Quantum Correlations in Many-Body Systems

ISCQI – 2011, IOP Bhubaneswar

Ujjwal Sen
HRI, Allahabad

Quantum Correlations in Many-Body Systems

2nd half

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Outline

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1. Beyond bipartite entanglement

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1. Beyond bipartite entanglement

2. Beyond entanglement

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1. Beyond bipartite entanglement

2. Beyond entanglement (but within QIC)

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3. Dynamics of entanglement

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1. Beyond bipartite entanglement

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Outline

1. Multipartite entanglement

2. Beyond entanglement (but within QIC)

3. Dynamics of entanglement

Multiparty entanglement

- Many notions available.

Multiparty entanglement

- Many ***notions*** available.
- However, not all r computable.

Multiparty entanglement

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a. Geometric measure

Multiparty entanglement

- Many ***notions*** available.
- However, not all r computable.

a. Geometric measure

Wei, Goldbart, PRA'03
Balsone, DellAnno, DeSiene, Illuminatti, PRA'08
+ ……

Multiparty entanglement

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- However, not all r computable.

a. Geometric measure
b. Global measure

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Meyer, Wallach, JMP'02

+ ……

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c. Generalized geometric measure

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A. Sen(De), US, PRA'10

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Geometric measure

$|\mathbf{s}\rangle$ → quantum state of two parties

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$|\mathbf{a}\rangle\, |\mathbf{b}\rangle$ → a product state of the two parties

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1 – sq of mod of inner product is a “distance”

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1 – sq of mod of inner product is a "distance"

Minimize that distance over all prod states

Geometric measure

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Minimize that distance over all prod states

This minimum distance quantifies entanglement.

Geometric measure

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$|\mathbf{s}\rangle \rightarrow$ quan

$|\mathbf{a}\rangle\ |\mathbf{b}\rangle$. parties

Minimization usually difficult.

1 – sq c stance”

Minimize that prod states

This minimum distance quantifies multisite entanglement.

Geometric measure

|**s**⟩ → quan

|**a**⟩ |**b**⟩ . parties

1 – sq d stance"

Minimize that prod states

Minimization usually difficult.
But possible sometimes by
using symmetries, …

This minimum distance quantifies multisite entanglement.

Geometric measure

$|\mathbf{s}\rangle \rightarrow$ quan

Sometimes differently normalized …

Minimize that

prod states

This minimum distance quantifies multisite entanglement.

Geometric measure

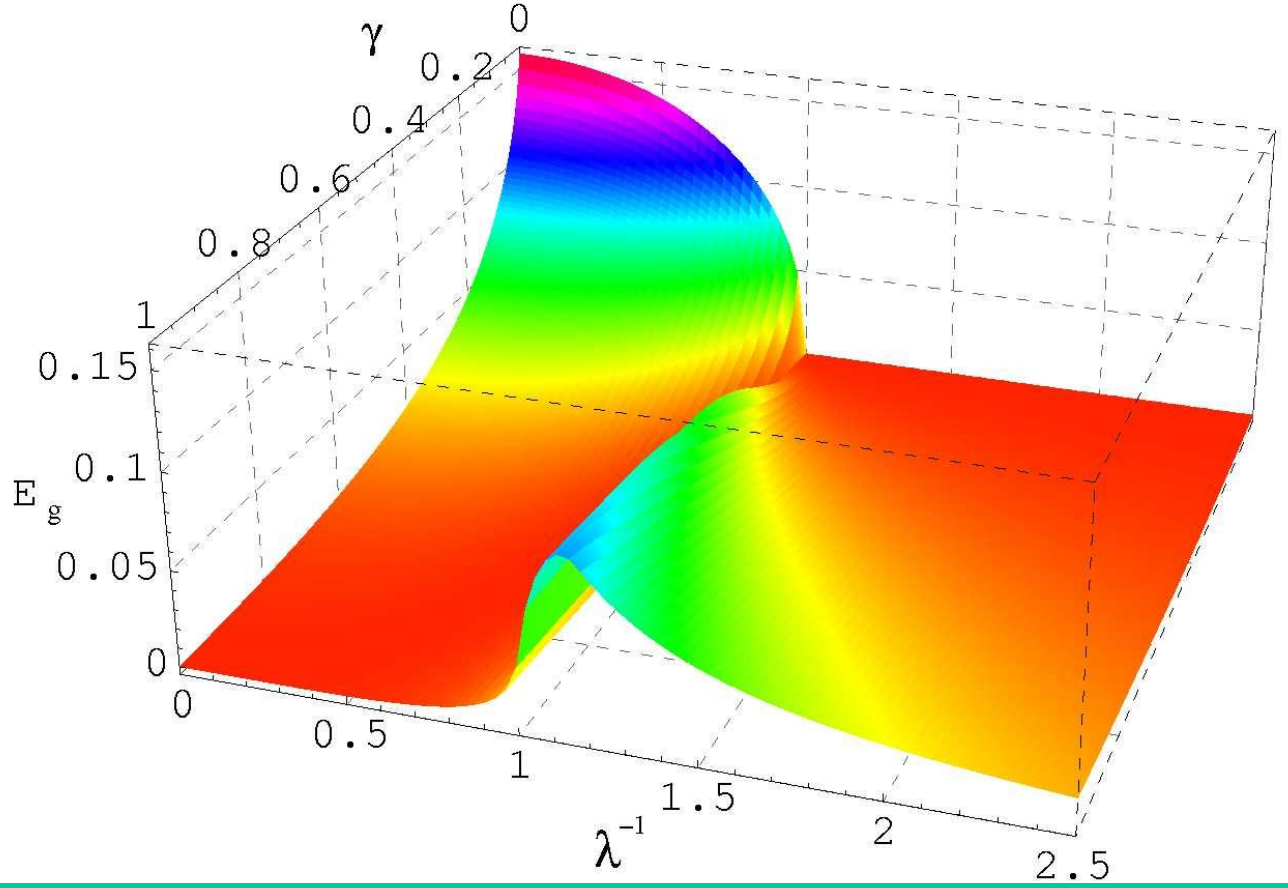
|s⟩ → quan

Can be generalized to mixed states.
Computation even more difficult.

Minimize that prod states

This minimum distance quantifies multisite entanglement.

Multisite entanglement detects QPT



Wei, Das, Mukhopadhyay, Vishveshwara, Goldbart, PRA'05

Multiparty entanglement

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a. Geometric measure
b. Global measure
c. Generalized geometric measure

Global measure

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Average local entropy

Global measure

$|\mathbf{s}\rangle$ → quantum state of two parties

Average local entropy

1 - Average of local purities

Global measure

$|s\rangle$ → quantum state of two parties

Average local entropy

1 - Average of local purities

local purity = trace of sq of local density

Global measure

$|\mathbf{s}\rangle$ → quantum state of two parties

Average local entropy

1 - Average of local purities

local purity = trace of sq of local density

This average linearized entropy quantifies entanglement.

Global measure

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But now more options! Many local purities!

Global measure

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Average local entropy

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types of

But now more options! Many local purities!

Global measure

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1 - Average of local purities

local purity = trace of sq of local density

But now more options! Many local purities!

E.g., G(2,3) = 1 – av. of local purities of 2-site densities that r 3-1 spins apart).

Global measure

$|s\rangle$ → quantum state of n parties

Average local entropy

1 - Average of local purities

local purity = trace of sq of local density

But now more options! Many local purities!

E.g., G(2,3) = 1 – av. of local purities of 2-site densities that r 3-1 spins apart).

3rd neighbor

Global measure

|s⟩ → quantur

Average local

1 - Average o

local purity =

Similarly,
G(3,n,m), etc.

But now more options! Many local purities!

E.g., G(2,3) = 1 – av. of local purities of 2-site densities that r 3-1 spins apart).

3rd neighbor

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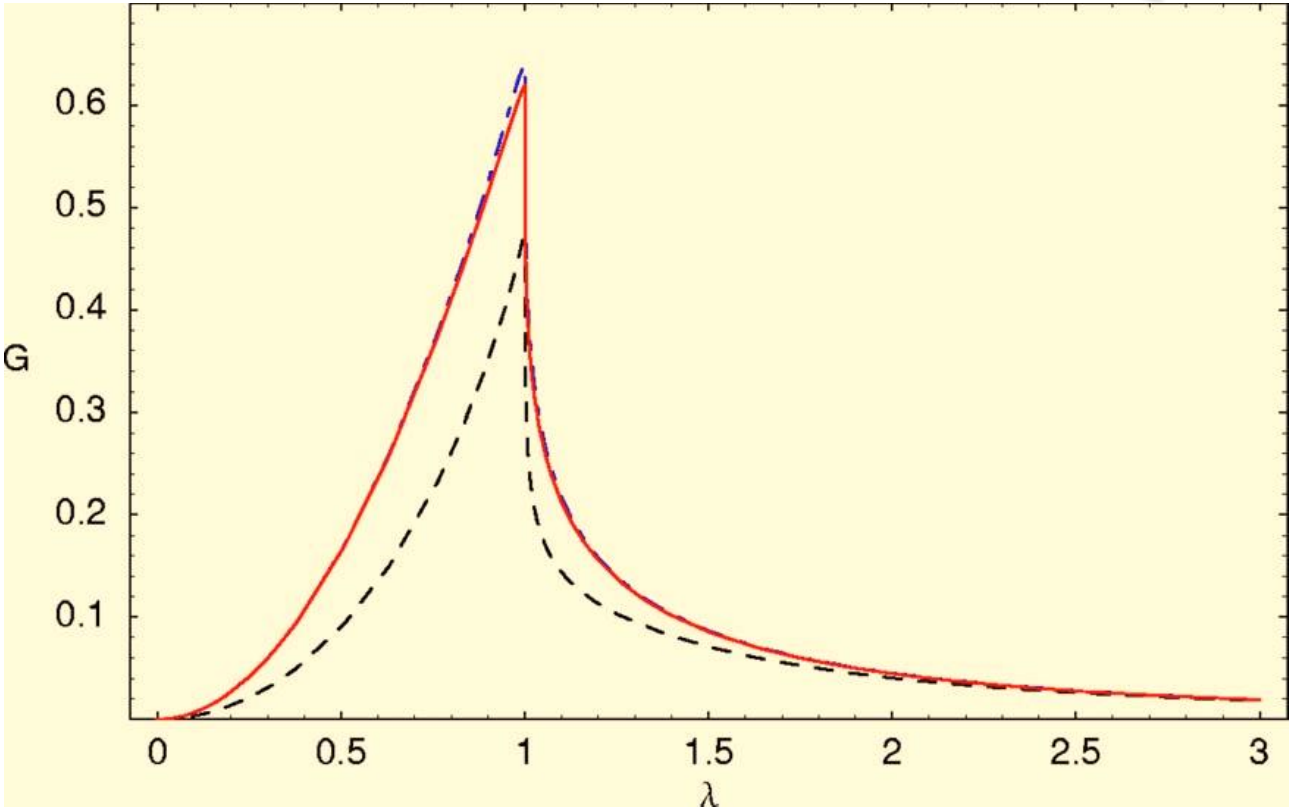
local purity = trace of sq of local density

But now more options! Many local purities!

E.g., G(2,3) = 1 – av. of local purities of 2-site densities that r 3-1 spins apart).

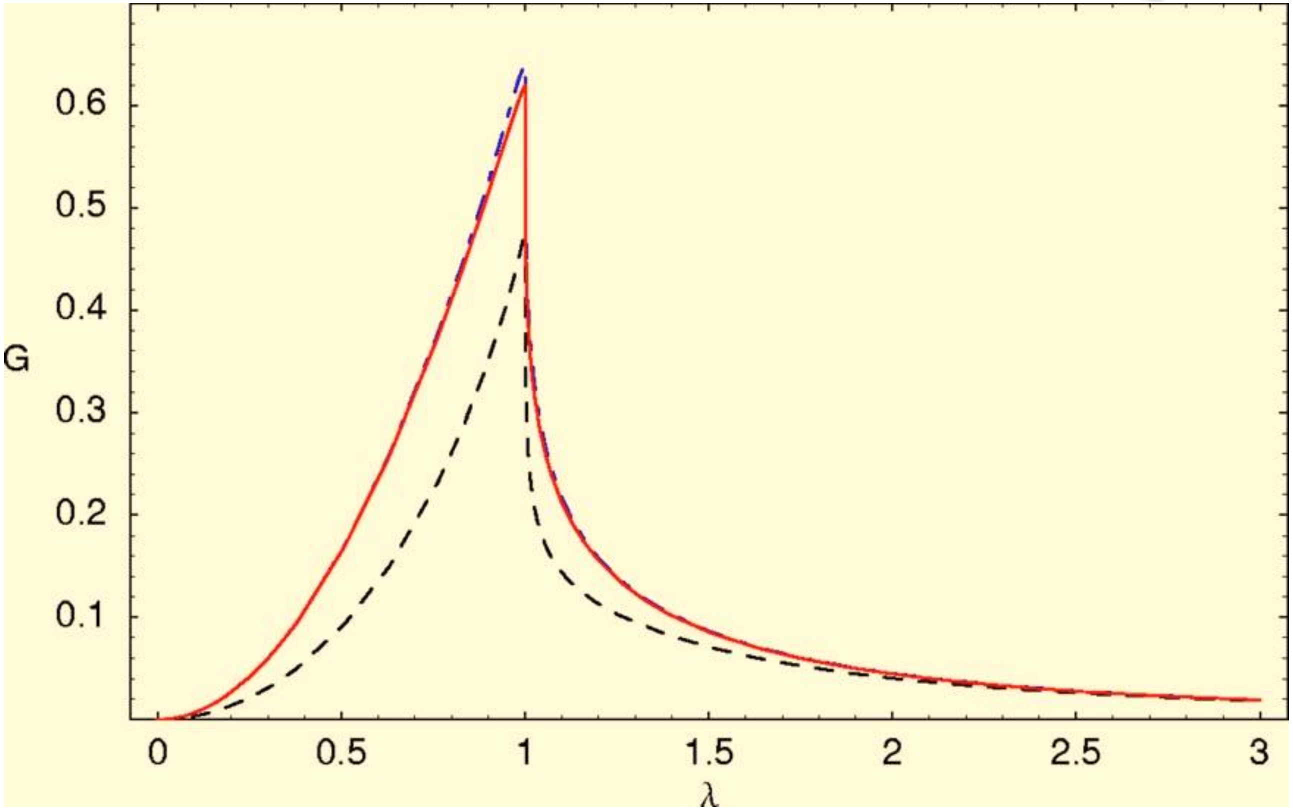
Bipartite case: Only G(1,1) exists.

Global measure detects QPT



deOliviera, Rigolin, deOliviera, PRA'06

Global measure detects QPT



Dashed black = G(2,1)
Solid red = G(2,15)
Dotted dashed blue = av of the G(2,L)

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Generalized

Geometric measure

Generalized

Geometric measure

^

$|s\rangle$ → quantum state of n parties

Generalized

Geometric measure

|**s**⟩ → quantum state of n parties

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Generalized

Geometric measure

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1 – sq of mod of inner product is a "distance"

"genuine"

Minimize that distance over all

This minimum distance quantifies multisite entanglement.

Generalized

Geometric measure

rties

te of the n parties

uct is a "distance"

Minimize that distance over all

This minimum distance quantifies multisite entanglement.

Efficiently computable for arbitrary pure states of arb # of parties and arb dimensions

"genuine"

Generalized

Geometric measure

For any pure state, we get
a single real number that quantifies
genuine multisite entanglement.

n parties

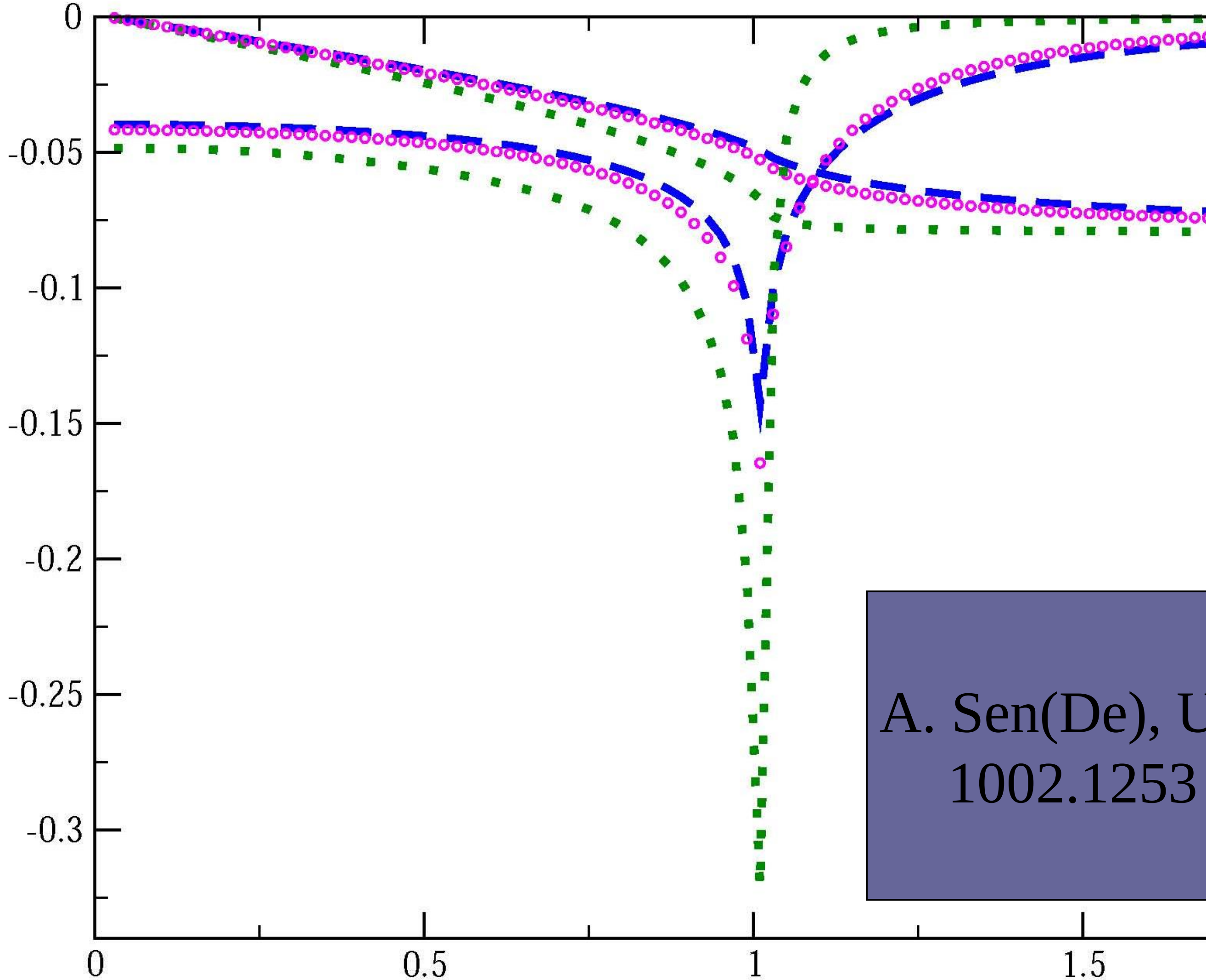
"distance"

"genuine"

Minimize that distance over all

This minimum distance quantifies multisite entanglement.

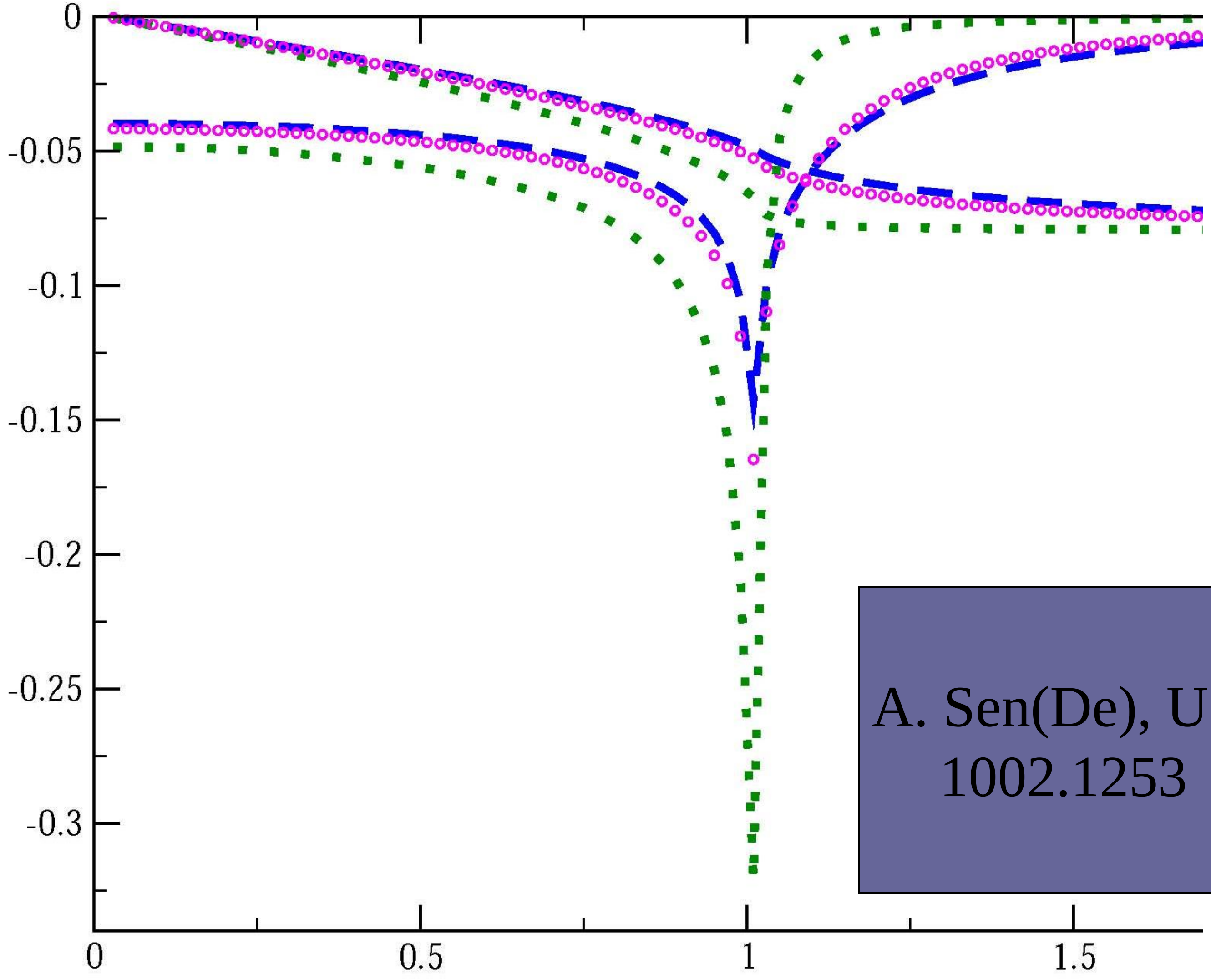
GGM detects QPT



A. Sen(De), US, 1002.1253

GGM –1/2

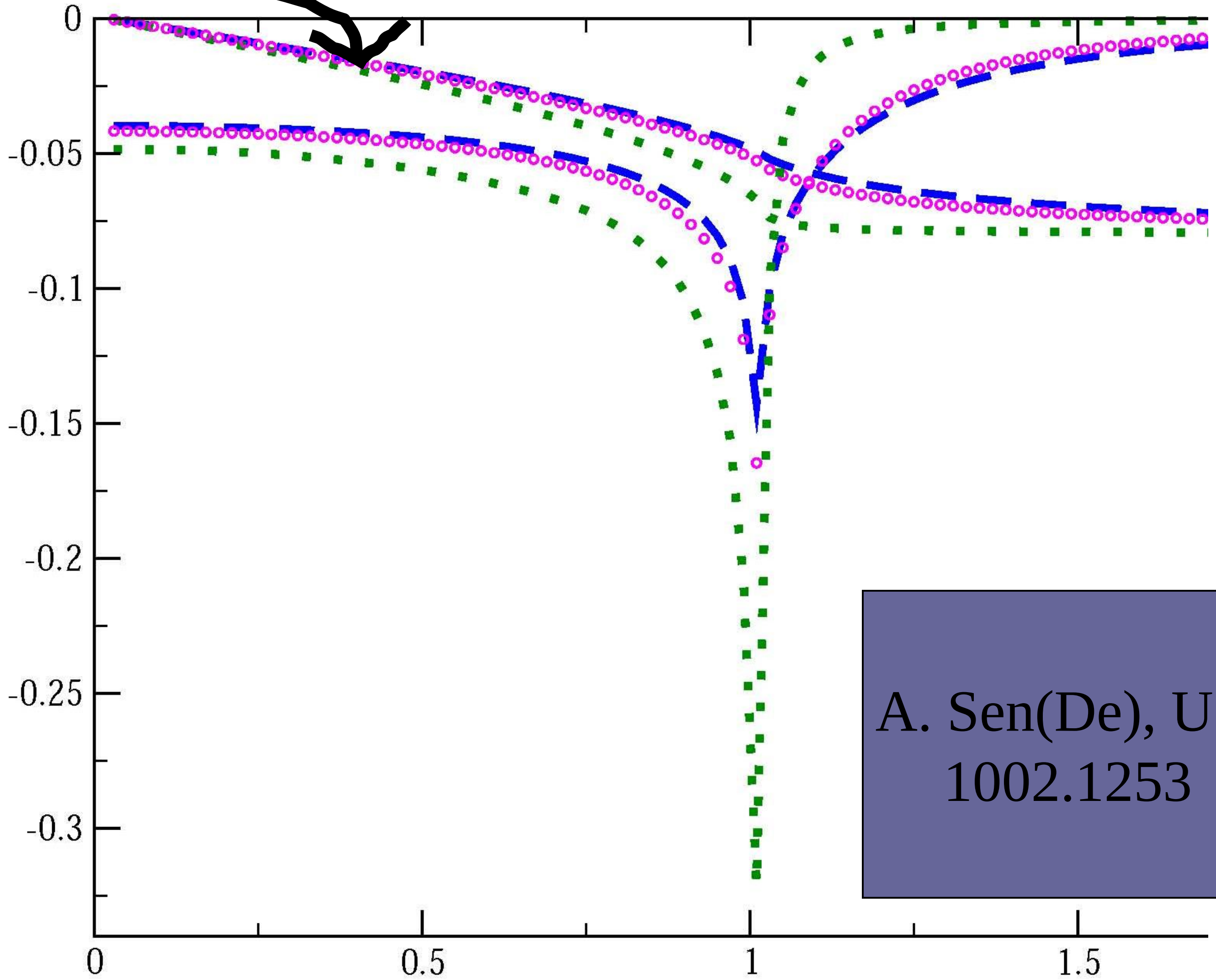
GGM detects QPT



A. Sen(De), US, 1002.1253

GGM detects QPT

GGM –1/2



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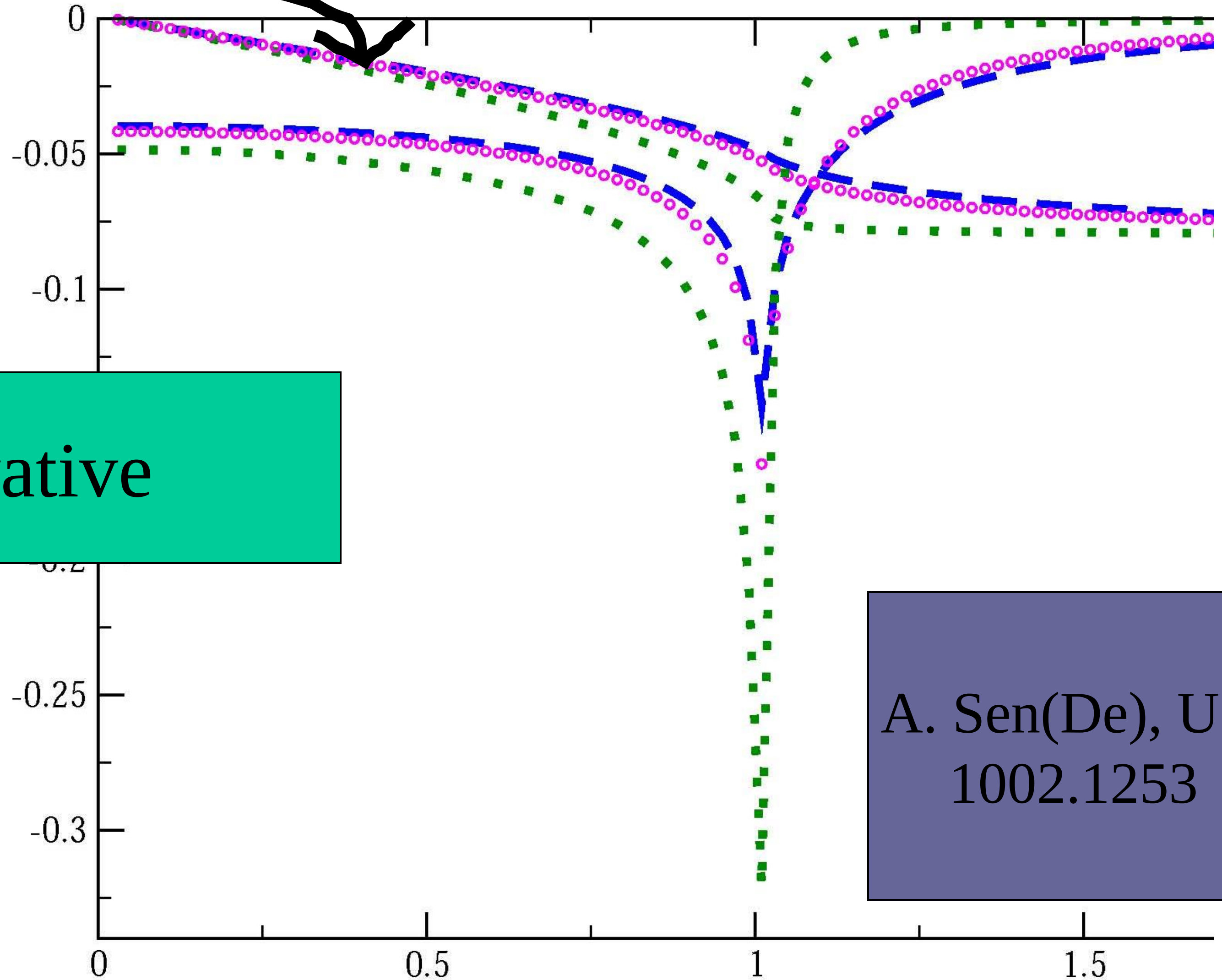

derivative

A. Sen(De), US, 1002.1253

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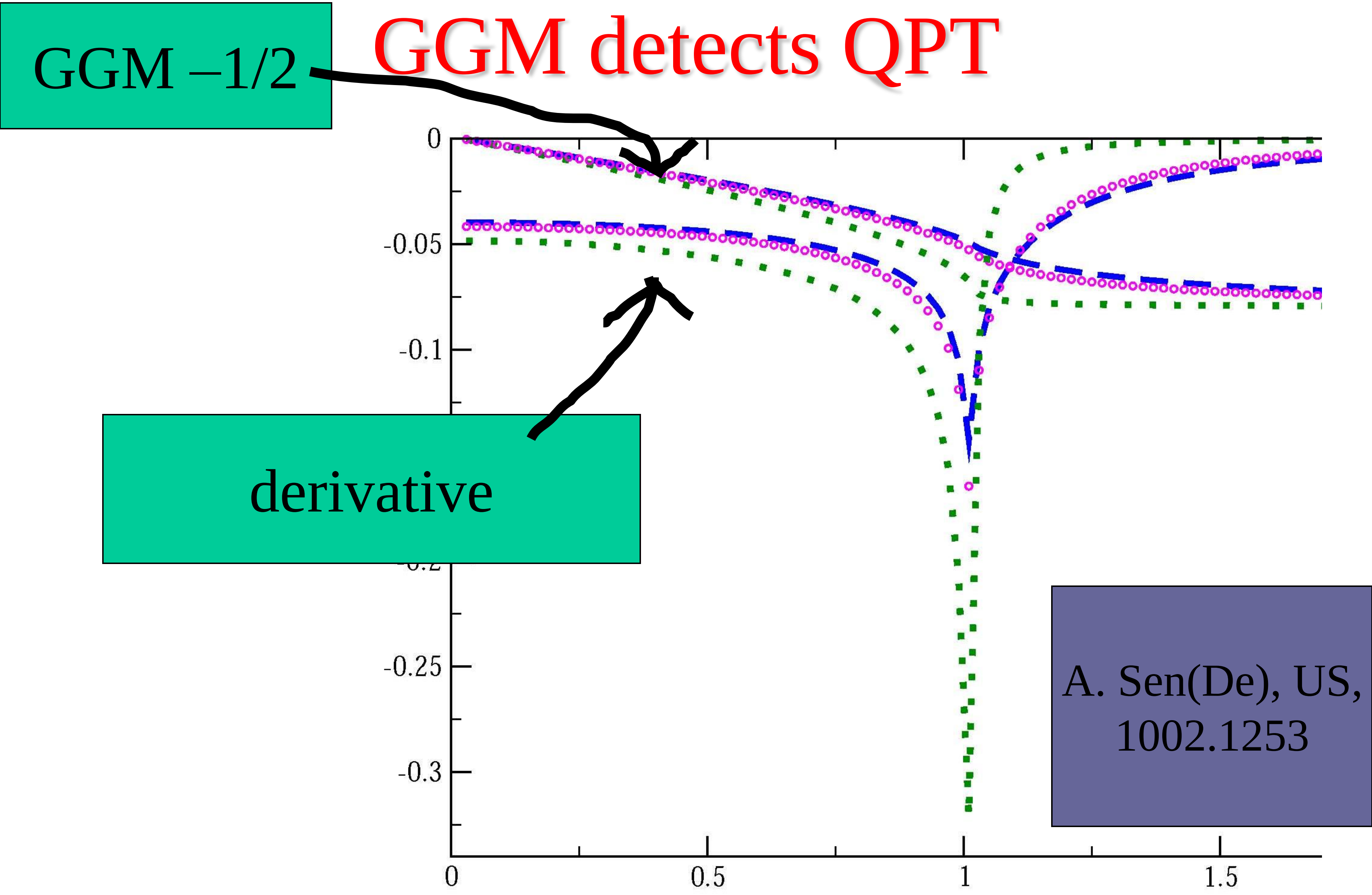

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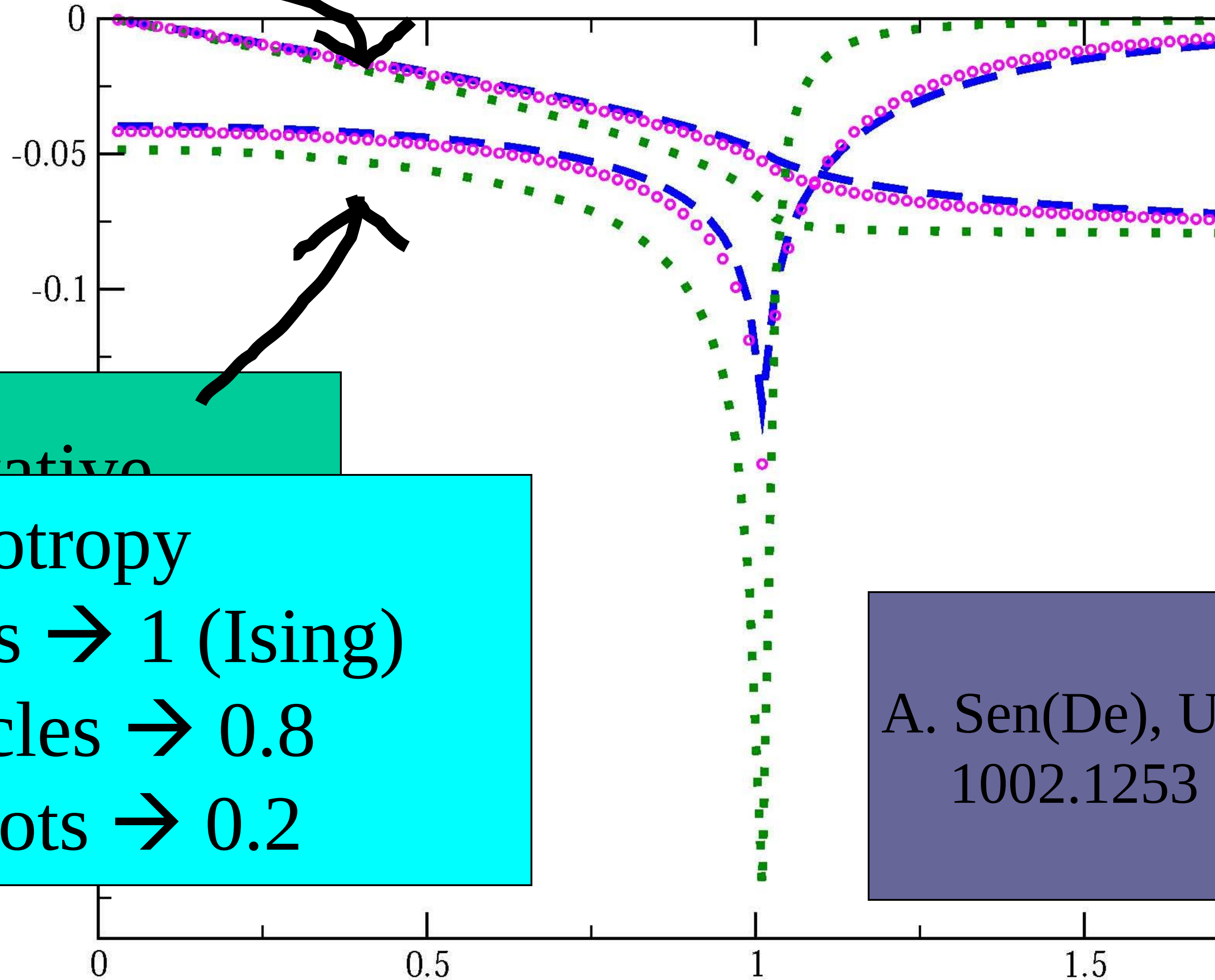
GGM –1/2



derivative

anisotropy
Blue dashes → 1 (Ising)
Pink circles → 0.8
Green dots → 0.2

A. Sen(De), US, 1002.1253

Outline

1. Beyond bipartite entanglement

2. Beyond entanglement (but within QIC)

3. Dynamics of entanglement

R there q correlations not explained by entanglement?

R there q correlations not explained by entanglement?

Is there basis for such a query?

R there q correlations not explained by entanglement?

Is there basis for such a query?

Many!

R there q correlations not explained by entanglement?

Is there basis for such a query?

Many!

E.g. →

Distinguishable states = Orthogonal vectors

Physics

Mathematics

True only when all operations r allowed.

Distinguishable states = Orthogonal vectors

Physics

Mathematics

Not so in general!

In particular, expected that distinguishing entangled states will be difficult with LOCC.

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However …

"Quantum Nonlocality without entanglement"

- Sets of orthogonal product states may be locally indistinguishable!

Bennett, DiVincenzo, Fuchs, Mor, Rains, Shor, Smolin, Wootters, PRA'99
Bennett, DiVincenzo, Mor, Shor, Smolin, Terhal, PRL'99
…

- Two orthogonal states r always distinguishable

- Two multiparty orthogonal states r always distinguishable locally

- Two multiparty orthogonal states r always distinguishable locally, irrespective of the entanglement content!

"No nonlocality in two states"

- Two multiparty orthogonal states r always distinguishable locally, irrespective of the entanglement content!

"No nonlocality in two states"

- Two multiparty orthogonal states r always distinguishable locally, irrespective of the entanglement content!

Walgate, Short, Hardy, Vedral, PRL'00

. . .

"More nonlocality with less entanglement"

- Local indistinguishability of a set of orthogonal states may increase with decrease in their entanglement content.

M. Horodecki, Sen(De), US, K. Horodecki, PRL'03

"More nonlocality with less entanglement"

We infer that there r other forms of
quantum correlations
not captured in the
entanglement-separability paradigm.

M. Horodecki, Sen(De), US, R. Horodecki, PRL'03

“More nonlocality with less entanglement”

Other indications as well!
Won’t go in those directions.

M. Horodecki, Sen(De), US, K. Horodecki, PRL’03

Around 2000 …

Hendersen, Vedral, JPhysA’01
Ollivier, Zurek, PRL’02
introduced
Quantum Discord

Around 2000 …

Around the same time,
Quantum Work Deficit
was also introduced.

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Oppenheim, MPR Horodeccy, PRL'02

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Oppenheim, MPR Horodeccy, PRL'02
MKPR Horodeccy, Oppenheim, Sen(De), US, Synak-Radtke, PRA'05

Around 2000 …

Both quantum discord and quantum work deficit used in many-body physics.

Oppenheim, MPR Horodeccy, PRL’02

MKPR Horodeccy, Oppenheim, Sen(De), US, Synak-Radtke, PRA’05

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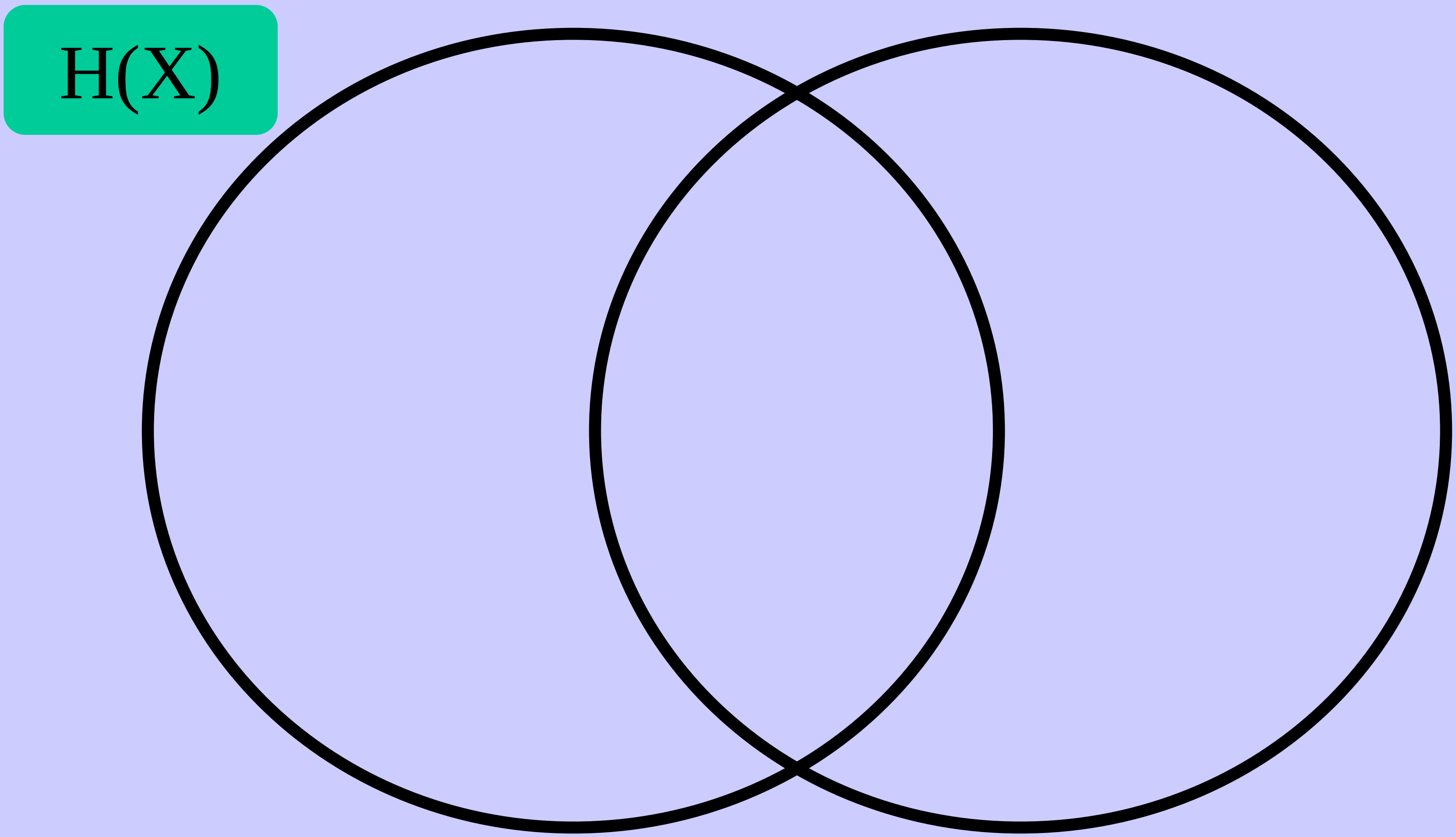
Oppenheim, MPR Horodeccy, PRL'02

MKPR Horodeccy, Oppenheim, Sen(De), US, Synak-Radtke, PRA'05

Defining quantum discord



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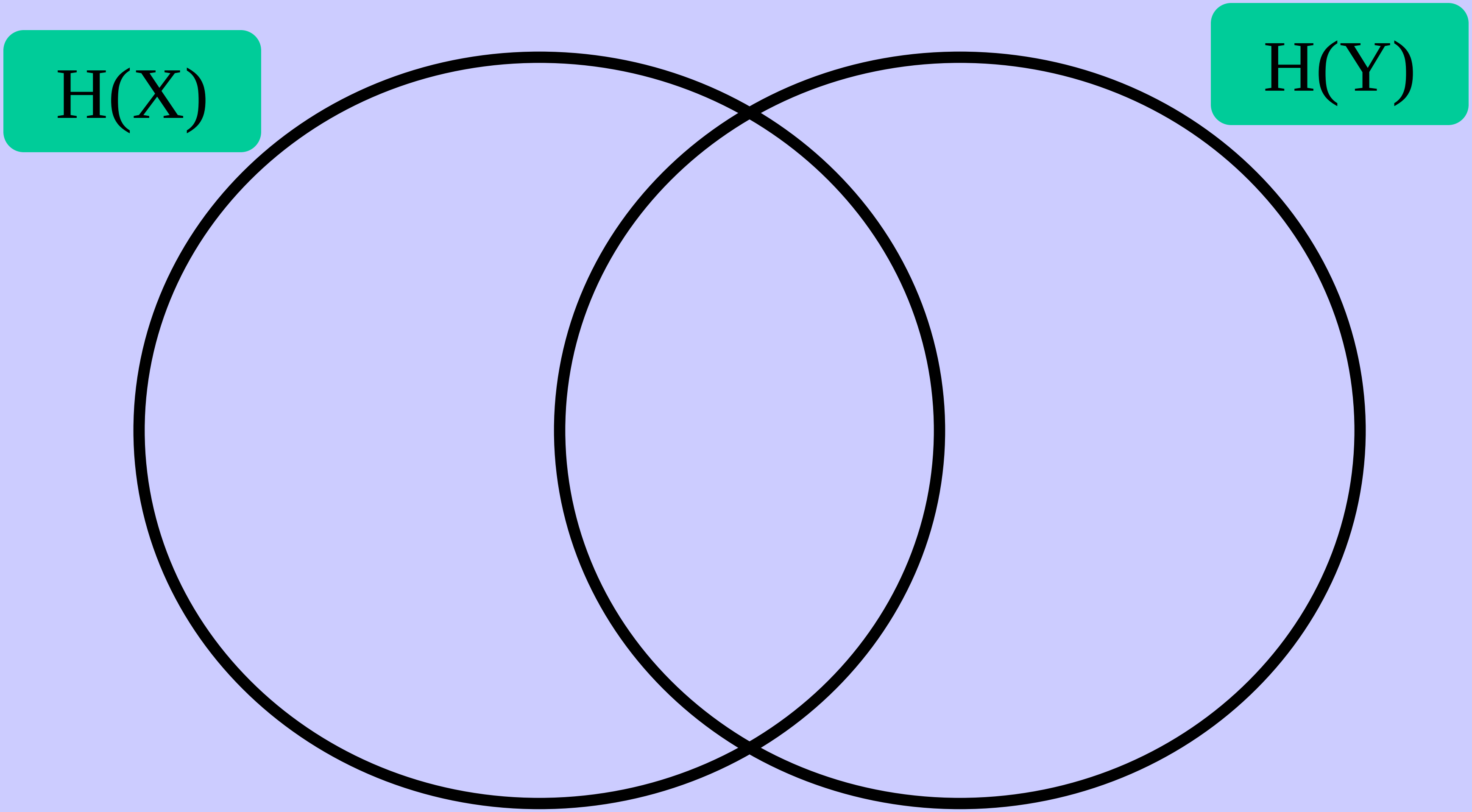

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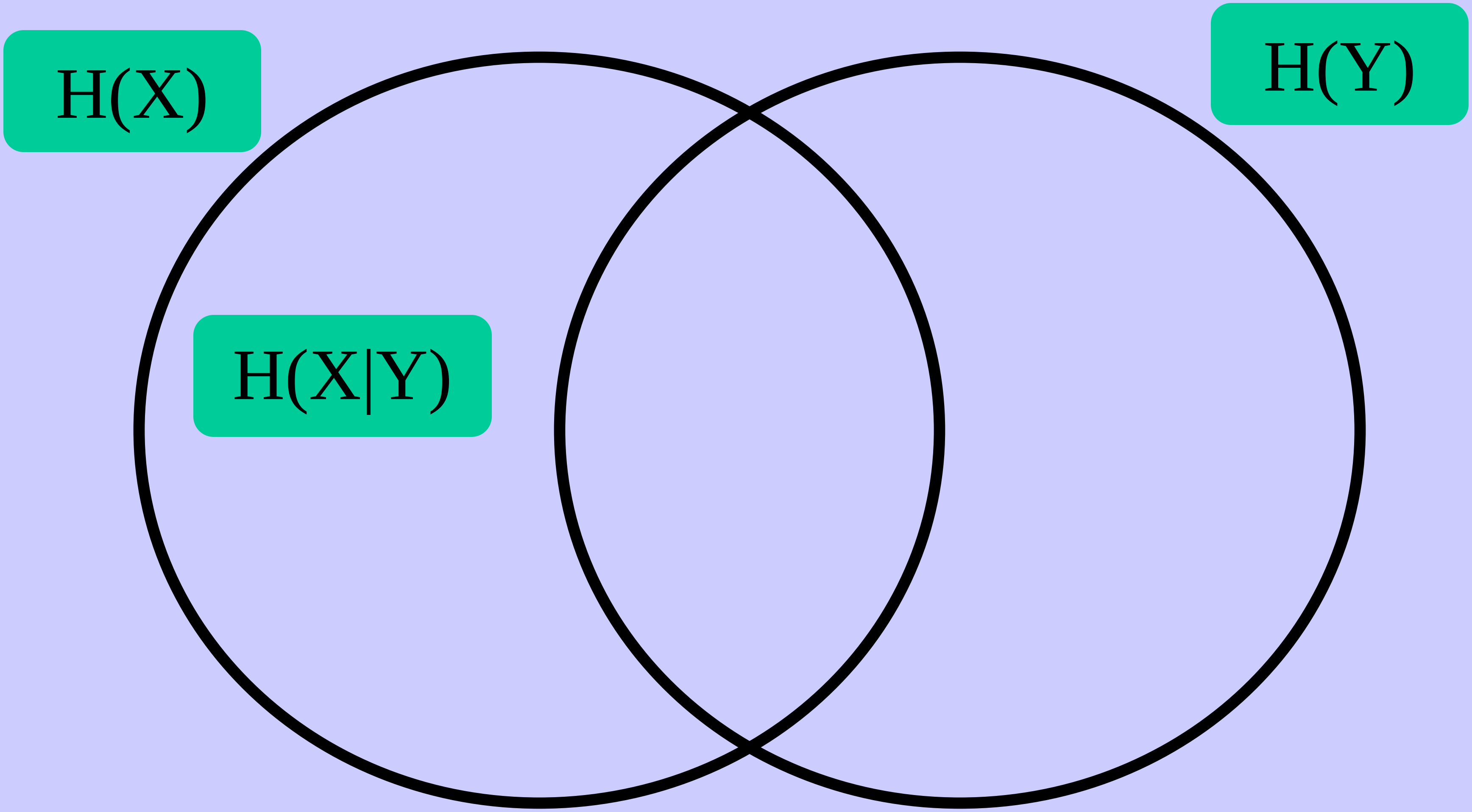

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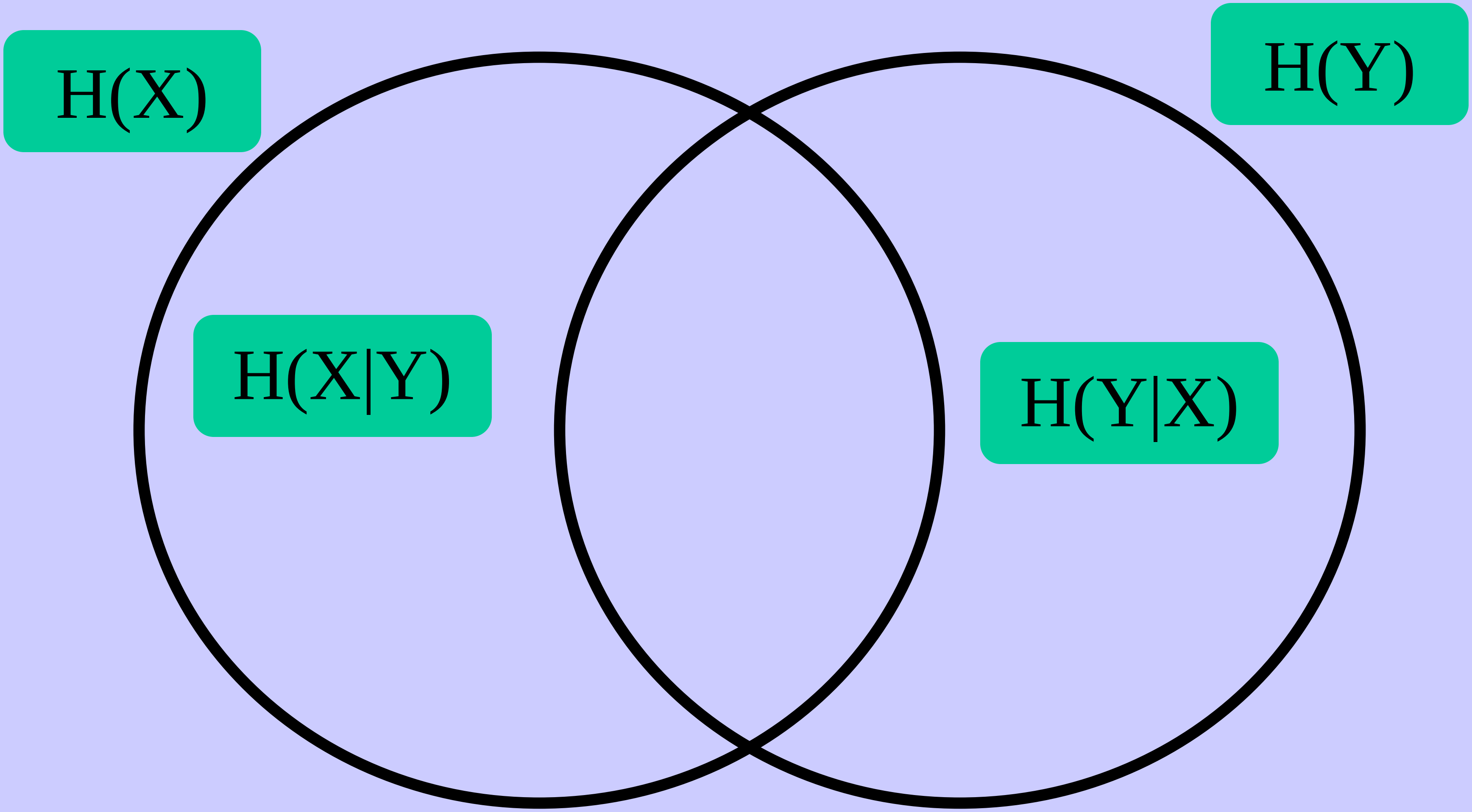

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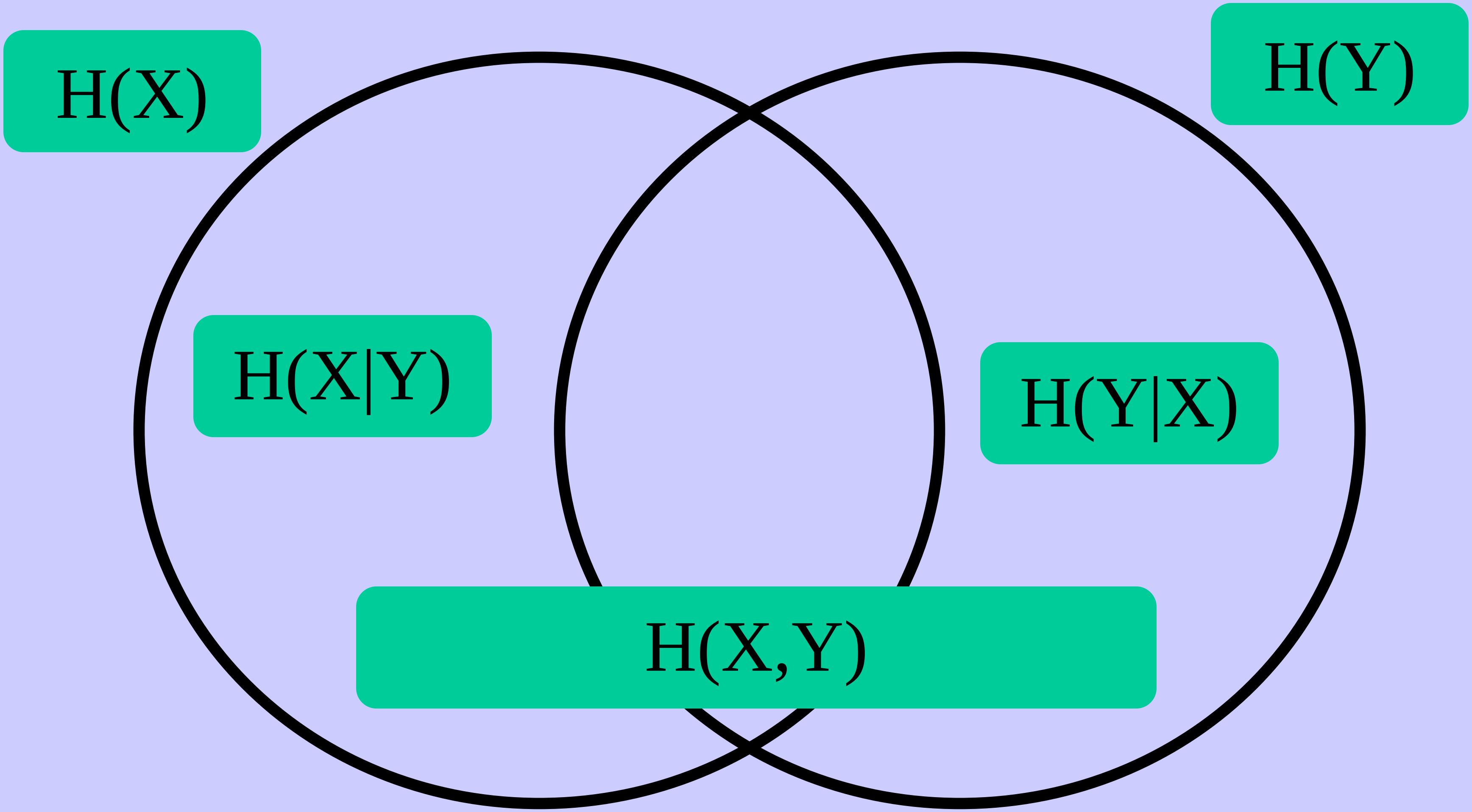

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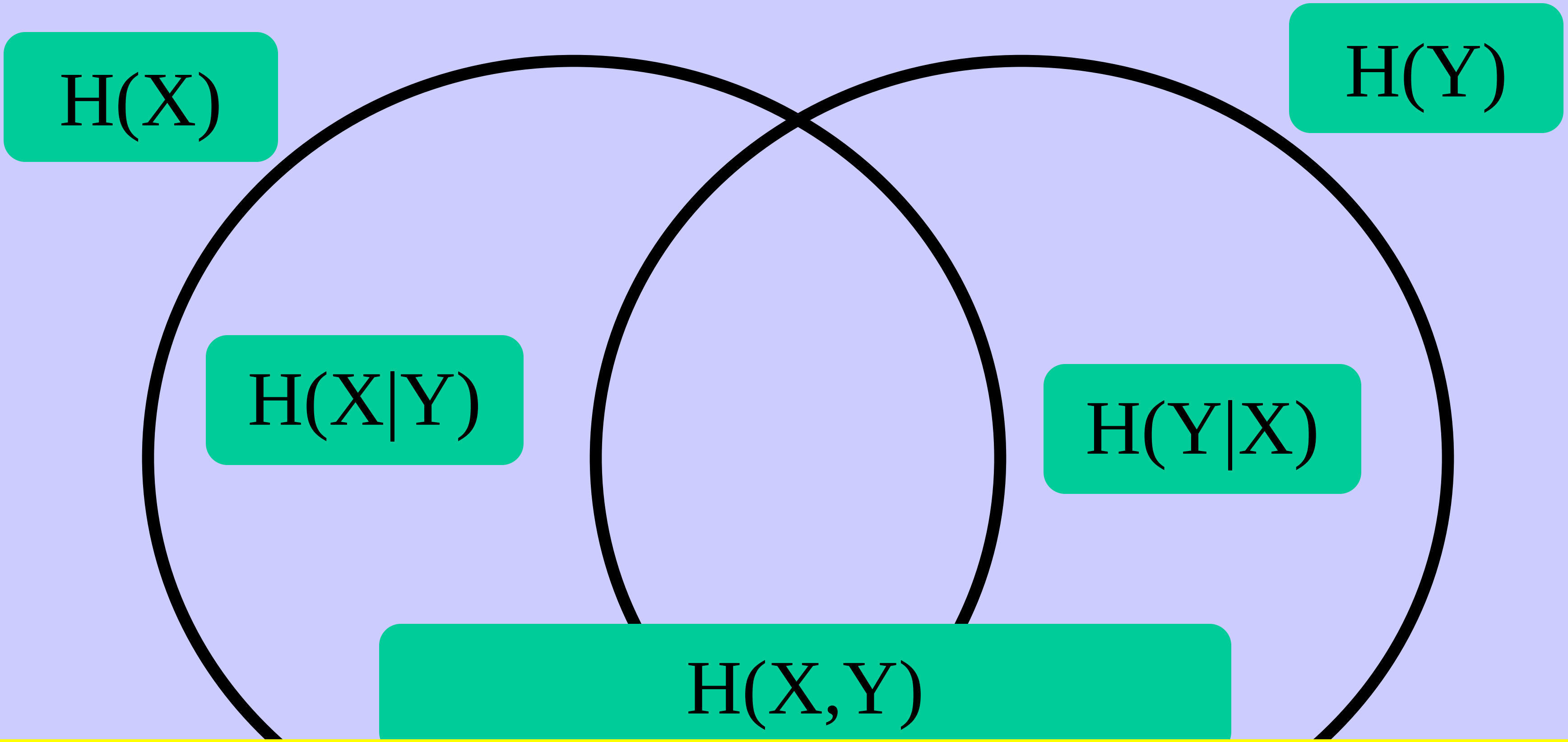
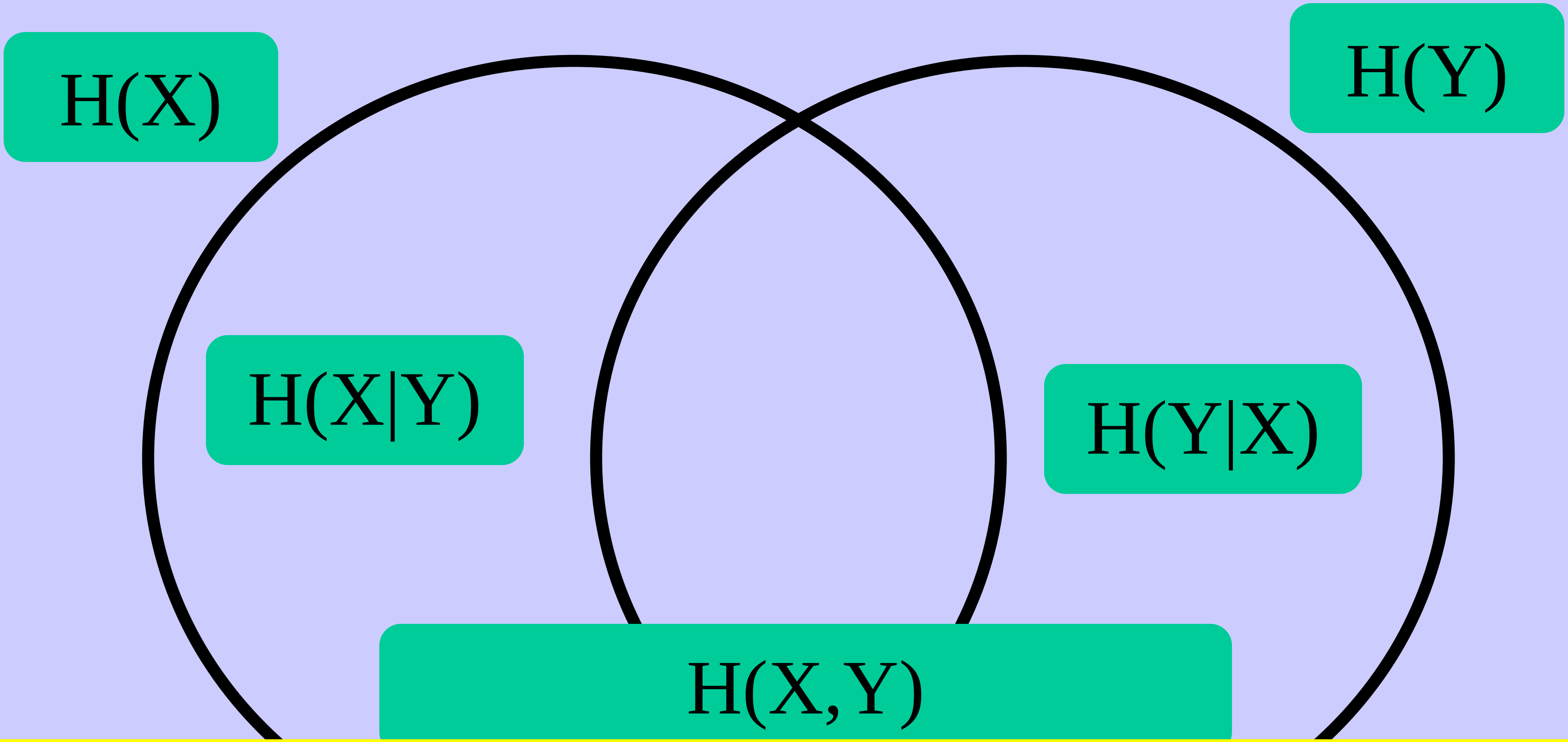


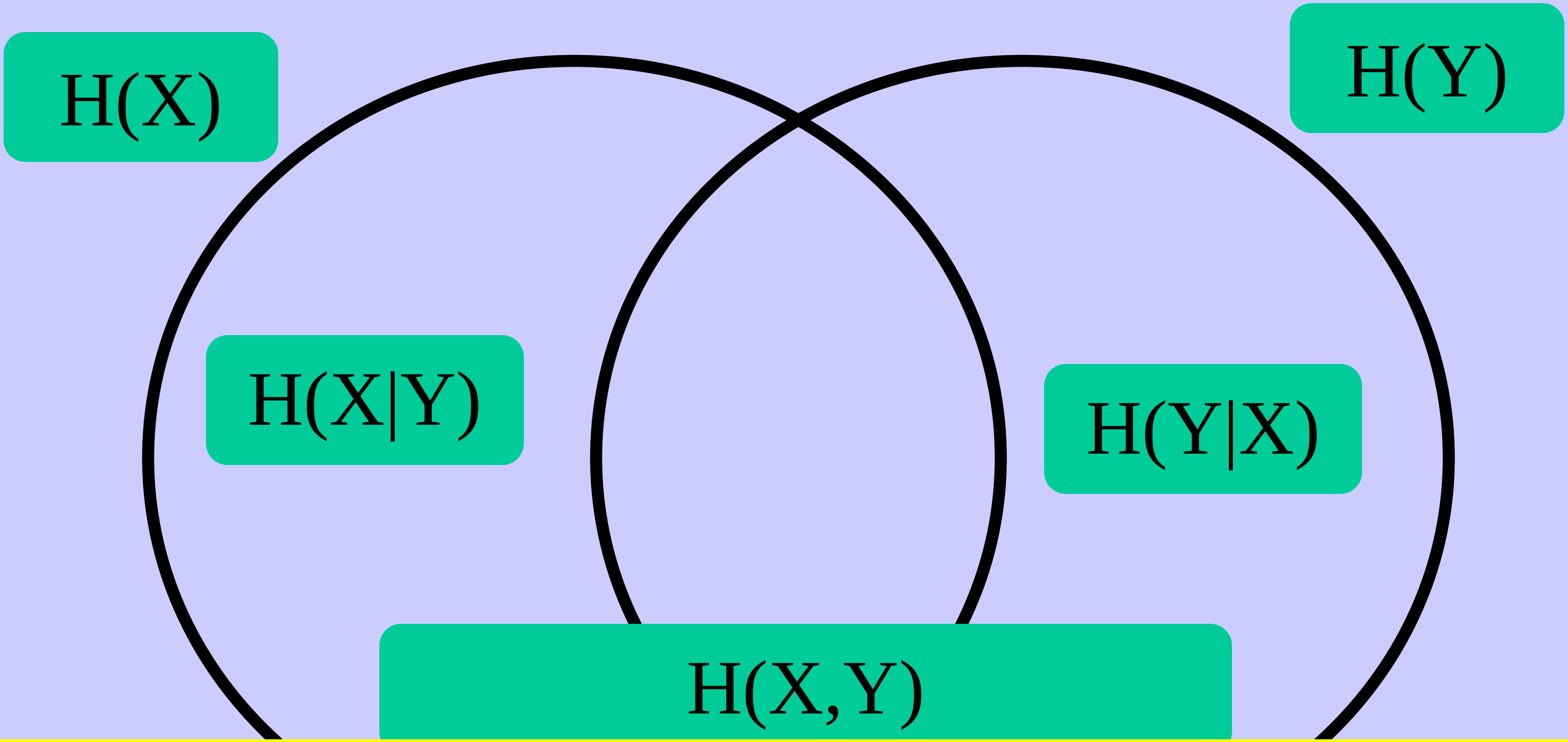
Common area of the two circles =

Defining quantum discord



H(X) + H(Y) – H(X,Y)

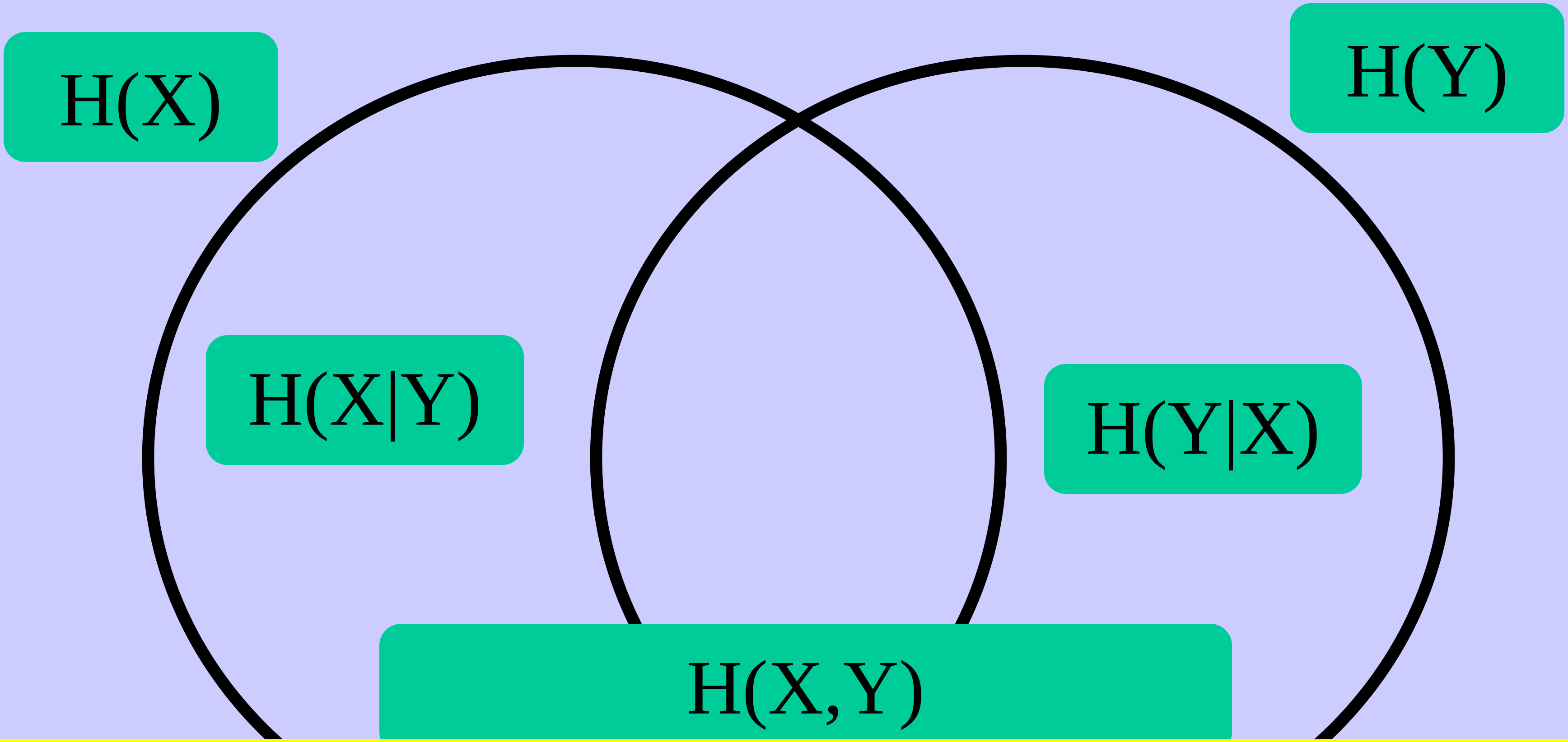
Defining quantum discord



$$H(X) + H(Y) - H(X,Y)$$

or

Defining quantum discord



or

$$H(X) + H(Y) - H(X,Y)$$

$$H(X) - H(X|Y)$$

Defining quantum discord

Equivalent representations of
mutual information
for classical random variables.

H(X,Y)

H(X) + H(Y) – H(X,Y)

or

H(X) – H(X|Y)

Defining quantum discord

Quantizing them produces
inequivalent quantities
for bipartite quantum states.

H(X,Y)

$$H(X) + H(Y) - H(X,Y)$$

or

$$H(X) - H(X|Y)$$

Defining quantum discord

Quantizing them produces
inequivalent quantities
for bipartite quantum states.

The difference is called Discord.

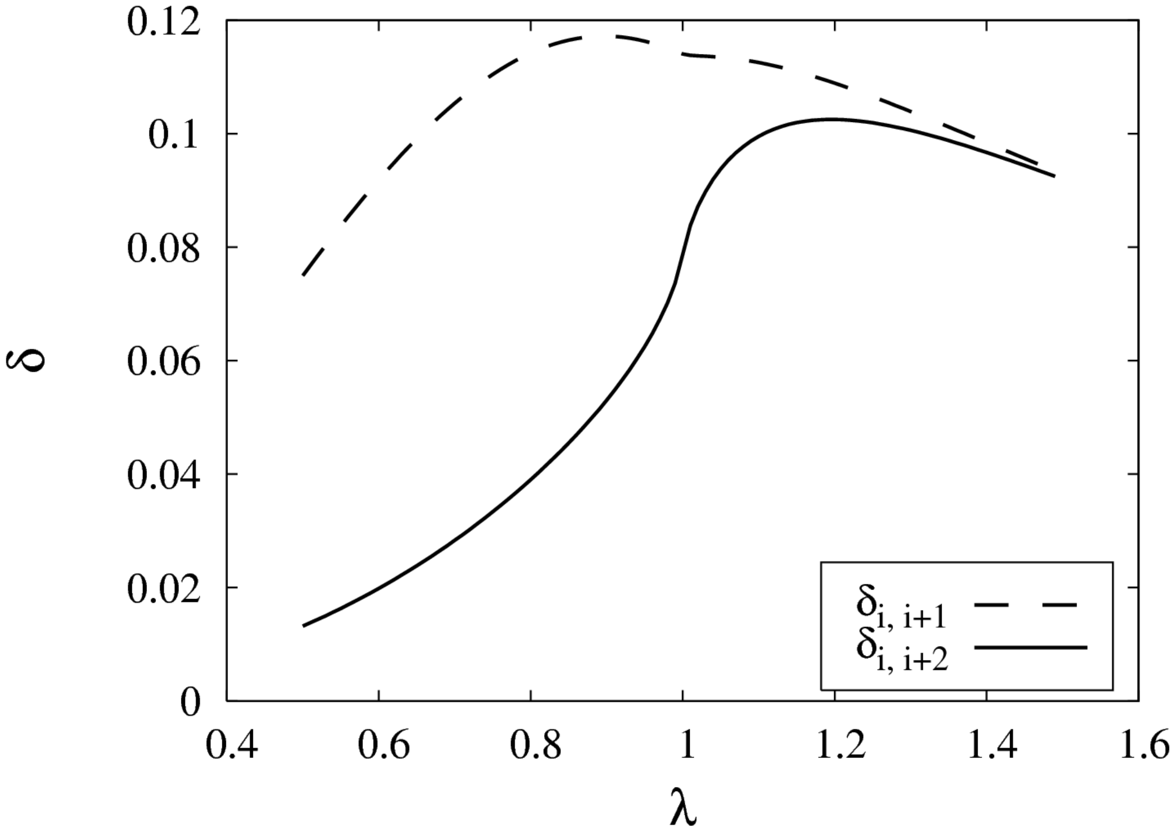
H(X,Y)

H(X) + H(Y) – H(X,Y)

or

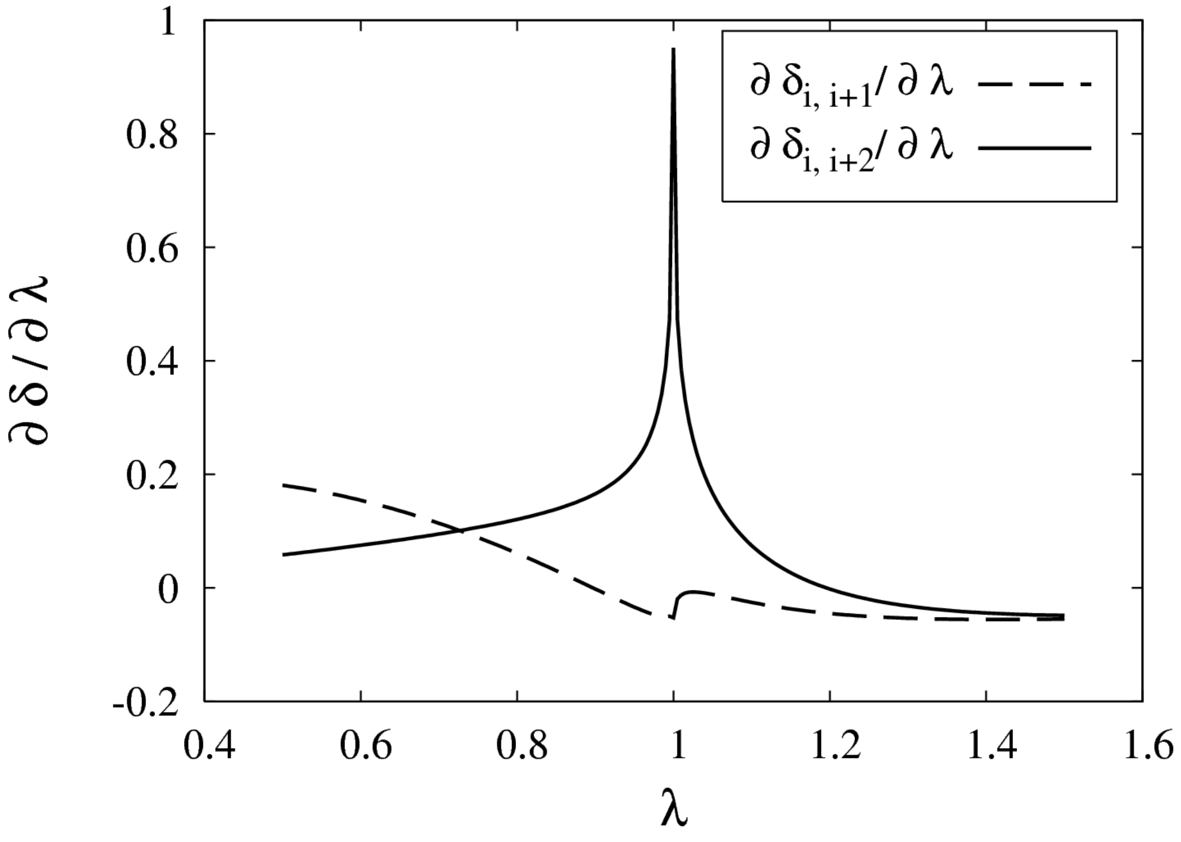
H(X) – H(X|Y)

Discord detects QPT



Dillenschneider, PRB’08

Discord detects QPT



Dillenschneider, PRB’08

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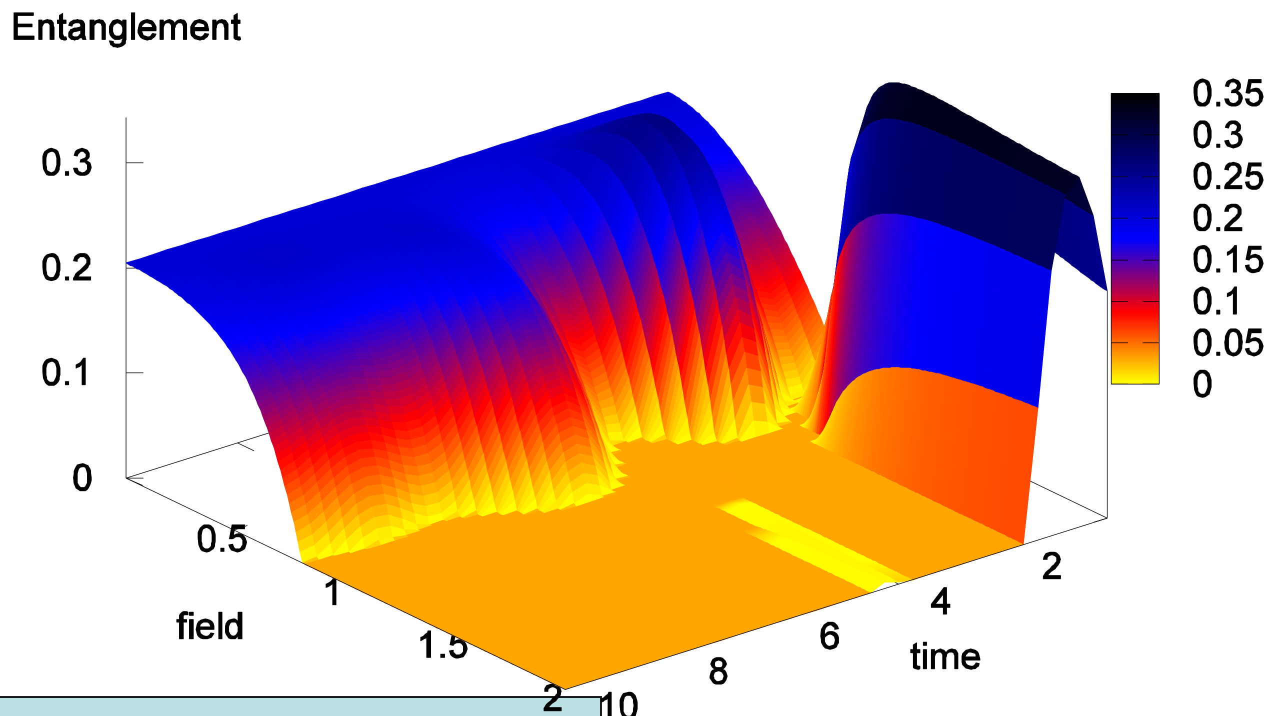
Dynamics of entanglement

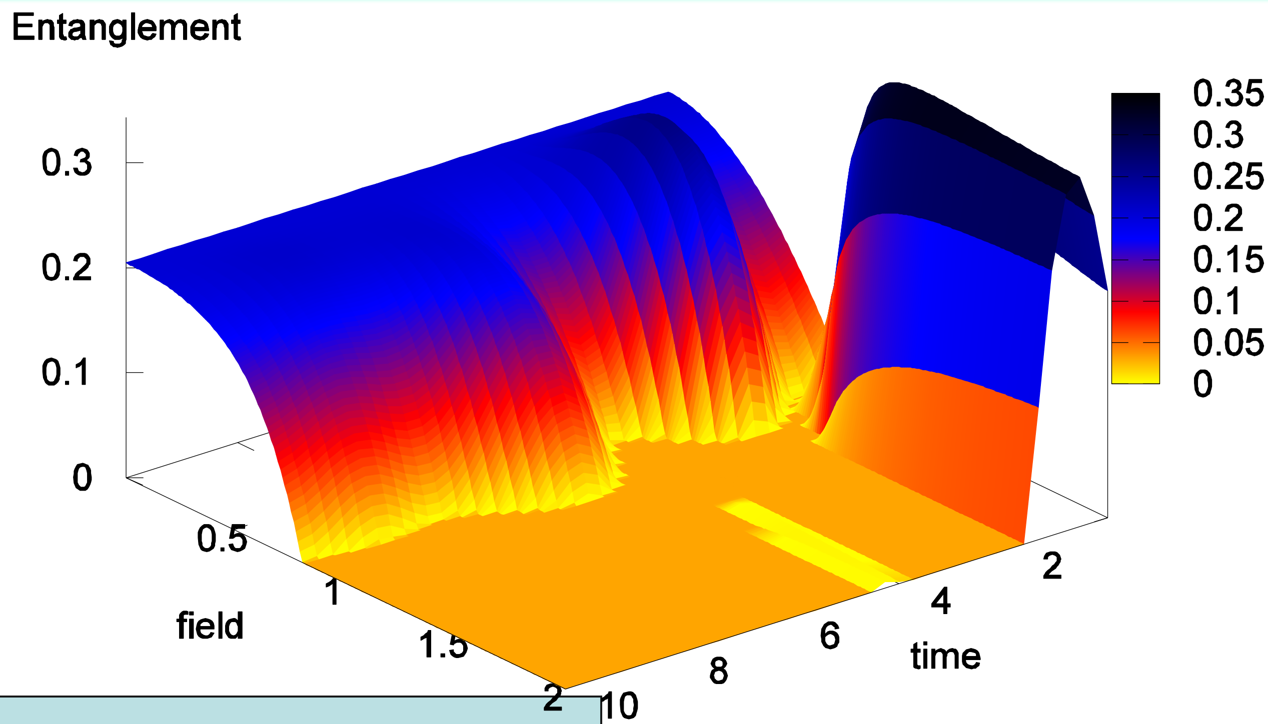
Entanglement in dynamics of many body systems



A “river” of separable states.
Time is a control parameter.

Entanglement in dynamics of many body systems

anisotropic XY model



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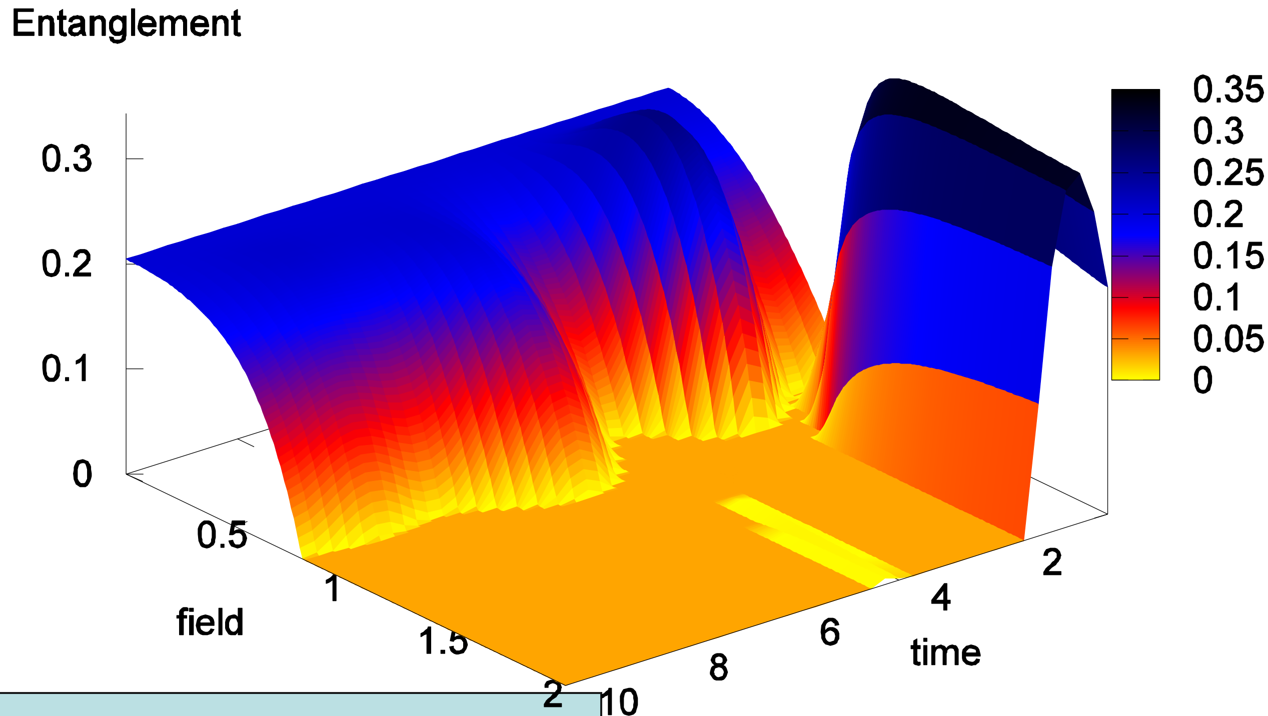
Time is a control parameter.

Entanglement in dynamics of many body systems

anisotropic XY model
anisotropy = 1/2



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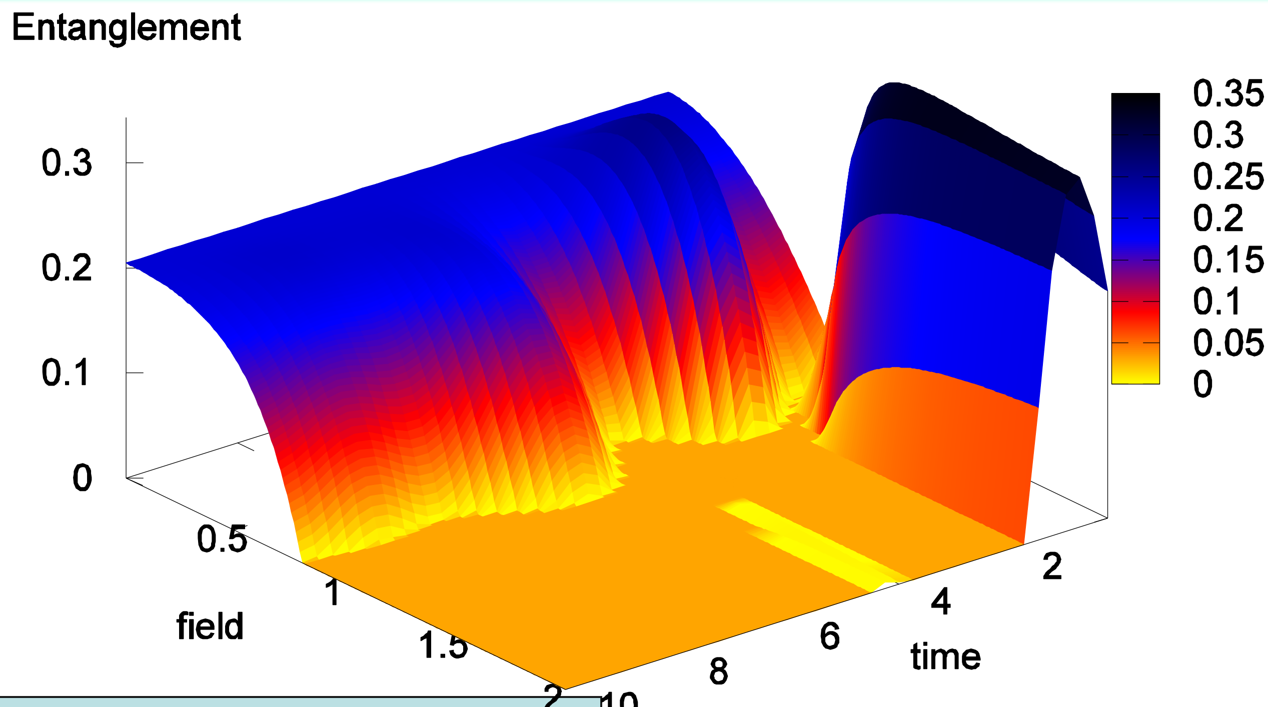
Entanglement in dynamics of many body systems

anisotropic XY model
time-dependent transverse field



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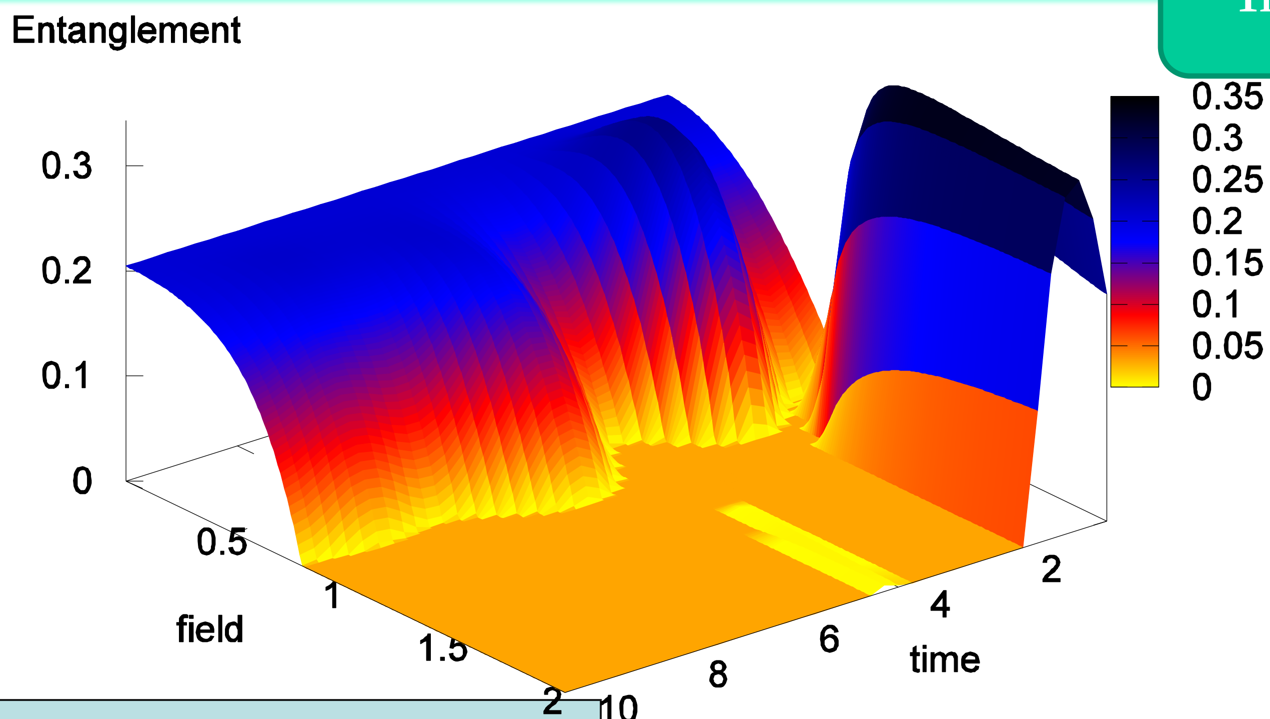
Entanglement in dynamics of many body systems

anisotropic XY model
time-dependent transverse field

Initial kick



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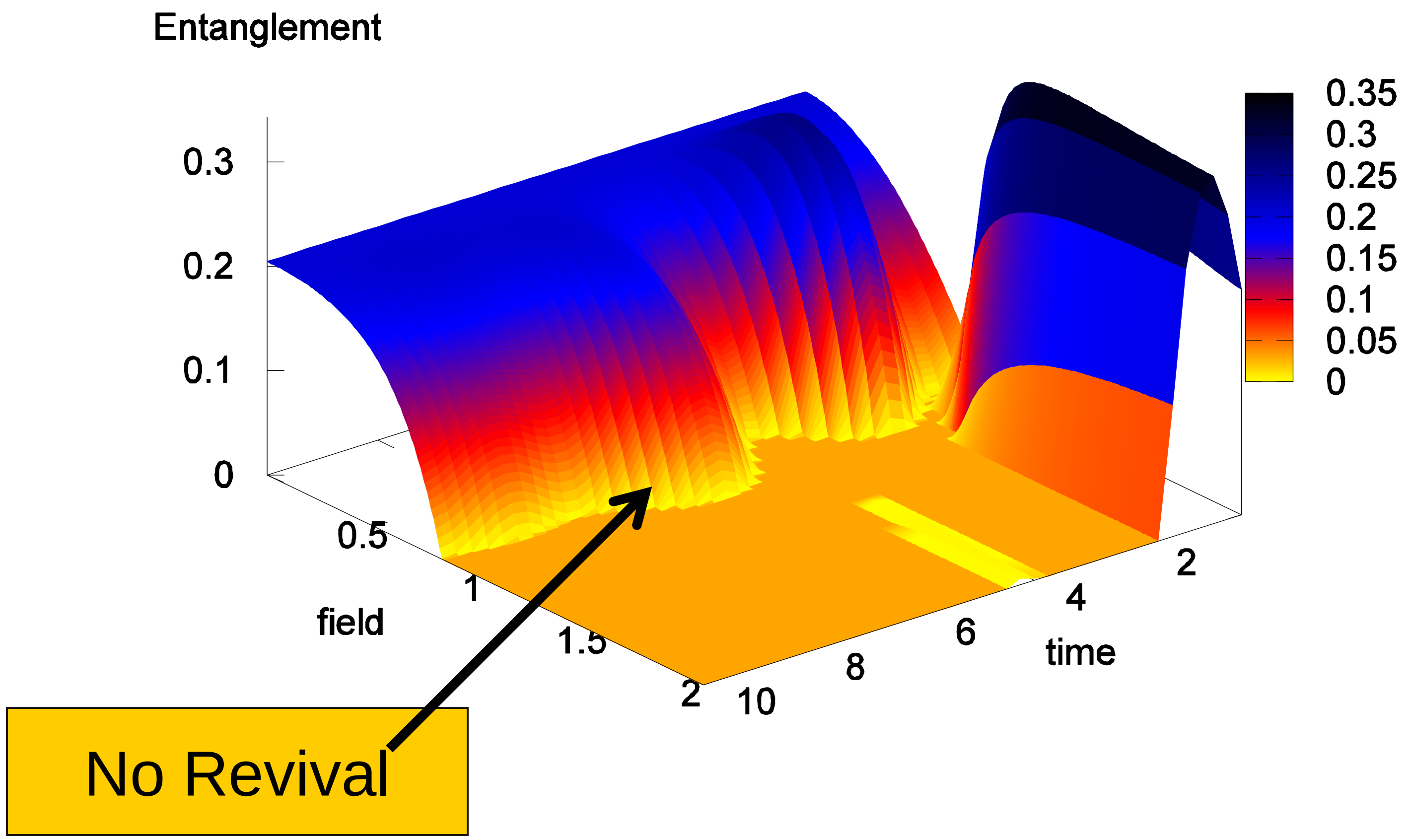
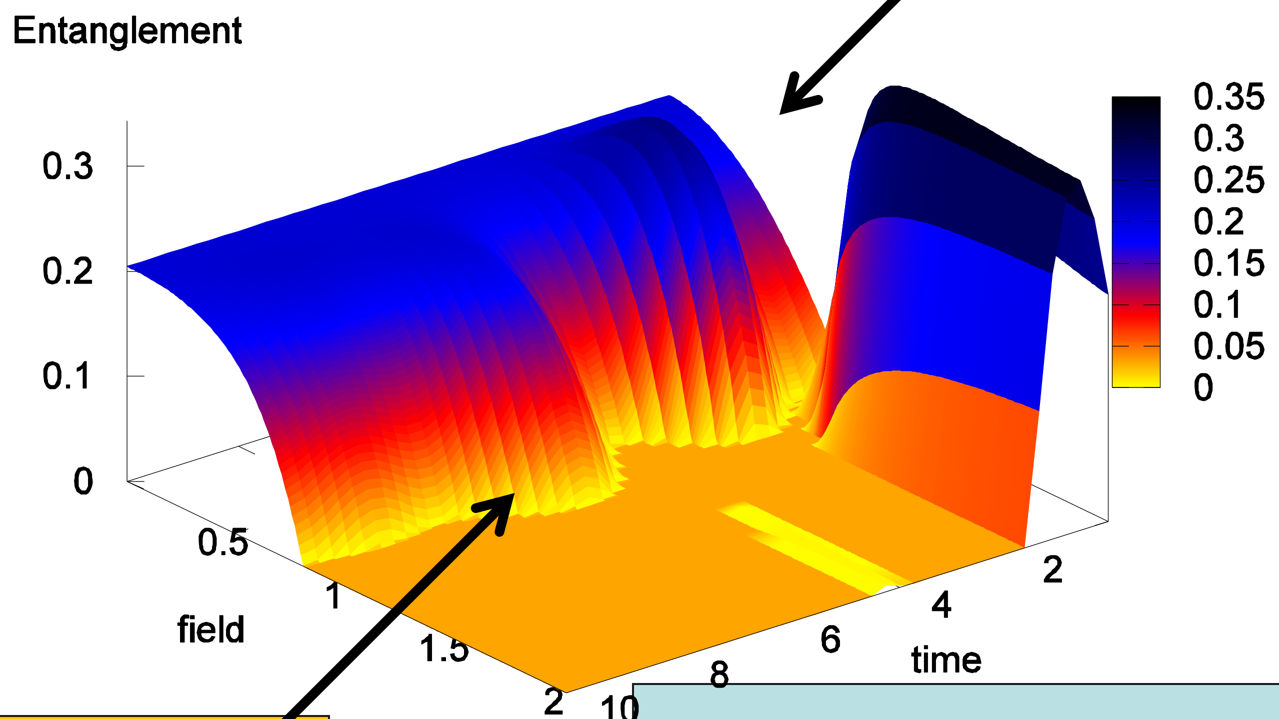


Entanglement in dynamics of many body systems



Collapse & Revival

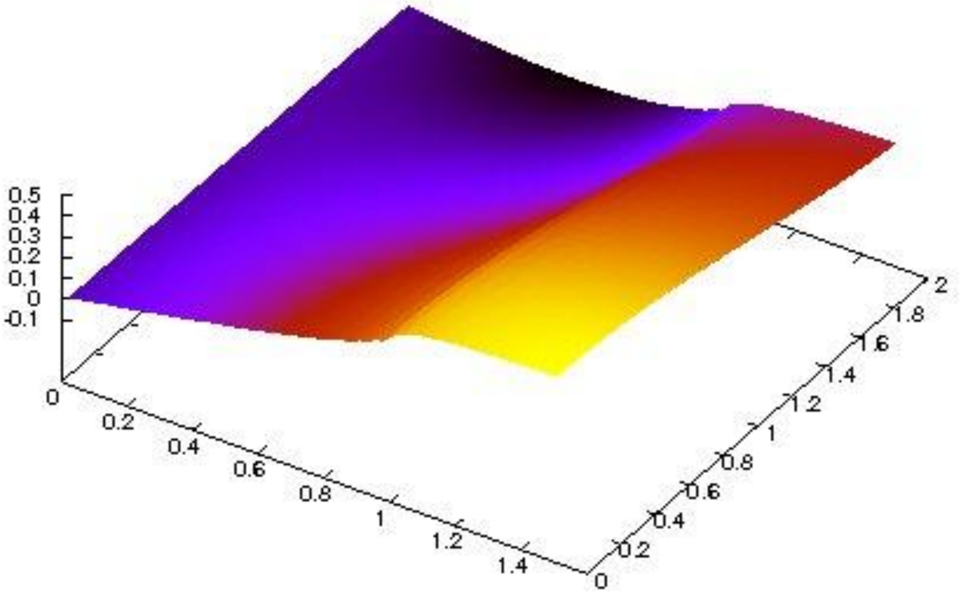
No Revival

"Dynamical Phase Transition"

Sen(De), US, Lewenstein, PRA'05

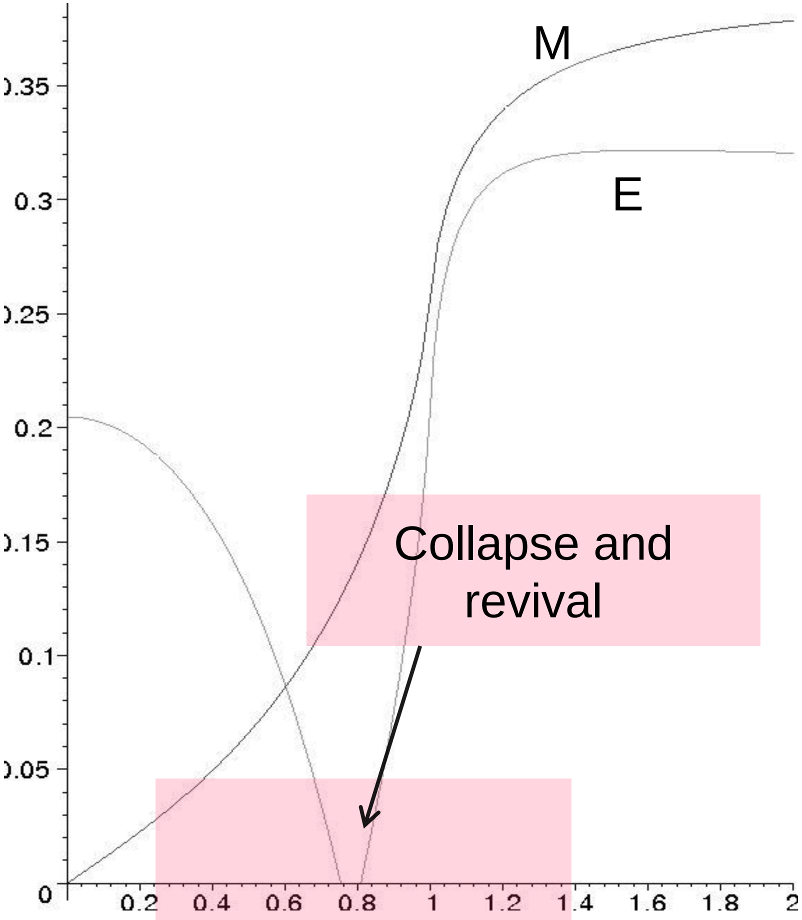
What abt magnetization?

- No dynamical phase transition



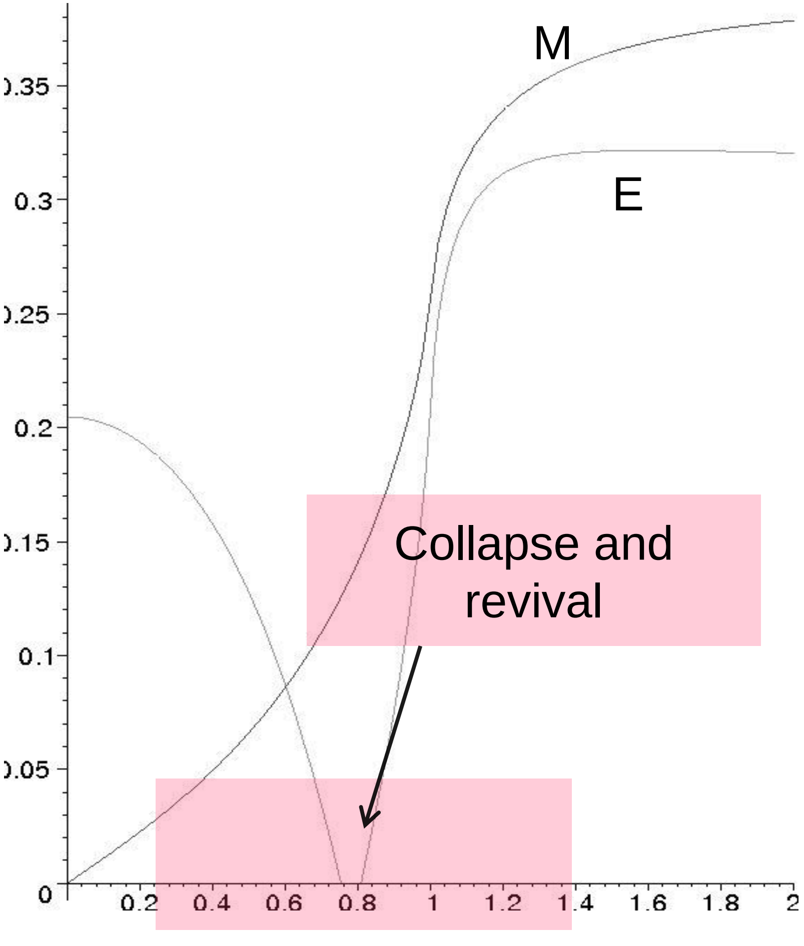
Entanglement: Dynamical Phase Transition

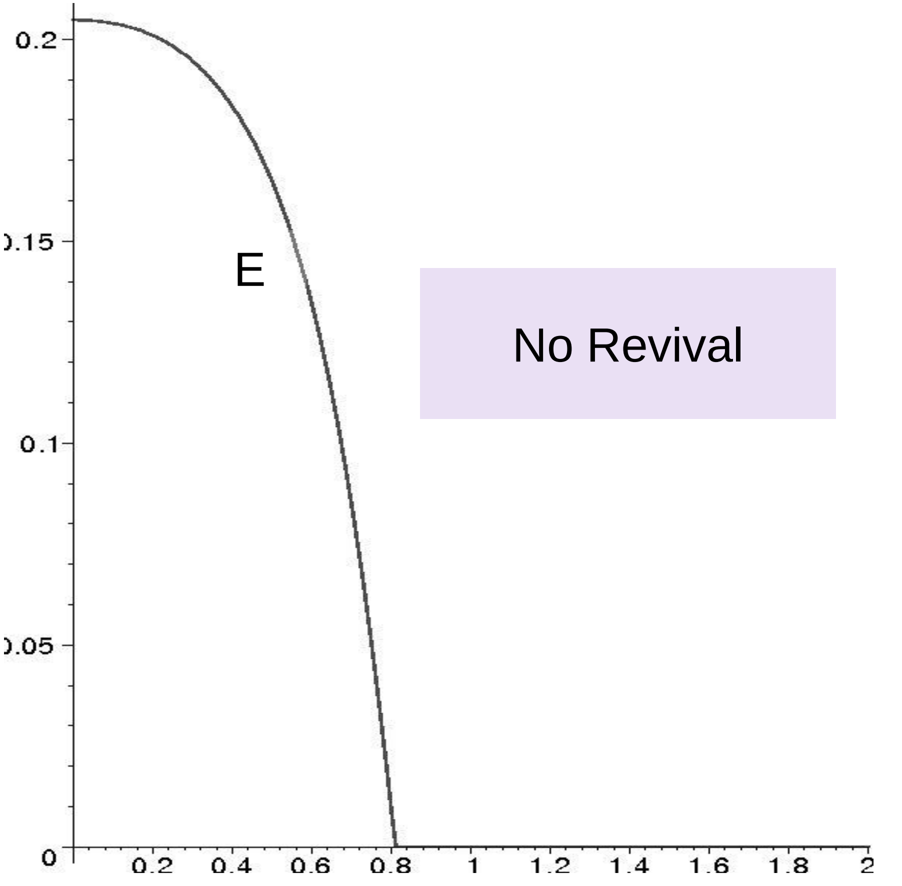
- Short time



Entanglement: Dynamical Phase Transition

- Short time: Ent. vs field
- Relatively long time

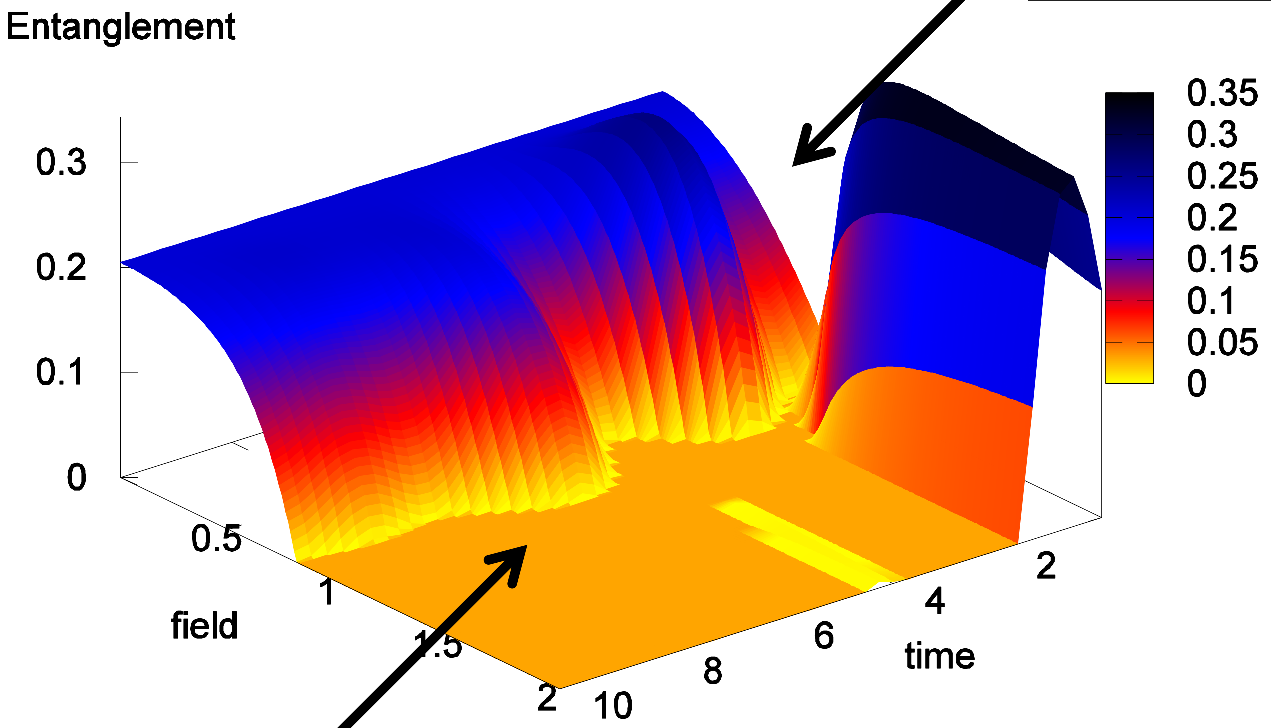




Why Dynamical Phase Transition?

Dhar, R. Ghosh, Sen(De), US, 1011.5309

Entanglement in dynamics many body systems



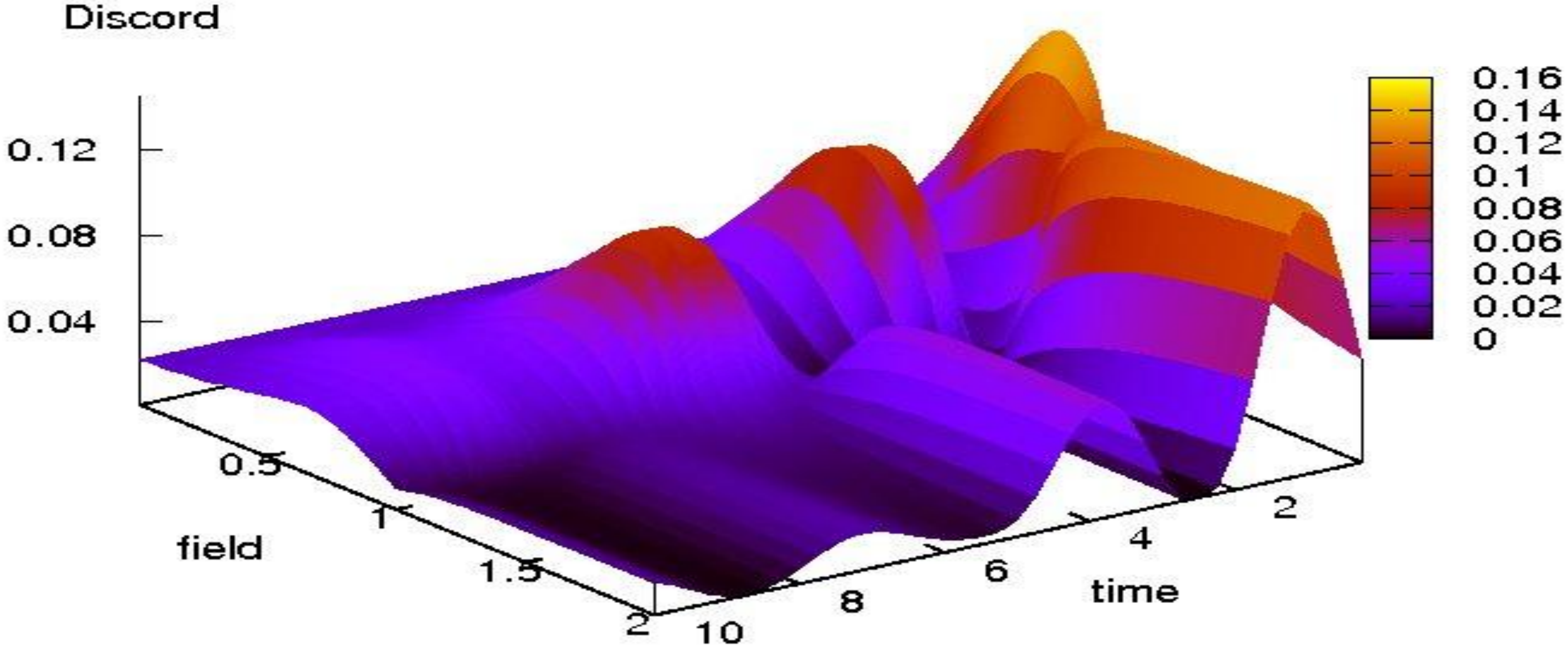
Collapse & Revival

No Revival

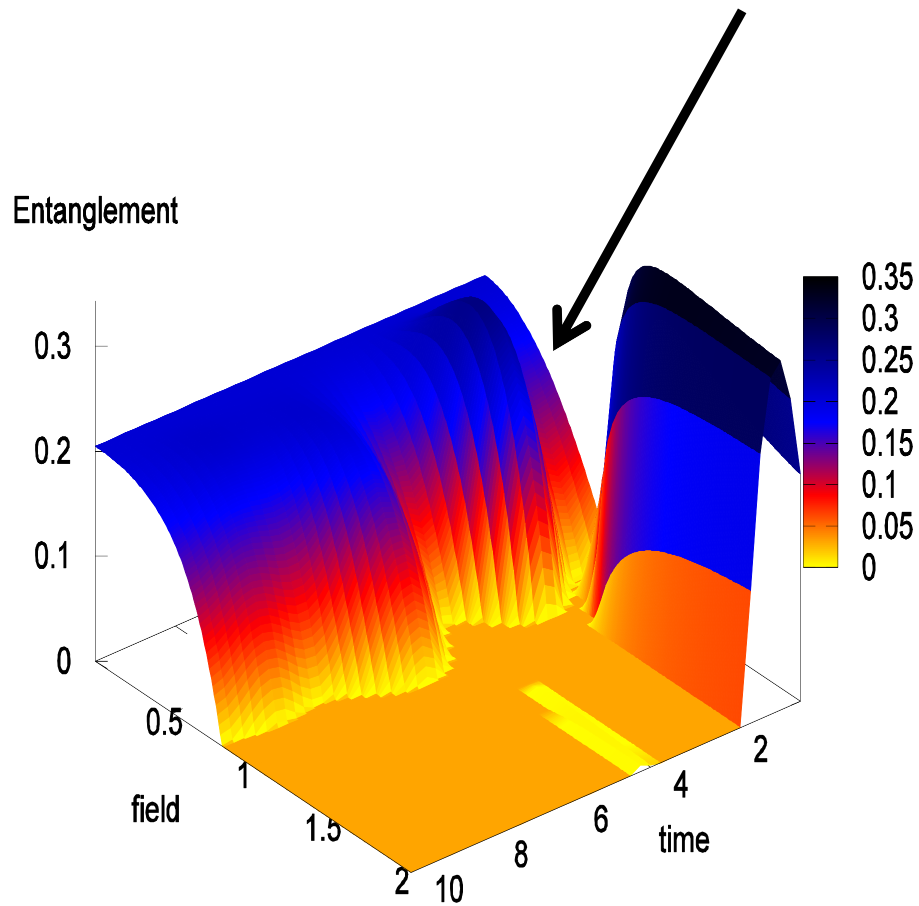
"Dynamical Phase Transition"
Sen(De), US, Lewenstein, PRA'05

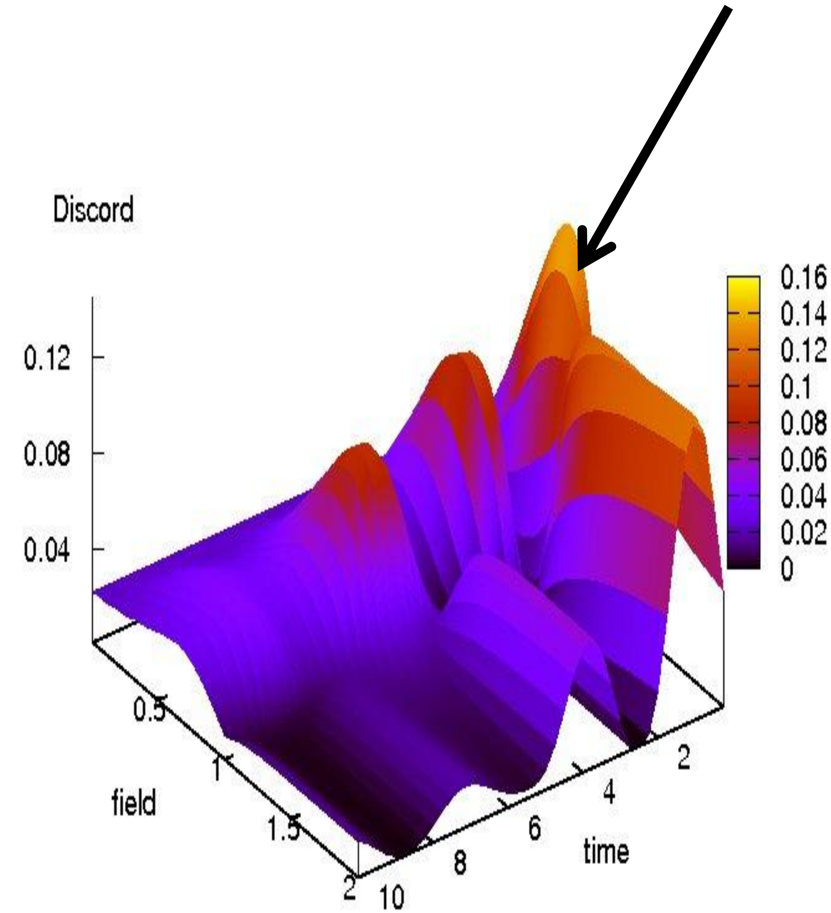
Why revival after collapse?

- Discord



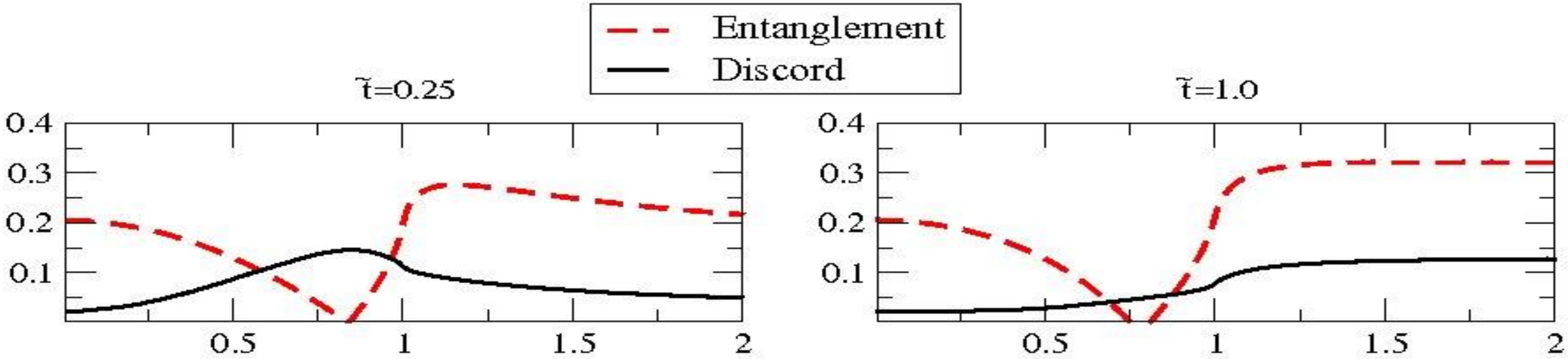
Why revival after collapse?





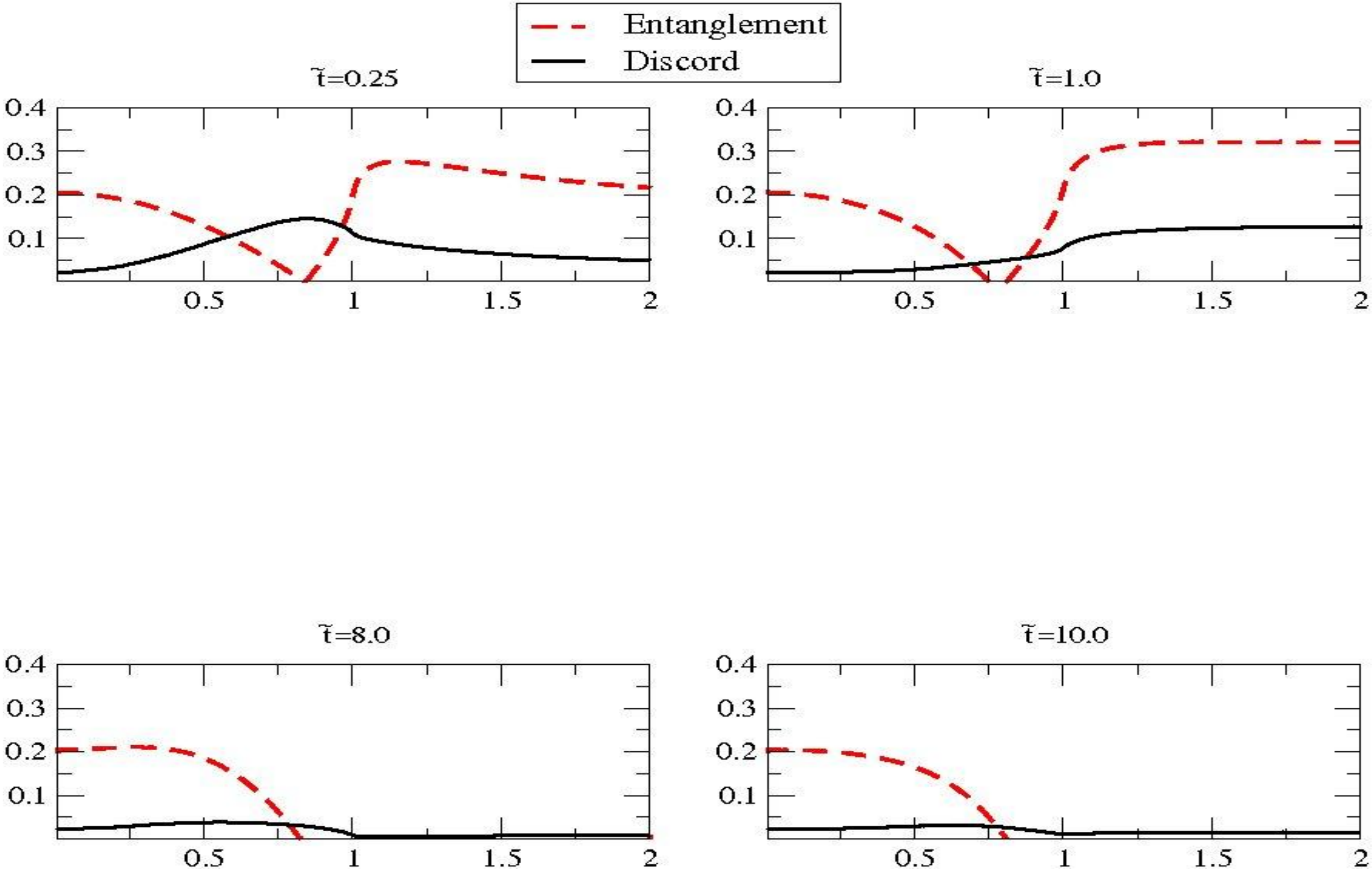
Entanglement vs. Discord

- Zooming in



Entanglement vs. Discord

- Zooming in



Thesis:

Discord surge heralds entanglement revival

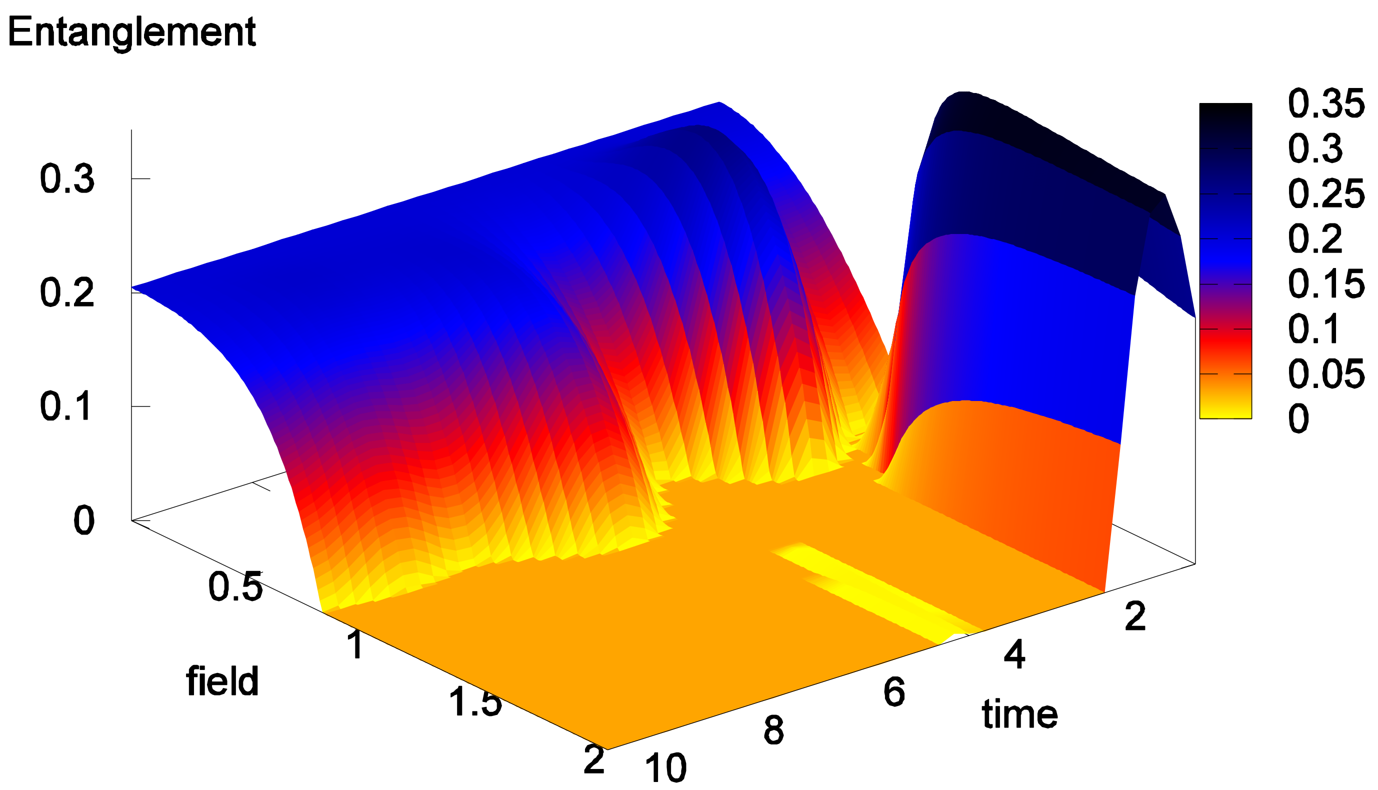
For a fixed t,

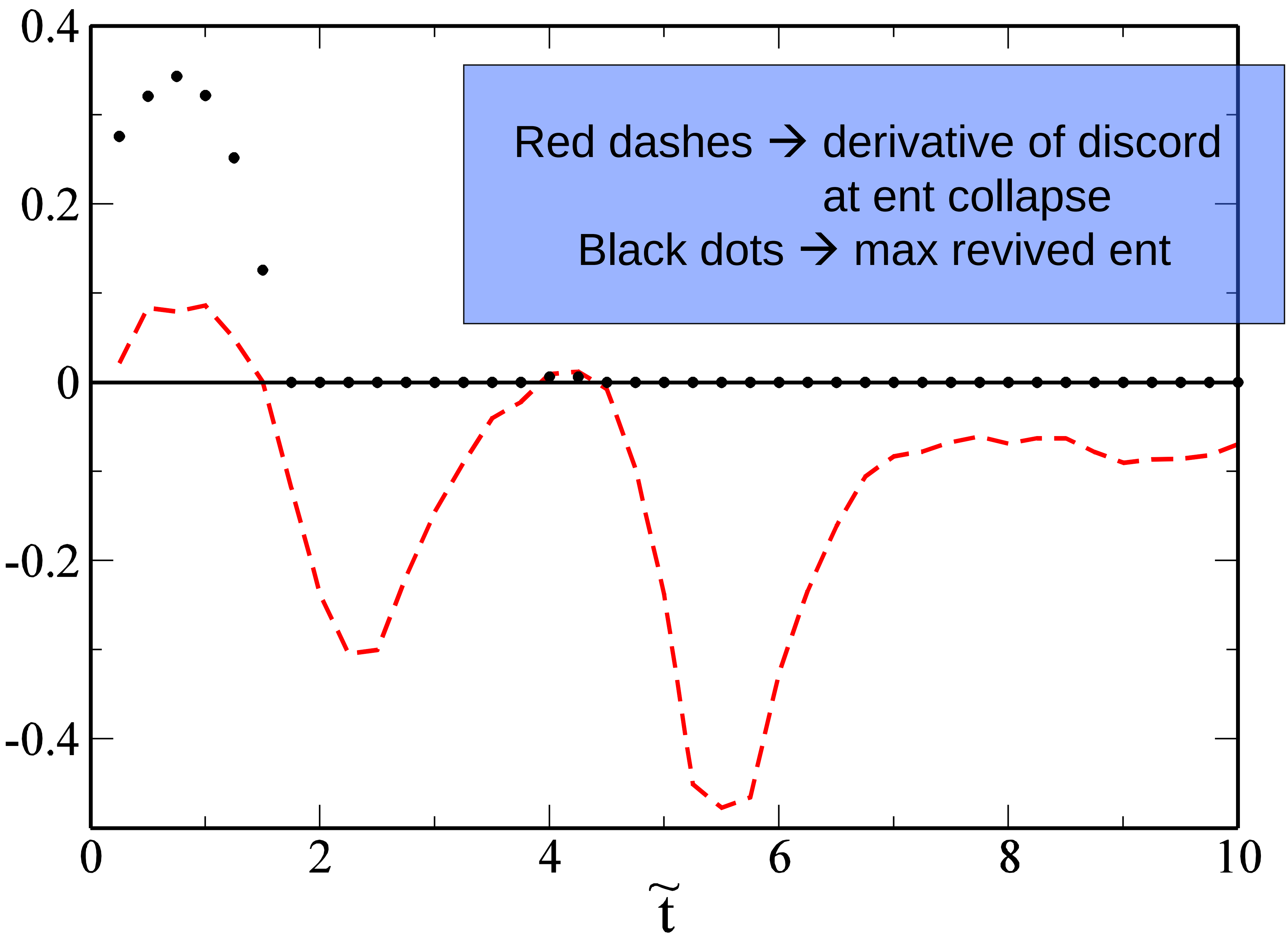
Increasing discord at entanglement collapse

implies

revival of entanglement.

Entanglement





Red dashes → derivative of discord
at ent collapse
Black dots → max revived ent
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10
$\tilde{t}$

Upshot

Bridge being built between many-body physics and quantum information science.

Upshot

Bridge being built between many-body physics and quantum information science.

Many secrets remain to be uncovered …

More work done

- Adv. Phys. **56**, 243 (2007)
- Rev. Mod. Phys. **80**, 517 (2008)


Thank you!

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