Monte Carlo studies of the spontaneous rotational symmetry breaking in a matrix model with the complex action

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Collaboration with K.N. Anagnostopoulos and J. Nishimura

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1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model) ⇒ Promising candidate for the constructive definition of superstring theory.

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115.

$$S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right).$$

- Dimensional reduction of $\mathcal{N} = 1$ 10d Super-Yang-Mills (SYM) theory to 0d.
  A_μ (10d vector) and ψ (10d Majorana-Weyl spinor) are $N \times N$ matrices .
- Evidences for spontaneous breakdown of SO(10) symmetry to SO(4).
  J. Nishimura and F. Sugino, hep-th/0111102, H. Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.
- Complex action is crucial for spontaneous breakdown of rotational symmetry:
  J. Nishimura and G. Vernizzi, hep-th/0003223.
- Difficulty of Monte Carlo simulation :
  The determinant (from integrating out fermions) is complex.

2 Simplified IKKT model

Simplified model with spontaneous rotational symmetry breakdown,

J. Nishimura, hep-th/0108070.

$$S = \underbrace{\frac{N}{2}\mathrm{tr}\, A_\mu^2}_{=S_b} \underbrace{-\bar{\psi}^f_\alpha (\Gamma_\mu)_{\alpha\beta} A_\mu \psi^f_\beta}_{=S_f}$$

- A_μ: $N \times N$ hermitian matrices ($\mu = 1, \cdots, 4$)
  $\bar{\psi}^f_\alpha, \psi^f_\alpha$: N-dim vector ($\alpha = 1, 2, f = 1, \cdots, N_f$), N_f = (number of flavors).

$$\Gamma_1 = i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \ \Gamma_2 = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \Gamma_3 = i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \ \Gamma_4 = \sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- SU(N) symmetry and SO(4) rotational symmetry.

- Partition function:

$$Z = \int dA e^{-S_B} (\det \mathcal{D})^{N_f} = \int dA e^{-S_0} e^{i\Gamma}, \text{ where}$$
$$\mathcal{D} = \Gamma_\mu A_\mu = (2N \times 2N \text{ matrices}), \quad e^{-S_0} = e^{-S_B} |\det \mathcal{D}|^{N_f}.$$

Analytical studies of the model

Solvable at $N \to \infty$ using random matrix theory (RMT) technique.

$$\langle \frac{1}{N} \mathrm{tr}\, A_\mu^2 \rangle = \begin{cases} 1 + r + \mathrm{o}(r), & (\mu = 1,2,3) \\ 1 - r + \mathrm{o}(r), & (\mu = 4), \end{cases}$$

for small $r = N_f/N$.

Spontaneous breakdown of SO(4) symmetry to SO(3).

For the phase-quenched partition function $Z_0 = \int dA e^{-S_0}$,

$\langle \frac{1}{N} \mathrm{tr}\, A_\mu^2 \rangle = 1 + r/2$ for $\mu = 1,2,3,4$.

The phase plays a crucial role in the spontaneous rotational symmetry breakdown.

Gaussian expansion analysis up to 9th order:

T. Okubo, J. Nishimura and F. Sugino, hep-th/0412194.

Spontaneous breakdown of SO(4) to SO(2) at finite r.

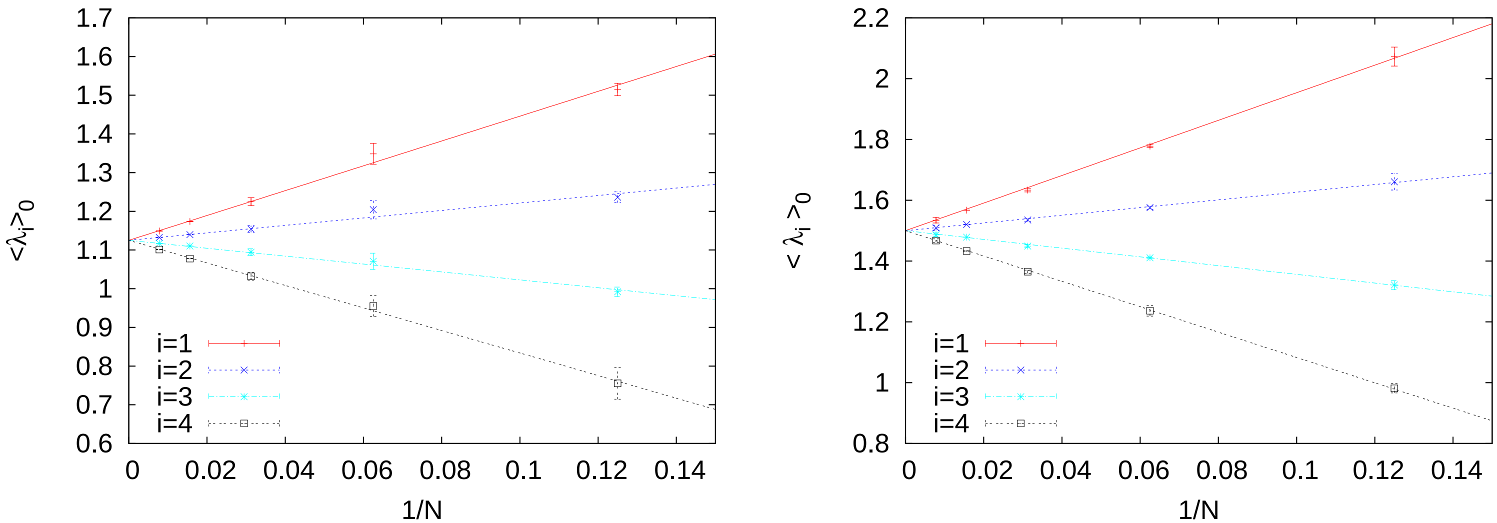
3 Monte Carlo studies of the model

Hybrid Monte Carlo (HMC) simulation of the phase-quenched model

HMC simulation of the partition function Z_0 with the phase omitted.

Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

λ_i $(i = 1, 2, 3, 4)$: eigenvalues of $T_{\mu\nu}$ $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4)$



Results for $r = \frac{1}{4}$ (left) and $r = 1$ (right).

$$\lambda_1 = \cdots = \lambda_4 \to 1 + \frac{r}{2} \ (\text{as } N \to \infty).$$

Factorization method

An approach to the complex action problem in Monte Carlo simulation.

K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041,
J. Ambjorn, K. N. Anagnostopoulos, J. Nishimura and J. J. M. Verbaarschot, hep-lat/0208025.

Overlap problem: Discrepancy of a distribution function between the phase-quenched model Z_0 and the full model Z.

Force the simulation to sample the important region for the full model.

Standard reweighting method:

$$\langle \lambda_i \rangle = \frac{\langle \lambda_i \cos \Gamma \rangle_0}{\langle \cos \Gamma \rangle_0},$$ **where** $\langle * \rangle_0 =$ **(V.E.V. for the phase-quenched model** Z_0**).**

(Number of configurations required) $\simeq e^{\mathrm{O}(N^2)}$. $\Rightarrow$ **complex-action problem.**

$\tilde{\lambda}_i \stackrel{\text{def}}{=} \lambda_i / \langle \lambda_i \rangle_0$: deviation from 1 $\Rightarrow$ effect of the phase.

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle = \frac{1}{C} \rho_i^{(0)}(x) w_i(x),$$

where

$$C = \langle \cos \Gamma \rangle_0, \quad \rho_i^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle_0, \quad w_i(x) = \langle \cos \Gamma \rangle_{i,x},$$
$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i)].$$

Resolution of the overlap problem: The system is forced to visit the configurations where $\rho_i(x)$ is important.

Monte Carlo evaluation of $\langle \tilde{\lambda}_i \rangle$

Direct evaluation:

$$\langle \tilde{\lambda}_i \rangle = \int_0^{\infty} dx\, x \rho_i(x) = \frac{\int_0^{\infty} dx\, x \rho_i^{(0)}(x) w_i(x)}{\int_0^{\infty} dx\, \rho_i^{(0)}(x) w_i(x)}.$$

Difficult because $w_i(x) \simeq 0$ at large N.

The errorbar must be very small ($w_i(x) = 0.04 \pm 0.05$ no longer makes sense).

$w_i(x) > 0 \Rightarrow \langle \tilde{\lambda}_i \rangle$ is the minimum of $\mathcal{F}_i(x)$:

$$\mathcal{F}_i(x) = (\text{free energy density}) = -\frac{1}{N^2} \log \rho_i(x).$$

We solve $\mathcal{F}_i'(x) = 0$, namely

$$\frac{1}{N^2} f_i^{(0)}(x) = -\frac{d}{dx}\left(\frac{1}{N^2} \log w_i(x)\right).$$

Result for $r = N_f/N = 1$

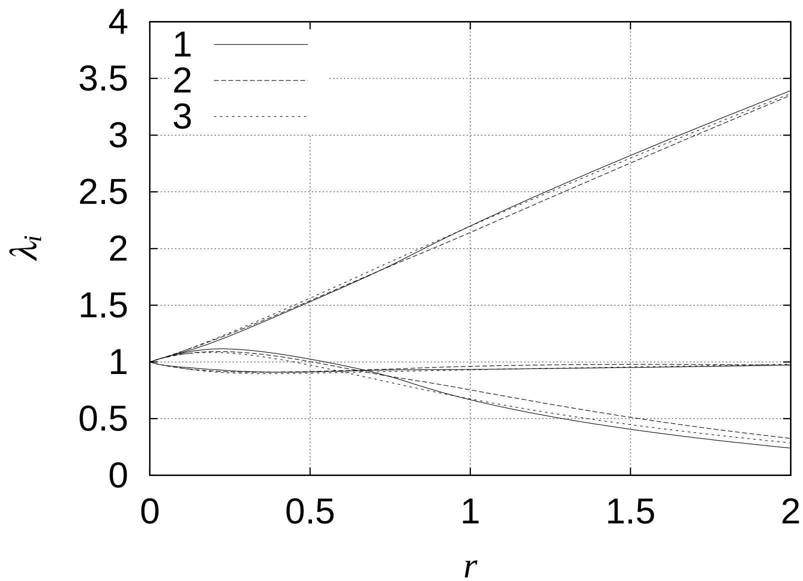
Result for 9th-order Gaussian expansion:

T. Okubo, J. Nishimura and F. Sugino, hep-th/0412194.

$\tilde{\lambda}_{i=1} \simeq 1.4,\ \tilde{\lambda}_{i=2} \simeq 1.4,\ \tilde{\lambda}_{i=3} \simeq 0.7,\ \tilde{\lambda}_{i=4} \simeq 0.5.$

Spontaneous breakdown of the rotational symmetry SO(4) → SO(2).

Quoted from Figure 4 (right) of hep-th/0412194.



Both $\frac{1}{N^2}\log w_i(x)$ and $\frac{1}{N^2} f_i^{(0)}(x)$ scale at large N as

$$\frac{1}{N^2}\log w_i(x) \to \Phi_i(x), \quad \frac{1}{N^2} f_i^{(0)}(x) \to F_i(x).$$

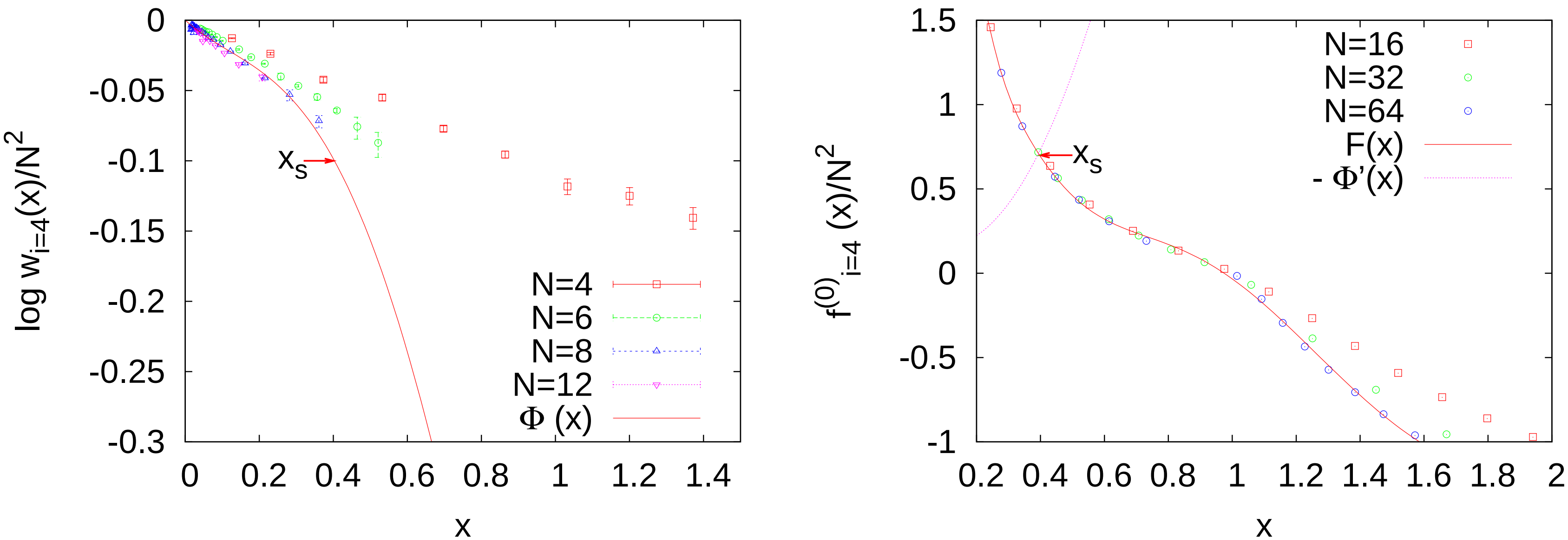
The minimum of "free energy density" is obtained by

$$F_i(x) + \Phi'(x) = 0.$$

Fitting of $F_i(x)$:

$$F_i(x) \simeq a_{i,0} + (a_{i,1}x + \frac{b_{i,1}}{x}) + \cdots + (a_{i,4}x^4 + \frac{b_{i,4}}{x^4}).$$

$i = 4$ case



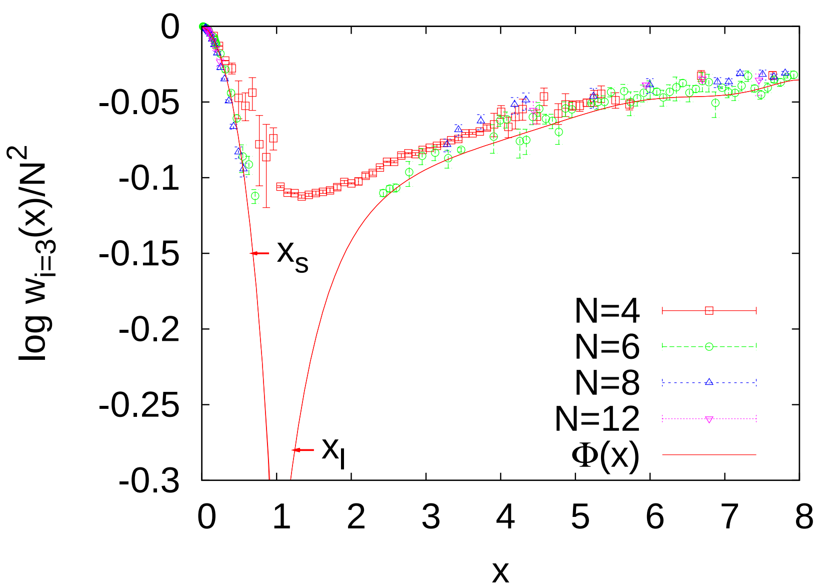
$\Phi_{i=4}(x)$ **decreases monotonously** $\Rightarrow$ **One extremum of "free energy density"**

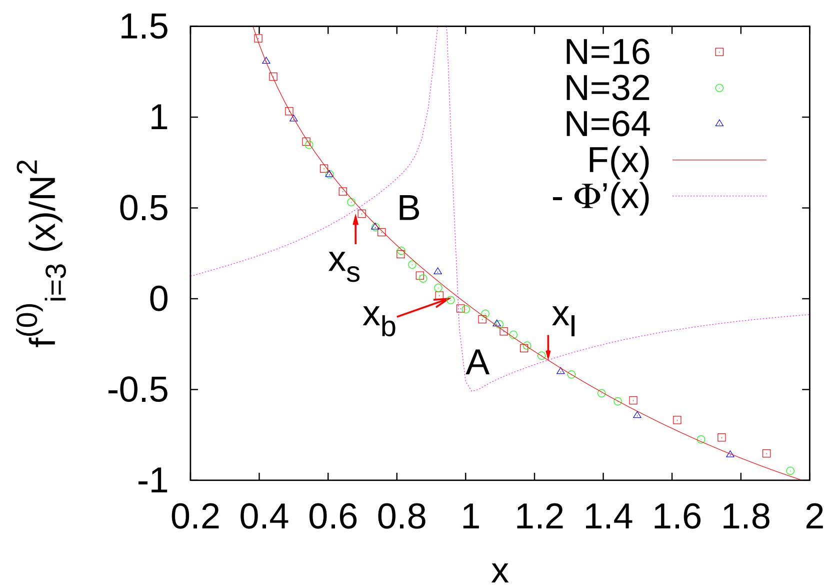
$\Rightarrow$ **single-peak structure of** $\rho_{i=4}(x)$ **at** $x_s \simeq 0.4$.

$\Phi_i(x)$ **: fitted by 4-th order polynomial.**

$$\langle \tilde{\lambda}_{i=4} \rangle \simeq 0.4.$$

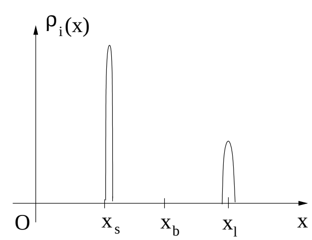
$i = 3$ case





Three extrema of "free energy density" $\Rightarrow$ double-peak structure of $\rho_{i=3}(x)$.

$x_s \simeq 0.7$, $x_l \simeq 1.2$ $(x_s < x_b < x_l)$.



Which peak is the higher, x_s or x_l?

Extrapolation of $\Phi_i(x)$:

$$\Phi_i(x) \simeq \begin{cases} \phi_{i,s}(x) = c_{i,0} + c_{i,1}x + \cdots + c_{i,4}x^4, & (x < x_s), \\ \phi_{i,l}(x) = d_{i,0} + d_{i,1}x + \cdots + d_{i,8}x^8, & (x > x_l), \\ \frac{\phi_{i,s}(x)e^{-\mathcal{C}(x-\alpha)} + \phi_{i,l}(x)e^{\mathcal{C}(x-\alpha)}}{e^{-\mathcal{C}(x-\alpha)} + e^{\mathcal{C}(x-\alpha)}}, \\ (x_s < x < x_l). \end{cases}$$

At $x = \alpha$, $\phi_{i,s}(x) = \phi_{i,l}(x)$.

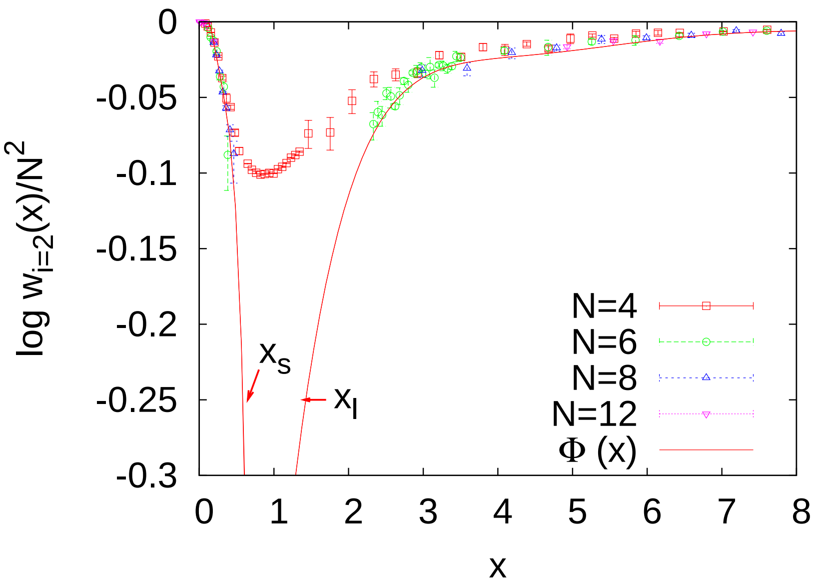
- $\frac{1}{N^2}(\log \rho_i(x_l) - \log \rho_i(x_b)) = \int_{x_b}^{x_l} dx (F_i(x) + \Phi_i'(x)) =$ **(A's area).**
- $\frac{1}{N^2}(\log \rho_i(x_s) - \log \rho_i(x_b)) = -\int_{x_s}^{x_b} dx (F_i(x) + \Phi_i'(x)) =$ **(B's area).**

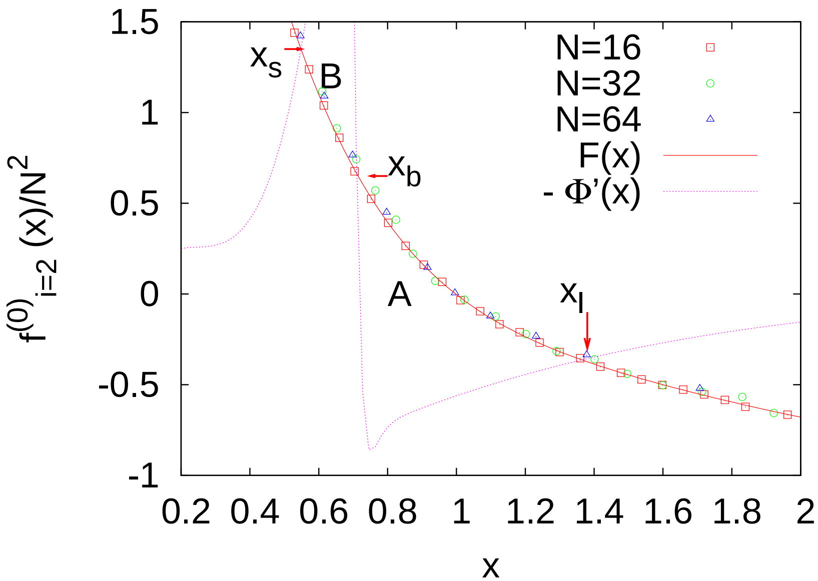
Difference of the height:

$$\begin{aligned} \Delta_i &= \frac{1}{N^2}(\log \rho_i(x_l) - \log \rho_i(x_s)) = (\Phi_i(x_l) - \Phi_i(x_s)) + \int_{x_s}^{x_l} dx F_i(x) \\ &= \text{(A's area)-(B's area)} \simeq -0.10. \end{aligned}$$

The higher peak lies at $x_s \Rightarrow \langle \tilde{\lambda}_{i=3} \rangle \simeq 0.7$.

$i = 2$ case





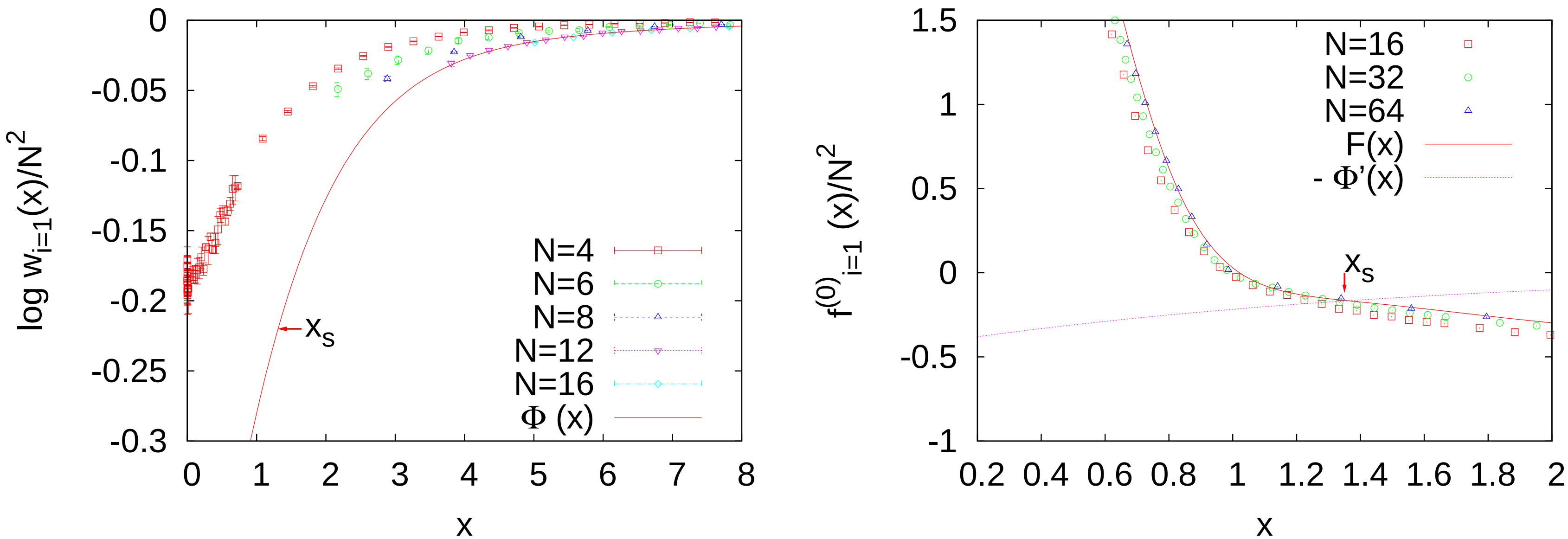
Three extrema of "free energy density" $\Rightarrow$ double-peak structure of $\rho_{i=2}(x)$.

$x_s \simeq 0.6$, $x_l \simeq 1.4$ $(x_s < x_b < x_l)$.

$\Phi_{i=2}(x)$ is fitted similarly to $\Phi_{i=3}(x)$.

$\Delta_{i=2} \simeq 0.12 \Rightarrow$ The higher peak lies at $x_l \Rightarrow \langle \tilde{\lambda}_{i=2} \rangle \simeq 1.4$.

$i = 1$ case



$\Phi_{i=1}(x)$ increases monotonously $\Rightarrow$ One extremum of "free energy density"

$\Rightarrow$ single-peak structure of $\rho_{i=1}(x)$ at $x_s \simeq 1.4$..

$\Phi_i(x)$: fitted by 4-th order polynomial.

$$\langle \tilde{\lambda}_{i=1} \rangle \simeq 1.4.$$

VEV's $\langle \tilde{\lambda}_{i=1,2,3,4} \rangle$ are consistent with 9th order Gaussian expansion method.

Spontaneous breakdown of the rotational symmetry $SO(4) \rightarrow SO(2)$.

4 Conclusion

Monte Carlo simulation of the simplified IKKT model via factorization method.

Simulation of the $r = 1$ case $\rightarrow$ symmetry breakdown of SO(4) to SO(2).

Future problems

- **Application of the multi-canonical method to matrix models.**
- **Simulation of the 6,10-dimensional IKKT model**

  It costs $O(N^6)$ CPU time.

  However, the effect of the phase may be milder than this simplified model.