Controlling quantum state and its coherence

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D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770

Motivation

$$|\psi\rangle = \sum_i k_i \, |i\rangle$$

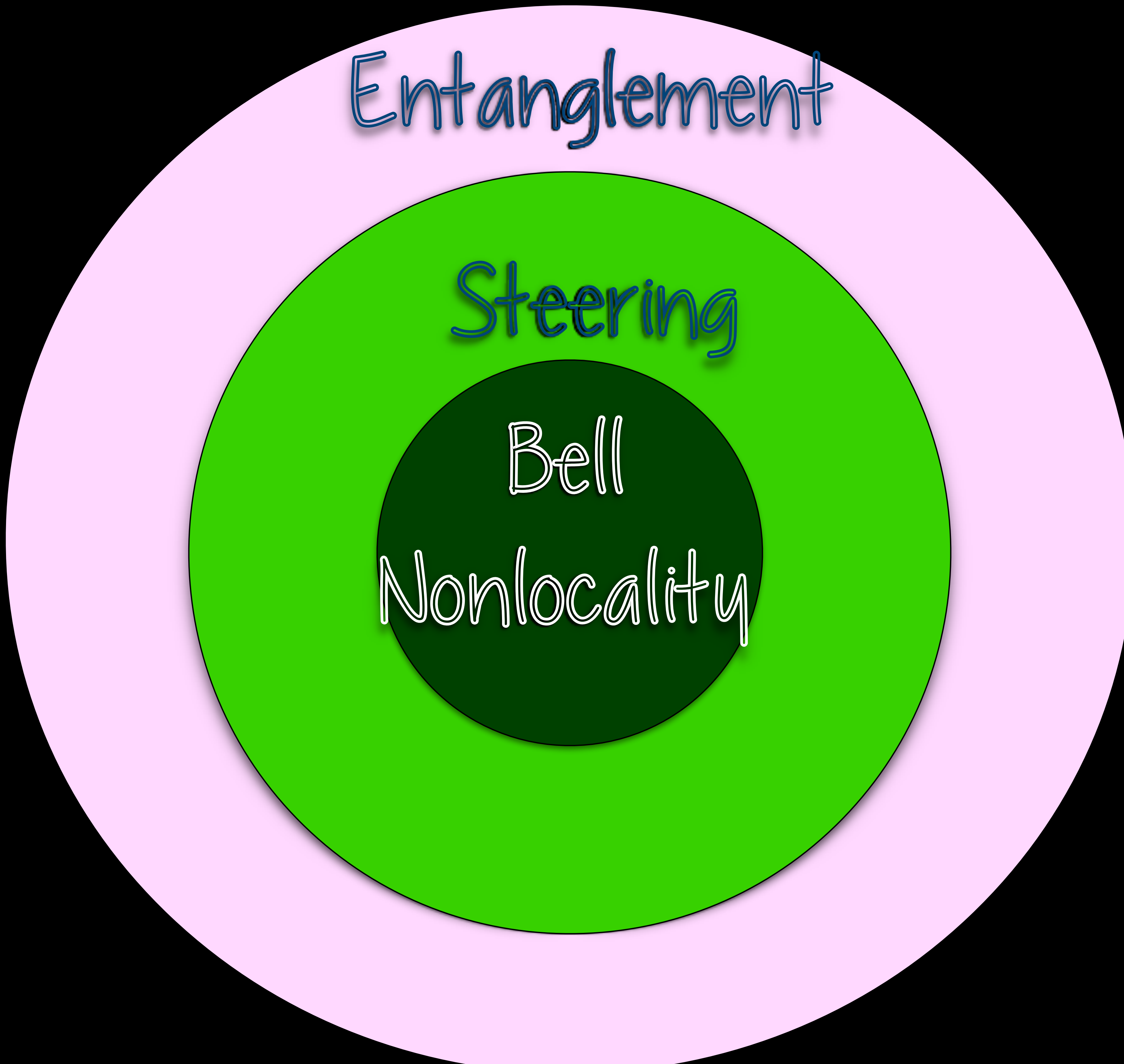
STATE $\rightleftharpoons$ SUPERPOSITION $\rightleftharpoons$ COHERENCE

CONTROLLING STATE $\overset{?}{\rightleftharpoons}$ CONTROLLING COHERENCE

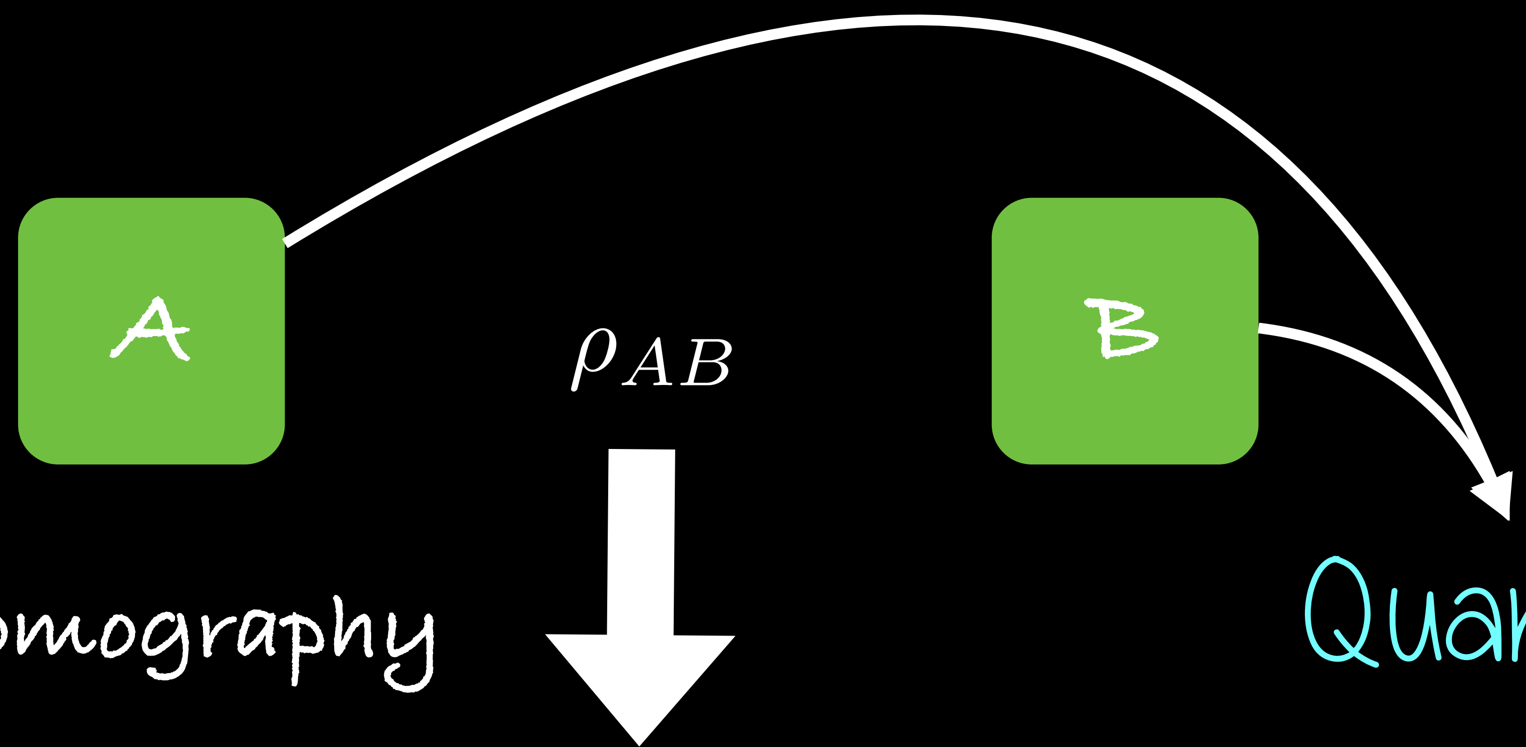
Outline of talk

- Quantum correlations
- Uncertainty relations
- Quantum coherence
- Steering of quantum state
- Steering of quantum coherence
- Example: Pure state and mixed state

Quantum Correlations



Entanglement



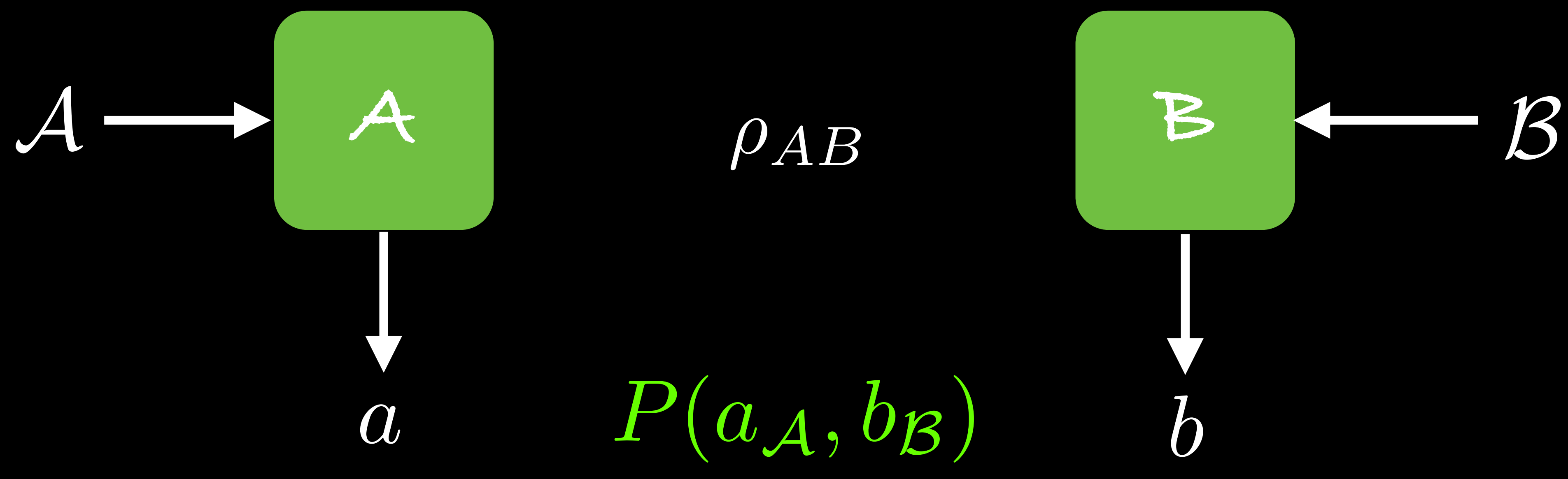
State tomography

Quantum

$$\rho_{AB} \neq \sum_i p_i \, \rho_i^A \otimes \rho_i^B$$

It is the weakest form of non-local correlation

Bell-nonlocal correlation



It violates any Bell-CHSH inequality

It is the strongest form of non-local correlation

Steering

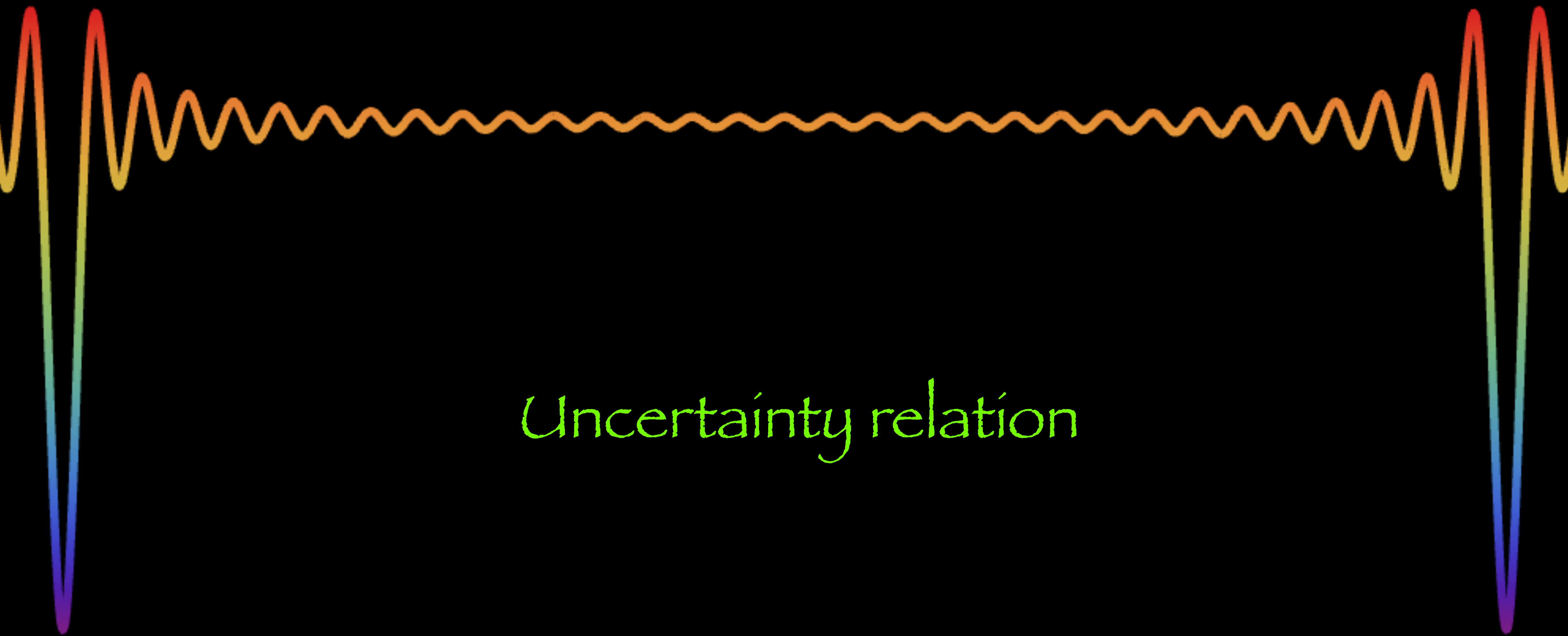
Steerable, if Alice can control the state of Bob's system

S. J. Jones, H. M. Wiseman, and A. C. Doherty, Phys. Rev. A **76**, 052116 (2007).

Quantum Correlations

Entanglement $<$ Steering $<$ Bell-nonlocal

S. J. Jones, H. M. Wiseman, and A. C. Doherty, Phys. Rev. A 76, 052116 (2007).



Uncertainty relation

Uncertainty relation

Two non-commuting observables can not be measured simultaneously with arbitrary precision.

Coarse grained form : Heisenberg uncertainty relation (HUR) and entropic form of uncertainty relation (EUR).

Fine-grained form : Fine-grained uncertainty relation (FUR).

HUR

$$\Delta\mathcal{A}\,\Delta\mathcal{B} \;\geq\; \frac{|\langle[\mathcal{A},\mathcal{B}]\rangle|}{2}$$

Here, uncertainty is measured by standard deviation which is coarse grained measure of uncertainty.

$$\Delta\mathcal{A} = \sqrt{\sum_a a^2\,p_a \;-\; \sum_a (a\,p_a)^2}$$

W. Heisenberg, Z. Phys. 43, 172 (1927); E. H. Kennard, Z. Phys. 44, 326 (1927).

EUR

$$\mathcal{H}(\mathcal{A}) + \mathcal{H}(\mathcal{B}) \geq \log \frac{1}{c}$$

Here, uncertainty is measured by Shannon entropy.

$$\mathcal{H}(\mathcal{A}) = -\sum_a p_a \log p_a$$

and the complementarity, 1/c, is defined as

$$c = \max_{a,b} |\langle a|b\rangle|^2$$

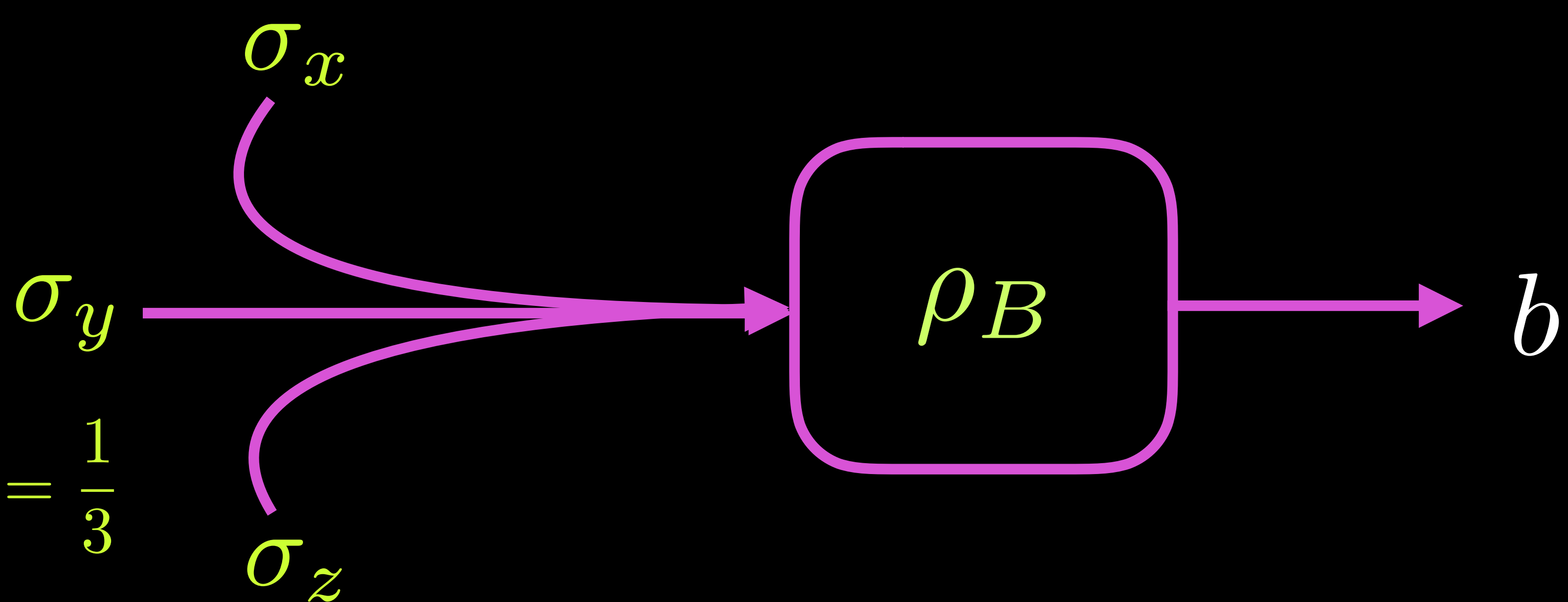
H. Maassen, and J. B. M. Uffink, Phys. Rev. Lett. 60, 1103 (1988).

Fine-grained Uncertainty relation

Here, uncertainty is measured by probability of a particular measurement outcome or a combination of measurement outcomes, i.e., fine-graining of all possible outcomes.

J. Oppenheim, and S. Wehner, Science 330, 1072 (2010).

FUR



$P(\sigma_i) = \frac{1}{3}$

$0 \equiv |\uparrow\rangle$

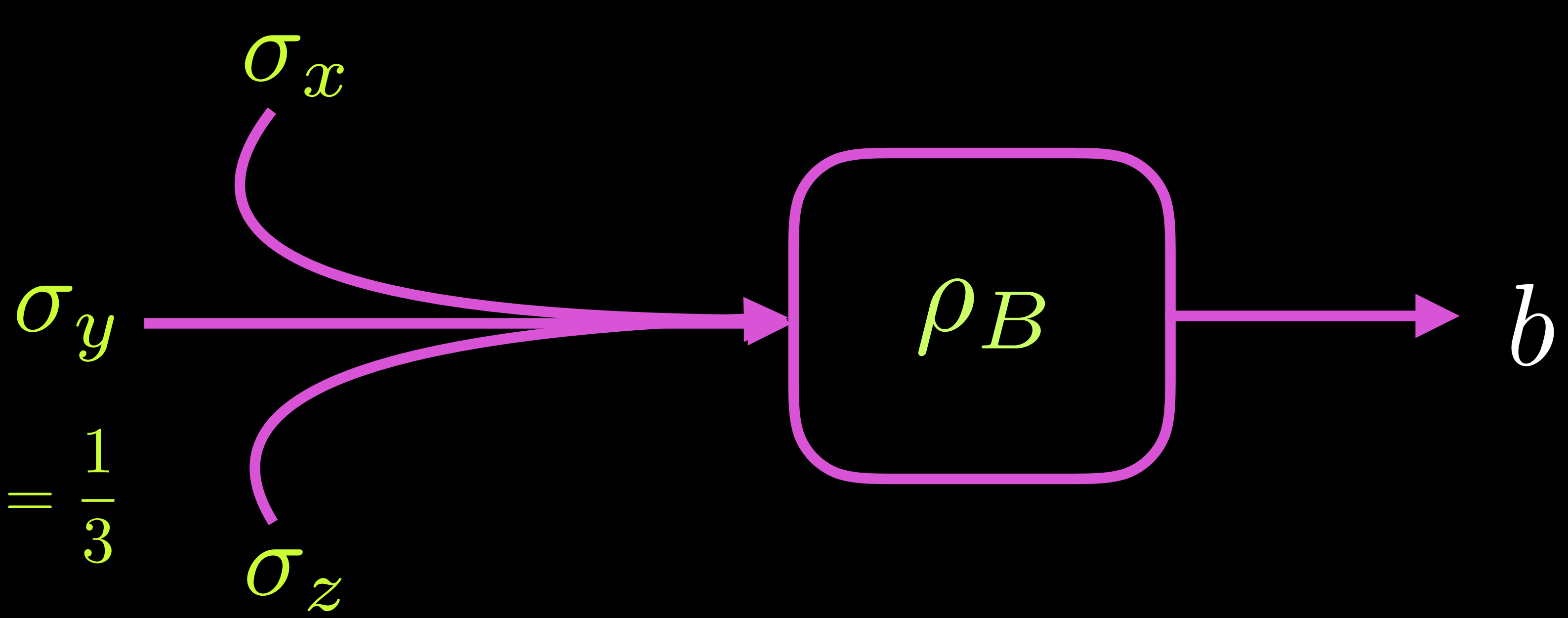
$1 \equiv |\downarrow\rangle$

$$P_{\text{Success}} = \sum_{i=1}^{3} P(\sigma_i)\, P(b_{\sigma_i} = 0) \leq P_{\text{Success}} = \max_{\rho_B} P_{\text{Success}}$$

J. Oppenheim, and S. Wehner, Science 330, 1072 (2010).

FUR

$$\rho_{\text{Certain}} = \frac{1}{2}\left(I + \frac{1}{\sqrt{3}}\left(\sigma_x + \sigma_y + \sigma_z\right)\right)$$

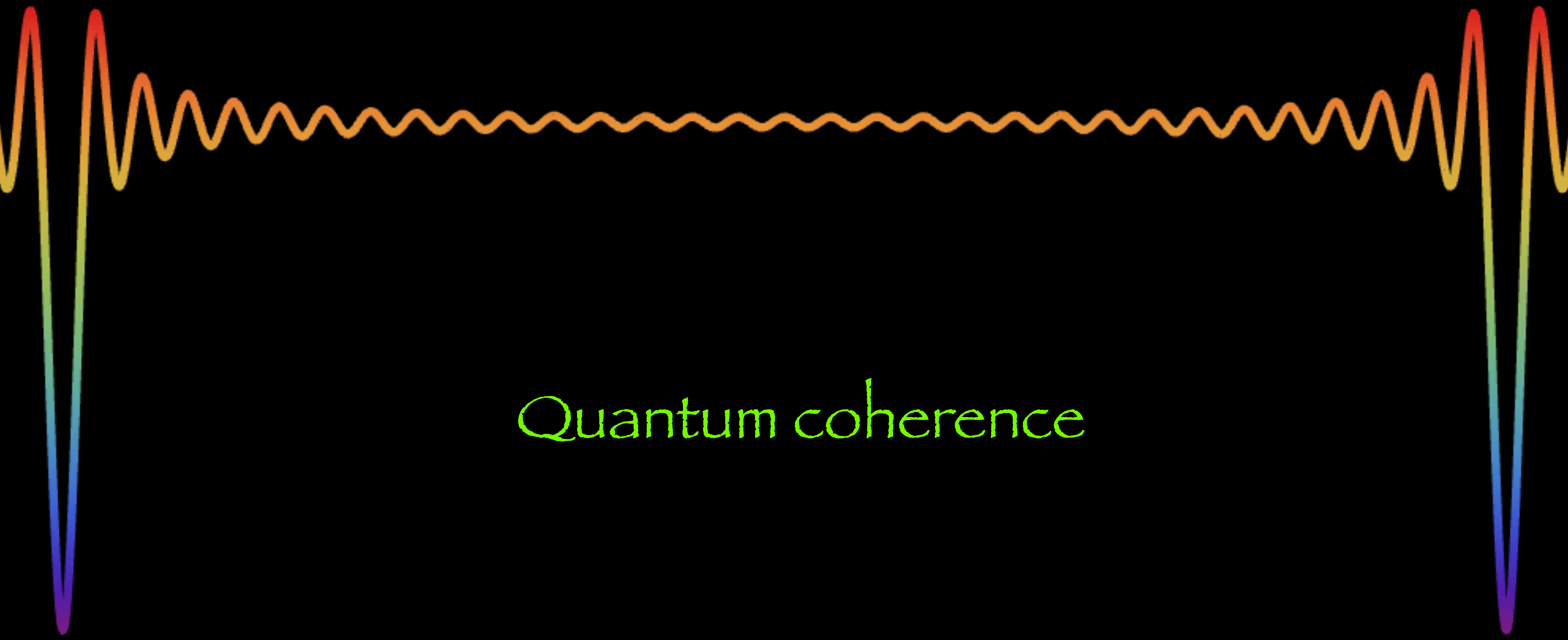


$P(\sigma_i) = \frac{1}{3}$

$0 \equiv |\uparrow\rangle$

$1 \equiv |\downarrow\rangle$

$$P(0_{\sigma_x}) + P(0_{\sigma_y}) + P(0_{\sigma_z}) \leq \frac{3}{2} + \frac{3}{2\sqrt{3}}$$



Quantum coherence

Quantum coherence

Zero Coherence: All off-diagonal terms are zero.

$$\rho_B = \begin{pmatrix} x_1 & & & \rho_B(i,j|i \neq j) = 0 & \\ & x_2 & & & \\ & & x_3 & & \\ \rho_B(i,j|i \neq j) = 0 & & & \ddots & \\ & & & & x_n \end{pmatrix}$$

Measures of Quantum coherence

l_1-norm:

$$C^{l_1}(\rho_B) = \sum_{i,j,i\neq j} |\rho_B(i,j)|$$

Relative entropy of coherence:

$$C^E(\rho_B) = \mathcal{S}(\rho_B^D) - \mathcal{S}(\rho_B)$$

ρ_B^D : diagonal matrix formed with diagonal element of ρ_B.

T. Baumgratz, M. Cramer, and M. B. Plenio, Phys. Rev. Lett. 113, 140401 (2014).

Measures of Quantum coherence

Skew information: the coherence of the state in the basis of eigenvectors of the observable

$$C^S_{\mathcal{B}}(\rho_B) = -\frac{1}{2}Tr\left[\sqrt{\rho_B}, \mathcal{B}\right]^2$$

S. Luo, Phys. Rev. Lett. 91, 180403 (2003); Theor. Math. Phys. 143, 681 (2005).

Quantum coherence

Is it possible to measure quantum coherence with arbitrary precision in all possible mutually non-commuting basis, simultaneously?

Coherence complementarity relations

Coherence complementarity relations

$$C_x^{l_1}(\rho) + C_y^{l_1}(\rho) + C_z^{l_1}(\rho) \leq \sqrt{6}$$

$$C_x^{E}(\rho) + C_y^{E}(\rho) + C_z^{E}(\rho) \leq 2.23$$

$C_k^{E\,(l_1)}(\rho)$: is calculated by writing the state in basis σ_k.

$$\rho_{\max}^{C} = \frac{1}{2}\left(I + \frac{1}{\sqrt{3}}\left(\sigma_x + \sigma_y + \sigma_z\right)\right)$$

D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770

Coherence complementarity relations

$$C_x^S(\rho) + C_y^S(\rho) + C_z^S(\rho) \leq 2$$

$C_k^S(\rho)$: is measured in basis σ_k .

$$\rho_{\max}^C = \frac{1}{2}\left(I + \frac{1}{\sqrt{3}}\left(\sigma_x + \sigma_y + \sigma_z\right)\right)$$

D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770

Quantum Steering

Quantum steering

EPR paradox

Entanglement is used to put question about incompleteness of quantum physics by Einstein, podolsky and Rosen.

Steering

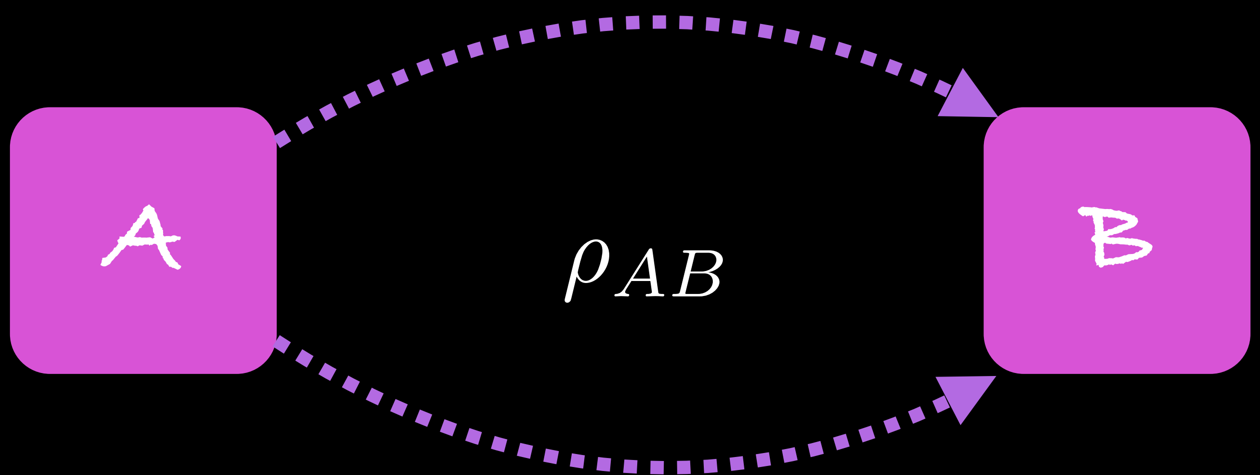
Schrodinger re-expressed EPR paradox as the power to control of one system by distantly located system.

A. Einstein, D. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).

E. Schrodinger, Proc. Cambridge Philos. Soc. 31, 553 (1935); 32, 446 (1936).

EPR-Steering : Physical interpretation

$$\mathcal{R} = \sigma_z$$



$$\mathcal{S} = \sigma_x$$

Steerability : Alice's control on the state of Bob's system, i.e., Bob can know his system with higher precision than allowed by uncertainty principle.

EPR-Steering : Physical interpretation

$\mathcal{R} = \sigma_z$

A

$$|\Psi\rangle_{AB} = \frac{|\uparrow\downarrow\rangle_{AB} - |\downarrow\uparrow\rangle_{AB}}{2}$$

B

$\mathcal{S} = \sigma_x$

Alice's measurement outcome	State of Bob's system
$\vert\uparrow\rangle_{z\,(x)}$	$\vert\downarrow\rangle_{z\,(x)}$
$\vert\downarrow\rangle_{z\,(x)}$	$\vert\uparrow\rangle_{z\,(x)}$

Bob's Uncertainty of system B is zero when Alice communicates her results

EPR-Steering : Mathematical interpretation

Steerability : Absence of local hidden state (LHS) model for Bob's system.

$$\rho_{AB} \neq \sum_{\lambda} P(\lambda)\, \rho_{\lambda}^{A} \otimes \rho_{\lambda,\, Q}^{B}$$

$$P(a_{\mathcal{A}},\, b_{\mathcal{B}}) \neq \sum_{\lambda} P(\lambda)\, P(a_{\mathcal{A}}|\lambda)\, P_{Q}(b_{\mathcal{B}}|\lambda)$$

S. J. Jones, H. M. Wiseman, and A. C. Doherty, Phys. Rev. A **76**, 052116 (2007).

Local Hidden State model

LHS model

Local : Alice prepares system B in a state quantumly uncorrelated with other systems possessed by Alice.

Hidden : Bob has no information about the state of B.

$$\rho_{AB} = \sum_{\lambda} P(\lambda)\, \rho_{\lambda}^{A} \otimes \rho_{\lambda,\,Q}^{B}$$

Result : Once Alice sends the system B to Bob, Alice does not have any control on the state of system B.

Steering criterion : Intuition

When Alice and Bob share steerable state, Alice can reduce Bob's uncertainty about his system by controlling its state.

$P(b_{\mathcal{B}}\|a_{\mathcal{A}})$	It should violate some local uncertainty relation satisfied by $P(b_{\mathcal{B}})$

Steerability of quantum state

Fine-grained steering criteria

$$P(a_{\mathcal{A}}, b_{\mathcal{B}}) = \sum_{\lambda} P(\lambda) P(a_{\mathcal{A}}|\lambda)\, P_Q(b_{\mathcal{B}}|\lambda)$$

$$+$$

$$q_{min} \sum_i p_i \leq \sum_i p_i\, q_i \leq q_{\max} \sum_i p_i$$

$$\alpha$$

$$\mathbf{P(b_{\sigma_x}) + P(b_{\sigma_y}) + P(b_{\sigma_z}) \leq \frac{3}{2} + \frac{3}{2\sqrt{3}}}$$

$$P(b_{\sigma_x}|a_{\mathcal{A}_1}) + P(b_{\sigma_y}|a_{\mathcal{A}_2}) + P(b_{\sigma_z}|a_{\mathcal{A}_3}) \leq \frac{3}{2} + \frac{3}{2\sqrt{3}}$$

T. Pramanik, M. Kaplan, and A. S Majumdar, Phys. Rev. A 90, 050305(R) (2014).

Steerability of pure entailed state

$$\rho_P = \sqrt{\alpha}\,|00\rangle + \sqrt{1-\alpha}\,|11\rangle$$

All pure entangled states are maximally steerable

$$P(b_{\sigma_x}|a_{\mathcal{A}_1}) + P(b_{\sigma_y}|a_{\mathcal{A}_2}) + P(b_{\sigma_z}|a_{\mathcal{A}_3}) = 3$$

T. Pramanik, M. Kaplan, and A. S Majumdar, Phys. Rev. A 90, 050305(R) (2014).

Steerability of werner state

$$\rho_W = p\,\rho_S + (1-p)\,\frac{I}{4}$$

$$\rho_S = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Steerability of werner state

$$\rho_W = p\,\rho_S + (1-p)\,\frac{I}{4}$$

$p > \dfrac{1}{3}$: The state is entangled.

$p > \dfrac{1}{2}$: The state is steerable with infinite measurement settings.

$p > \dfrac{1}{\sqrt{3}}$: The state is steerable with three measurement settings.

$p > \dfrac{1}{\sqrt{2}}$: The state is Bell nonlocal and steerable with two settings.

Steerability of quantum coherence

Steerability of quantum coherence

A

η_{AB}

B

$\Pi^a_{\sigma_i}$

$\eta_{B|\Pi^a_{\sigma_i}}$

Steerable, if it violates coherence complementarity relation

Steerability of quantum coherence

$$C_x^{l_1}(\eta_{B|\Pi^a_{y\,(z)}}) + C_y^{l_1}(\eta_{B|\Pi^a_{z\,(x)}}) + C_z^{l_1}(\eta_{B|\Pi^a_{x\,(y)}}) > \sqrt{6}$$

$$C_x^{E}(\eta_{B|\Pi^a_{y\,(z)}}) + C_y^{E}(\eta_{B|\Pi^a_{z\,(x)}}) + C_z^{E}(\eta_{B|\Pi^a_{x\,(y)}}) > 2.23$$

$$C_x^{S}(\eta_{B|\Pi^a_{y\,(z)}}) + C_y^{S}(\eta_{B|\Pi^a_{z\,(x)}}) + C_z^{S}(\eta_{B|\Pi^a_{x\,(y)}}) > 2$$

D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770.

Steerability of pure entailed state

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D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770.

Steerability of werner state

$$\rho_W = p\,\rho_S + (1-p)\,\frac{I}{4}$$

$p > 0.82$: Steerable under l_1-norm.

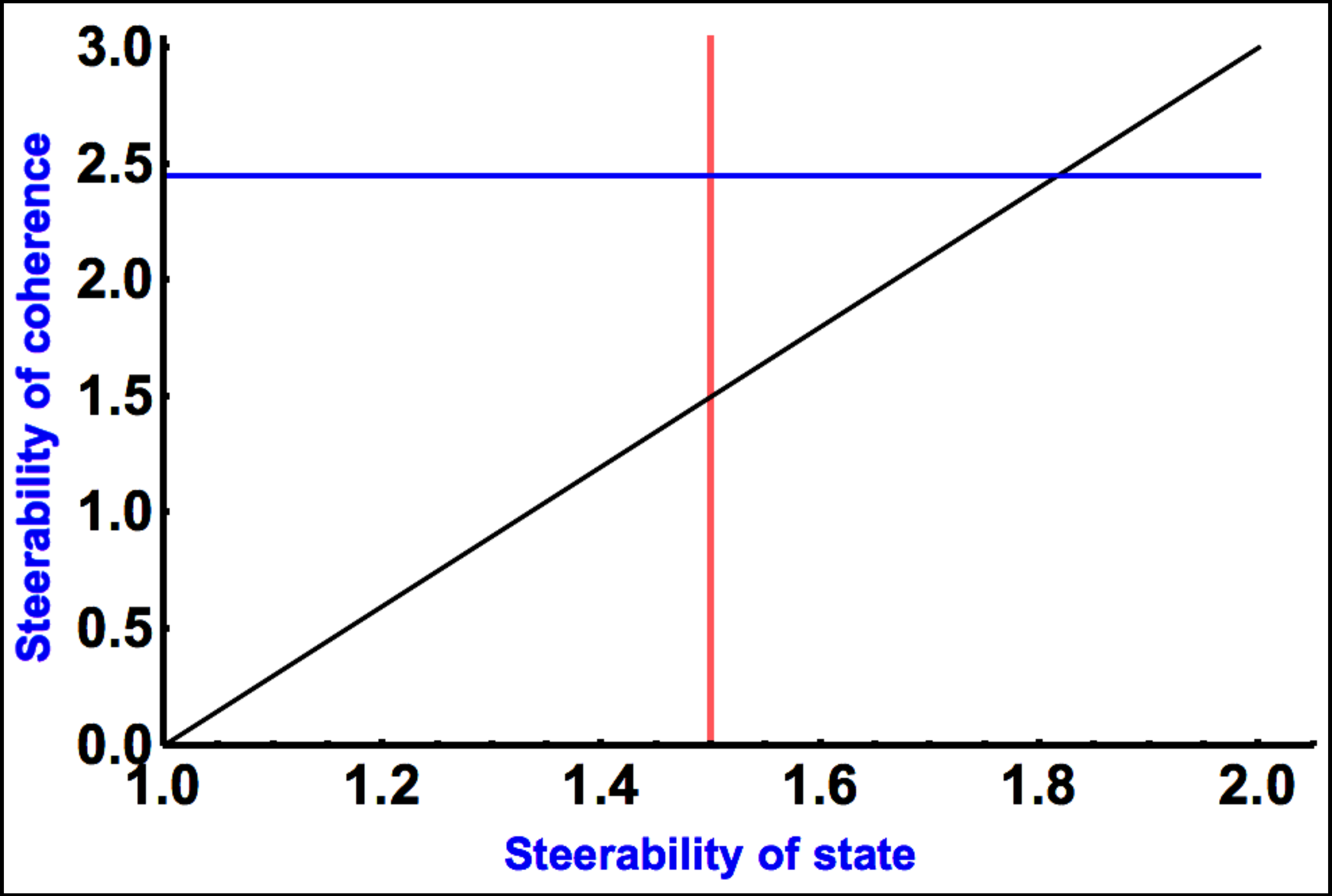
$p > 0.91$: Steerable under relative entropy coherence

$p > 0.94$: Steerable under skew information

State Steerability : $p > 0.58$

D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770.

Steerability of werner state



D. Mondal, T. Pramanik, and A. K. Pati, arXiv: 1508.03770.

Summary

- Steering is a kind of non-local correlation where one of the systems is not trusted as quantum system.
- Fine-grained steering criterion overcomes the limitations of coarse grained form of steering criteria.
- Coherence complementary relation : No single quantum state is fully coherence under all non-commuting basis.
- With three measurement settings, Werner state is steerable (state property) for p > 0.58.
- With three measurement settings, Werner state is steerable (coherence property) for p > 0.82.

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Thank You