Wave-Particle Duality for Multi-Slit Interference

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International School & Conference on Quantum Information-2016

Outline

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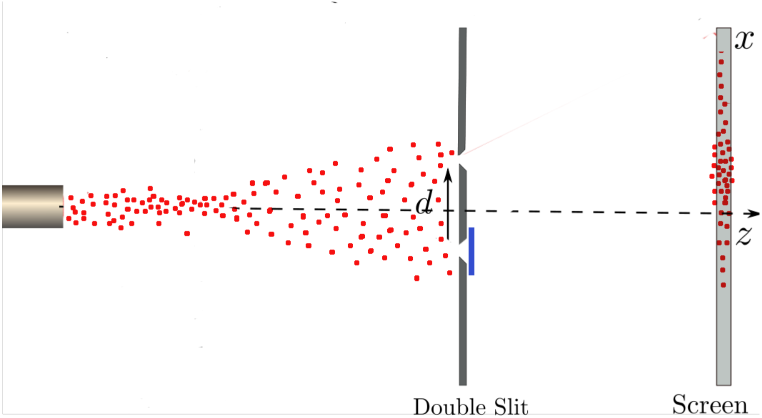
1. Two-Slit Experiment and Complementarity
2. Three-Slit interference
3. N-slit interference

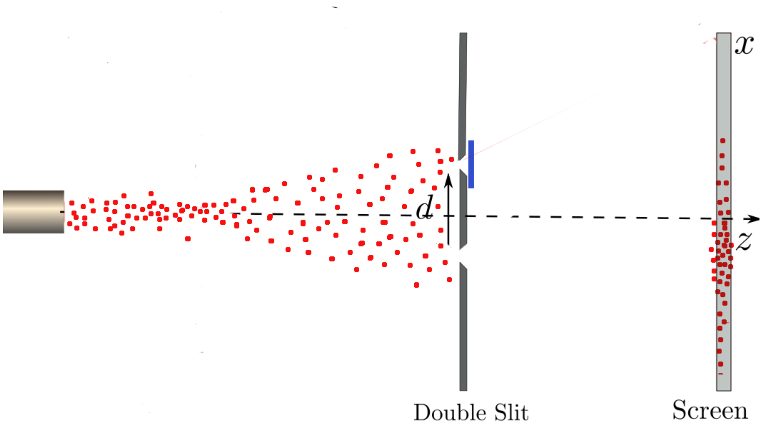
The Two-Slit Experiment.

Slit 2 closed



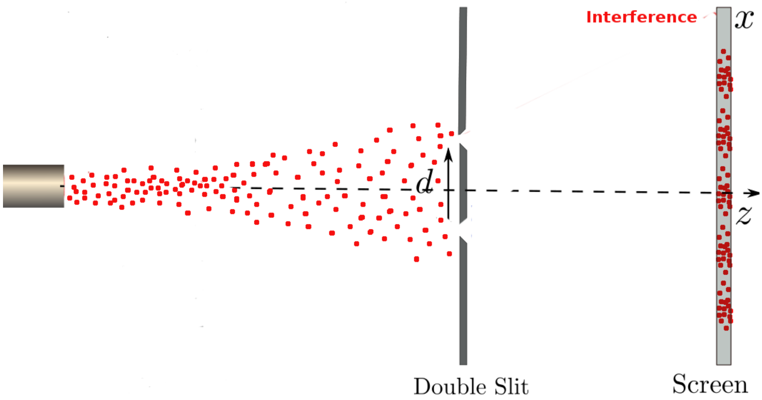
The Two-Slit Experiment.

Slit 1 closed



The Two-Slit Experiment.

Both slits open



Two-slit experiment with electrons

Tonomura, Endo, Matsuda, Kawasaki, Ezawa, *Am. J. Phys.* **57**(2) (1989).

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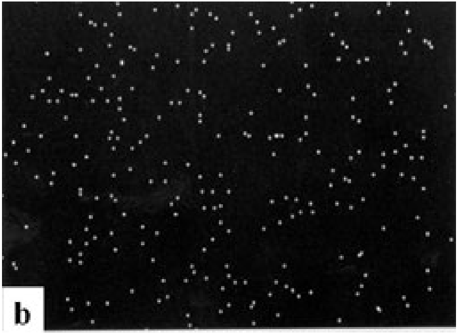

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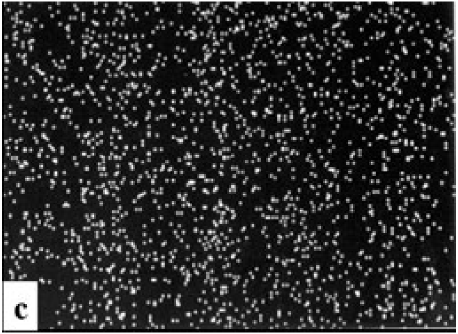

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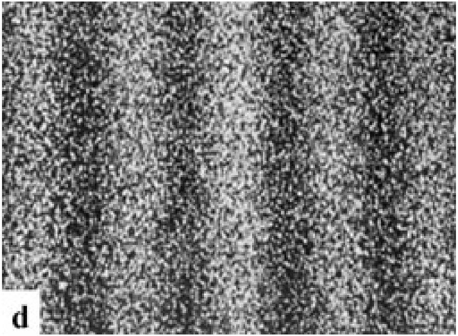
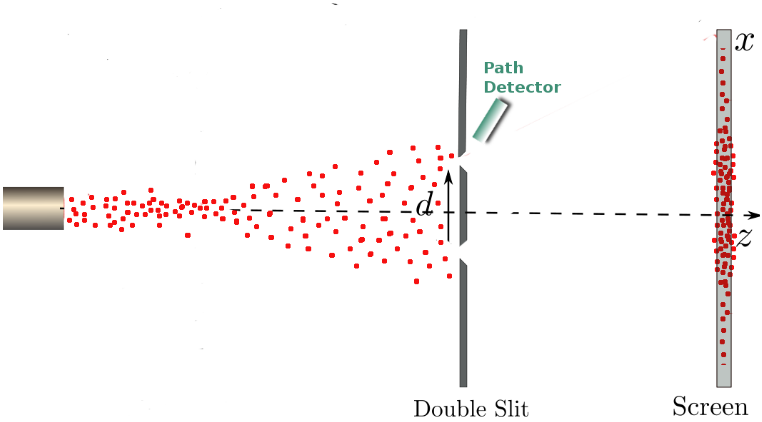


Which slit did the electron pass through?

Getting the "*Welcher-Weg*" (which-way) information



Bohr's Complementarity Principle.

Niels Bohr in 1928

Certain physical concepts are complementary. If two concepts are complementary, an experiment that clearly illustrates one concept will obscure the other complementary one....

- An experiment that illustrates the particle properties of light will not show any of the wave properties of light.
- an experiment that illustrates the wave properties of light will not show any of the particle nature of light.

In the two-slit experiment, the **"which-way"** information and the existence of **interference** pattern are mutually exclusive.

Either Wave Nature OR Particle Nature

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Either Wave Nature OR Particle Nature

Observing Wave & Particle nature simultaneously

What happens if one tries to observe both wave and particle nature at the same time?


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Complementarity in the double-slit experiment: Quantum nonseparability and a quantitative statement of Bohr's principle

William K. Wootters and Wojciech H. Zurek
Phys. Rev. D **19**, 473 – Published 15 January 1979

Trying to observe particle nature, blurs the interference

Quantitative Wave-Particle Duality

D.M. Greenberger, A. Yasin, *Phys. Lett. A* **128**, 391 (1988).

$\psi = ae^{ikx} + be^{-ikx}$

Physics Letters A

Volume 128, Issue 8, 18 April 1988, Pages 391–394

Simultaneous wave and particle knowledge in a neutron interferometer

Daniel M. Greenberger, Allaine Yasin

City College of the City of New York, New York, NY 10031, USA

Probability density on the screen

$$|\psi|^2 = |a|^2 + |b|^2 + 2|a||b|\cos(kx+\phi)$$

$\mathcal{V} \rightarrow$ Visiblity of interference

$$\mathcal{V} \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2|a||b|}{|a|^2 + |b|^2}$$

$\mathcal{P} \rightarrow$ Predictability of the path

$$\mathcal{P} = \frac{|a|^2 - |b|^2}{|a|^2 + |b|^2}$$

$$\mathcal{P}^2 + \mathcal{V}^2 \leq 1$$

A quantitative statement of wave-particle duality

Predictability and Visiblity cannot be 1 at the same time.

Refinement:

Two interferometric complementarities

Gregg Jaeger, Abner Shimony, and Lev Vaidman

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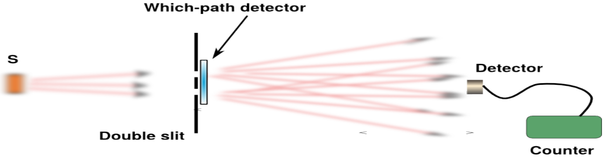

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Particle goes through lower slit $\rightarrow$ Path-detector state $|d_2\rangle$

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Duality relation How much of wave and particle natures can be seen simultaneously

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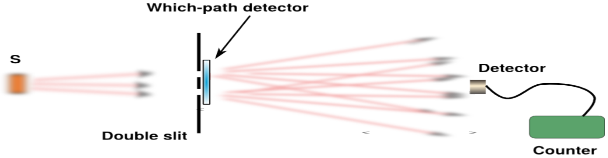

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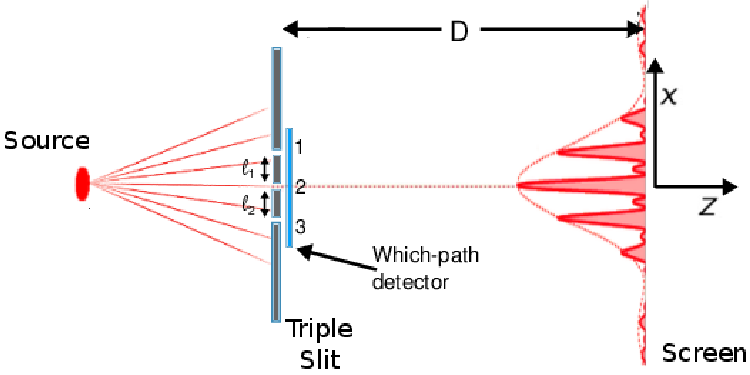
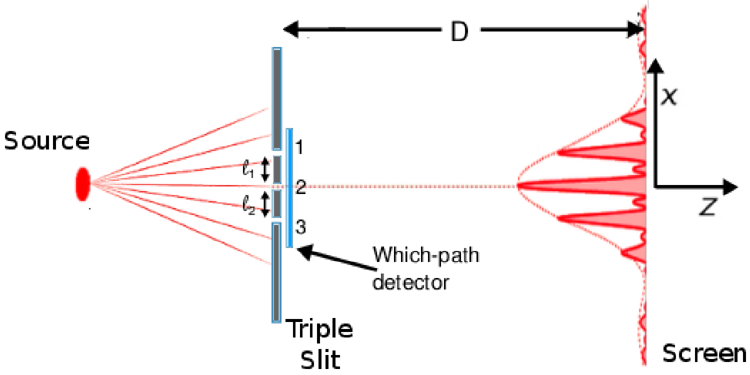
Triple-slit interference



Duality relation for 3-slit interference?

Triple-slit interference



Duality relation for 3-slit interference?

Attempts to find duality relation for 3-slit interference

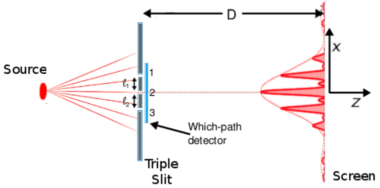
- G. Jaeger, A. Shimony, L. Vaidman, "Two interferometric complementarities," *Phys. Rev. A* **51**, 54 (1995).
- S. Dürr, "Quantitative wave-particle duality in multibeam in terferometers," *Phys. Rev. A* **64**, 042113 (2001).
- G. Bimonte, R. Musto, "Comment on 'Quantitative wave-particle duality in multibeam interferometers'," *Phys. Rev. A* **67**, 066101 (2003).
- G. Bimonte, R. Musto, "On interferometric duality in multibeam experiments" *J. Phys. A: Math. Gen.* **36**, 11481 (2003). (2003).
- B-G. Englert et al., "Wave-particle duality in multi-path interferometers: General concepts and three-path interferometers," *Int. J. Quantum Inform.* **6**, 129 (2008).
- M. Zawisky, M. Baron, R. Loidl, "Three-beam interference and which-way information in neutron interferometry," *Phys. Rev. A* **66**, 063608 (2002).
- Alfredo Luis, Phys. Rev. A 78, 025802 (2008).
- D. Kaszlikowski, L.C. Kwek, M. Zukowski, B-G. Englert, Phys. Rev. Lett. 91, 037901 (2003).

Other recent works on 3-slit interference

- U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, G. Weihs, "Ruling Out Multi-Order Interference in Quantum Mechanics", *Science* **329**, 418-421 (2010).
- H.D. Raedt, K. Michielsen, K. Hess, "Analysis of multipath interference in three-slit experiments", *Phys. Rev. A* **85**, 012101 (2012).
- R. Sawant, J. Samuel, A. Sinha, S. Sinha, U. Sinha, "Nonclassical paths in quantum interference experiments," *Phys. Rev. Lett.* **113**, 120406 (2014).

Finding which way the particle went

State of the particle emerging from the triple slit



$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|\psi_1\rangle|d_1\rangle + |\psi_2\rangle|d_2\rangle + |\psi_3\rangle|d_3\rangle)$$

$|d_1\rangle, |d_2\rangle, |d_3\rangle \rightarrow$ states of path-detector

$|d_3\rangle \Rightarrow$ Particle went through slit 3
$|d_2\rangle \Rightarrow$ Particle went through slit 2
$|d_1\rangle \Rightarrow$ Particle went through slit 1

Problem of finding out which-slit the particle went through

$\downarrow$ reduces to

Problem of distinguishing between $|d_1\rangle, |d_2\rangle, |d_3\rangle$

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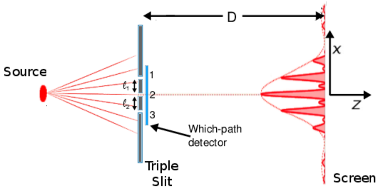

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Unambiguous Quantum State Discrimination (UQSD)

If two states $|d_1\rangle, |d_2\rangle$ are orthogonal
An operator exists:

$$\boldsymbol{A}|d_1\rangle = a_1|d_1\rangle, \boldsymbol{A}|d_2\rangle = a_2|d_2\rangle$$

By measuring $\boldsymbol{A}$ one can tell if the state is $|d_1\rangle$ or $|d_2\rangle$

If $|d_1\rangle, |d_2\rangle$ are not orthogonal: no such operator exists

One cannot distinguish between two non-orthogonal states 100%
There will always be some error

Unambiguous Quantum State Discrimination

Yields two kinds of measurement results, at random:

1. One can distinguish 100% between the two states

2. One cannot distinguish between the two states at all

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Unambiguous Quantum State Discrimination

Let an ancilla system interact with the d-system

$$
\begin{aligned}
\mathbf{U}_a|d_1\rangle|a_0\rangle &= \alpha|p_1\rangle|a_1\rangle + \beta|q\rangle|a_2\rangle \\
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$$

$\langle p_1|p_2\rangle = 0 \qquad \langle a_1|a_2\rangle = 0$ $\qquad |\beta|^2 = |\langle d_1|d_2\rangle|, \quad |\alpha|^2 = 1 - |\langle d_1|d_2\rangle|$

It can be shown that such an interaction always exists.

Probability of failure = $|\beta|^2 = |\langle d_1|d_2\rangle|$

Probability of success = $|\alpha|^2 = 1 - |\langle d_1|d_2\rangle|$

$|d_1\rangle, |d_1\rangle$ can be distinguished without error with a maximum probability

$$P = 1 - |\langle d_1|d_2\rangle|$$

This should be a natural definition of distinguishability!

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A new definition of distinguishability

Which slit a particle went through in a two-slit interference experiment can be determined with the same probability with which one can distinguish between $|d_1\rangle, |d_2\rangle$.

New distinguishability

$$\mathcal{D}_Q = 1 - |\langle d_1|d_2\rangle|$$

Contrast this with Englert's distinguishability $\mathcal{D} = \sqrt{1 - |\langle d_1|d_2\rangle|^2}$

UQSD has been generalized to N non-orthogonal states $|d_1\rangle, |d_2\rangle, |d_3\rangle, \ldots |d_N\rangle$ $\quad$ $|d_k\rangle$ occurs with a probability p_k

Probability to unambiguously tell which of the N states is a given state

$$P_N \leq 1 - \frac{1}{N-1}\sum_{i\neq j}\sqrt{p_i p_j}|\langle d_i|d_j\rangle|$$

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Distinguishability for 3-slit interference

Probability to unambiguously tell which of the 3 states, $|d_1\rangle, |d_2\rangle, |d_3\rangle$ is a given state

$$P_3 \leq 1 - \left(\sqrt{p_1 p_2}|\langle d_1|d_2\rangle| + \sqrt{p_2 p_3}|\langle d_2|d_3\rangle| + \sqrt{p_1 p_3}|\langle d_1|d_3\rangle|\right)$$

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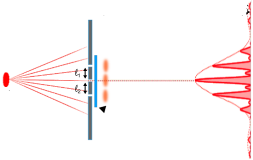
Wave-packet dynamics

State of the particle when it comes out of triple-slit



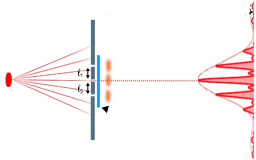
$$\Psi(x,0) = A\left(\sqrt{p_1}|d_1\rangle e^{-\frac{(x-\ell_1)^2}{4\epsilon^2}} + \sqrt{p_2}|d_2\rangle e^{-\frac{x^2}{4\epsilon^2}} + \sqrt{p_3}|d_3\rangle e^{-\frac{(x+\ell_2)^2}{4\epsilon^2}}\right)$$

$$\Psi(x,0) \xrightarrow{\text{Time evolution}} \xrightarrow{H=\frac{p_x^2}{2m}} \Psi(x,t)$$ Particle reaches screen

$$\begin{aligned}|\Psi(x,t)|^2 &= |A|^2\left(e^{-\frac{x^2}{2\sigma^2}}\left(p_1 e^{-\frac{\ell_1^2-2x\ell_1}{2\sigma^2}} + p_2 + p_3 e^{-\frac{\ell_2^2+2x\ell_2}{2\sigma^2}}\right)\right.\\ &\quad +2\sqrt{p_1p_2}|\langle d_1|d_2\rangle|e^{-\frac{2x^2+\ell_1^2-2x\ell_1}{4\sigma^2}}\cos\left(\frac{x\ell_1\hbar t}{4m\Omega^2}\right) + 2\sqrt{p_2p_3}|\langle d_2|d_3\rangle|e^{-\frac{2x^2+\ell_2^2+2x\ell_2}{4\sigma^2}}\cos\left(\frac{x\ell_2\hbar t}{4m\Omega^2}\right)\\ &\quad \left. + 2\sqrt{p_1p_3}|\langle d_1|d_3\rangle|e^{-\frac{2x^2+\ell_1^2+\ell_2^2+2x(\ell_2-\ell_1)}{4\sigma^2}}\cos\left(\frac{x(\ell_1+\ell_2)\hbar t}{4m\Omega^2}\right)\right),\end{aligned}$$

Wave-packet dynamics

State of the particle when it comes out of triple-slit



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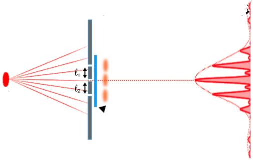

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\end{aligned} \tag{1}$$

Visibility of interference

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}},$$ Visibility

From our expression for interference, we get

$$\mathcal{V} \leq \frac{3\left(\sqrt{p_1 p_2}|\langle d_1|d_2\rangle| + \sqrt{p_1 p_3}|\langle d_1|d_3\rangle| + \sqrt{p_2 p_3}|\langle d_2|d_3\rangle|\right)}{2 + \sqrt{p_1 p_2}|\langle d_1|d_2\rangle| + \sqrt{p_1 p_3}|\langle d_1|d_3\rangle| + \sqrt{p_2 p_3}|\langle d_2|d_3\rangle|}.$$

Using $\mathcal{D}_Q \equiv 1 - (\sqrt{p_1 p_2}|\langle d_1|d_2\rangle| + \sqrt{p_2 p_3}|\langle d_2|d_3\rangle| + \sqrt{p_1 p_3}|\langle d_1|d_3\rangle|)$ we get

$$\mathcal{V} + \frac{2\mathcal{D}_Q}{3-\mathcal{D}_Q} \leq 1 \quad \text{or} \quad \mathcal{D}_Q + \frac{2\mathcal{V}}{3-\mathcal{V}} \leq 1$$

A new duality relation for 3-slit interference [1]

[1] M.A. Siddiqui, T. Qureshi, Prog. Theor. Exp. Phys. 2015, 083A02 (2015)

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Outline

1. Two-Slit Experiment and Complementarity
2. Three-Slit interference
3. N-slit interference

N-slit (or N-path) interference

in the presence of a path-detector

$$|\Psi\rangle = c_1|\psi_1\rangle|d_1\rangle + c_2|\psi_2\rangle|d_2\rangle + c_3|\psi_3\rangle|d_3\rangle + \cdots + c_N|\psi_N\rangle|d_N\rangle$$

Distinguishability for N-path interference

$$\boxed{\mathcal{D}_Q = 1 - \frac{1}{N-1}\sum_{j\neq k}\sqrt{p_j p_k}|\langle d_j|d_k\rangle|}$$

$\sum_{j,k=1}^{N}$ Two-slit interference from j'th and k'th slits $\rightarrow$ N-slit interference

Calculation of visibility difficult

Density matrix

$$\rho = |\Psi\rangle\langle\Psi|$$

Reduced density matrix of the particle

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Measure of Coherence

Reduced density matrix of the particle

$$\rho_s \equiv Tr_{path-detector}|\Psi\rangle\langle\Psi| = \sum_{j=1}^{n}\sum_{k=1}^{n} c_j c_k^* \langle d_k|d_j\rangle\, |\psi_j\rangle\langle\psi_k|$$

PRL 113, 140401 (2014) PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2014

Quantifying Coherence

T. Baumgratz, M. Cramer, and M. B. Plenio
Institut für Theoretische Physik, Albert-Einstein-Allee 11, Universität Ulm, 89069 Ulm, Germany
(Received 24 February 2014; revised manuscript received 18 July 2014; published 29 September 2014)

l_1-norm of coherence:

$$C_{l_1}(\rho) = \sum_{j\neq k} |\rho_{jk}|$$

Shown to be a good measure of coherence

Minimum value is zero. Maximum value not fixed.

Coherence as a measure of wave-nature

We introduce a quantity Coherence

$$\mathcal{C}(\rho) \equiv \frac{1}{N-1} \sum_{j \neq k} |\rho_{jk}|$$

(is basis dependent)

Coherence values: $0 \leq \mathcal{C} \leq 1$.

For a maximally coherent state
$|\Psi\rangle = \frac{1}{\sqrt{N}}\left(|\psi_1\rangle + |\psi_2\rangle + |\psi_3\rangle + \cdots + |\psi_N\rangle\right)$

$$\mathcal{C} = 1$$

For a completely diagonal density matrix

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Wave-particle duality in N-path interference

Initial state

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Distinguishability

$$\mathcal{D}_Q = 1 - \frac{1}{N-1}\sum_{j\neq k}|c_j c_k||\langle d_j|d_k\rangle|$$

Coherence

$$\mathcal{C} = \frac{1}{N-1}\sum_{j\neq k}|\langle\psi_j|\rho_s|\psi_k\rangle| = \frac{1}{N-1}\sum_{j\neq k}|c_j||c_k||\langle d_k|d_j\rangle|$$

$$\boxed{\mathcal{C} + \mathcal{D}_Q = 1}$$

First ever duality relation for N-slit interference [2]

[2] M.N. Bera, T. Qureshi, M.A. Siddiqui, A.K. Pati, Phys. Rev. A 92, 012118 (2015)

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General duality relation in various limits

N=3 (Three-slit interference)

Path-distinguishability becomes

$\mathcal{D}_Q = 1 - (|c_1 c_2||\langle d_1|d_2\rangle| + |c_2 c_3||\langle d_2|d_3\rangle| + |c_1 c_3||\langle d_1|d_3\rangle|)$

Coherence reduces to

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Relation between coherence and ideal interference visibility by

$$\mathcal{C} = \frac{2\mathcal{V}}{3-\mathcal{V}}.$$

The duality relation $\mathcal{D}_Q + \mathcal{C} = 1$ reduces to

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Exactly the same as the duality relation derived for the 3-slit interference

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General duality relation in various limits

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Path-distinguishability becomes

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Coherence reduces to

$$\mathcal{C} = 2|c_1 c_2||\langle d_1|d_2\rangle|$$

But $|c_1 c_2||\langle d_1|d_2\rangle|$ is also equal to the visibility!

The duality relation $\mathcal{D}_Q + \mathcal{C} = 1$ reduces to

$$\boxed{\mathcal{D}_Q + \mathcal{V} = 1}$$

A new duality relation for two-slit interference

- For $c_1 = c_2 = \frac{1}{\sqrt{2}}$, $\mathcal{D}_Q + \mathcal{V} = 1$ reduces to $\mathcal{D}^2 + \mathcal{V}^2 = 1$ Englert's relation
- For $|\langle d_1|d_2\rangle| = 1$, $\mathcal{D}_Q + \mathcal{V} = 1$ reduces to $\mathcal{P}^2 + \mathcal{V}^2 \leq 1$ Greenberger's relation

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Conclusions

- Wave-nature and particle nature can be seen at the same time, although to a limited degree
- For 3-slit interference, fringe visibility and path distinguishablity obey a new duality relation $\mathcal{V} + \frac{2\mathcal{D}_Q}{3-\mathcal{D}_Q} \leq 1$
- For N-slit interference, wave-nature is quantified by quantum *coherence* $\mathcal{C}$. The duality relation is the simplest $\mathcal{C} + \mathcal{D}_Q = 1$

Three slit interference: A duality relation
M.A. Siddiqui, T. Qureshi, *Prog. Theor. Exp. Phys.* 2015, 083A02 (2015).

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M.N. Bera, T. Qureshi, M.A. Siddiqui, A.K. Pati, *Phys. Rev. A* 92, 012118 (2015).

Understanding Quantitative Wave-Particle Duality
T. Qureshi, *arXiv:1501.02195 [quant-ph]*

Quantum twist to complementarity: A duality relation
T. Qureshi, *Prog. Theor. Exp. Phys. (Letters)* 2013, 041A01 (2013).