

# Quantum Simulation of arbitrary Hamiltonians with superconducting qubits

Colin Benjamin (NISER, Bhubaneswar)

Collaborators: A. Galiutdinov, E. J. Pritchett,  
M. Geller, A. Sornborger and P. C. Stancil (UGA)  
J. M. Martinis (UCSB)

# Outline

- Quantum Simulation
- Superconducting simulator
  - Quantum statics: Hamiltonian mapping
  - Quantum dynamics
- Application to-
  - i. Random real Hamiltonian
  - ii. Molecular collisions

# Introduction

- Definition: Quantum simulation is a process in which a quantum computer simulates another quantum system(Lloyd, '96).
- Corollary: A classical computer can also simulate quantum systems .

# Superconducting simulator(1)

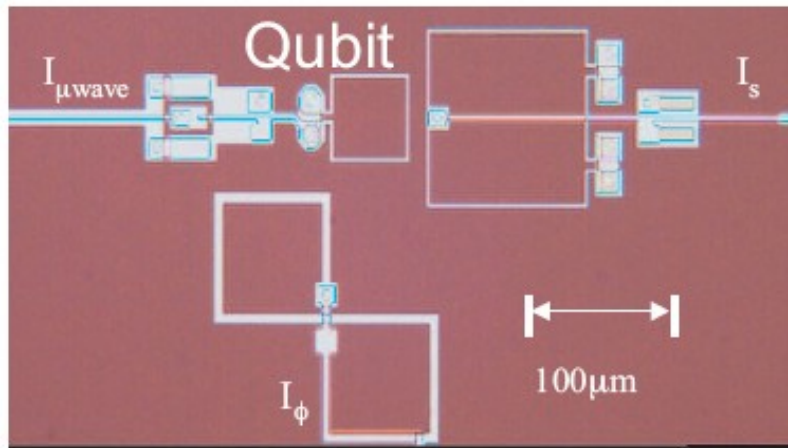
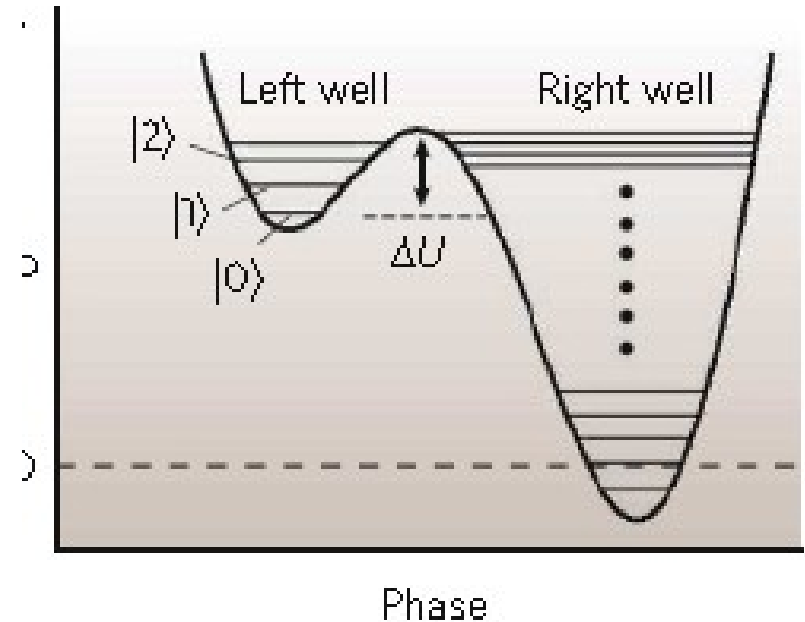


Figure 4: Single superconducting qubit device with associated microwave control and readout circuitry.



# Superconducting simulator(2)

Hamiltonian

$$H_{qc} = \sum_i \begin{pmatrix} 0 & 0 \\ 0 & \bar{\epsilon} \end{pmatrix}_i + \sum_i \begin{pmatrix} 0 & 0 \\ 0 & \delta\epsilon_i(t) \end{pmatrix}_i + \sum_{ij} g_{ij}(t)\Phi_i\Phi_j,$$

$$\text{with } \Phi_i = (a\mathbf{I} + b\sigma^z + c\sigma^x)_i,$$

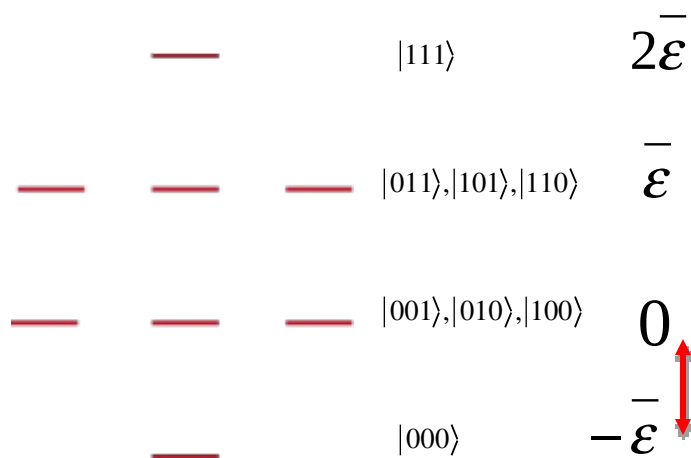
Rescaled energies

$$H_{qc} = \sum_i \begin{pmatrix} 0 & 0 \\ 0 & \delta\epsilon_i(t) \end{pmatrix}_i + \sum_{ij} g_{ij}(t)\Phi_i\Phi_j.$$

# Superconducting simulator(3)

## The 1-excitation subspace

$$n = 3 \quad \{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$$

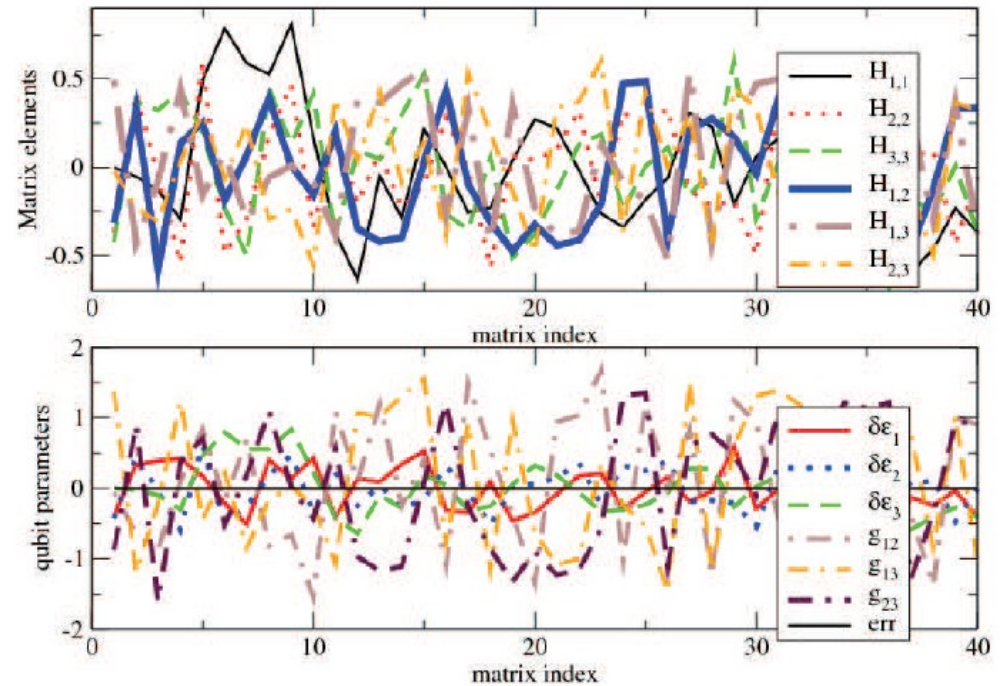


$$H_{\text{qc}} = \begin{pmatrix} \delta & \varepsilon + f_1 & & g_{23} & & g_{12} \\ & & & \delta & \varepsilon + f_2 & & g_{13} \\ & & & & & & \delta & \varepsilon + f_3 \end{pmatrix}$$

$f$ 's are function of  $g$ 's<sup>6</sup>

# Mapping(1)

- Random Hamiltonia
- Exact mapping:

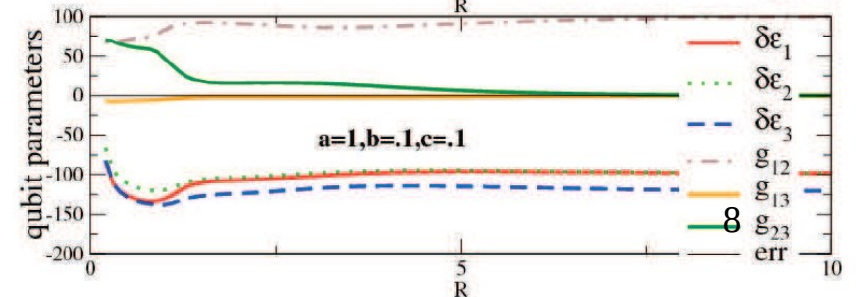
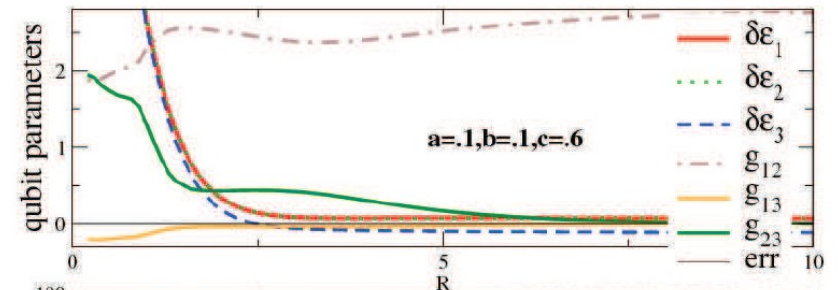
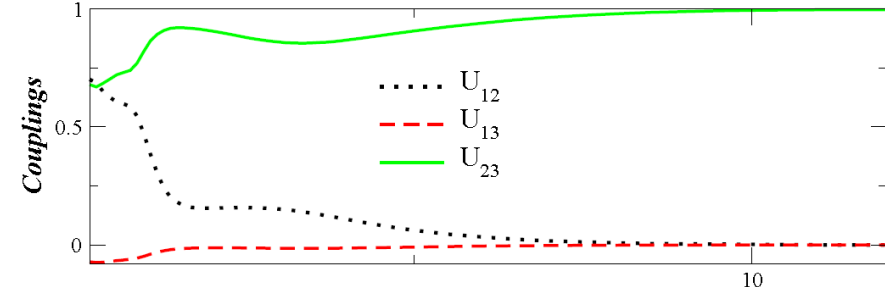
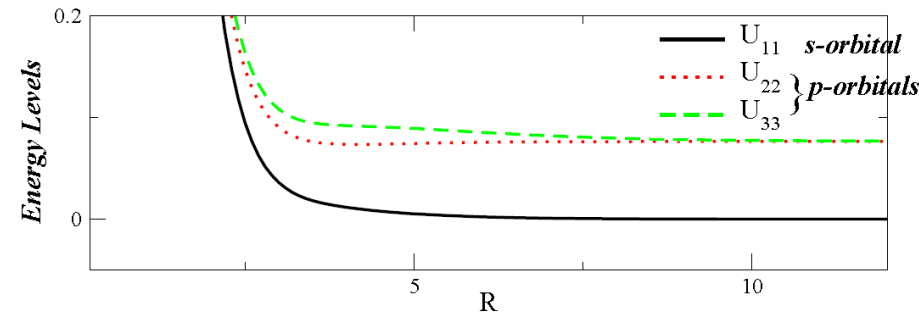


$$\text{err} = \|H_{\text{rand}} - H_{\text{qc}}\| = 0$$

# Mapping (2)

- Molecular collision
- $\text{Na}(3s) + \text{He} \rightarrow \text{Na}(3p) + \text{He}$
- $\text{err} = \|H'/\lambda - H_{\text{qc}}\| = 0$

$$\lambda = \|H'(t)\| / [\bullet 0.1\text{GHz}]$$



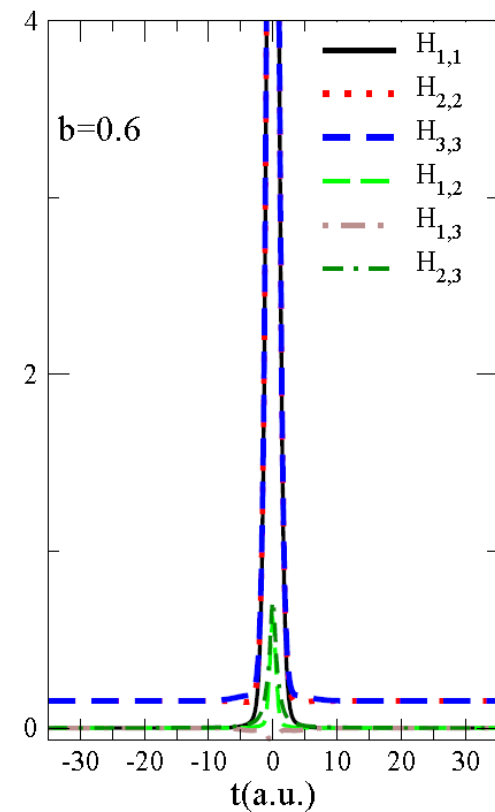
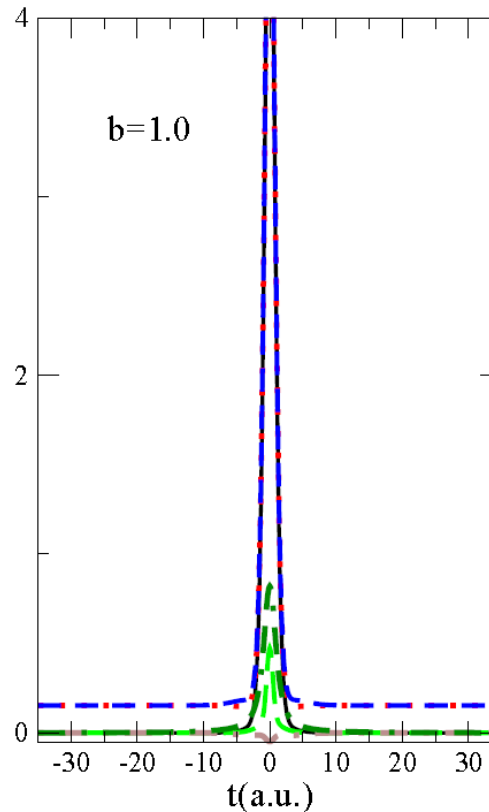


# Quantum dynamics: Molecular Collision(1)

## Time dependent Hamiltonian

distance to  
time transformation

$$R^2 = b^2 + v^2 t^2, v=1$$



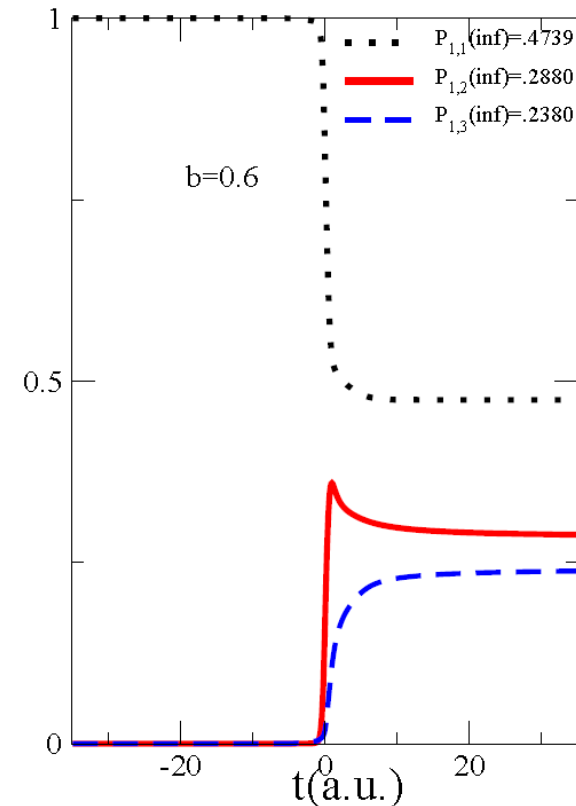
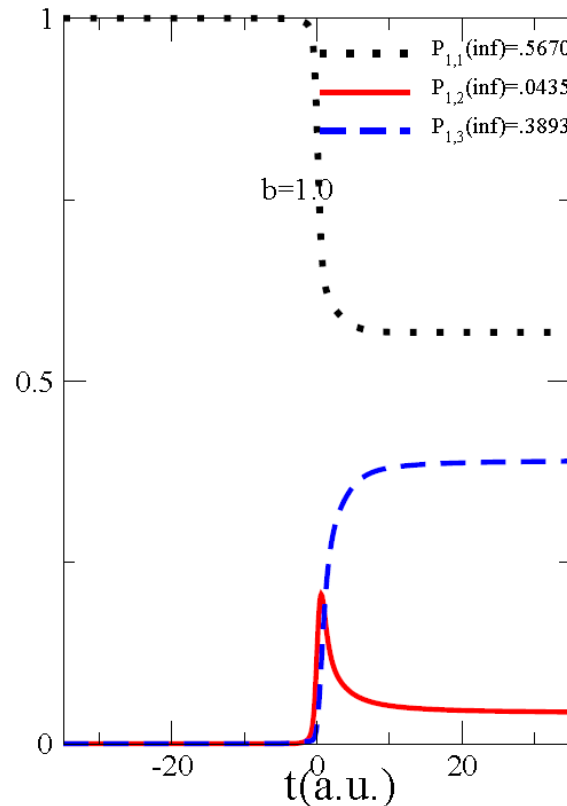
# Quantum dynamics: Molecular Collision(2)

## Scattering Probabilities

$$i \frac{da(t)}{dt} = H'(t)a(t)$$

$$a_i(t \rightarrow -\infty) = \delta_{ij}$$

$$P_{ij} = |a_j(\infty)|^2$$



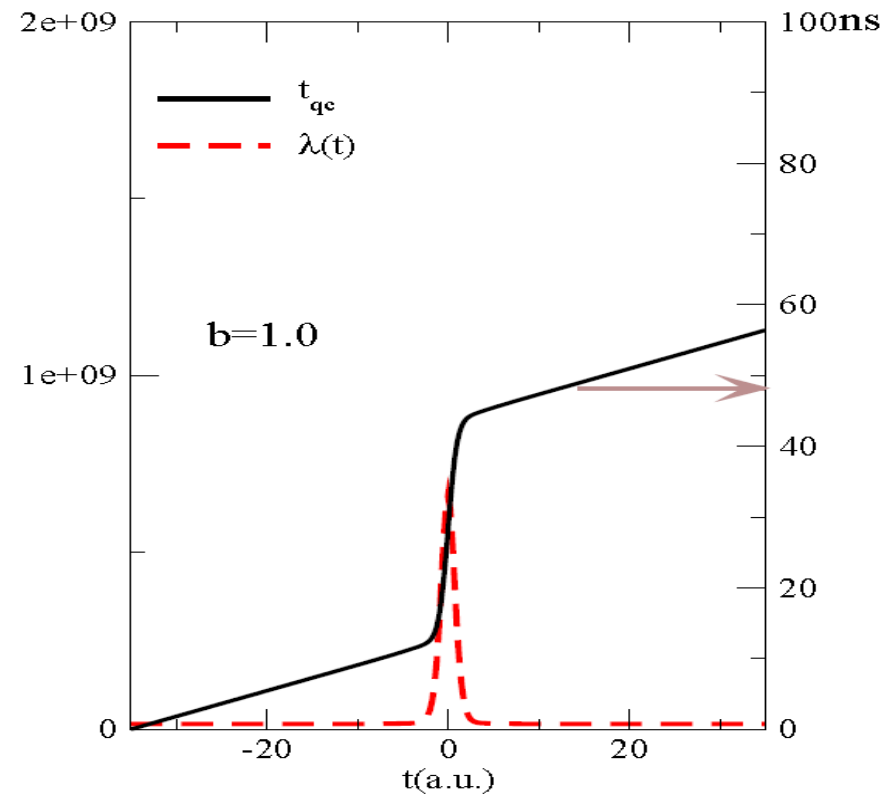
# Quantum dynamics: Quantum Computer(1)

## Energy-time rescaling

- $$a(t) = e^{-iH't} a(-\infty)$$
$$= e^{-i(H'/\lambda)(\lambda t)} a(-\infty)$$

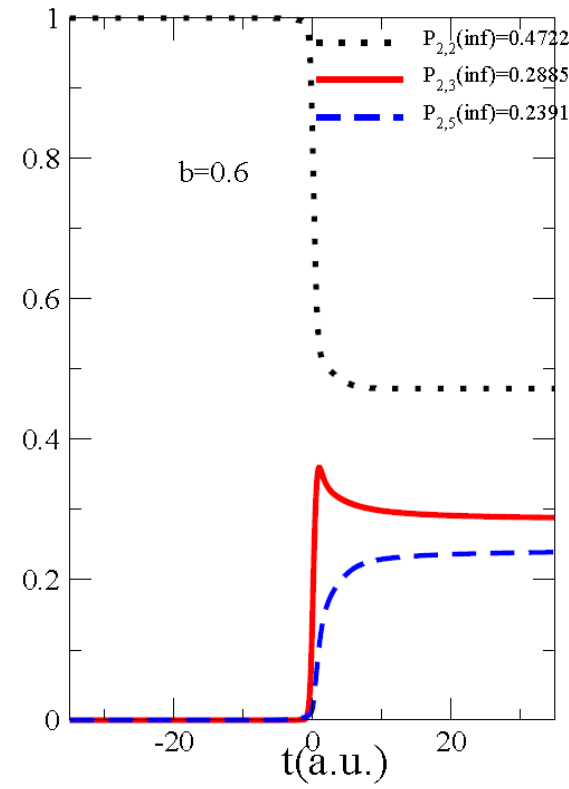
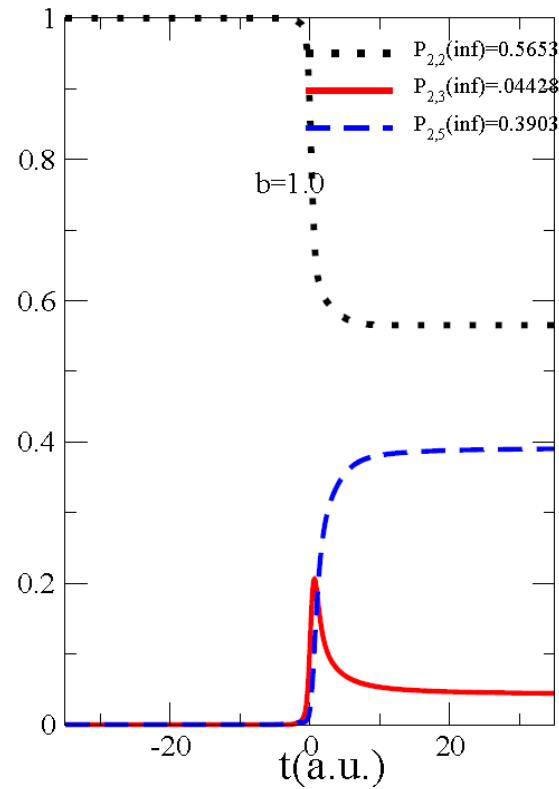
$$\rightarrow a(t_{qc}) = e^{-iH_{qc} t_{qc}} a(0)$$

- $$dt_{qc}/dt = \lambda,$$
$$t_{qc}(-\infty) = 0$$



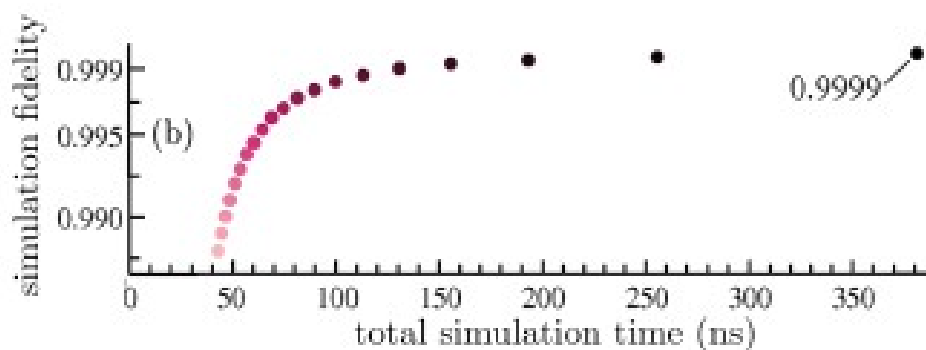
# Quantum dynamics: Quantum Computer(2)

## Scattering probabilities

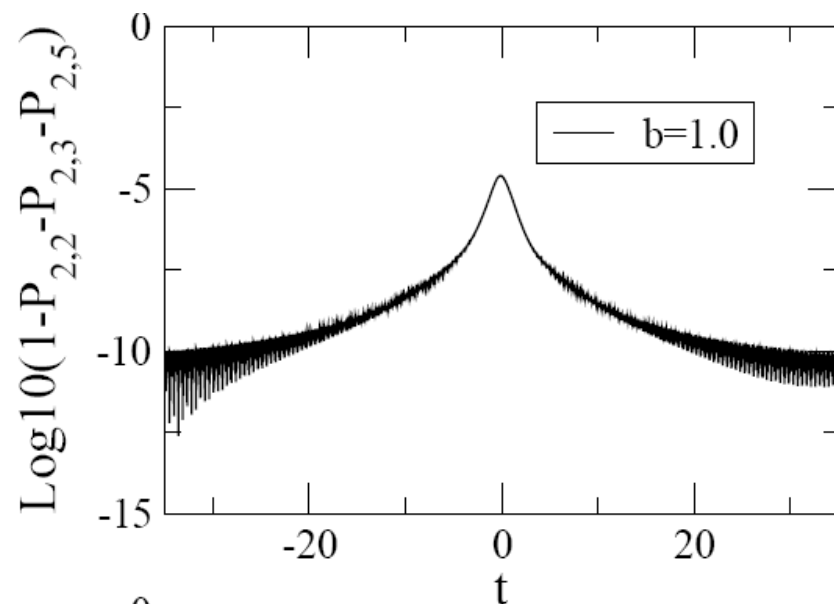


# Quantum Computer: Fidelity and Leakage

## Simulation fidelity



## Leakage



# Conclusion

- proposed a superconducting quantum simulator
- can simulate any random Hamiltonian
  - mapping error: zero
- example simulation of a molecular collision (electron orbital scattering)
  - 99% fidelity