Rotating Attractors - one entropy function to rule them all

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ISM06, Puri, 17.12.06

talk based on: hep-th/0606244 (Astefanesei , K. G., Jena, Sen,Trivedi);
hep-th/0507096 (K.G., Iizuka, Jena & Trivedi);
hep-th/0506177 (Sen)
and work in progress....

Plan

☞ Study rotating attractors using Sen's entropy function

☞ Motivation

- $\exists$?
- mininal requirements for the attractor mechanism ?
- learn more about of the Entropy of non-SUSY blackholes?
- technical points:
  - $\rightarrow$ entropy function formalism still apply with less symmetry?
  - $\rightarrow$ only need near horizon geometry

What are blackhole attractors?

- ☞ Context = Theory with gravity, gauge fields, neutral scalars
  - ❑ generically appear as (part of) low energy limit of string theory
- ☞ scalars (or moduli) encode geometry of compactified dimensions
- ☞ Attractor mechanism = scalars' values fixed at Blackhole's horizon
- ☞ independent of values at infinity
- ☞ So horizon area depends only on gauge charges $\Rightarrow$ Entropy depends only on charges
- ☞ works for Extremal ($T = 0$) blackholes

Hand waving

- ☞ number of microstates of extremal blackhole determined by quantised charges
  - ❑ entropy can not vary continuously
- ☞ but the moduli vary continuously
- ☞ resolution: horizon area independent of moduli
  - ❑ moduli take on fixed values at the horizon determined by charges
- ☞ No mention of SUSY

Outline

- ☞ Go through examples of application of entropy function
  - ❑ without rotation
  - ❑ with rotation
  - ❑ Time permitting: blackring
- ☞ Study Lagrangians which generically appear as (the bosonic part of) certain low energy limits of string theory

Outline

- ☞ Go through examples of application of entropy function

  - ❏ without rotation:

- – Only need near horizon geometry
- – Assume extremal ($T = 0$) $\leftrightarrow$ $AdS_2 \times S^2$ near horizon symmetries
- – Equations of motion $\Leftrightarrow$ Extremising an Entropy function
- – Value of the Entropy function at extremum = Wald Entropy of Blackhole
- – need to solve algebraic equations
- – Argument is independent of SUSY

Outline

☞ Go through examples of application of entropy function

- ❑ without rotation
- ❑ with rotation:

– Only need near horizon geometry

– Assume $AdS_2 \times U(1)$ near horizon symmetries

– Equations of motion $\Leftrightarrow$ Extremising an Entropy function

– need to solve differential equations

– Entropy function at extremum = Wald Entropy

– Argument is independent of SUSY

Outline

☞ Go through examples of application of entropy function

- ❏ without rotation
- ❏ with rotation
- ❏ Time permitting: a blackring in 5-d

– Only need near horizon geometry
– Assume $AdS_2 \times S^1 \times S^2$ near horizon symmetries
– not the most general ansatz
– Equations of motion $\Leftrightarrow$ Extremising an Entropy function
– Entropy function at extremum $=$ Wald Entropy
– need to solve algebraic equations
– End up with $AdS_3 \times S^2$

Step 1

☞ First we look at simple 4-dimensional spherically symmetric black holes

☞ later we will compare it with more complicated cases.

Entropy Function (Sen)

Set up:

- ☞ **Gravity, p-form gauge fields, massless neutral scalars**
- ☞ **$\mathcal{L}$ gauge and coordinate invariant - in particular there may be higher derivative terms**
- ☞ **Extremal $=$ $AdS_2 \times S^2$ Near horizon geometry**
- ☞ **Entropy function:**
  - ❑ **First we consider, f, the Lagrangian density evaluated at the horizon:**

$$f[e^i, p^i, R_{AdS_2}, R_{S_2}, \varphi_s] = \int_H \sqrt{-g}\mathcal{L}$$

$$q_i = \frac{\partial f}{\partial e^i}$$

Entropy Function (Sen)

Set up:

- Gravity, p-form gauge fields, neutral scalars
- $\mathcal{L}$ gauge and coordinate invariant - in particular there may be higher derivative terms
- Extremal = $AdS_2 \times S^2$ Near horizon geometry
- **Entropy function:**
  - First we consider, f, the Lagrangian density evaluated at the horizon.
  - Now take the Legendre transform of f w.r.t the electric charges:

$$\mathcal{E} = 2\pi \left(q_i e^i - \int_H \sqrt{-g}\mathcal{L} \right)$$

$$\mathcal{E} = \mathcal{E}[q_i, p^i, R_{AdS_2}, R_{S_2}, \varphi_s]$$

Entropy Function (Sen)

$$\mathcal{E} = 2\pi \left(q_i e^i - \int_H \sqrt{-g}\mathcal{L} \right)$$

Results:

- ☞ **equations of motion $\Leftrightarrow$Extremising $\mathcal{E}$**
- ☞ **Wald Entropy = Extremum of $\mathcal{E}$**
- ☞ **Fixing q_i and p^i fixes everything else completely**

Caveats

Entropy function, $\mathcal{E}$, might have flat directions

$\Rightarrow$ The near horizon geometry is not completely determined by extremisation of $\mathcal{E}$
$\Rightarrow$ There may be a dependence of the near horizon geometry on the moduli

But since these are flat directions

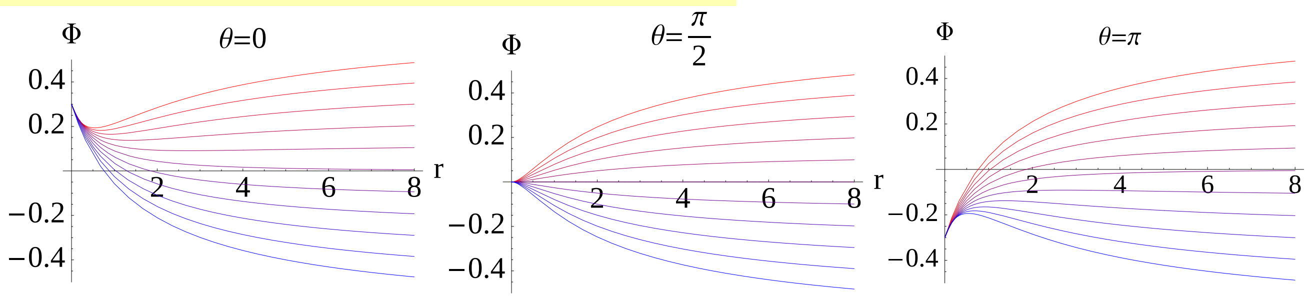
✔ the entropy is still independent of the moduli

☞ Generalised attractor mechanism

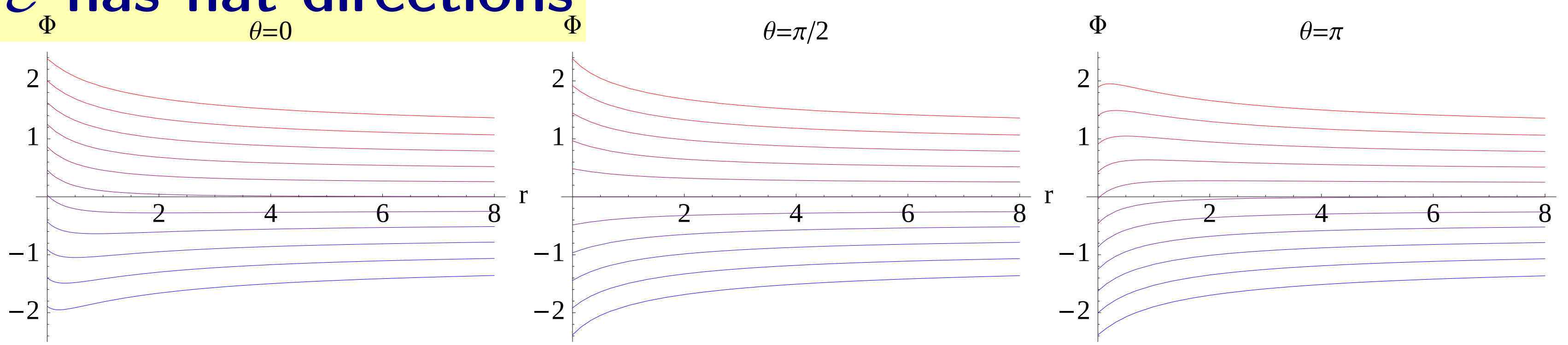
Also note that we have assumed that a blackhole solution exists which may not always be the case.

Summary of rotating attractors

- **Scalars constant on horizon (non-dyonic)**
  - $S_{BH} = 2\pi\sqrt{(V_{eff})^2 + J^2}$ $\qquad\qquad \partial_\Phi V_{eff} = 0$
- **Scalars vary over horizon ($\Phi = \Phi(\theta)$) (dyonic)**
  - $\mathcal{E}$ has no flat directions



  - $\mathcal{E}$ has flat directions



Simple example: spherically symmetric case

$$\begin{aligned}\mathcal{L} &= R - h_{rs}(\vec{\Phi}) g^{\mu\nu} \partial_\mu \Phi_s \partial_\nu \Phi_r - f_{ij}(\vec{\Phi}) g^{\mu\rho} g^{\nu\sigma} F^{(i)}_{\mu\nu} F^{(j)}_{\rho\sigma} \\ &\quad - \frac{1}{2} \tilde{f}_{ij}(\vec{\Phi}) \epsilon^{\mu\rho\nu\sigma} F^{(i)}_{\mu\nu} F^{(j)}_{\rho\sigma}\end{aligned}$$

Simple example: spherically symmetric case

$$\begin{aligned}\mathcal{L} &= R - h_{rs}(\vec{\Phi}) g^{\mu\nu} \partial_\mu \Phi_s \partial_\nu \Phi_r - f_{ij}(\vec{\Phi}) g^{\mu\rho} g^{\nu\sigma} F^{(i)}_{\mu\nu} F^{(j)}_{\rho\sigma} \\ &\quad - \frac{1}{2} \tilde{f}_{ij}(\vec{\Phi}) \epsilon^{\mu\rho\nu\sigma} F^{(i)}_{\mu\nu} F^{(j)}_{\rho\sigma}\end{aligned}$$

The effect of the $\tilde{f}_{ij}$ term is basically

$$q_i \to q_i - 4 \tilde{f}_{ij} p^j$$

and for simplicity we will neglect it

Simple example: spherically symmetric case

$$\mathcal{L} = R - h_{rs}(\vec{\Phi}) g^{\mu\nu} \partial_\mu \Phi_s \partial_\nu \Phi_r - f_{ij}(\vec{\Phi}) g^{\mu\rho} g^{\nu\sigma} F^{(i)}_{\mu\nu} F^{(j)}_{\rho\sigma}$$

Ansatz: $AdS_2 \times S^2$ **near horizon geometry**

$$\begin{aligned} ds^2 &= v_1 \left(-r^2 dt^2 + dr^2/r^2 \right) + v_2 d\Omega_2^2 \\ F^i_{rt} &= e^i \qquad F^i_{\theta\phi} = \frac{p^i}{4\pi} \sin\theta \\ \Phi_r &= u_r \, (\textbf{const.}) \end{aligned}$$

Entropy function

Wish to calculate:

$$\mathcal{E} = 2\pi(q_i e^i - f) = 2\pi\left(q_i e^i - \int d\theta d\phi \sqrt{-g}\mathcal{L}\right)$$

Calculate the action:

$$f[\vec{e},\vec{p},\vec{u},v_1,v_2] = (4\pi)(v_1 v_2)\left(\frac{2}{v_2} - \frac{2}{v_1} + f_{ij}(u_r)\left(\frac{2e^i e^j}{v_1^2} - \frac{2p^i p^j}{(4\pi)^2 v_2^2}\right)\right)$$

Calculate the conjugate variables:

$$q_i = \frac{\partial f}{\partial e^i} = (16\pi)(v_1 v_2) f_{ij}(u_r)\left(\frac{e^j}{v_1^2}\right)$$

$\Rightarrow$

$$e^j = \left(\frac{1}{16\pi}\right)\left(\frac{v_1}{v_2}\right) f^{ik} q_k$$

Entropy function

Finally

$$\mathcal{E}[\vec{q},\vec{p},\vec{u},v_1,v_2] = 2\pi\left(8\pi(v_2-v_1)+\left(\frac{v_1}{v_2}\right)V_{eff}\right)$$

Notation

$$V_{eff} = \frac{1}{2\pi}\left[p^i f_{ij}(\vec{u})p^j + \frac{1}{16}q_i f^{ij}(\vec{u})q_j\right]$$

Entropy function

Finally

$$\mathcal{E}[\vec{q},\vec{p},\vec{u},v_1,v_2] = 2\pi\left(8\pi(R_S^2 - R_{AdS}^2) + \left(\frac{R_{AdS}^2}{R_S^2}\right)V_{eff}\right)$$

Notation

$$V_{eff} = \frac{1}{2\pi}\left[p^i f_{ij}(\vec{u})p^j + \frac{1}{16}q_i f^{ij}(\vec{u})q_j\right]$$

Equations of Motion

Then the equations of motion are equivalent to extremising the entropy function:

$$\frac{\partial \mathcal{E}}{\partial \Phi_I} = 0 \;\;\Rightarrow\;\; \frac{\partial V_{eff}}{\partial \Phi_I} = 0$$
$$\frac{\partial \mathcal{E}}{\partial v_1} = 0 \;\;\Rightarrow\;\; 8\pi - v_2^{-1} V_{eff}(\Phi_I) = 0$$
$$\frac{\partial \mathcal{E}}{\partial v_2} = 0 \;\;\Rightarrow\;\; -8\pi + v_1 v_2^{-2} V_{eff}(\Phi_I) = 0$$

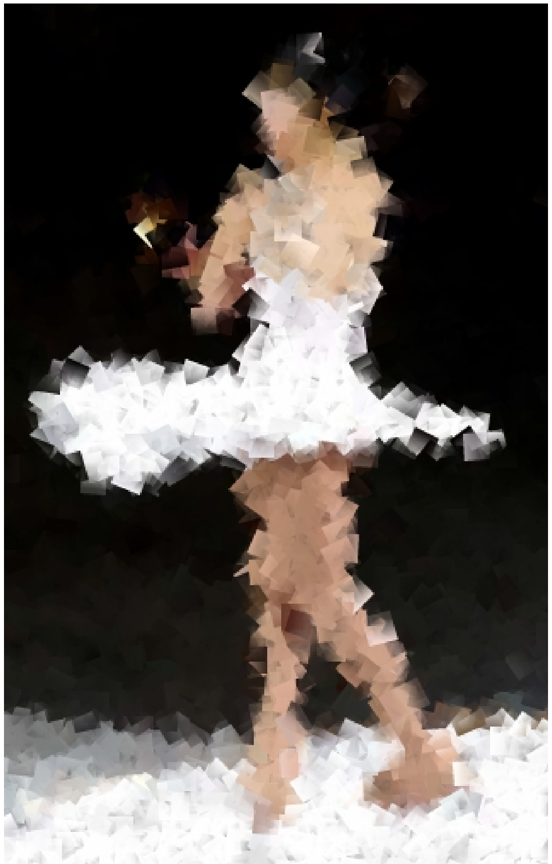
So

$$v_1 = v_2 = 8\pi V_{eff}$$

and

$$S_{BH} = 2\pi V_{eff}$$

Rotation



What is the generalisation of an $AdS_2 \times S^2$ near horizon geometry for rotating blackholes?

☞ **Take a hint from the near horizon geometry of extremal Kerr Blackholes (Bardeen, Horowitz)**

- ❑ $SO(2,1) \times U(1)$

Recall: $SO(2,1) \times S^2$ Ansatz

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + v_2 d\theta^2 + v_2 \sin^2\theta \, d\phi^2$$

$$\varphi_s = u_s$$

$$\frac{1}{2} F^{(i)}_{\mu\nu} dx^\mu \wedge dx^\nu = e^i dr \wedge dt + \frac{p^i \sin\theta}{4\pi} d\theta \wedge d\phi$$

$SO(2,1) \times U(1)$ Ansatz

$$ds^2 = v_1(\theta)\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + \beta^2 d\theta^2 + v_2(\theta)\sin^2\theta\,(d\phi - \alpha r dt)^2$$

$$\varphi_s = u_s(\theta)$$
$$A^i = e^i r dt + b^i(\theta)(d\phi - \alpha r\, dt)$$

Horizon has spherical topology $\Rightarrow v_2(\theta)$ **at poles** ~ 1

$$p^i = \int d\theta d\phi F^{(i)}_{\theta\phi} = 2\pi(b^i(\pi) - b^i(0))\,.$$

$SO(2,1)\times U(1)$ Ansatz

$$ds^2 = v_1(\theta)\left(-r^2dt^2+\frac{dr^2}{r^2}\right)+\beta^2d\theta^2+v_2(\theta)\sin^2\theta\,(d\phi-\alpha r dt)^2$$

$$\varphi_s = u_s(\theta)$$

$$\tfrac{1}{2}F^{(i)}_{\mu\nu}dx^\mu\wedge dx^\nu = (e^i-\alpha b^i(\theta))dr\wedge dt + b^{i\prime}(\theta)d\theta\wedge(d\phi-\alpha r\,dt)$$

Horizon has spherical topology $\Rightarrow v_2(\theta)$ **at poles** ~ 1

$$p^i = \int d\theta d\phi F^{(i)}_{\theta\phi} = 2\pi(b^i(\pi)-b^i(0))\,.$$

$SO(2,1) \times U(1)$ Ansatz

$$ds^2 = \Omega^2 e^{2\psi}\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + \beta d\theta^2 + e^{-2\psi}\left(d\phi - \alpha r dt\right)^2$$

$$\varphi_s = u_s(\theta)$$

$$\tfrac{1}{2}F^{(i)}_{\mu\nu}dx^\mu \wedge dx^\nu = (e^i - \alpha b^i(\theta))dr \wedge dt + b^{i\prime}(\theta)d\theta \wedge (d\phi - \alpha r\, dt)$$

Horizon has spherical topology $\Rightarrow e^{-2\psi}$ **at poles** $\sim \sin^2\theta$

$$p^i = \int d\theta d\phi F^{(i)}_{\theta\phi} = 2\pi(b^i(\pi) - b^i(0))\,.$$

Symmetries

☞ One way to see that the ansatz has $SO(2,1) \times U(1)$ symmetries is to check that it is invariant under the Killing vectors, ∂_ϕ and

$$L_1 = \partial_t, \quad L_0 = t\partial_t - r\partial_r, \quad L_{-1} = \frac{1}{2}\left(\frac{1}{r^2} + t^2\right)\partial_t - (tr)\partial_r + \frac{\alpha}{r}\partial_\phi\,.$$

☞ can also be seen by thinking of ϕ as a compact dimension and find that the resulting geometry has a manifest $SO(2,1)$ symmetry with the conventional generators.

Entropy function:

We define

$$f[\alpha, \beta, \vec{e}, \Omega(\theta), \psi(\theta), \vec{u}(\theta), \vec{b}(\theta)] := \int d\theta d\phi \sqrt{-g}\mathcal{L}$$

☞ **The equations of motion are:**

$$\frac{\partial f}{\partial \alpha} = J \qquad \frac{\partial f}{\partial \beta} = 0 \qquad \frac{\partial f}{\partial e^i} = q_i \qquad \frac{\delta f}{\delta b^i(\theta)} = 0$$

$$\frac{\delta f}{\delta \Omega(\theta)} = 0 \quad \frac{\delta f}{\delta \psi(\theta)} = 0 \quad \frac{\delta f}{\delta u_s(\theta)} = 0$$

Entropy function:

Equivalently we let

$$\mathcal{E}[J,\vec{q},\vec{b}(\theta),\beta,v_1(\theta),v_2(\theta),\vec{u}(\theta)] = 2\pi\,(J\alpha+\vec{q}\cdot\vec{e}-f)$$

☞ **The equations of motion:**

$$\frac{\partial\mathcal{E}}{\partial\alpha}=0 \qquad \frac{\partial\mathcal{E}}{\partial\beta}=0 \qquad \frac{\partial\mathcal{E}}{\partial e^i}=0 \qquad \frac{\delta\mathcal{E}}{\delta b^i(\theta)}=0$$

$$\frac{\delta\mathcal{E}}{\delta v_1(\theta)}=0 \quad \frac{\delta\mathcal{E}}{\delta v_2(\theta)}=0 \quad \frac{\delta\mathcal{E}}{\delta u_s(\theta)}=0$$

Examples

- ☞ **Kerr, Kerr-Newman, constant scalars (non-dyonic)**
- ☞ **Dyonic Kaluza Klein blackhole (5-d$\circlearrowleft\longrightarrow 4$-d).**
  -(Rasheed)
- ☞ **Blackholes in toroidally compactified heterotic string theory**
  -(Cvetic,Youm;Jatkar,Mukherji,Panda)

Two derivative Lagrangians

$$\mathcal{L} = R - h_{rs}(\vec{\Phi}) g^{\mu\nu} \partial_\mu \Phi_s \partial_\nu \Phi_r - f_{ij}(\vec{\Phi}) g^{\mu\rho} g^{\nu\sigma} F^{(i)}_{\mu\nu} F^{(j)}_{\rho\sigma}$$

$$\begin{aligned}
\mathcal{E} &\equiv 2\pi (J\alpha + \vec{q}\cdot\vec{e} - \int d\theta d\phi \sqrt{-\det g}\, \mathcal{L}) \\
&= 2\pi J\alpha + 2\pi \vec{q}\cdot\vec{e} - 4\pi^2 \int d\theta \Bigg[2\Omega^{-1}\beta^{-1}\Omega'^2 - 2\Omega\beta - 2\Omega\beta^{-1}\psi'^2 \\
&\quad + \frac{1}{2}\alpha^2 \Omega^{-1}\beta e^{-4\psi} - \beta^{-1}\Omega h_{rs}(\vec{u}) u'_r u'_s \\
&\quad + 2 f_{ij}(\vec{u}) \left\{ \beta\Omega^{-1} e^{-2\psi}(e^i - \alpha b^i)(e^j - \alpha b^j) - \beta^{-1}\Omega e^{2\psi} b^{i\prime} b^{j\prime} \right\} \Bigg] \\
&\quad + 8\pi^2 \left[\Omega^2 e^{2\psi} \sin\theta (\psi' + 2\Omega'/\Omega) \right]_{\theta=0}^{\theta=\pi} .
\end{aligned}$$

Equations of motion

Notation:

$$\chi^i = e^i - \alpha b^i$$

Ω equation:

$$-4\beta^{-1}\Omega''/\Omega + 2\beta^{-1}(\Omega'/\Omega)^2 - 2\beta - 2\beta^{-1}(\psi')^2 - \frac{1}{2}\alpha^2\Omega^{-2}\beta e^{-4\psi}$$
$$-\beta^{-1}h_{rs}u'_r u'_s + 2f_{ij}\left\{-\beta\Omega^{-2}e^{-2\psi}\chi^i\chi^j - \alpha^{-2}\beta^{-1}e^{2\psi}\chi^{i\prime}\chi^{j\prime}\right\} = 0,$$

ψ equation:

$$4\beta^{-1}(\Omega\psi')' - 2\alpha^2\Omega^{-1}\beta e^{-4\psi}$$
$$+2f_{ij}\left\{-2\beta\Omega^{-1}e^{-2\psi}\chi^i\chi^j - 2\alpha^{-2}\beta^{-1}\Omega e^{2\psi}\chi^{i\prime}\chi^{j\prime}\right\} = 0,$$

u_s **equation:**

$$2\left(\beta^{-1}\Omega h_{rs}u'_s\right)' + 2\partial_r f_{ij}\left\{\beta\Omega^{-1}e^{-2\psi}\chi^i\chi^j - \alpha^{-2}\beta^{-1}\Omega e^{2\psi}\chi^{i\prime}\chi^{j\prime}\right\} \\ - \beta^{-1}\Omega\left(\partial_r h_{ts}\right)u'_t u'_s = 0,$$

b **equation:**

$$-\alpha\beta f_{ij}\Omega^{-1}e^{-2\psi}\chi^j - \alpha^{-1}\beta^{-1}\left(f_{ij}\Omega e^{2\psi}\chi^{j\prime}\right)' = 0$$

β **equation:**

$$\int d\theta I(\theta) = 0$$

where

$$I(\theta) = -2\Omega^{-1}\beta^{-2}(\Omega')^2 - 2\Omega + 2\Omega\beta^{-2}(\psi')^2 + \frac{1}{2}\alpha^2\Omega^{-1}e^{-4\psi} \\ + \beta^{-2}\Omega h_{rs}u'_r u'_s + 2f_{ij}\left\{\Omega^{-1}e^{-2\psi}\chi^i\chi^j + \alpha^{-2}\beta^{-2}\Omega(\theta)e^{2\psi(\theta)}\chi^{i\prime}\chi^{j\prime}\right\}$$

Charges:

$$q_i = 8\pi \int d\theta \, \left[f_{ij} \beta \Omega^{-1} e^{-2\psi} \chi^j \right] ,$$

$$J = \quad 2\pi \int_0^\pi d\theta \left\{ \alpha \Omega^{-1} \beta e^{-4\psi} - 4\beta f_{ij} \Omega^{-1} e^{-2\psi} \chi^i b^j \right\}$$

Solutions

☞ Equations can be solved for some simple cases

❑ Kerr, Kerr-Newmann, constant scalars

☞ Check known solutions fitted into the frame work:

❑ KK blackholes, Toroidal compactification of Heterotic string theory

Kaluza-Klein Blackholes

$$\mathcal{L} = R - 2(\partial\varphi)^2 - e^{2\sqrt{3}\varphi}F^2$$

☞ **Charges = Q, P, J**

☞ **2 types of extremal blackholes**

- ❑ **Both have $SO(2,1) \times U(1)$ near horizon geometry**
- ❑ **non-SUSY**

1. Ergo branch

- ☞ $|J| > PQ$
- ☞ **Ergo-sphere**
- ☞ $S = 2\pi\sqrt{J^2 - P^2Q^2}$
- ☞ $\mathcal{E}$ **has flat directions**

2. Ergo-free branch

- ☞ $|J| < PQ$
- ☞ **no Ergo-sphere**
- ☞ $S = 2\pi\sqrt{P^2Q^2 - J^2}$
- ☞ $\mathcal{E}$ **has no flat directions**

Blackholes in Heterotic String Theory on T^6

☞ **Charges** $= Q_1,\ Q_2,\ Q_3,\ Q_4, P_1,\ P_2,\ P_3,\ P_4,\ J,$

❑ **(Actually** 56 P**'s and** Q**'s)**

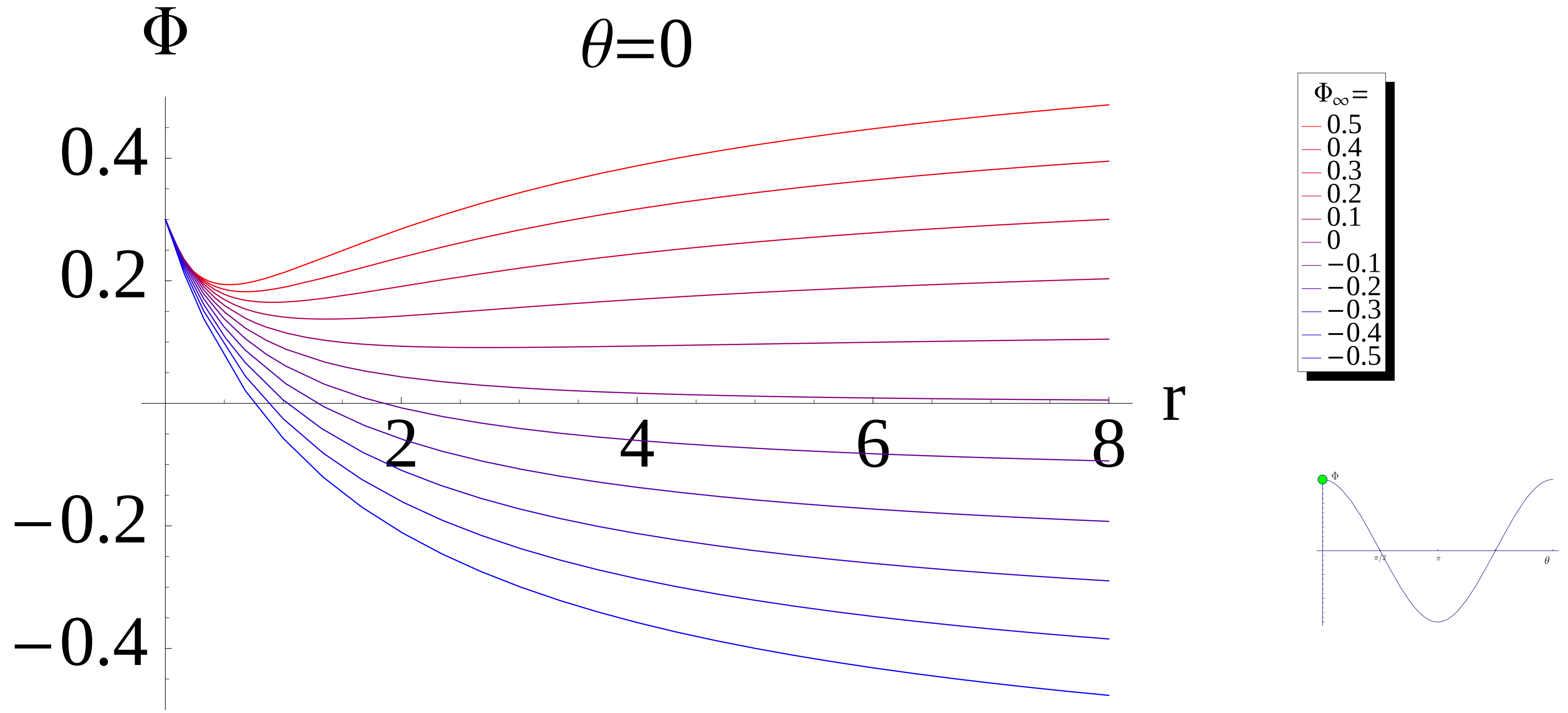
☞ **Duality invariant quartic**

$$\begin{aligned} D &= (Q_1Q_3+Q_2Q_4)(P_1P_3+P_2P_4) \\ &\quad -\frac{1}{4}(Q_1P_1+Q_2P_2+Q_3P_3+Q_4P_4)^2 \end{aligned}$$

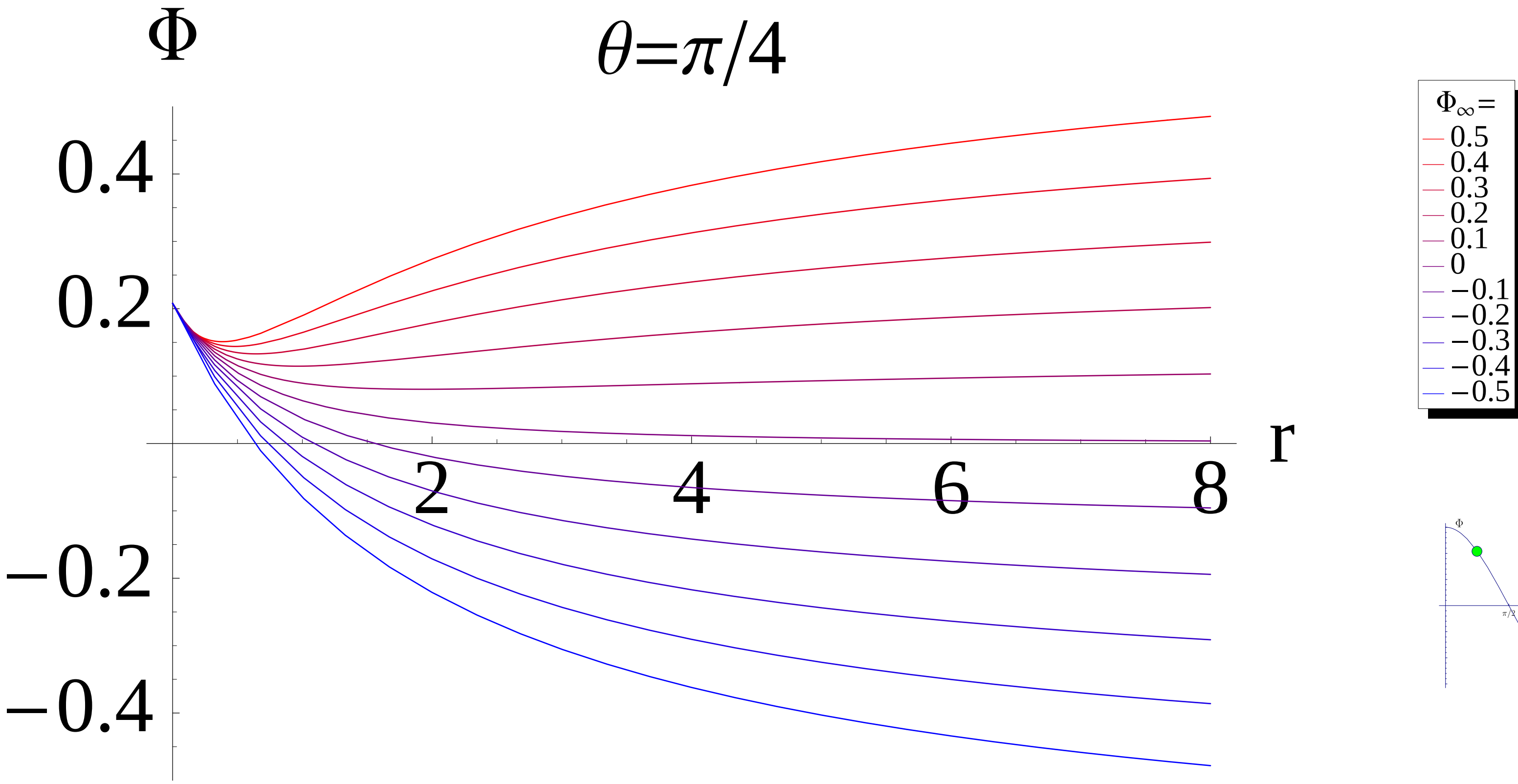
1. **Ergo branch**
   - ☞ **Ergo-sphere**
   - ☞ $S = 2\pi\sqrt{J^2+D}$
   - ☞ $\mathcal{E}$ **has flat directions**

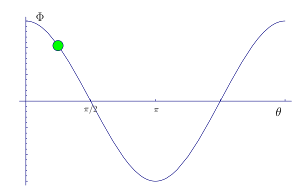
2. **Ergo-free branch**
   - ☞ **no Ergo-sphere**
   - ☞ $S = 2\pi\sqrt{-J^2-D}$
   - ☞ $\mathcal{E}$ **has no flat directions**

Ergo-free branch



Ergo-free branch





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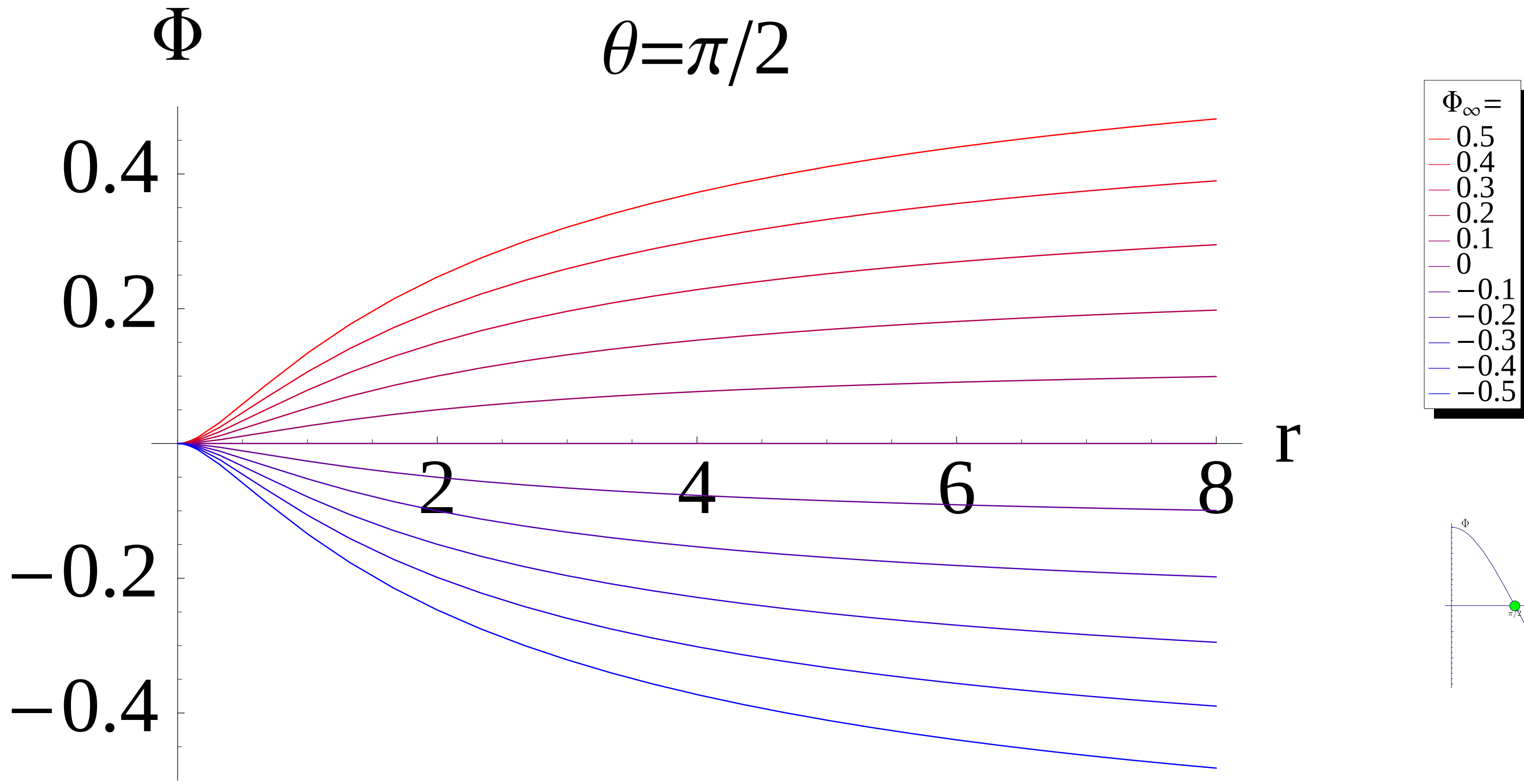
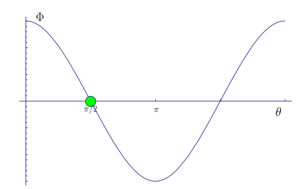
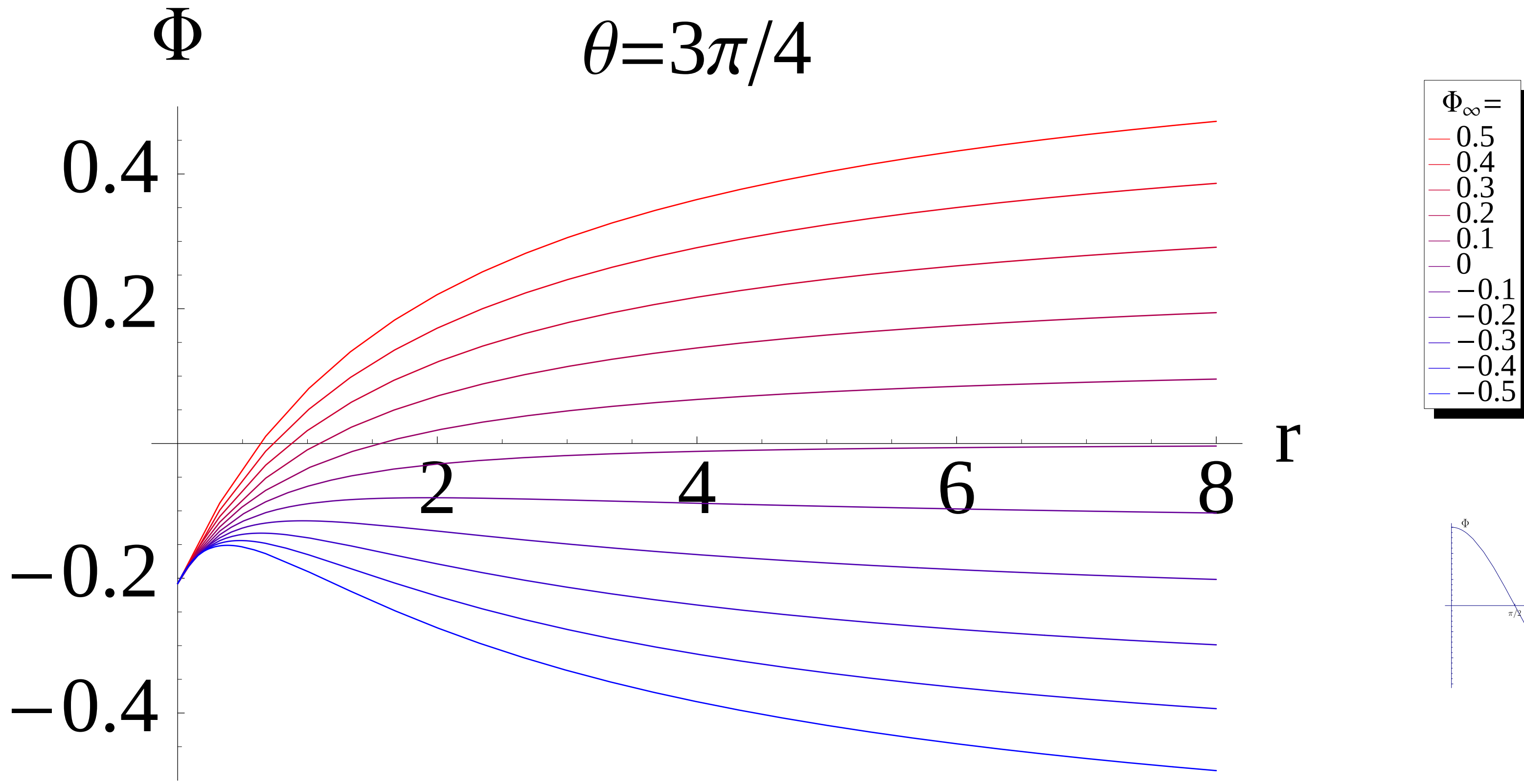
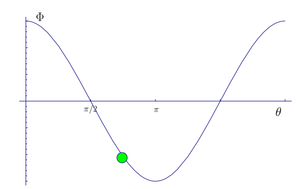


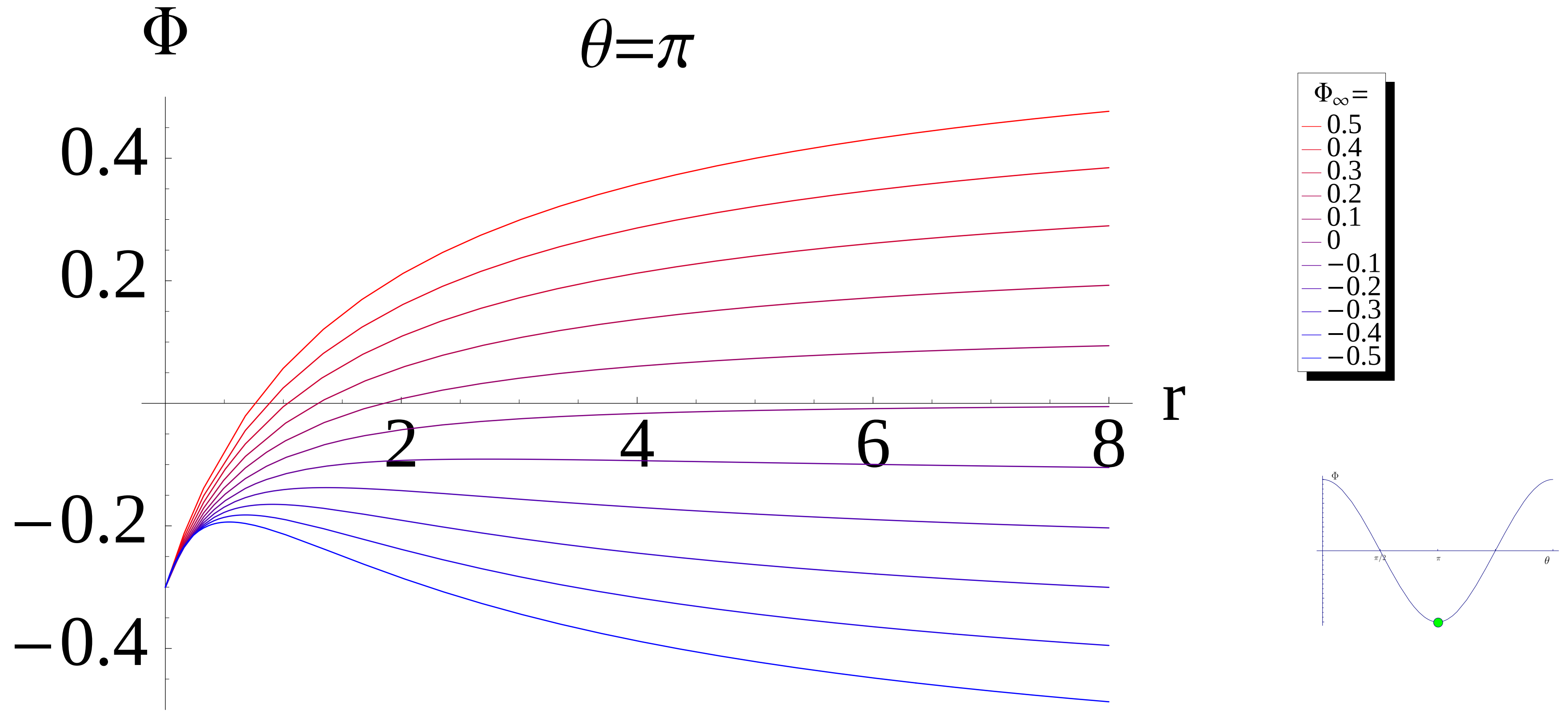


Ergo-free branch

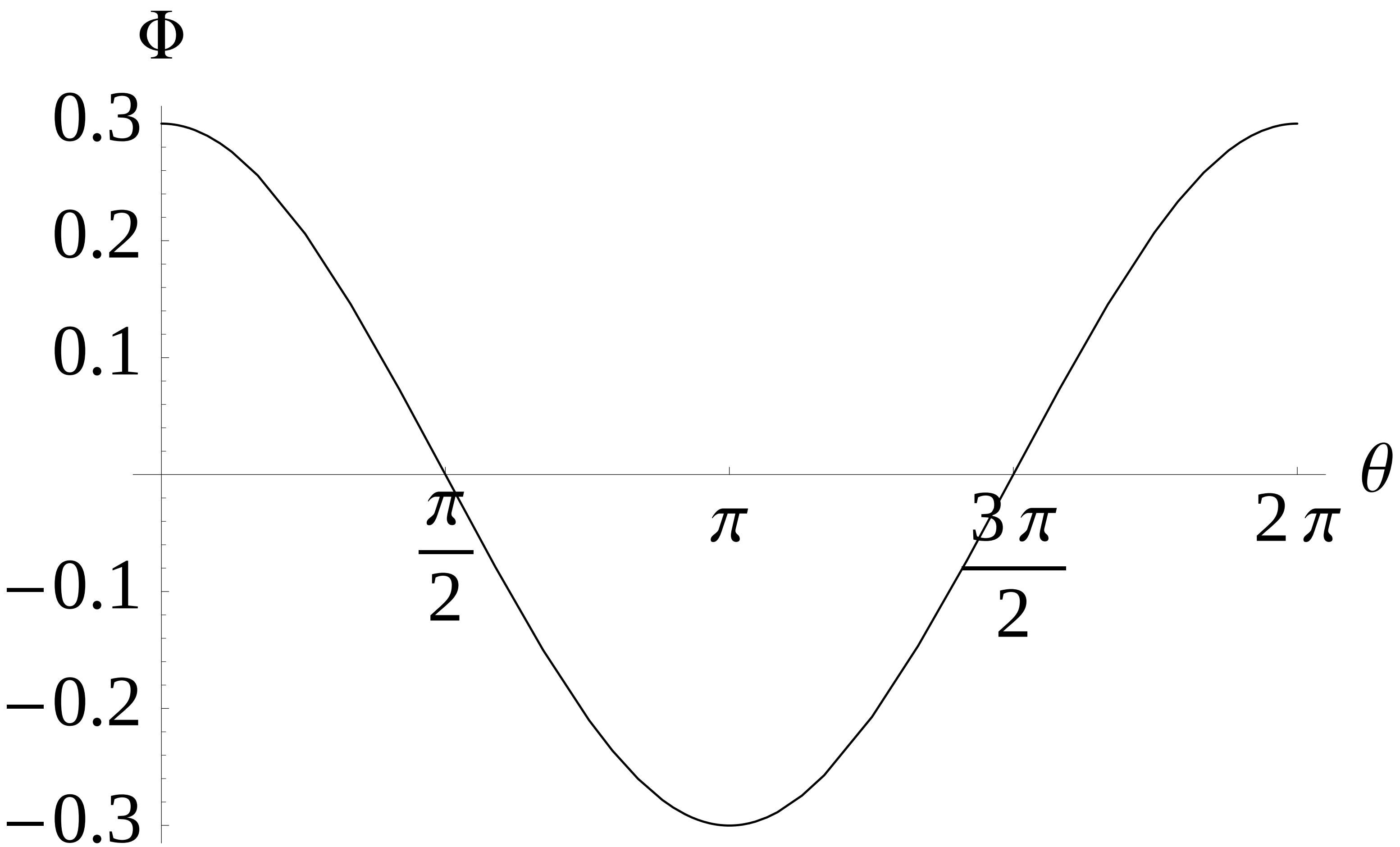




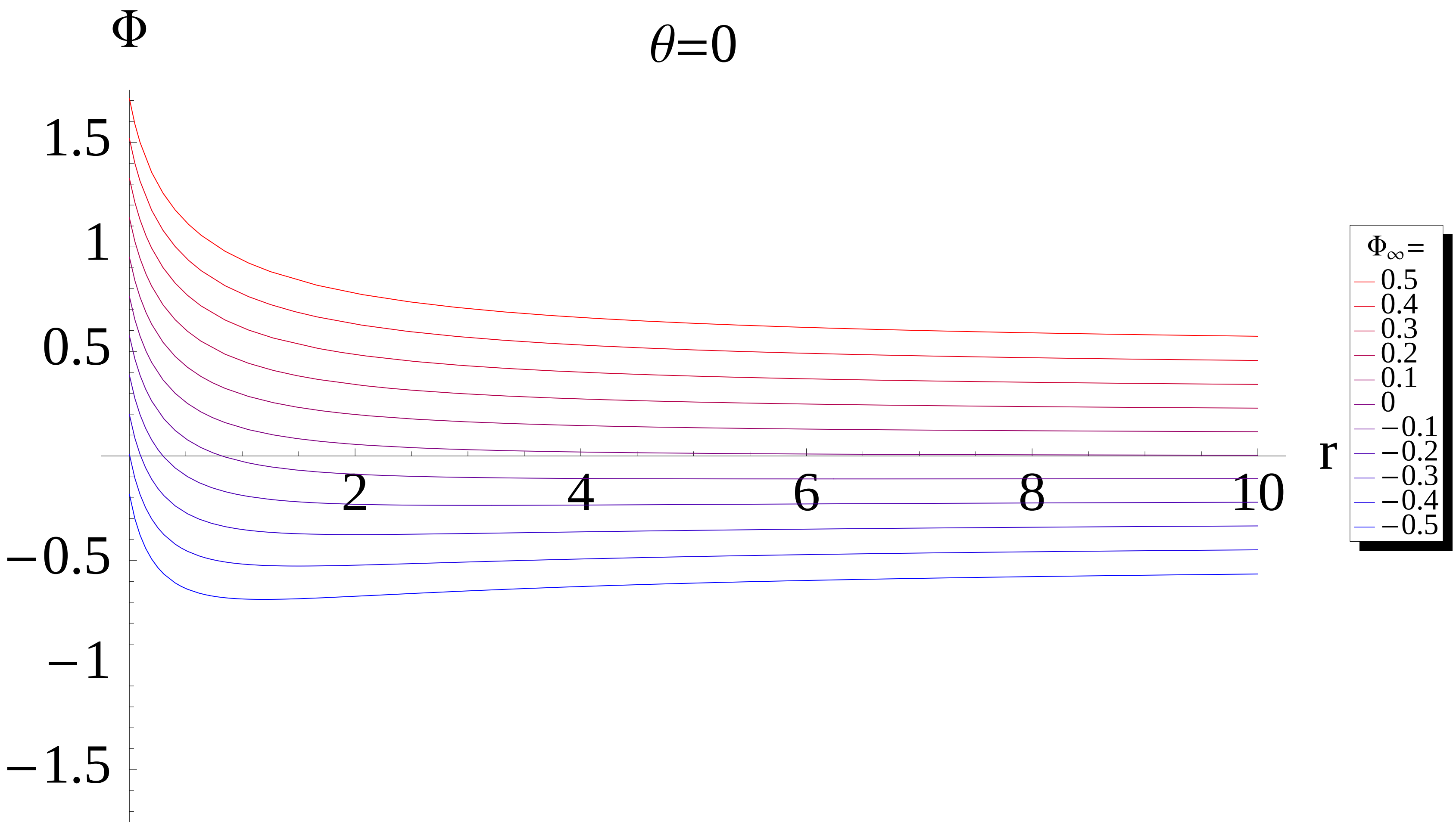
Ergo-free branch



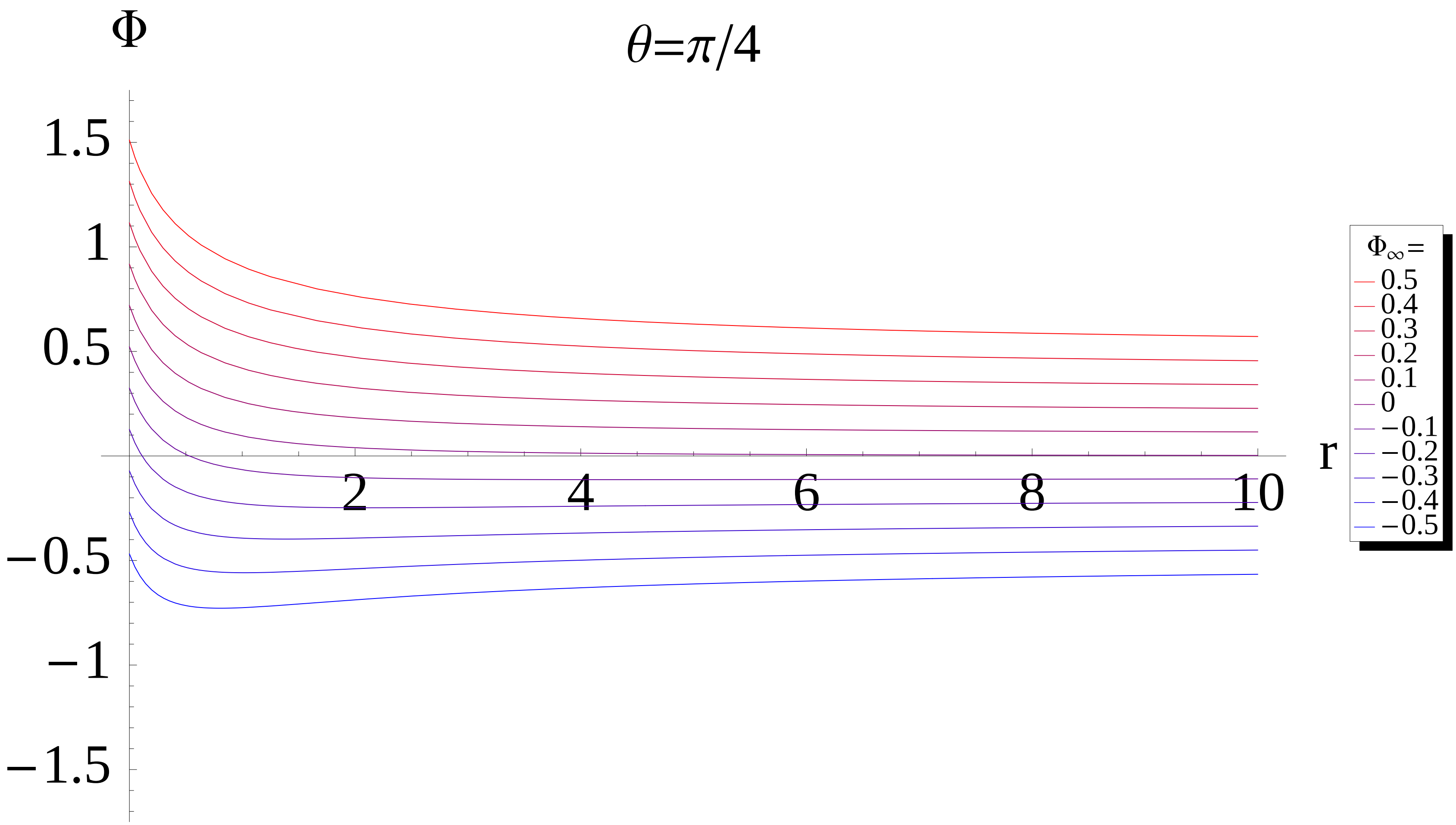
Scalar Field at Horizon



Ergo-branch

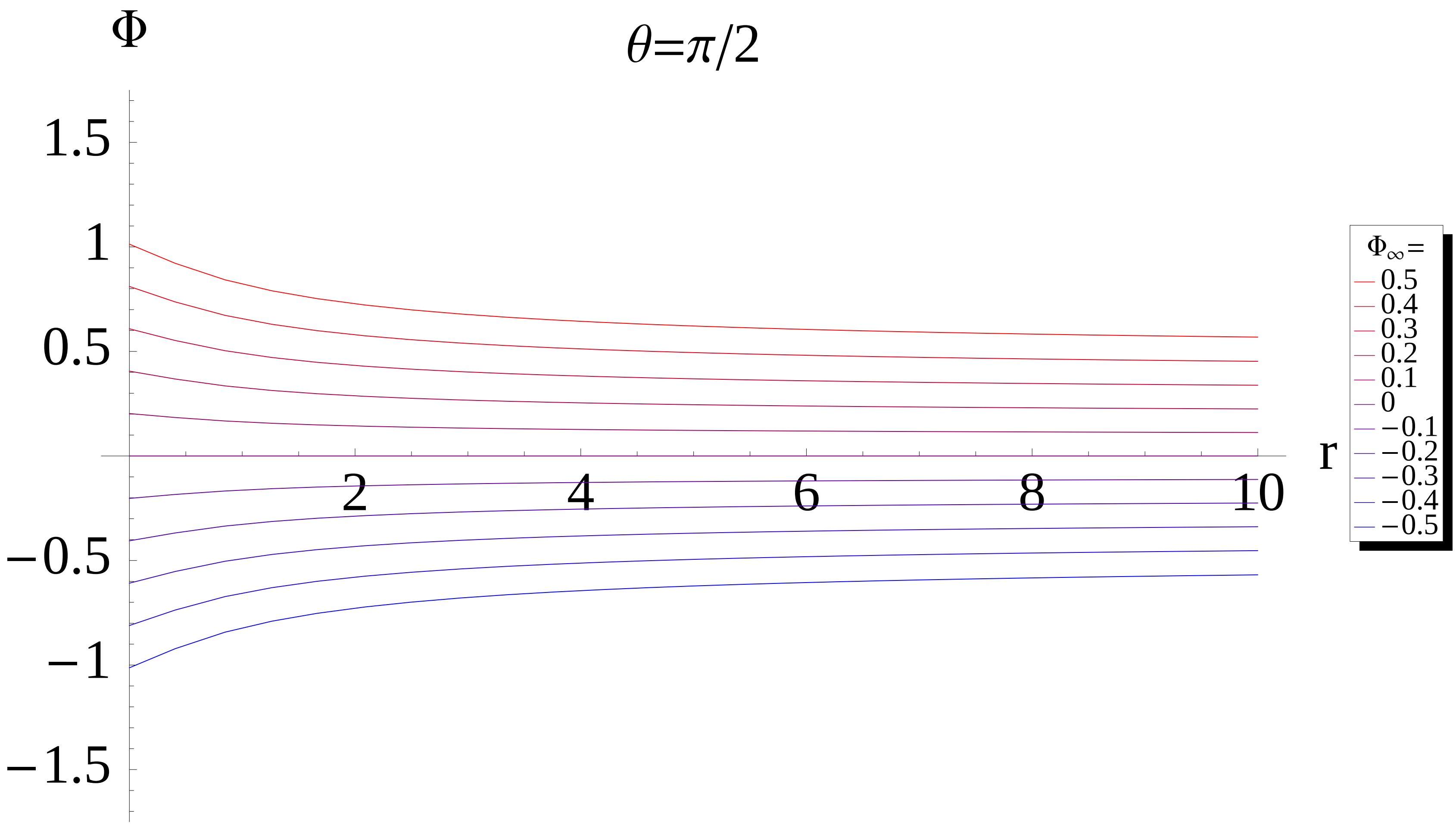


Ergo-branch

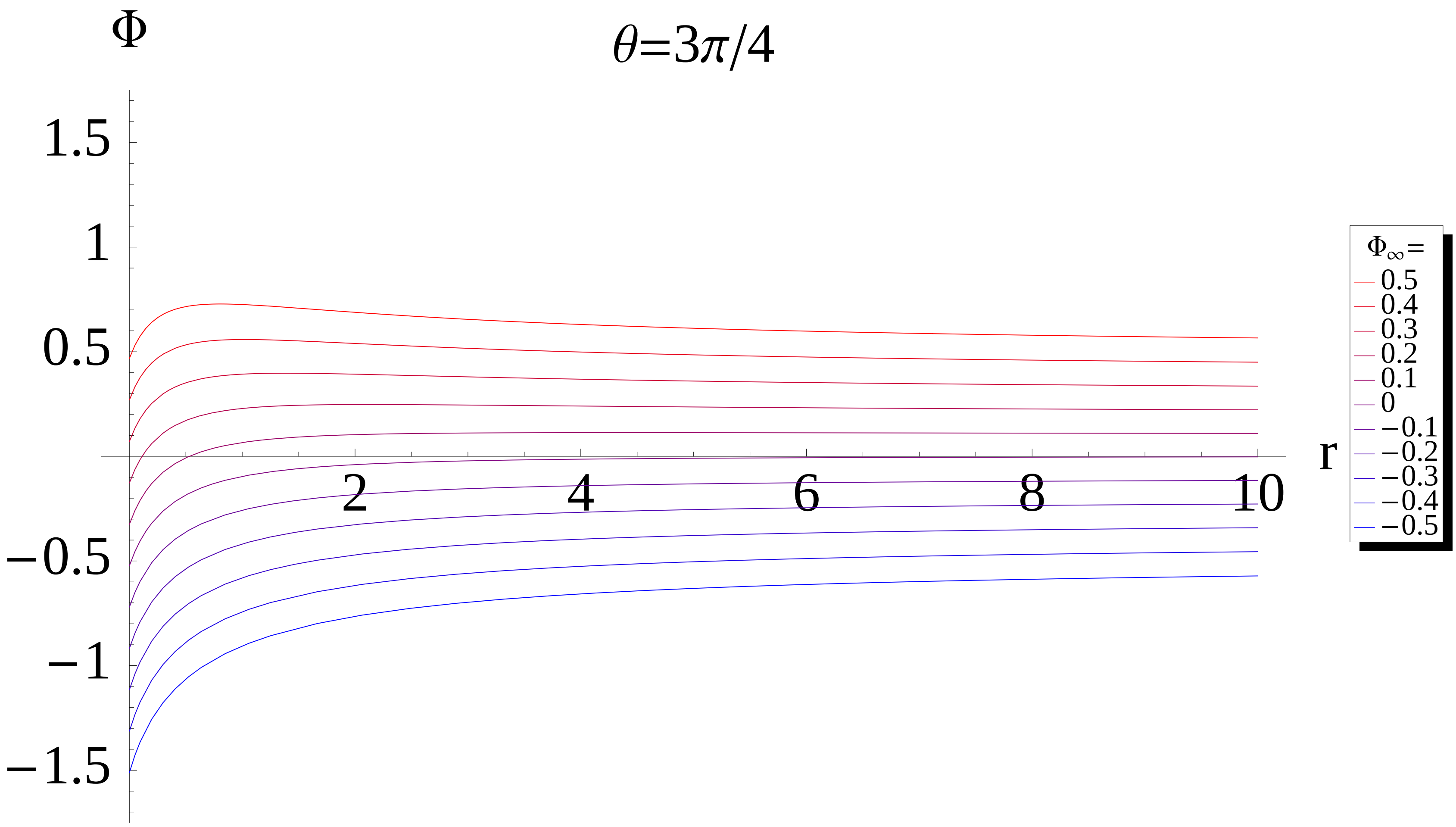


Φ
$\theta=\pi/4$
1.5
1
0.5
−0.5
−1
−1.5
2
4
6
8
10
r
$\Phi_\infty=$
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5

Ergo-branch

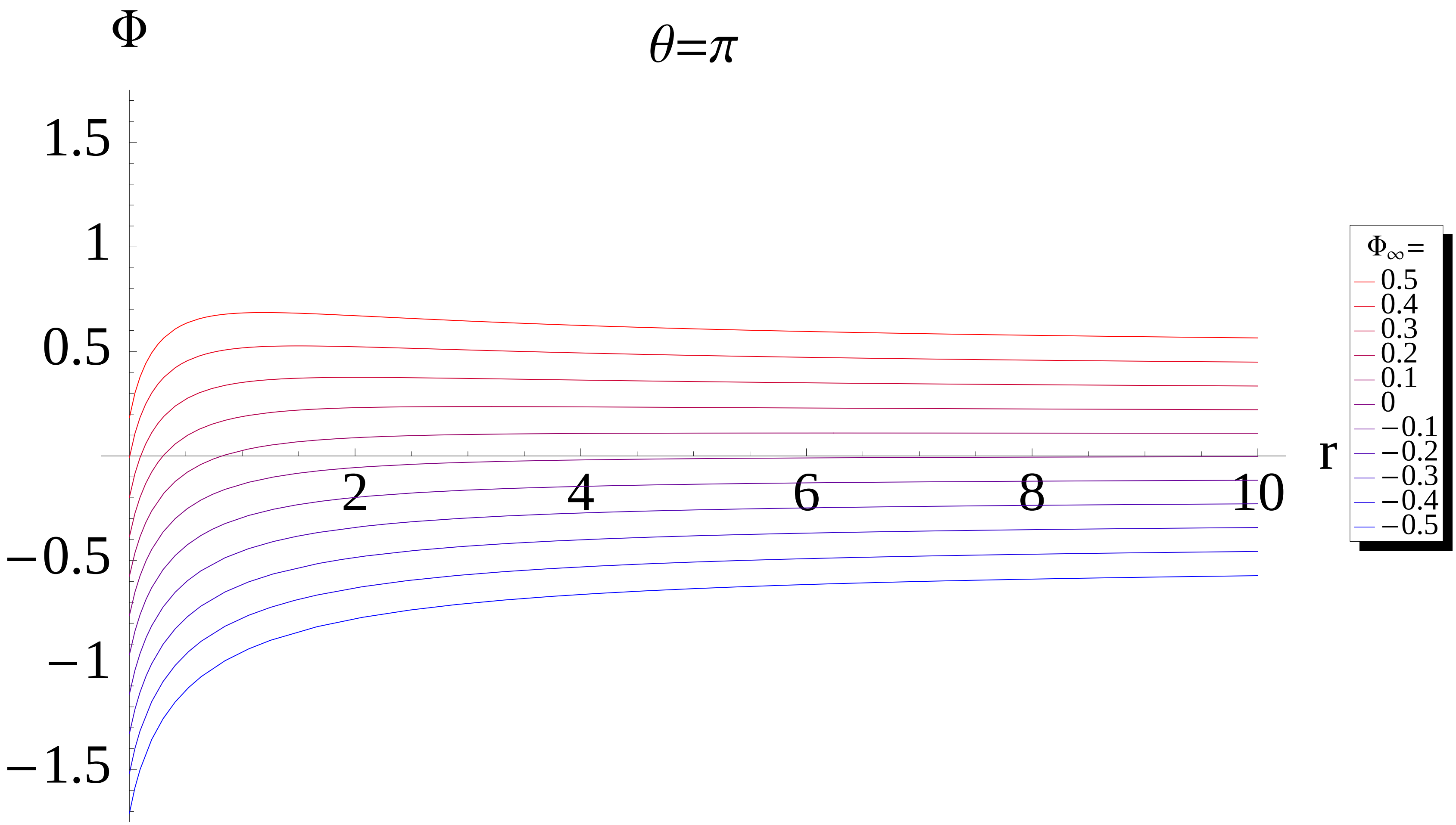


Ergo-branch

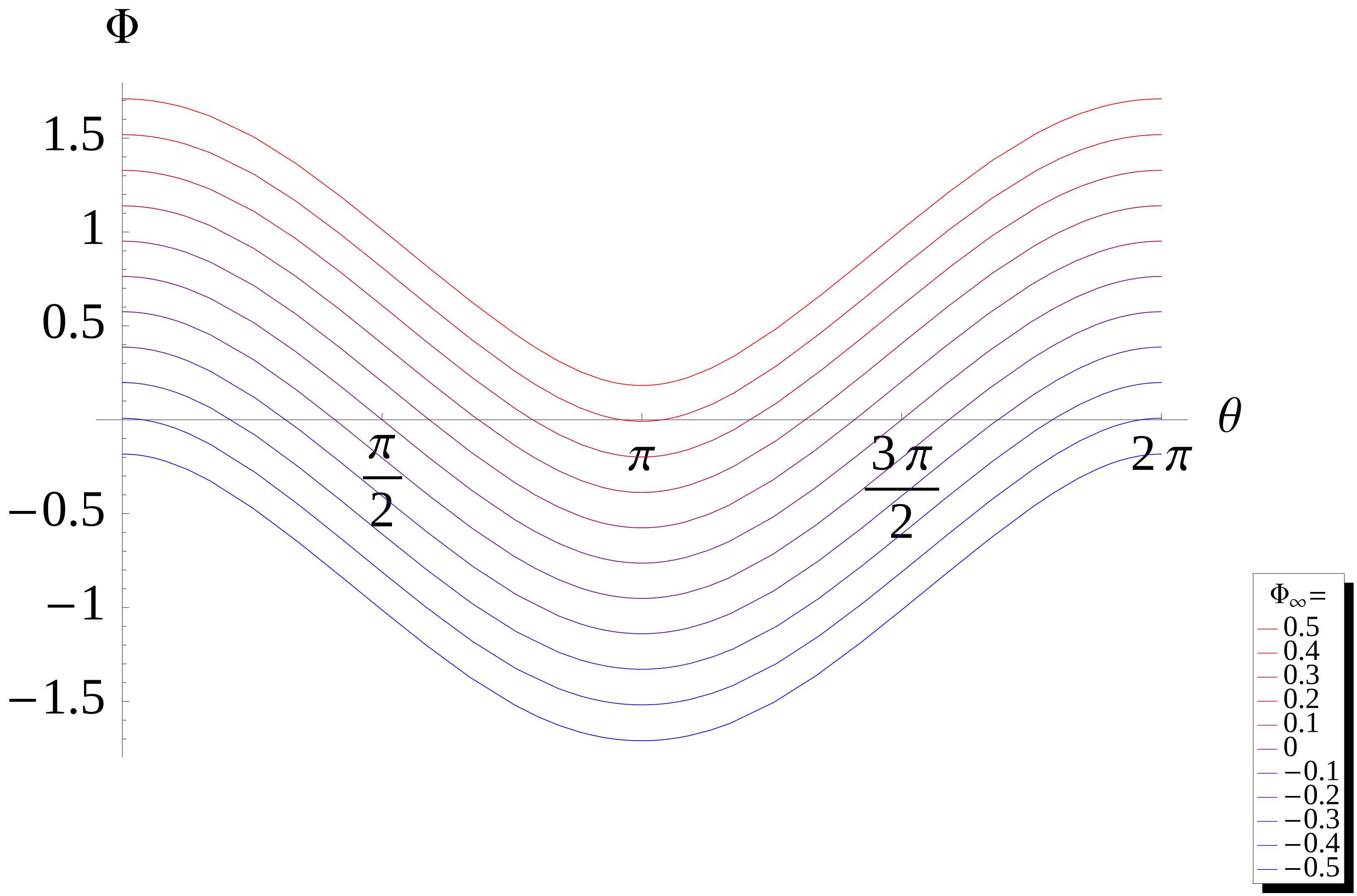


$\theta=3\pi/4$
Φ
r
1.5
1
0.5
−0.5
−1
−1.5
2
4
6
8
10
$\Phi_\infty=$
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5

Ergo-branch



Scalar Field at Horizon



Black ring attractors (5-d)

Consider 5-d Lagrangian with massless uncharged scalars coupled to $U(1)$ gauge fields with Chern-Simons terms:

$$\mathcal{L} = R - h_{ab}(\vec{\Phi})\partial_\mu \Phi^a \partial^\mu \Phi^b - f_{ij}(\vec{\Phi}) F^i_{\mu\nu} F^{j\,\mu\nu} - c_{ijk}\epsilon^{\mu\nu\alpha\beta\gamma} F^i_{\mu\nu} F^j_{\alpha\beta} A^k_\gamma$$

Action is not gauge invariant $\longrightarrow$ Entropy function formalism does not apply

- ☞ similar to BTZ black hole with gravitational Chern-Simons and/or gauge Chern-Simons term
- ☞ compactify ψ (Sen, Sahoo)
- ✔ can apply formalism to dimensionally reduced action
- ☞ Related work: (Kraus, Larsen), (Dabholkar, Iizuka, Iqubal, Sen, Shigemori)

Black ring attractors (5-d)

Ansatz: $AdS_2 \times S^1 \times S^2$

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + w^2 (d\psi + e\, r dt)^2 + v_2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right),$$

$$\Phi_s = u_s(\theta)$$
$$A^i = e^i r dt + A^i_\psi (d\psi + e\, r dt) - p^i \cos\theta d\phi,$$

☞ **dimensionally reduce**

☞ **calculate entropy function for our ansatz**

Black ring attractors (5-d)

Ansatz: $AdS_2 \times S^1 \times S^2$

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + w^2 \left(d\psi + e\, r dt\right)^2 + v_2 \left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

$$\Phi_s = u_s(\theta)$$

$$\tfrac{1}{2} F^{(i)}_{\mu\nu} dx^\mu \wedge dx^\nu = (e^i + e A^i_\psi) dr \wedge dt + p^i \sin\theta\, d\theta \wedge d\phi$$

☞ **dimensionally reduce**

☞ **calculate entropy function for our ansatz**

Calculation

☞ **First we consider, f, the (dimensionally reduced) action evaluated at the horizon**

$$f[v_1, v_2, w, e, \vec{A}_\psi, \vec{p}, \vec{e}, \vec{u}] = \int_H \sqrt{-g}\mathcal{L}$$

☞ **The equations of motion are**

$$\frac{\partial f}{\partial v_1} = 0 \quad \frac{\partial f}{\partial v_2} = 0 \quad \frac{\partial f}{\partial w} = 0$$

$$\frac{\partial f}{\partial A^i_\psi} = 0 \quad \frac{\partial f}{\partial u_a} = 0 \quad \frac{\partial f}{\partial e^i} = q_i \quad \frac{\partial f}{\partial e} = q$$

Entropy function

☞ **To calculate the entropy function,**

$$\mathcal{E} = 2\pi(qe + \vec{q}\cdot\vec{e} - f),$$

it is convenient to eliminate A^i_ψ, e and $\vec{e}$ using their equations of motion

☞ **Except in special cases, we find, from the A^i_ψ equation of motion, $F^i_{tr} = 0$**

Entropy function

After some algebra, we obtain the entropy function

$$\mathcal{E} = 32\pi^3 w \left(v_2 - v_1 + \frac{v_1}{v_2} \left(V_{eff} + \frac{\frac{3}{4}\hat{q}^2}{w^4} \right) \right),$$

where

$$V_{eff} = f_{ij}(\vec{u}) p^i p^j$$
$$\hat{q} \sim \left(q - \tfrac{1}{4} d^{ij} q_i q_j \right)$$

☞ **and** d^{ij} **is proportional to inverse of** $c_{ijk}p^k$**.**

Solution

Extremising the entropy function we find

$$\mathcal{E} = 128\pi^3 \sqrt{\frac{\hat{q}(V_{eff})^{\frac{3}{2}}}{6}}\Bigg|_{\partial V=0} = 32\pi^3 w v_2 = \frac{A}{4G_N}$$

with

$$v_1 = v_2 = \tfrac{4}{3} V_{eff}\Big|_{\partial V=0}$$

$$w^4 = \frac{\frac{9}{4}\hat{q}^2}{V_{eff}}\Bigg|_{\partial V=0}$$

and

$$e^2 w^2 = v_1 \Rightarrow AdS_3 \times S^2$$

Puzzles

- ☞ **What is the relationship between the ergosphere and the uniqueness of the Entropy function?**

- ☞ **Under what conditions do these solutions actually exist ?**

- ☞ **Why are these (bosonic) symmetries sufficient for the attractor mechanism ?**
  - ❏ AdS_2/CFT, AdS_3/CFT

- ☞ **What does this really tell us about the entropy of non-SUSY blackholes?**

Thank you