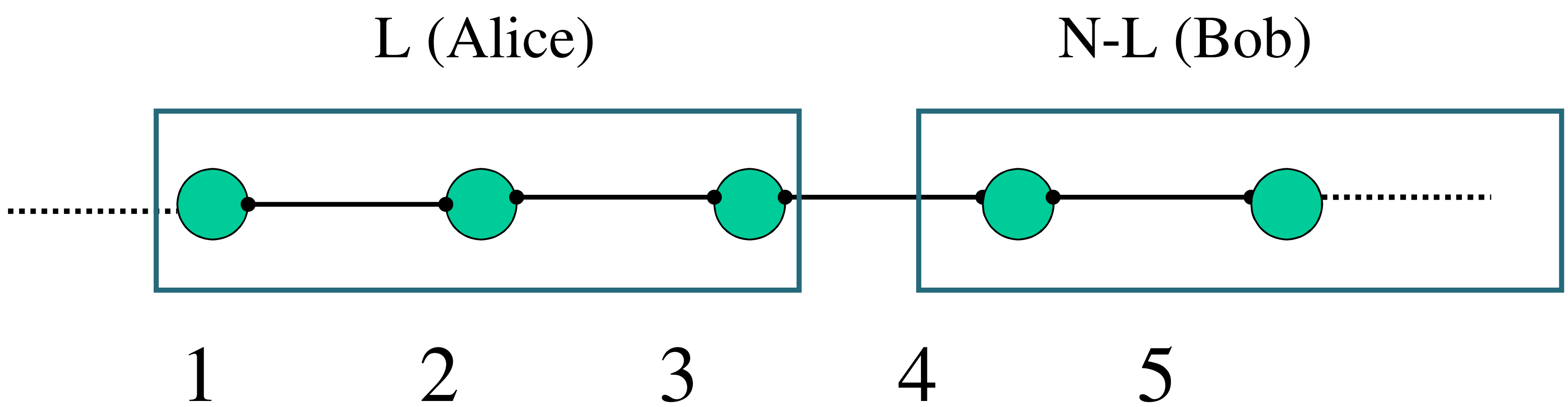
independent of block-size

Area Law: 1D

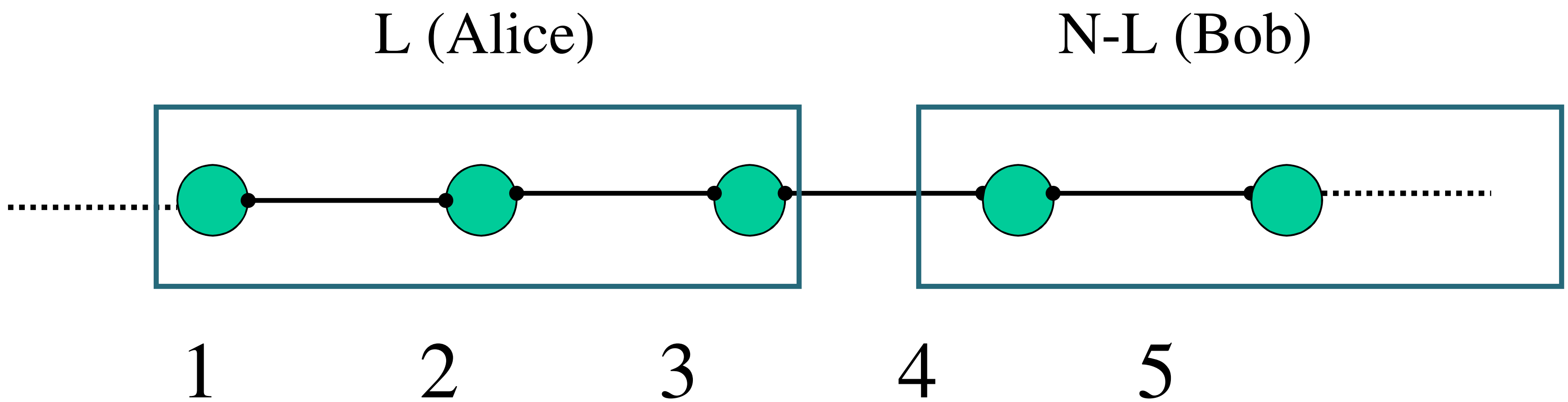


Block entanglement: $E(|\Psi\rangle_{L:N-L})$

$$\boxed{E(|\Psi\rangle_{L:N-L}) = S(\rho_L) \sim \ln L}$$

at criticality

Area Law:1D



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at cr

log divergence

Lot of progress in different directions.

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A case study:

Frustrated systems

What is frustration?

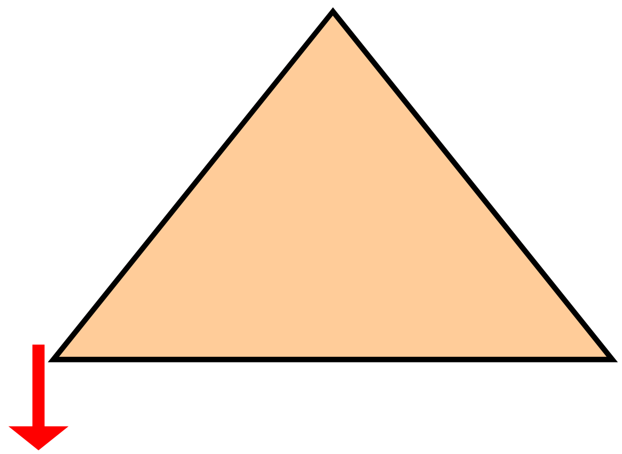
Definition

&

Quantification

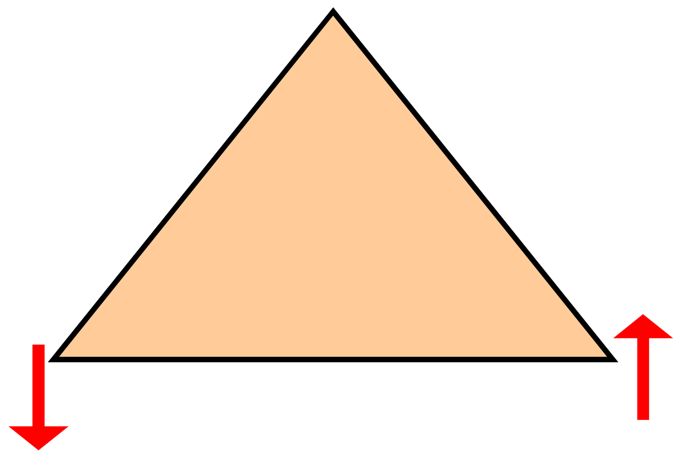
Frustration

Ising model: $H=J\,\Sigma\sigma^z_i\,\sigma^z_j$ with $J>0$



Frustration

Ising model: $H = J \sum \sigma^z_i \sigma^z_j$ with $J > 0$



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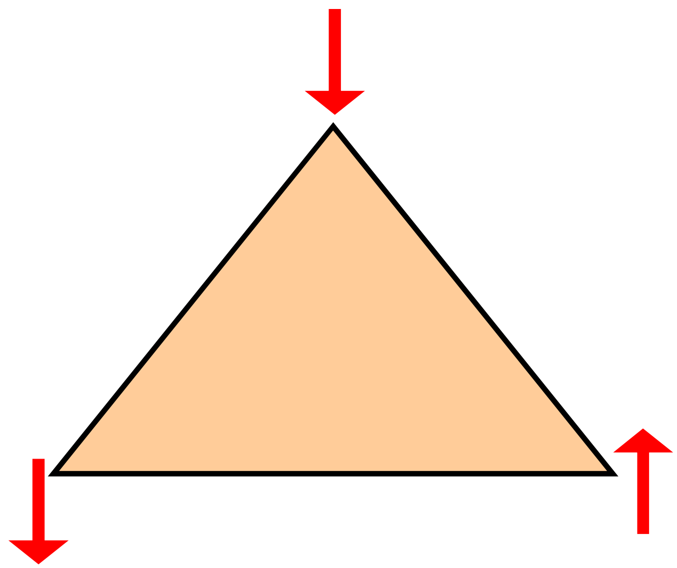

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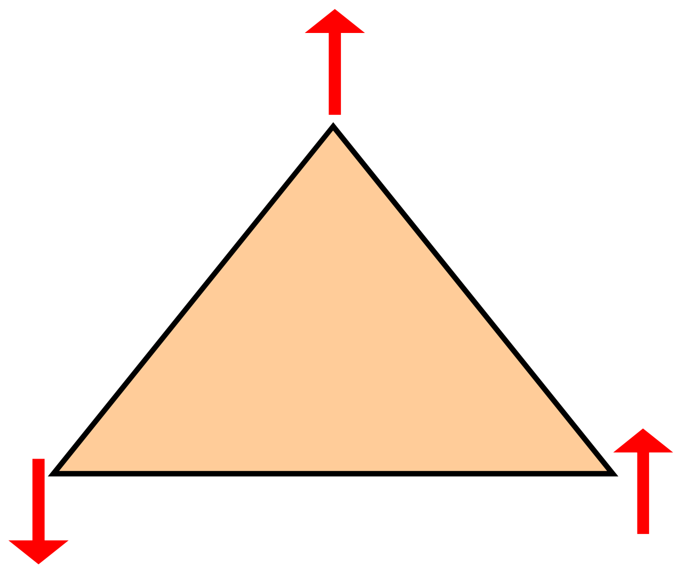

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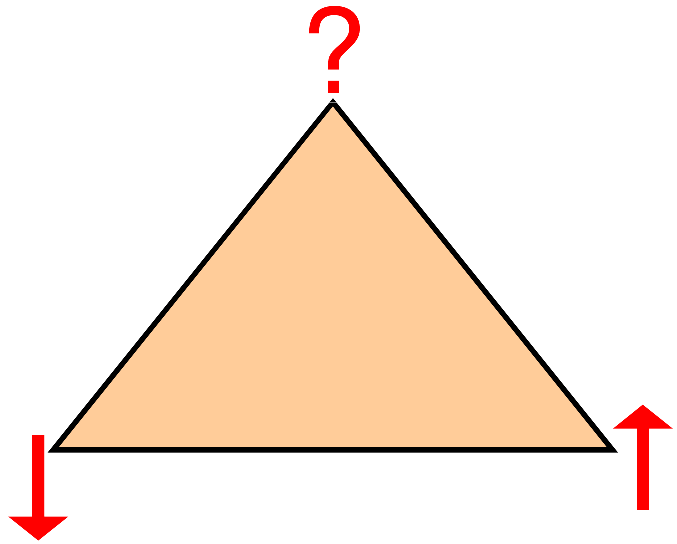

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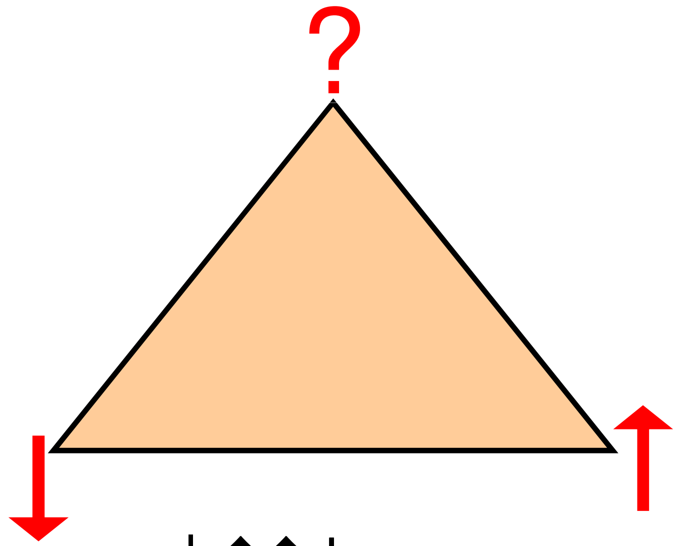

Ground states: $|\uparrow\uparrow\downarrow\rangle$,

$|\uparrow\downarrow\uparrow\rangle$,

$|\downarrow\uparrow\downarrow\rangle$, ...

Frustration degree

- Given H, $|\Gamma\rangle$,

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  replace one-body, two-body etc. in H by Ising ones,

  i.e. by σ^z_i or $\sigma^z_i\sigma^z_j$ etc.

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$$H^I = \sum_k \overbrace{H^k_f}^{\text{Frustrated part}} + \sum_l \overbrace{H^l_{nf}}^{\text{Non-Frustrated part}}$$

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$$\boxed{\Phi = \text{avg}\ \frac{\sum_k \langle\Gamma|H^k_f|\Gamma\rangle}{\sum_l |\langle\Gamma|H^l_{nf}|\Gamma\rangle|}}$$

Frustration

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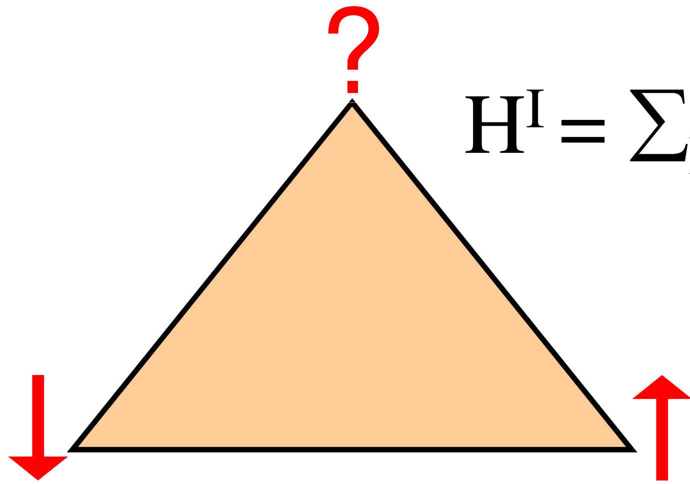

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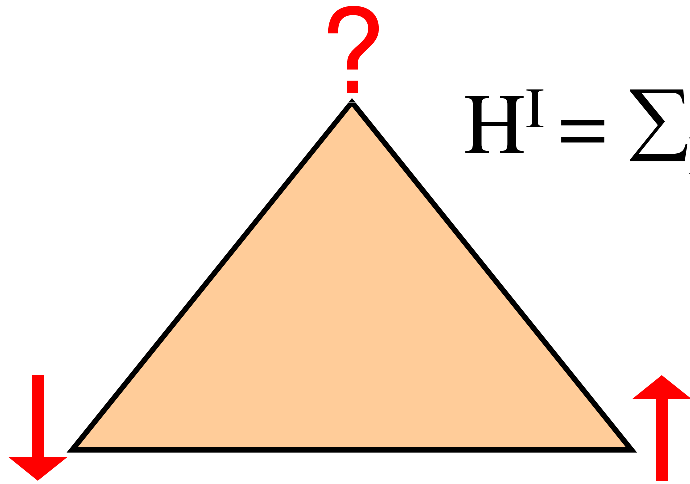

$$\sigma^z_1 \sigma^z_2 | \uparrow\uparrow\downarrow\rangle \rightarrow 1$$

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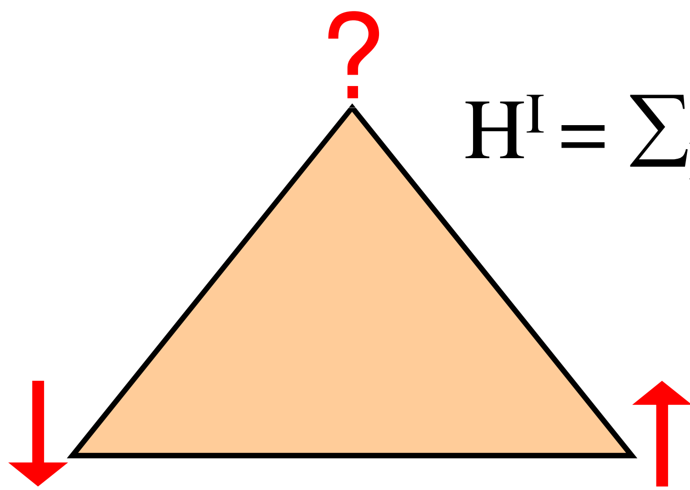

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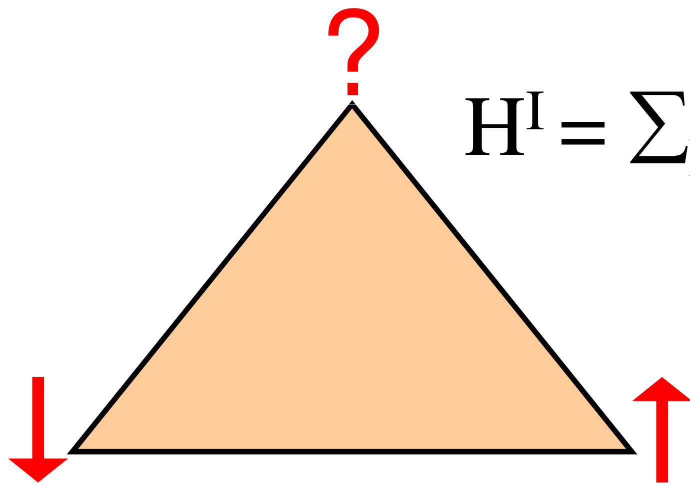

$$\sum_k \langle\Gamma|H^k_f|\Gamma\rangle \quad \sigma^z_1 \sigma^z_2 |\uparrow\uparrow\downarrow\rangle \to 1$$

Frustration

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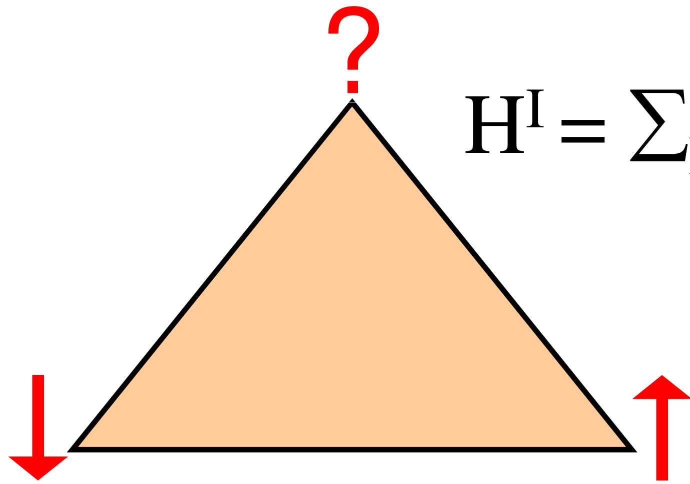

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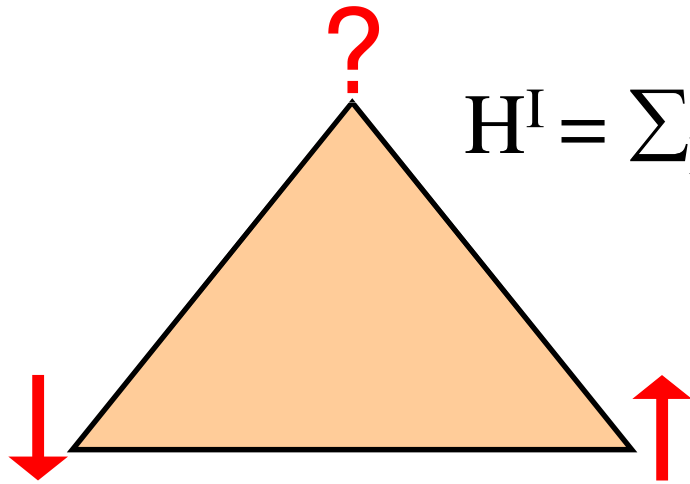

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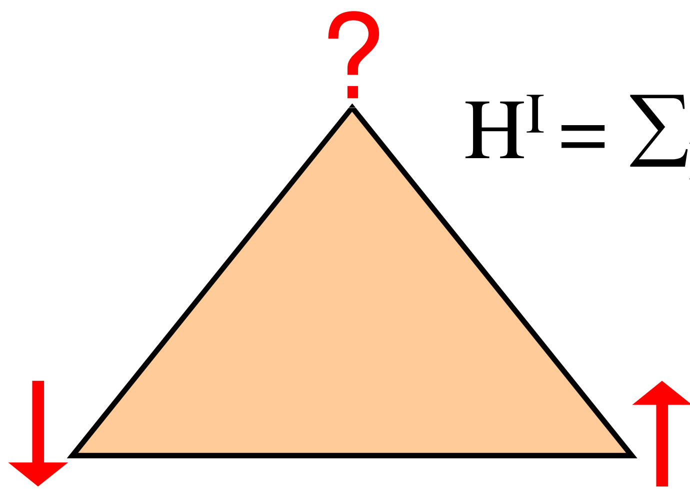

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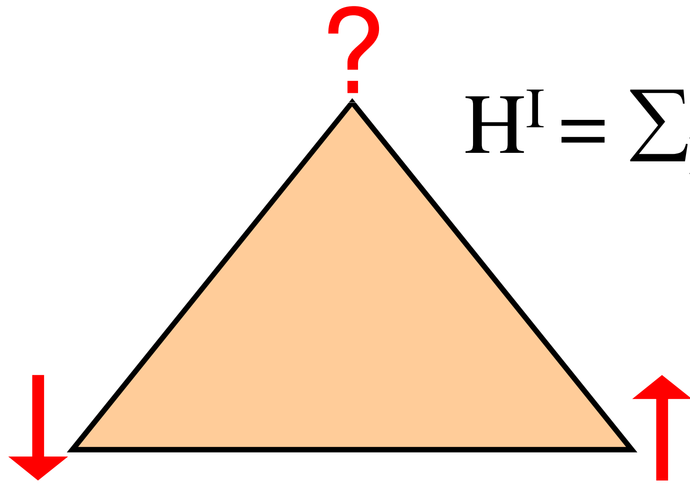

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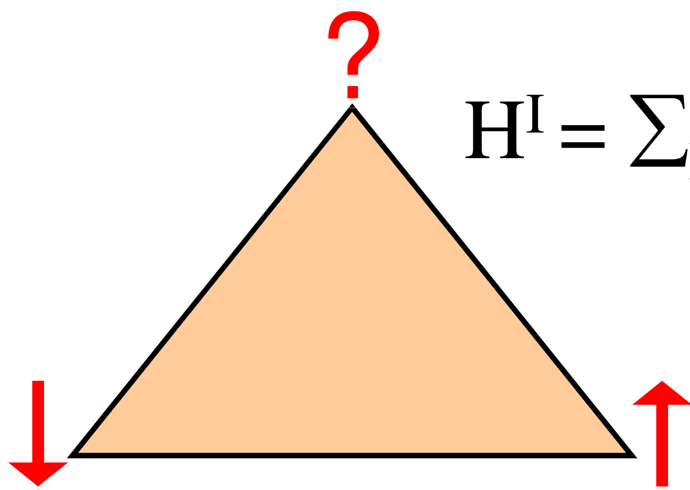

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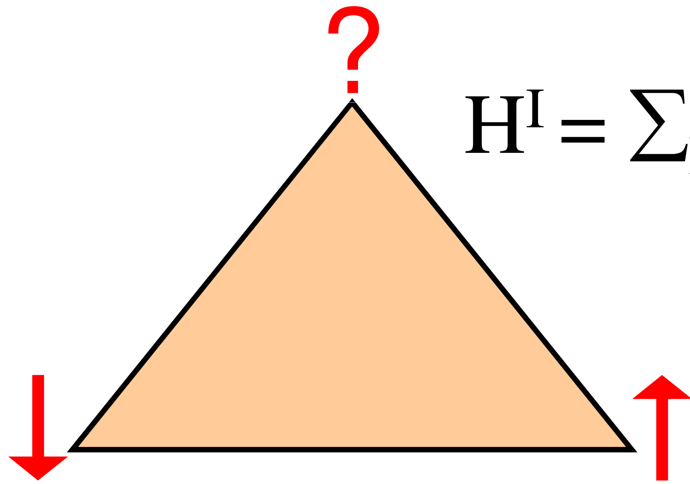

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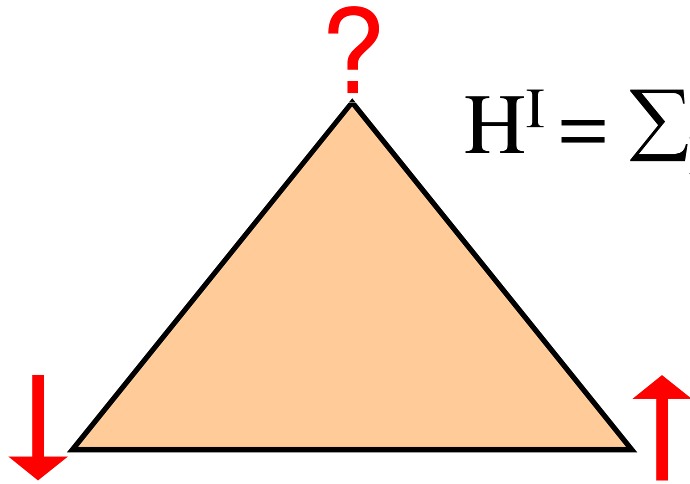

$$H^I = \sum_k H^k_f + \sum_l H^l_{nf}$$

$$\Phi = 1/2$$

$$\sum_l |\langle \Gamma | H^l_{nf} | \Gamma \rangle|$$

Cooling/Quenching Method

- Initial state:

$$\boxed{|\Phi\rangle_{\mathbf{in}} \equiv |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes |\psi\rangle_3 \otimes \ldots \otimes |\psi\rangle_N}$$

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- Project $|\Phi\rangle_{in}$ onto the ground state space of the model.

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- Maximize $E_{N/2:N/2}(|\Phi\rangle_f)$ over all choices of the initial state.

Main Thesis

- Highly frustrated systems do not follow any area law

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while

- Weakly frustrated systems follow the same area law as nonfrustrated systems away from criticality.

A. Sen(De), US, J. Dziarmaga, A. Sanpera, M. Lewenstein, PRL'08

Area Law
for frustrated systems

1. *Long range Ising model*

2. *Majumdar Ghosh model*

3. *Shastry-Sutherland model*

4. *Ising chain with NN interactions*

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$$H = J\,\Sigma\sigma^{z}_{i}\sigma^{z}_{j} \quad \text{with } J>0$$

$$\boxed{\Phi \approx 1}$$

Long range Ising model

$H = J \sum \sigma^z_i \sigma^z_j$ with $J > 0$

After quenching:

$\vert\psi\rangle$ = superposition of all vectors with $m \vert 0\rangle$s and $m \vert 1\rangle$s

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$H = J \sum \sigma^z_i \sigma^z_j$ with $J>0$

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$E_{k:2m-k} = \frac{1}{2} \log k$

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log divergence

Area law

Clear departure from area law

- Long range Ising model: “Infinite” dimensions

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Note: Effect due to frustration.

Not due to long-range interactions.

Area law

Clear departure from area law

- Long range Ising model: “Infinite” dimensions

- Possible area law: $k^{1-1/d}$ with $d \rightarrow \infty$

Note: Effect due to frustration.
Not due to long-range interactions.
Ising with J<0 : constant block entanglement.

Area Law
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Majumdar-Ghosh model

$H = J_1 \Sigma \sigma_i \sigma_{i+1} + J_2 \Sigma \sigma_i \sigma_{i+2}$ with $J_1, J_2 > 0$; $J_2 = J_1/2$

Majumdar-Ghosh model

$H = J_1 \Sigma \sigma_i \sigma_{i+1} + J_2 \Sigma \sigma_i \sigma_{i+2}$

$\boxed{\Phi \approx 1/2}$

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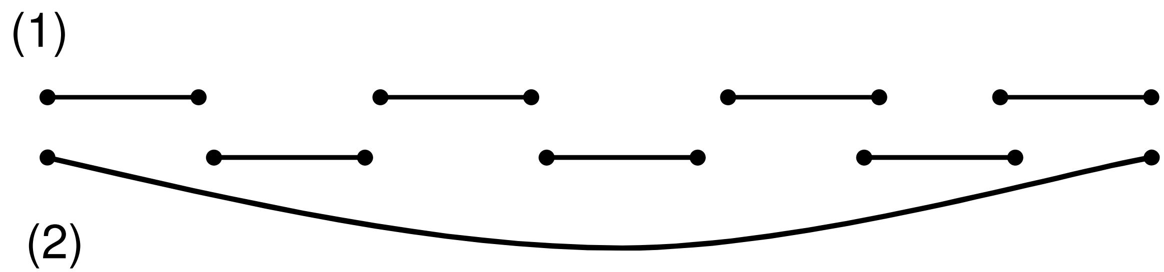
Ground state:



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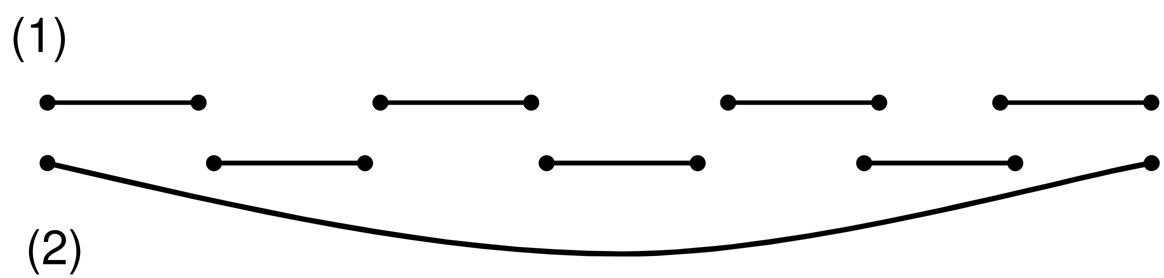

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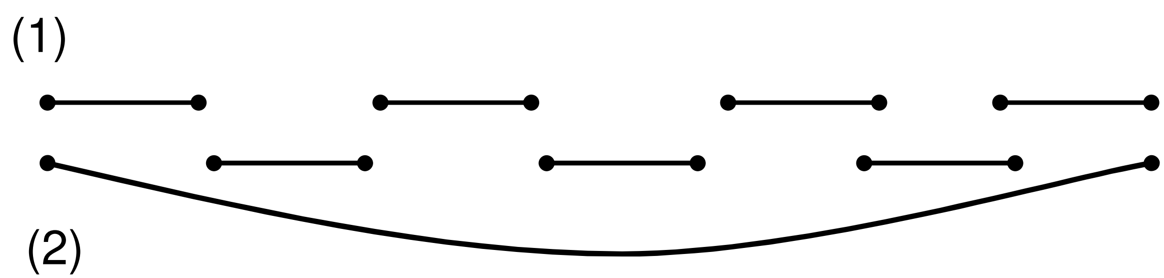

After quenching:

$E \geq 2$ (even) or 1 (odd)

Majumdar-Ghosh model

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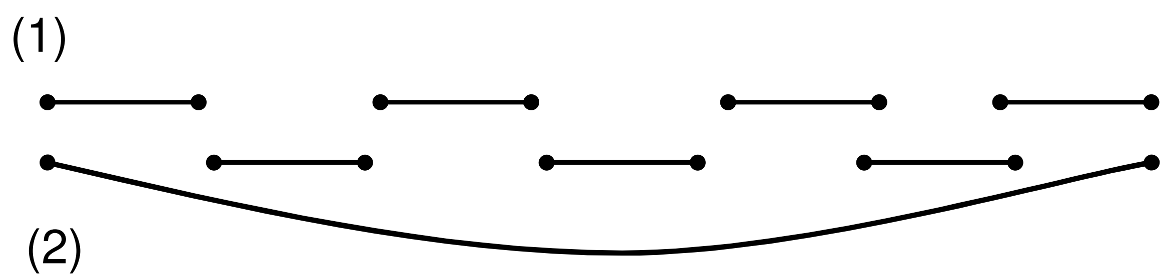

After quenching:

$E \geq 2$ (even) or 1 (odd)

$E \leq \log 5$ (even) or $\log 3$ (odd)

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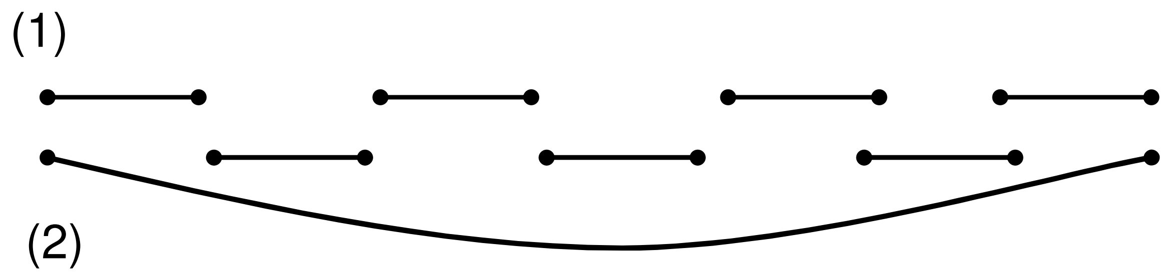

After quenching:

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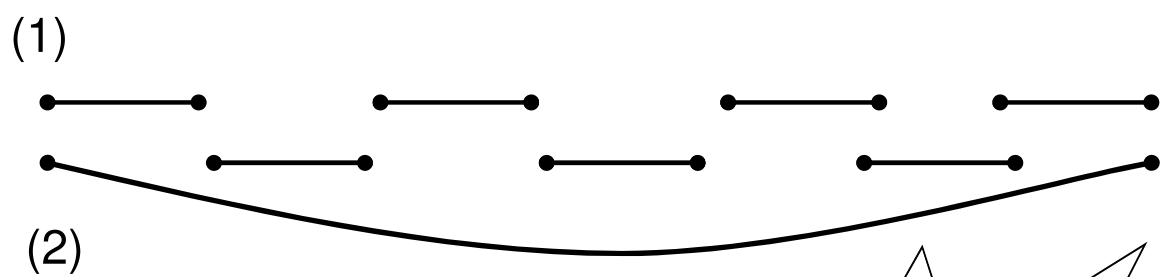
Numerically, $E = 2.3$ for 8 spins

Majumdar-Ghosh model

$H = J_1 \sum \sigma_i \sigma_{i+1} + J_2 \sum \sigma_i \sigma_{i+2}$ with $J_1, J_2 > 0$; $J_2 = J_1/2$

Ground state:



After quenching:

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$E \leq \log 5$ (ev… …l)

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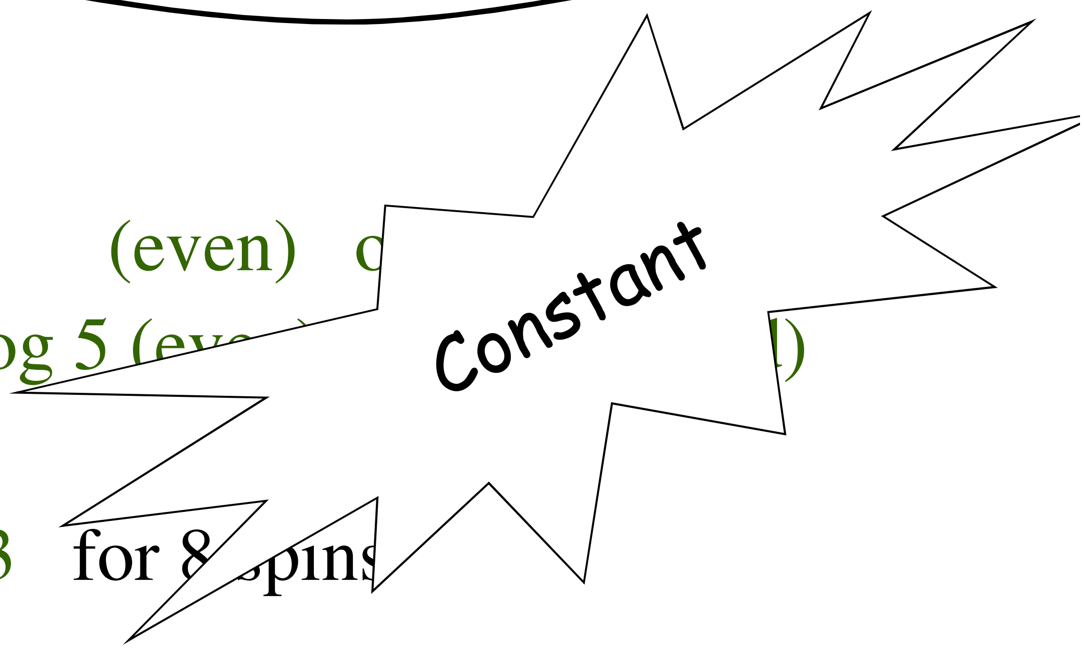

Area Law
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Shastry-Sutherland model

$$H=J \sum_{\langle ij\rangle} \sigma_i \sigma_j + J_d \sum_{\langle ij\rangle_d} \sigma_i \sigma_j \quad \text{with} \quad J_d > 2J$$

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$$\boxed{\Phi < 1/2}$$

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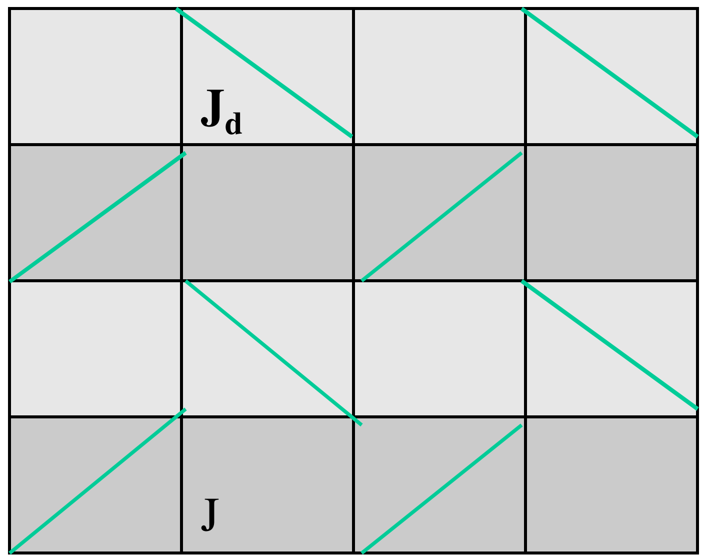

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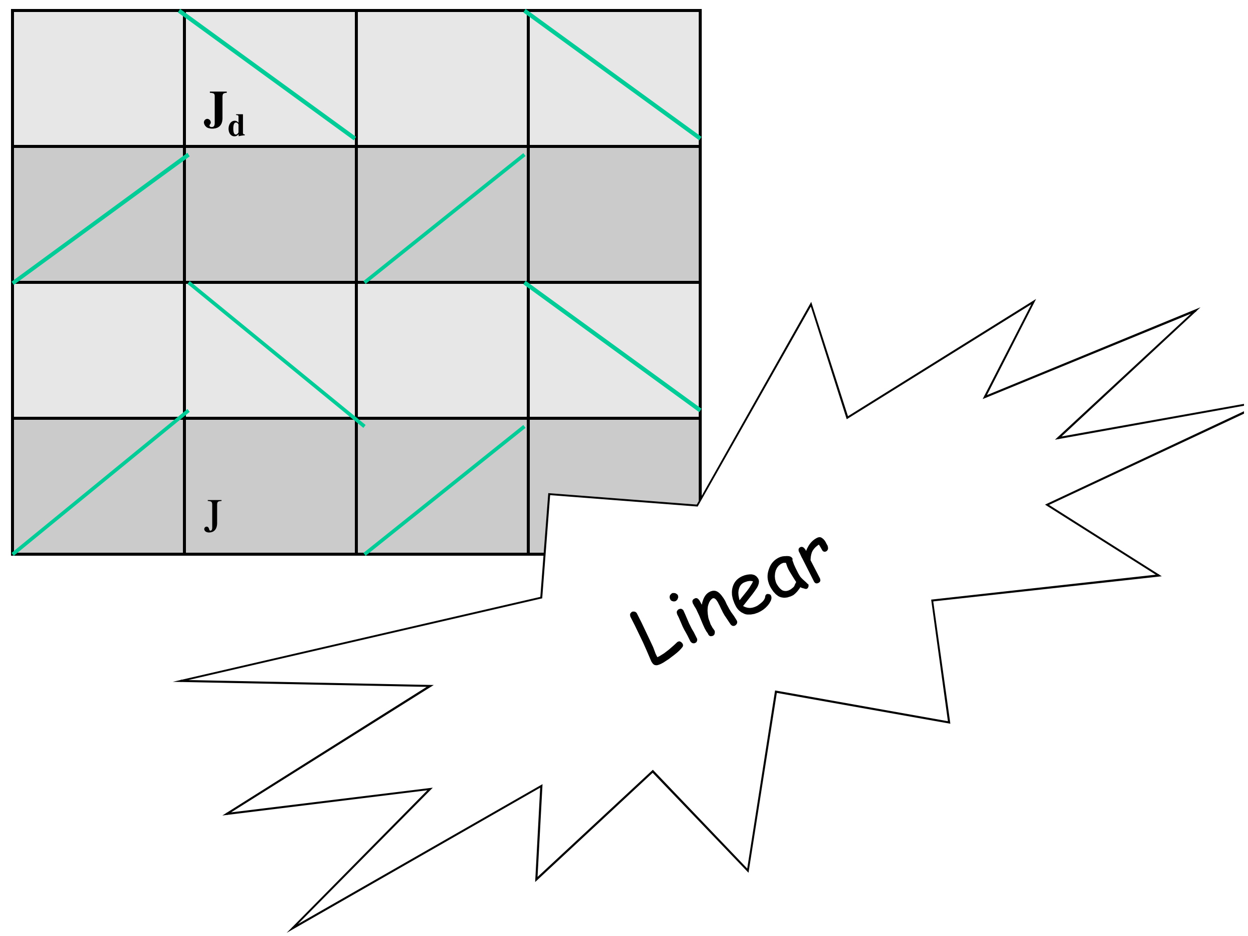

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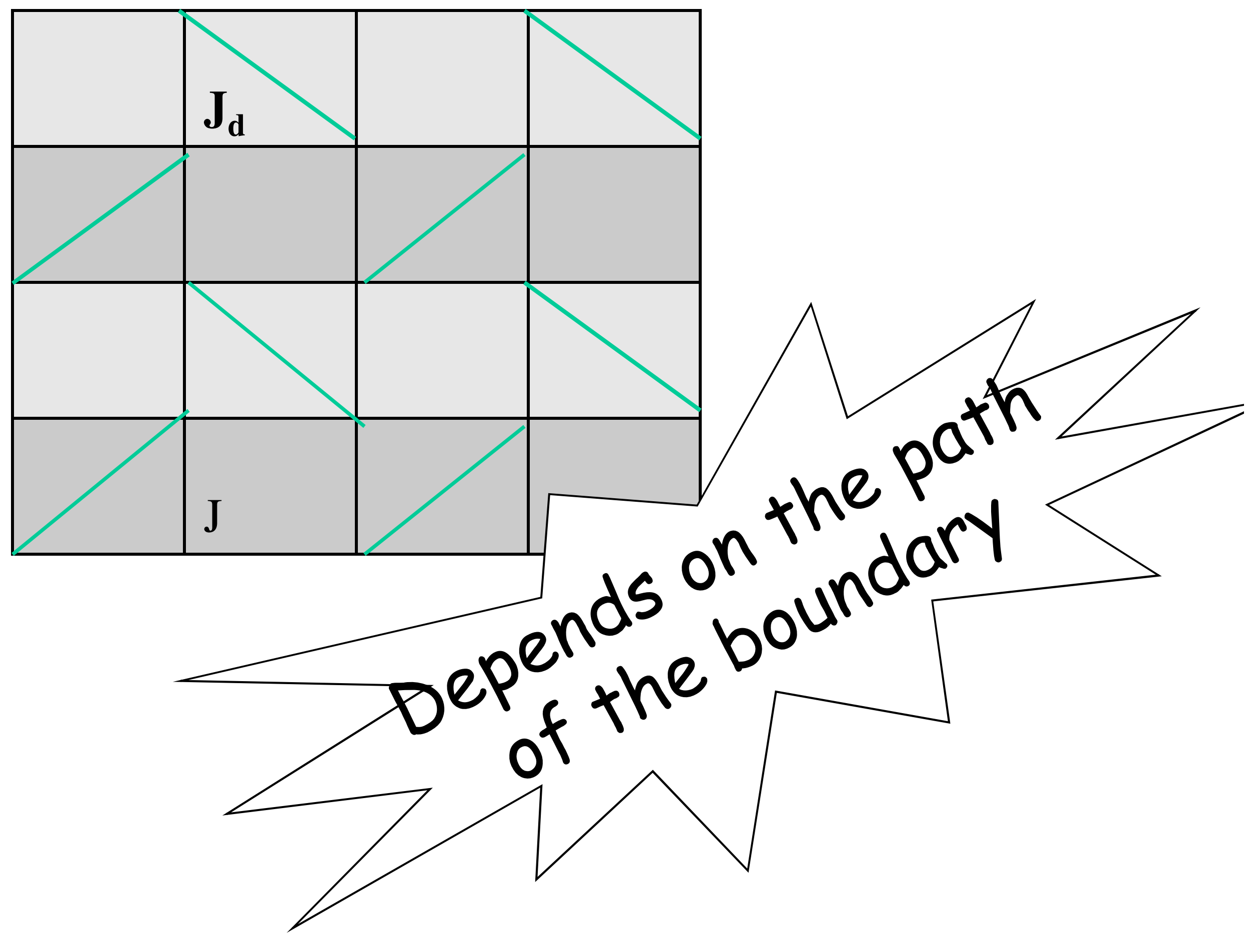

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Ising with NN

$H = J \sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j$ with all except one are negative.

Ising with NN

$$H = J \sum_{<ij>} \sigma^z_i \sigma^z_j$$ with all except one are negative.

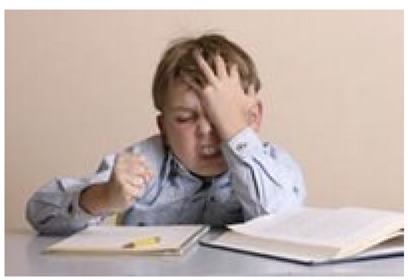
$$E_{k:2m-k} = \text{constant}$$

Area Law for frustrated systems

NO Area law

Area Law for frustrated systems

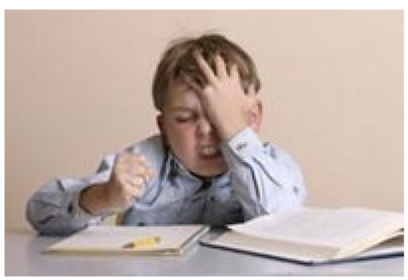
NO Area law



Area law

Area Law for frustrated systems

NO Area law

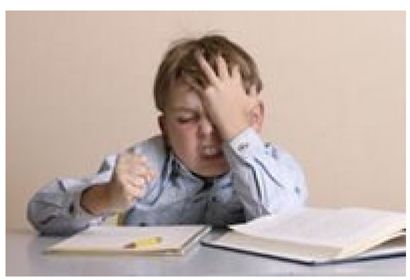


Area law

- Cooling technique

Area Law for frustrated systems

NO Area law



Area law

- Cooling technique
- Frustration degree

More work done

More work done

- Adv. Phys. **56**, 243 (2007)
- Rev. Mod. Phys. **80**, 517 (2008)

More work done

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And much more left …


Thank you!

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