Quantization of gravity, giants and sound waves

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Based on...

- GM 0502104
- A.Dhar, GM, N.Suryanarayana 0509164
- A.Dhar, GM, M.Smedback 0512312
- A.Dhar, GM 0603154
- GM, N.Suryanarayana 0606088
- A. Basu, GM 0608093
- GM, S.Minwalla, S.Raju, M.Smedback (work in progress)

Motivation

The formula

$$\ln \Omega = \frac{\mathrm{Area}}{4G_N}$$

proposes to describe the degrees of freedom of gravity.

AdS/CFT sometimes provides a boundary description of Ω, which may not be entirely satisfactory, since it may not answer the question of “where” the degrees of freedom are, and even “what” precisely the degrees of freedom are.

Motivation

Can we find configurations in the “bulk” which explain the degrees of freedom of black holes?

The other interesting, related, question is: when perturbative description of gravity (gravitons) breaks down, is there any bulk description that survives?

We will explore these questions in supersymmetric situations less complicated than that of black holes.

Contents

- Half-BPS configurations: Gravitons, giant gravitons and dual giant gravitons and their quantization [review]
- Half-BPS geometry: LLM solutions and their quantization using Kirillov's method
- Half-BPS D3-brane configurations account for all half-BPS geometries: bulk viewpoint
- Boundary story, using exact bosonization

Contents

- Other examples: 1/8 BPS sector of $AdS_5 \times S^5$
- BPS configurations in less supersymmetric backgrounds (AdS$_5 \times Y^5$ and AdS$_4 \times Y^7$).
- D1/D5 and spiky AdS/CFT

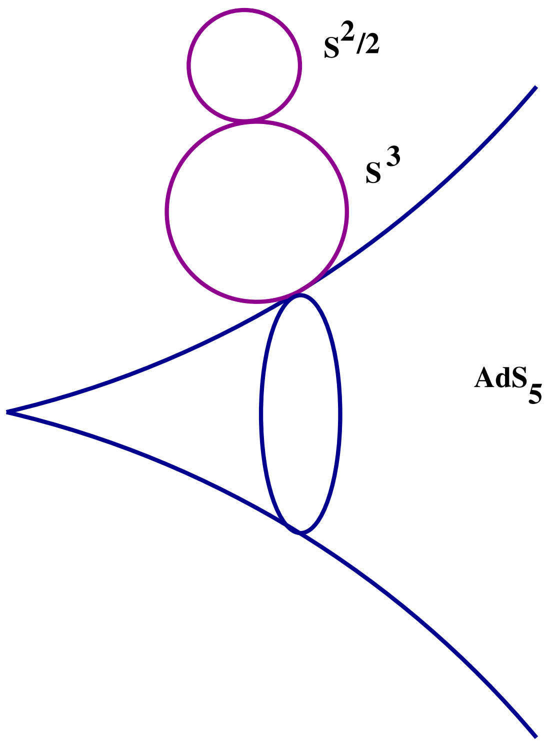
Half-BPS configurations

The $AdS_5 \times S^5$ metric

$$ds^2 = R^2(ds^2_{S^5} + ds^2_{AdS_5})$$

$$ds^2_{AdS_5} = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\,(d\Omega)^2$$

$$ds^2_{S^5} = \cos^2\theta\, d\phi^2 + d\theta^2 + \sin^2\theta(d\tilde{\Omega})^2$$



Half-BPS configurations

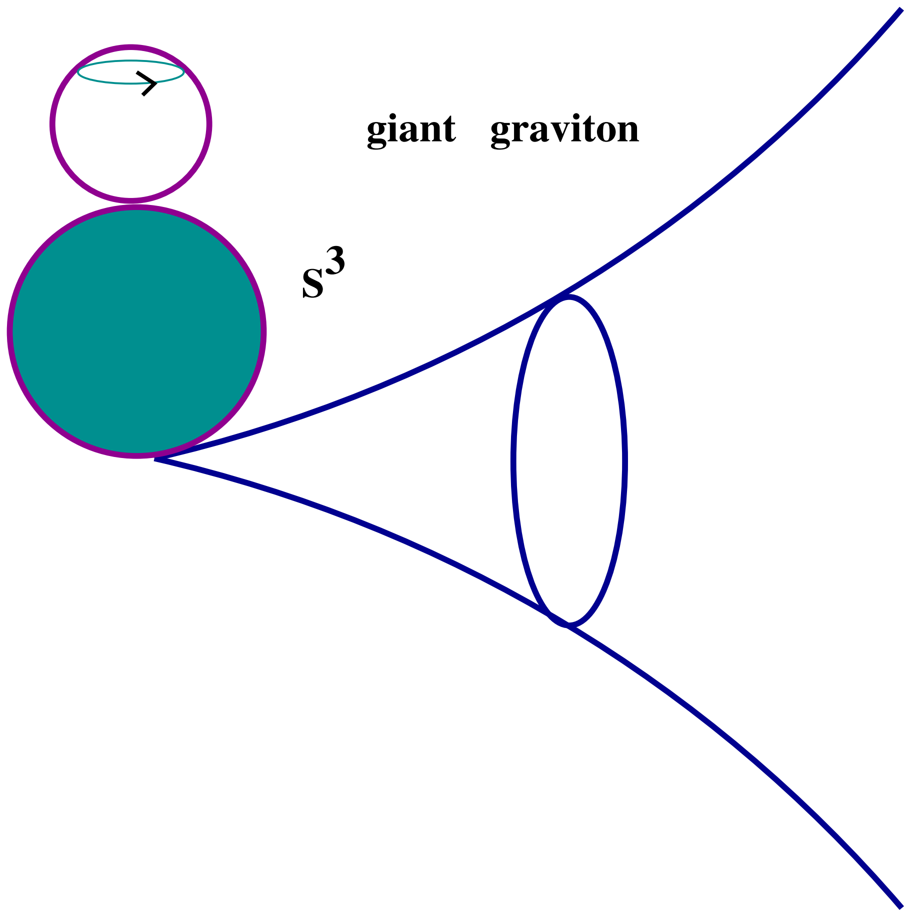
Graviton:

Kaluza-Klein particle (linear combination of metric and RR fluctuations) rotating at the speed of light in the ϕ direction.

$$H = p_\phi = n/R,\ n = 0, 1, 2, ..., \infty$$

The wavefunctions of these gravitons are given by $\psi_n \sim F_n(\rho, \theta) \exp[in\phi - int]$, where the F_n are localized functions which roughly behave as $(\cos\theta / \cosh\rho)^{|n|}$.

Half-BPS configurations



- D3 brane wrapping $S^3 \subset S^5$. Phase space $\theta, \phi, p_\theta, p_\phi$.
- Half-BPS condition $p_\theta = 0, p_\phi = \sin^2\theta$.

Half-BPS configurations

- Dirac constraints : 2D reduced phase with coordinates θ, ϕ

$$\{\sin^2\theta, \phi\}_{DB} = \{x_1, x_2\}_{DB} = 1/N$$

where $(x_1, x_2) = (\sin\theta\cos\phi, \sin\theta\sin\phi)$ maps a hemisphere to a unit Disc.

- Geometric quantization

$$H = p_\phi = N(x_1^2 + x_2^2) = n/l, \; n = 0, 1, 2, ..., N-1$$

- Harmonic oscillator with a truncated spectrum (since the phase space is a disc) $\Rightarrow$ Stringy Exclusion

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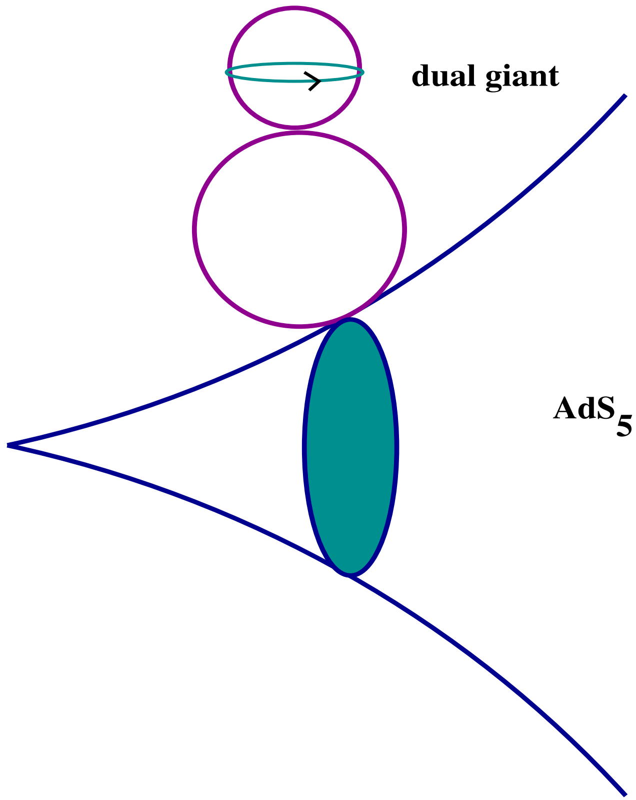
Das,Jevicki,Mathur 0008088

Dhar,GM,Suryanarayana 0509164

Half-BPS configurations



- D3-brane wrapping $S^3 \subset AdS_5$. 4D phase space $\rho, \phi, p_\rho, p_\phi$.
- Half-BPS: $p_\rho = 0, p_\phi = \sinh^2 \rho$

Half-BPS configurations

- Dirac constraints : 2D reduced phase with coordinates ρ, ϕ

$$\{\sinh^2\rho, \phi\}_{DB} = \{x_1, x_2\}_{DB} = 1/N$$

where $(x_1, x_2) = (\sinh\rho\cos\phi, \sinh\rho\sin\phi)$ maps $R^+ \times S^1 \mapsto R^2$.

- Geometric quantization

$$H = p_\phi = N(x_1^2 + x_2^2) = n/l,\ n = 0, 1, 2, ..., \infty$$

- Although the single-particle spectrum is infinite, there are only a finite number of dual giants $\Rightarrow$ Stringy Exclusion. This follows from the fact...

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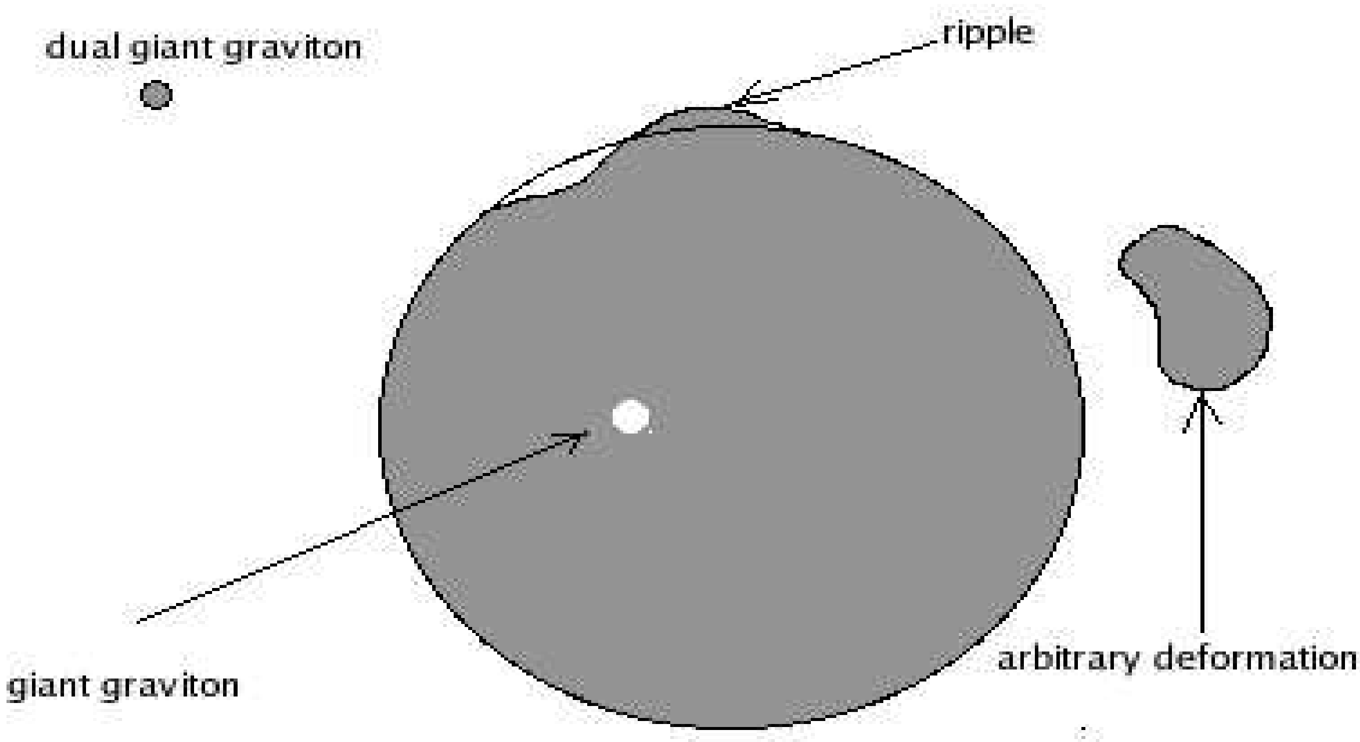
Half-BPS configurations

... that every time we cross a dual giant graviton in the direction of decreasing ρ, the 5-form flux decreases by 1. If we have more than N dual giants, we will have negative flux in the middle which is unphysical. (Strictly speaking, we must include back-reaction.)

Half-BPS geometries

- $AdS_5 \times S^5 +$ half-BPS configurations like giant/dual giant gravitons are expected to produce some new half-BPS geometries.
- ALL half-BPS geometries, preserving the same 16 susy's as the giant/dual giant gravitons, viz. $(\pm\frac{1}{2}, \pm\frac{1}{2} | \pm \frac{1}{2}, \pm\frac{1}{2}, \frac{1}{2})$, have been found by LLM (2004):
- $$\begin{aligned} ds^2 &= -\sqrt{\frac{y}{u(1-u)}}(dt + V_i[u]dx_i)^2 + \sqrt{\frac{u(1-u)}{y}}(dy^2 \\ &\quad + dx_i dx_i) + y\sqrt{\frac{1-u}{u}}(d\Omega_3)^2 + y\sqrt{\frac{u}{1-u}}(d\tilde{\Omega}_3)^2 \end{aligned}$$

Half-BPS geometries



$$u(x_1, x_2)^2 = u(x_1, x_2), \int dx_1 dx_2 \frac{u(x_1, x_2)}{2\pi} = 1. \Rightarrow W_\infty$$

Half-BPS geometries

Collective coordinate quantization of LLM geometry

Using W_∞ symmetry of the u-space can be used to give the following Kirillov action

$$\begin{aligned} S_{\mathrm{LLM}} &= \int \frac{dx_1 dx_2}{2\pi\hbar}\hbar \int_{\tilde{\Sigma}} dt\, ds\, u(\vec{x},t,s)\{\partial_\tau u, \partial_s u\}_{PB} - \int_{\Sigma} dt \tilde{H} \\ \tilde{H} &= \int \frac{dx_1 dx_2}{2\pi\hbar} u(\vec{x},t) \frac{x_1^2 + x_2^2}{2\hbar} \end{aligned}$$

Half-BPS geometries

This is the semiclassical limit of the second quantized action for N free fermions in a harmonic oscillator. u gets identified as fermion phase space density.

The same Kirillov action is reproduced by DBI $+$ CS action of multiple *non-overlapping* giant gravitons.

For arbitrary configuration of giant gravitons including overlapping ones, the action $S_{\rm gg}$ is that of second quantized *bosons*.

$S_{\rm LLM} = S_{\rm gg}$ under bosonization.

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The story at the boundary

- The half-BPS sector corresponds to a finite number (N) of fermions:

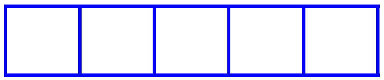
$$\mathrm{Tr}[\dot{Z}^2 - Z^2]$$

$$Z = \Phi_5 + i\Phi_6$$

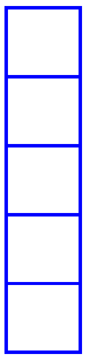
- The eigenvalues of Z behave as N fermions in a harmonic oscillator potential.

The story at the boundary

- Exact bosonization : All operators in the fermion theory can be written as $\psi_n^\dagger \psi_m$. These operators can be related to the Schur polynomials corresponding to

   $\longrightarrow$ dual giant

  or, alternatively, to

   $\longrightarrow$ giant

- ALL OPERATORS in the half-BPS sector in the boundary theory can be described by giant gravitons, alternatively by dual giant gravitons.

Dhar, GM, Suryanarayana 0509164

The story at the boundary

High energy behaviour

- Perturbative gravitons break down
- Three-point function of gravitons blows up for energies $E \gtrsim \sqrt{N}$. For $E \sim N, \Gamma_3 \sim e^N$.
- Three-point function of giant gravitons is nearly free at even high energies: for $E \sim N, \Gamma_3 \sim e^{-N}$.
- Conclusion: Gravity at high energy is described by giant gravitons, when perturbative gravitons do not make sense

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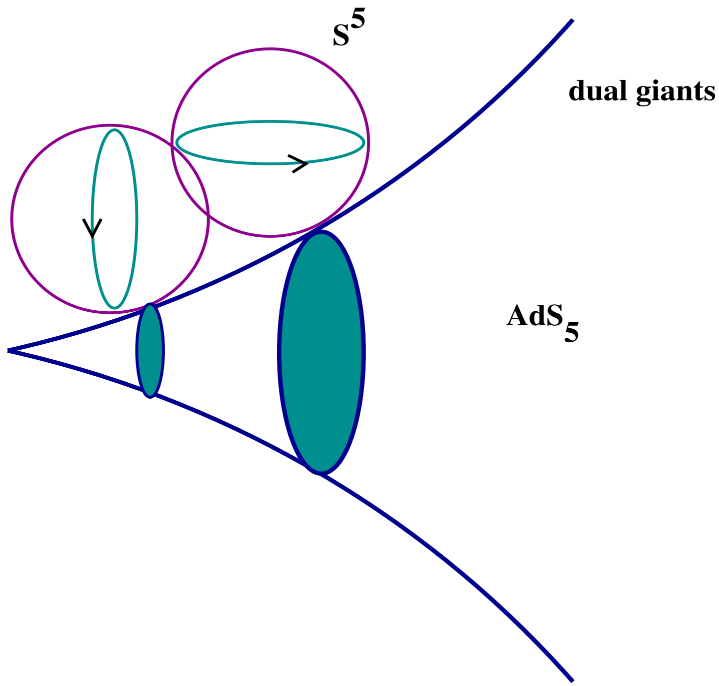
A.Dhar,GM, M.Smedback 0512312

1/8 BPS dual giants in $AdS_5 \times S^5$



- Phase space= $\rho, \theta, \phi, \chi_1, \chi_2, \chi_3$ and their momenta. Here the AdS radial coordinate ρ represents R^+ and the 5 angles parametrize S^5. [a]

[a] GM, N.V. Suaryanarayana 2006

1/8 BPS dual giants in $AdS_5 \times S^5$

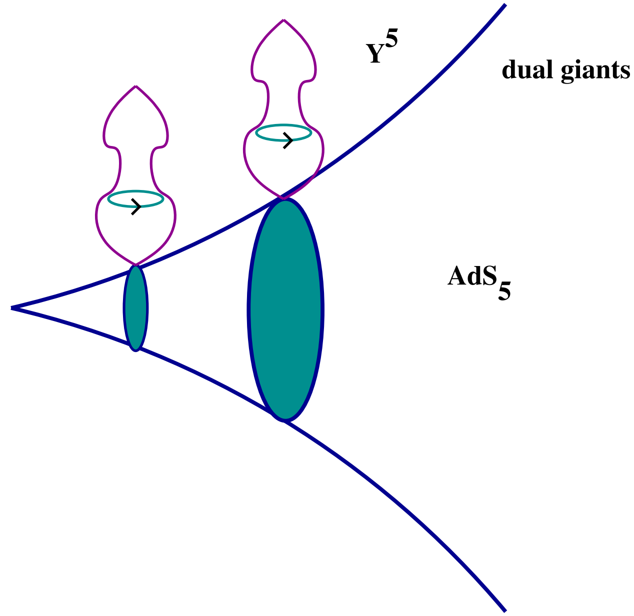
- By the BPS constraints (similar to the half-BPS dual giants), the 6D coordinate space becomes a phase space. The Dirac brackets imply that the space is symplectically C^3 and the Hamiltonian in the reduced phase space is that of a 3D SHO!
- Multiple dual giants are mutually BPS and correspond to free bosons.
- The stringy exclusion principle arises again from the “flux” argument: there cannot be more than N dual giant gravitons. Hence the system is that of $\leq N$ bosons in a 3D harmonic oscillator potential. This is exactly what the boundary theory gives.
- Conclusion: The entire spectrum of the 1/8-th BPS sector of the boundary theory is produced by $\leq N$ dual giant gravitons.

1/8 BPS dual giants in $AdS_5 \times S^5$

- The giant gravitons are more involved, but gives rise to the same spectrum
- This indicated that the two descriptions may be dual to each other, however a proof of the duality in a manner similar to the half-BPS case described above is still lacking.
- Although an LLM-like family of supergravity solutions is lacking here, in view of the boundary results it is almost evident that a quantization of the gravity solutions in the bulk would agree with the above quantization in terms of giants/dual giants.

Biswas, Gaiotto, Lahiri,Minwalla 06.....

Dual giants in $AdS_5 \times Y^5, AdS_4 \times Y^7$



- The quantization [a] of dual giant gravitons proceeds like in the previous case. The coordinate space $Y^{2n+1} \times R^+$ becomes the phase space because of the BPS constraint, with the symplectic structure the Kahler cone over the Sasaki-Einstein manifold.

[a] Basu, GM 0608093

Dual giants in $AdS_5 \times Y^5, AdS_4 \times Y^7$

- The geometric quantization gives N free holomorphic bosons on the Kahler cone.
- Thus, for $Y^5 = T_{1,1}$, the Kahler cone is given by four homogeneous coordinates $w_i : w_i w_i = 0$. The single particle wavefunctions are

$$\psi_{mnpq}(\vec{w}) = w_1^m w_2^n w_3^p w_4^q$$

- This agrees with the boundary theory wavefunctions.

D1-D5

The two-charge system

There are several descriptions of the degrees of freedom

- a. Boundary theory counting (Strominger-Vafa-Callan-Maldacena)
- b. Quantization of Lunin-Mathur-Maldacena-Maoz solutions [a]
- c. Supertubes [b]
- d. Probe strings (in progress) [c]
- etc.

[a] A.Basu, GM (unpublished); Rychkov 06...

[b] Marolf et al

[c] GM,S.Minwalla,S.Raju,M.Smedback 070mnnn

D1-D5

Puzzle:

- How are the various descriptions related?
- In particular, in a, the degrees of freedom seem to be D-brane fluctuations in the compact space, while in b, they seem to be fundamental strings which vibrate in the non-compact space (as well as in the compact space)

Spiky AdS/CFT

Indeed a similar puzzle exists in the half-BPS story in $AdS_5 \times S^5$since it seems that for some range of energies the same degrees of freedom are described by gravitons (fundamental strings) as well as by D3-branes.

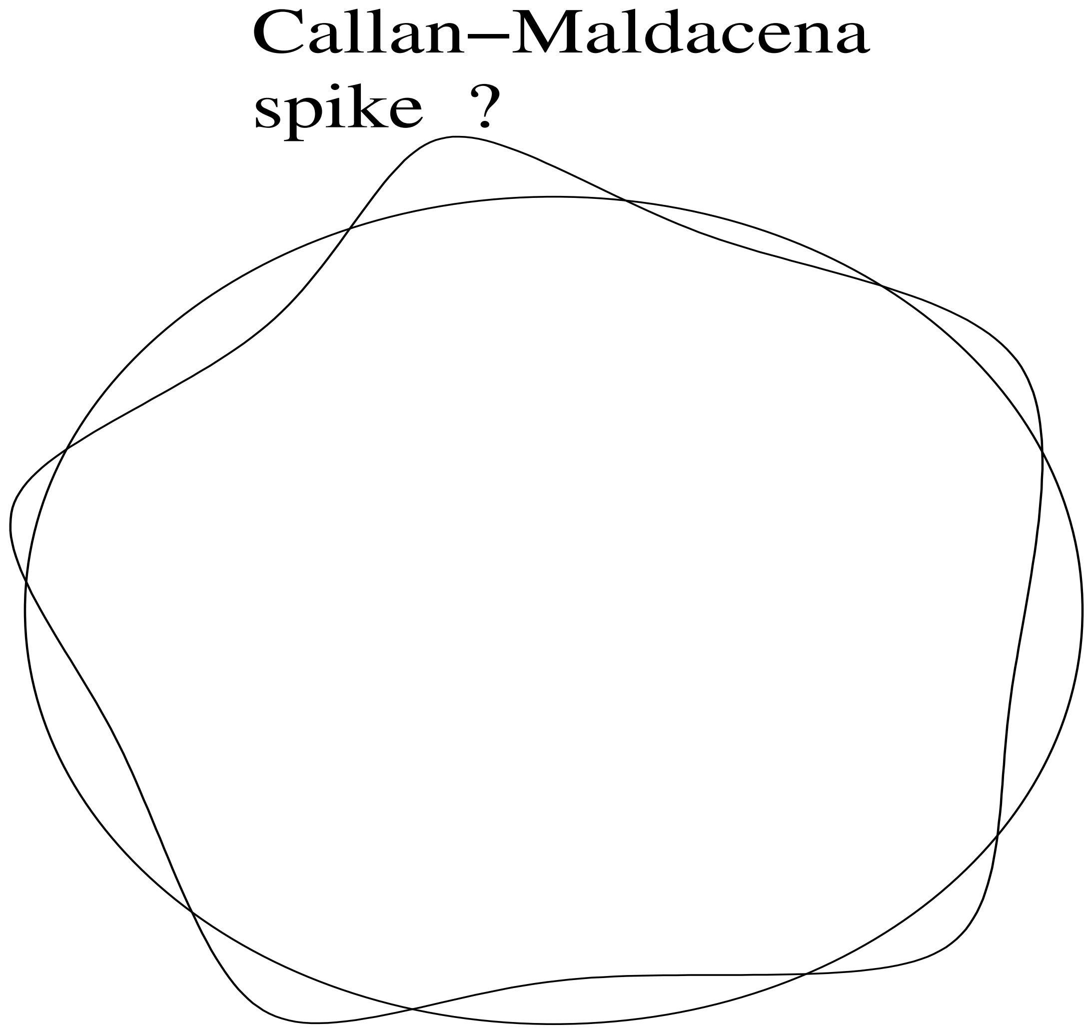
This problem can be cast in a way similar to ones studied earlier by Callan-Maldacena, Drukker et al, (see also Gomis et al).

In Callan-Maldacena D3-brane fluctuations (spikes) were shown to be accompanied by elecric fluxes on the brane which could be identified with fundamental strings.

In Drukker et al and Gomis et al, Wilson lines/loops were shown to be equivalently described by D3-branes or fundamental strings, the accuracy of the description depending on the number of Wilson lines/loops.

Spiky AdS/CFT

Half-BPS configurations in $AdS_5 \times S^5$



Conclusion

- The spectrum of the half-BPS sector of $AdS_5 \times S^5$ is described by giant gravitons, or equivalently, by dual giant gravitons. The duality between the two descriptions (and with the fermionic description of the CFT) can be derived by using exact bosonization in the boundary theory. The correlation functions of the latter are well-defined at high energies, unlike for gravitons.
- The spectrum of 1/8-BPS sector of $AdS_5 \times S^5$ is described by giant or dual giant gravitons. These are likely to be dual, though there are no proofs.
- The spectrum of half-BPS dual giant gravitons in $AdS_4 \times Y^7$ and $AdS_5 \times Y^5$ can be computed. In simple cases they can be shown to reproduce the entire spectrum.

Conclusion

- These are examples of “unreasonable efficiency” of the probe approximation since back-reations are not considered here. Even the stringy exclusion principle appears in a simple way. This represents finite N effects, way beyond the probe approximation.
- The description of perturbative gravity in terms of D3 branes is similar to spiky AdS/CFT.