Decoding geometry from gauge theory

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ISM06, Puri
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Hubeny, Liu, MR

Balasubramanian, Czech, Hubeny, Larjo, MR, Simón

1. *Motivation*
   - Geometry from gauge theory
2. *Detecting black hole structures*
   - The black hole singularity
   - Black hole event horizon formation
3. *Distinguishing geometries*
   - Of distinct signatures and resolving power
   - On ensembles & variances
   - Black holes versus microstates: Toy models
4. *Conclusion*

Motivation

Deciphering the hologram

- AdS/CFT correspondence provides holographic duality between gravitational physics and gauge dynamics.
- How does this holographic map encode geometric data of interest in classical gravity in terms of the gauge theoretic observables?

Relevance:

- Making precise the nature of the holographic map
- Understanding "Quantum Geometry"
- Implications for bulk locality.

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Geometry in gauge theory

Q: Encoding of geometry

What are the gauge theory observables that encode geometric data of a given spacetime?

- Causal structure
- Event Horizons
- Singularities

To be precise, given that,

Field theory in a particular state $\leftrightarrow$ Specific bulk geometry

what observables in the field theory should we look at to extract information about the above geometric quantities? Example

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Results for gauge theory encoding

Detecting geometric structures using CFT correlators

- The n-point functions of the boundary theory carry sufficient information to detect some of the characteristic features of the spacetime geometry.
- The main issue is identifying the precise observables of interest.

Plan of talk:

- Use of CFT correlators to detect geometric structures in the bulk.
- Distinguishing geometries using CFT observables.

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Black hole singularity in gauge theory

Singularity and analytic structure

Information regarding the black hole singularity is encoded in the field theory correlators.

Balasubramanian, Ross

Kraus, Ooguri, Shenker

Fidkowski, Kleban, Hubeny, Shenker

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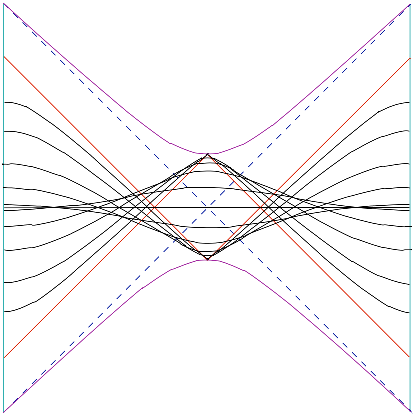
Singular correlators

- $\mathcal{O}$ is a gauge invariant local operator of dimension $\Delta \sim N$.
- Two point functions $\langle \mathcal{O}(t,\Omega)\,\mathcal{O}(t',\Omega)\rangle$ have a singularity for

$$t' = t - \frac{i\,\beta_{BH}}{2} + t_c$$

β_{BH} is the inverse Hawking temperature.

Black hole singularity in gauge theory



Spacelike geodesics to probing the singularity.

Black hole singularity in gauge theory

Some remarks on singularity probes

To extract the precise information corresponding to the singularity we need:

- Compute $\langle \mathcal{O}(t,\Omega)\,\mathcal{O}(t',\Omega)\rangle$
- Analytically continue the correlation function to detect the singularity.
- Cleaner understanding of the analytic structure by working directly in momentum space.

Festuccia, Liu

Black hole formation in gauge theory

Black hole collapse

Q: Can we use the AdS/CFT correspondence to probe black hole collapse?

- Identify gauge theoretic interpretation of horizon formation.
- Understand thermalization in field theory as the dual of black hole formation. Festuccia, Liu
- Long term goal: Prove field theory process corresponding to black hole formation and subsequent Hawking evaporation is unitary.

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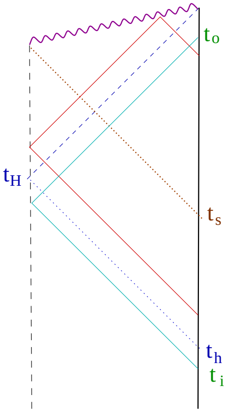
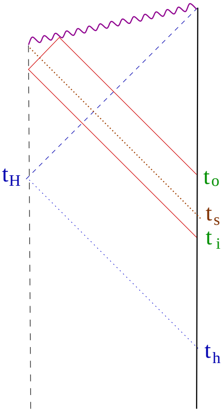
Black hole event horizon



(a)



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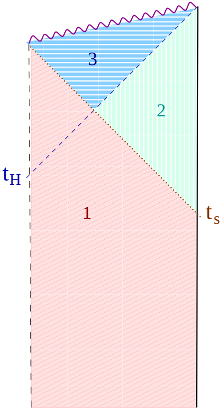
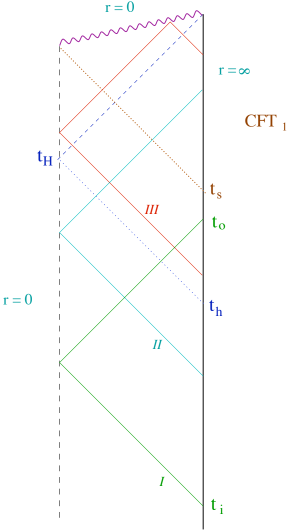


(c)

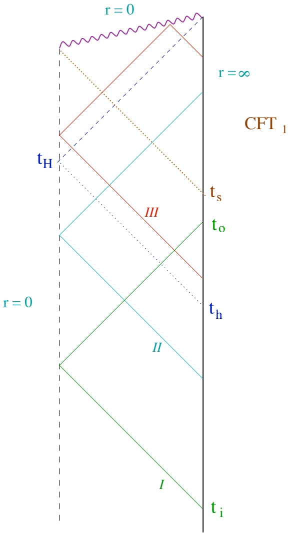
Black hole event horizon



- Spacetime is pure AdS for $t \leq t_s$; we change the state of the boundary at t_s.
- Probe the geometry by local operator correlation function $\langle\, \mathcal{O}(t_i, \Omega_i)\, \mathcal{O}(t_o, \Omega_o)\, \rangle$

Correlations summary

Black hole event horizon



- Want to detect the event corresponding to horizon formation, the instant t_H.
- Claim: we can detect this from the correlation functions.

Correlations summary

Generalities of bulk-cone singularities

Bulk-cone singularities

- Green's functions in curved spacetime $\mathcal{M}$ are singular at null separated points.

$$\mathcal{G}(X, Y) \to \infty \qquad \Longleftrightarrow \qquad ||X - Y||^2 \to 0$$

- Boundary correlation functions defined as limiting values of bulk correlators also inherit these singularities: bulk-cone singularities.

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Generalities of bulk-cone singularities

- CFT correlation functions on $\mathbf{S^3} \times \mathbf{R}$ in state $|*\rangle$

$$G(t_i, \Omega_i; t_o, \Omega_o) = \langle\, \mathcal{O}(t_i, \Omega_i)\, \mathcal{O}(t_o, \Omega_o)\, \rangle_*$$

- Singularity locus of $G(t_i, \Omega_i; t_o, \Omega_o)$ encodes features of the geometry: this is clear in the limit where we can use a geodesic approximation.
- With some assumptions one can use the singularity locus to infer the metric in the bulk. Hammersley

Event horizon formation

The probe of horizon formation

- For purely radial geodesics look at the singularity structure of $G(t_i, t_o) = \langle\, \mathcal{O}(t_i, \Omega)\, \mathcal{O}(t_o, -\Omega)\, \rangle_{\text{shell}}$.
- Interested in location of singularity given by $t_o(t_i)$.

Salient points

- For $t_o < t_h$, $G(t_i, t_o)$ is singular for $t_o = t_i + \pi R_{AdS}$.
- As $t_o > t_h$, this singularity is shifted to $t_o - t_i > \pi R_{AdS}$.
- Location of singularity $t_o(t_i)$ increases monotonically and $t_o(t_i) \to \infty$ as $t_i \to t_h$. Gao, Wald
- Divergence of return time signals event horizon formation.

Radial geodesics in collapse

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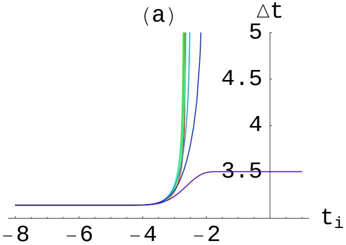
General story

- Expect divergent behaviour in geodesics carrying angular momentum since they are trapped in the black hole orbit.
- The singularity locus of $G(t_i, \Omega_i; t_o, \Omega_o)$ in the collapse geometry is quite involved.
- One can nevertheless extract t_h in the boundary theory.

Event horizon formation



Divergence in the time difference for various starting values of the time t_i. The curves are plotted for different values of the angular momenta. (a) corresponds to a think shell and (b) to a thick shell.

Bulk-cone locus details

Comments about bulk-cone singularities

Remarks:

- The structure of the bulk-cone singularity locus is tied to the bulk light-cone.
- Distinct geometries have sufficiently different light-cone structure, e.g., star $\neq$ black hole.

- Q: Can we distinguish different geometries by the structure of the bulk-cone singularity locus?
- A: This is dependent on the resolving power of the singularity locus: typical states are likely to behave similar to each other. Use analytic properties!

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On microstates and geometries

Microstate perspective

- Black hole microstates correspond to smooth, horizon free 'geometries'. Mathur, · · ·
- The microstate geometries differ from the black hole spacetime inside the horizon, being comprised of some spacetime foam.

Can correlation functions $\langle\, \mathcal{O}(x)\, \mathcal{O}(y)\, \rangle_*$ distinguish

- microstates from each other?
- individual microstates from thermal state?

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On microstates and geometries

For a system with large entropy S:

- Classical degeneracy expected to be broken by quantum effects with

$$\Delta E \sim e^{-S}$$

Balasubramanian, Marolf, Rozali

- Expect e^{-S} to govern characteristic scales of deviations between microstates $\Longrightarrow$ resolving power of e^{-S}.
- The relevant time scale from analysis of correlation functions in the canonical and micro-canonical ensembles

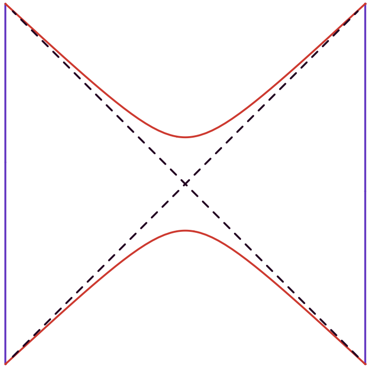
$$t_{dist} \sim e^{S}$$

On microstates and geometries

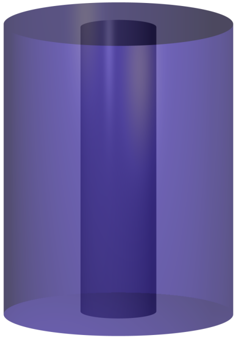
Macrostate perspective

- Black holes are characterized by non-trivial causal structure.
- Microstate geometries do not have complicated causal structure; the spacetime foam can however act coherently.
- Two spacetime boundaries for eternal black holes versus one boundary for microstates.

On microstates and geometries



Two boundaries for Schwarzschild-AdS.



Microstate geometry with single boundary.

On microstates and geometries

- AdS/CFT correspondence can be interpreted as an isomorphism between $\mathcal{H}_{bulk}$ and $\mathcal{H}_{CFT}$, for pure states and for density matrices.

$$|\mathrm{pure}\rangle_{bulk} \quad \leftrightarrow \quad |\mathrm{pure}\rangle_{CFT}$$

$$\rho_{bulk} \quad \leftrightarrow \quad \rho_{CFT}$$

- Easy to construct geometries dual to density matrices in the field theory, like black hole or Wheeler bags of gold.

Frievogel, Hubeny, Maloney, Myers, MR, Shenker

On microstates and geometries

- Eternal black holes in AdS correspond to the thermal density matrix.
- Should be able to tell apart the black hole from a microstate.
- Use the double boundary picture seriously – analytic continuation. Balasubramanian, Czech, Hubeny, Larjo, MR, Simón
- First deal with influence statistics our notions of distinguishability of microstates: the bane of ensemble equivalence.

Basis states in the microcanonical ensemble

Consider the microcanonical ensemble at energy E, with energy resolution $\mathcal{O}(\Delta E)$. We can choose to parameterize the states by

- Energy eignestates:

$$\mathcal{M}_{bas} = \{\ |s\rangle\ :\ H|s\rangle = e_s|s\rangle \quad ; \quad E \le e_s \le E + \Delta E\ \}$$

- Normalized superpositions of energy eigenstates:

$$\mathcal{M}_{sup} = \left\{ |\psi\rangle = \sum_s c_s^{\psi}|s\rangle \right\} \ , \qquad \sum_s |c_s^{\psi}|^2 = 1$$

Note that

$$\dim(\mathcal{M}_{sup}) = \dim(\mathcal{M}_{bas}) - 1 = e^S$$

Variances in the microcanonical ensemble

Consider some local operator $\mathcal{O}$ whose correlator we want to measure in the ensemble of microstates.

- $\mathcal{O}$ in general is not diagonal in the Hamiltonian basis $|\,\psi\,\rangle$.

$$|\psi\rangle = \sum_{\alpha} c_{\alpha}^{\psi}\,|\alpha\rangle$$

  $\{|\,\alpha\,\rangle\}$ is the eigen-basis of $\mathcal{O}$ with eigenvalues $\{o_{\alpha}\}$.

- We can easily calculate the mean value of the operator expectation value:

$$\langle\,\mathcal{O}\,\rangle_{\mathcal{M}_{sup}} = \int D\psi\,\langle\psi|\mathcal{O}|\psi\rangle$$

Variances in the microcanonical ensemble

- We can measure the spread of the ensemble $\mathcal{M}_{sup}$ over the eigenvectors of $\mathcal{O}$ using the usual notion of variance:

$$\mathrm{Var}[\mathcal{O}]_{\mathcal{M}_{sup}} = \langle \mathcal{O}^2 \rangle_{\mathcal{M}_{sup}} - \langle \mathcal{O} \rangle^2_{\mathcal{M}_{sup}}$$

- This is not the quantity we are interested in.
- We want to know the difference in probabilities of measuring a given eigenvalue of $\mathcal{O}$ in different states of $\mathcal{M}_{sup}$.

Variances in the microcanonical ensemble

Variance of observables

- Fix the eigenvalue if working with an operator $\mathcal{O}$ such that $[\mathcal{O}, H] \neq 0$.
- Obtain the variance due to the distribution of the states $|\psi\rangle$ in the ensemble (subject to standard uniform distribution).

Defining

$$c_\psi^k = \langle\psi|\mathcal{O}^k|\psi\rangle$$

$$\begin{aligned} \langle c^k\rangle_{\mathcal{M}_{sup}} &= \int D\psi\, c_\psi^k \\ \mathrm{Var}[c^k]_{\mathcal{M}_{sup}} &= \int D\psi\, (c_\psi^k)^2 - \langle c^k\rangle^2_{\mathcal{M}_{sup}} . \end{aligned}$$

Details of the variances

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Details of the variances

Variances in the microcanonical ensemble

Entropic suppression

The variance in the ensemble of superpositions is diminished by a factor of e^S in comparison to the variance in the ensemble comprising of energy eigenstates:

$$\mathrm{Var}(\mathcal{O})_{\mathcal{M}_{sup}} = \frac{1}{e^S + 1}\,\mathrm{Var}(\mathcal{O})_{\mathcal{M}_{bas}}$$

- $\mathcal{M}_{sup}$ gives us the worst case scenario for distinguishing microstates.
- We need to defeat the exponential suppression in order to be able to distinguish the microstates (apart from the usual statistical suppression).

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Canonical versus microcanonical ensemble

- Calculation in the microcanonical ensemble in general is non-trivial.
- Compare the canonical expectations to get an estimate of how the variance behave.
- This will certainly give us information about how the canonical ensemble differs from the microcanonical: distinguish pure states from mixed!

Two toy models

- Free chiral boson with $E \gg 1$ for statistics.
- D1-D5 system and the $M = 0$ BTZ black hole.

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I. The free field results

- For a free chiral boson one can show that

$$\frac{\sqrt{\mathrm{var}(\mathcal{O}(\tau))_{\mathcal{M}_{\mathrm{bas}}}}}{\langle \mathcal{O}(\tau) \rangle_{\mathcal{M}_{bas}}} \gg 1$$

for Euclidean time scale τ:

$$\tau \sim \frac{\beta}{2}$$

- The entropic suppression in $\mathcal{M}_{sup}$ makes its presence felt by increasing the relevant time scale:

$$\tau \sim \frac{3\beta}{2}$$

- The calculation is easy in the canonical ensemble as two-point functions are linear in the occupation numbers $\{N_n\}$.

I. The free field results

Moral from free field

- Simple probes (like $\mathrm{Tr}\left(X^{(i}X^{j)}\right)$ are able to distinguish microstates from thermal state at

$$\tau_{dist} \sim \beta \propto \frac{1}{S}$$

- Contrast this with usual Poincaré recurrence time $t_P \sim e^S$.

Some caveats

- The free field theory doesn't describe a black hole!
- Single chiral boson is incapable of encoding fractionation that is crucial to the picture of the microstate geometries.

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II. The D1-D5 system

- For the D1-D5 system at the orbifold point, we can calculate

$$\langle \{N_{n\mu}, N'_{n\mu}\} | \mathcal{A}^\dagger(t,\phi)\mathcal{A}(0,0) | \{N_{n\mu}, N'_{n\mu}\} \rangle$$

Balasubramanian, Kraus, Shigemori

- Like the free field case the correlation function is a linear function of the occupation numbers $\{N_{n\mu}, N'_{n\mu}\}$.
- Variances easy to estimate in the canonical ensemble from standard statistical distributions.

II. The D1-D5 system

Moral from fractionated free field

- Variances in the correlation function get large for

$$\tau \sim \log S$$

- Simple probes (like $\mathrm{Tr}\left(X^{(i}X^{j)}\right)$ are able to distinguish microstates in $\mathcal{M}_{bas}$ from thermal state at

$$\tau \sim S$$

- Folding in the exponential supression factor for $\mathcal{M}_{sup}$ we find that the relevant timescale is

$$\tau_{sup} \sim S^2$$

Salient points

Detecting geometric structures

- Exploiting the bulk causal structure we can infer properties of boundary correlators.
- In particular, we can obtain predictions of interesting features of correlation functions of a CFT state that has a good geometric dual.
- CFT correlation functions being multi-local in insertion points carry non-trivial information about prospects of being able to distinguish bulk geometries.

Salient points

Typicality versus thermality

- With large degeneracy of microstates distinguishing bulk geometries becomes hard due to statistics.
- The variances amongst members of the ensemble in their response to probes is highly suppressed.
- Enhance the variance by exploiting the coordinate space dependence of the correlation functions and appropriate analytic continuations.

Issues to be explored

Detecting geometric structures

- Quantifiable examples of bulk-cone singularities: D1-D5 system, LLM geometries.
- Explore coupling constant dependence: how does the singularity locus vary?
- Moral for the black hole singularity story; finite λ phase transitions?

Issues to be explored

Typicality versus thermality

- Generalize the discussion to systems containing honest black holes, D1-D5-$p_{L,R}$.
- Other interesting analytic continuations
- Estimate the minimum resolution for distinguishing microstates amongst each other.

Example: Reconstructing the fifth dimension

RG scale and spacetime dimension

Conventional lore: radial direction away from the boundary is the energy scale in the field theory.

Evidence

- Scaling properties in the boundary field theory.
- The UV/IR correspondence; small length scales in the field theory are related to large length scales in the geometry and vice-versa.

Validity

The identification is most robust near the boundary of AdS spacetime. Deep in the bulk this picture can be modified, *e.g.*, black holes.

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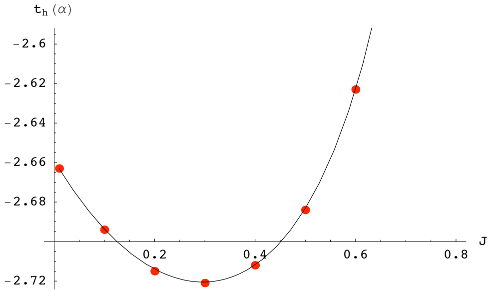
Angular momentum and bulk-cone singularity locus



- Variation of $t_h(J)$ as a function of angular mometum J. The value at $J = 0$ is the time that corresponds to the event horizon formation.

Return to event horizon formation

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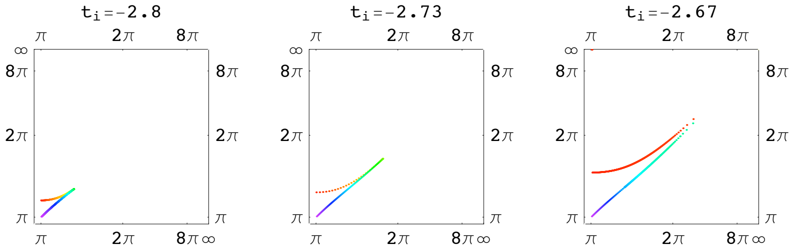
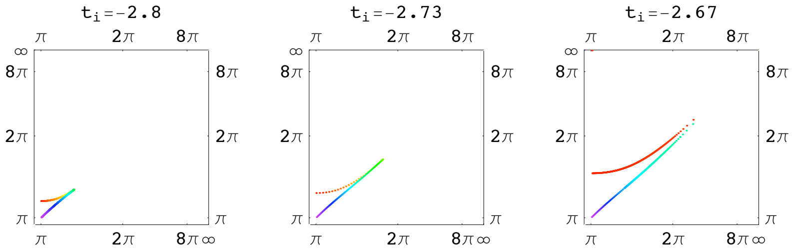


- $(\Delta\phi, \Delta t)$ for 3 values of t_i in a thin shell geometry (modeled by AdS-Vaidya metric).

Angular momentum and bulk-cone singularity locus



- The left endpoint of the upper branch corresponds to $J = 0$ and that for lower branch corresponds to $J = E$.
- For comparison $t_c \approx -2.72$ and $t_h \approx -2.66$.
- t_c is the minimal value of the starting time for which we see a divergent t_o.

Return to event horizon formation

Details of the variances in microcanonical ensemble

- Consider moments of the operator in state $|\psi\rangle \in \mathcal{M}_{sup}$

$$c_\psi^k = \langle\psi|\mathcal{O}^k|\psi\rangle$$

which can be succinctly encoded in a generating function:

$$C_\psi(\theta) = \sum_n \frac{\theta^n}{n!} c_\psi^n = \langle\psi|e^{\theta\mathcal{O}}|\psi\rangle$$

- The variance can be characterized in terms of ensemble averages of the generating function:

$$\langle C(\theta)\rangle_{\mathcal{M}_{sup}} = \int D\psi\, C_\psi(\theta)$$

$$\langle C_2(\theta_1,\theta_2)\rangle_{\mathcal{M}_{sup}} = \int D\psi\, C_\psi(\theta_1)\, C_\psi(\theta_2) - \langle C(\theta_1)\rangle_{\mathcal{M}_{sup}}\langle C(\theta_2)\rangle_{\mathcal{M}_{sup}}$$

Details of the variances in microcanonical ensemble

- We are going to be interested in the differing responses of individual states in the micro-canonical ensemble to the local correlation functions, which is characterized by:

$$\langle c^k \rangle_{\mathcal{M}_{sup}} = \left[\frac{d^k \langle C(\theta) \rangle_{\mathcal{M}_{sup}}}{d\theta^k} \right]_{\theta=0}$$

$$\mathrm{Var}[c^k]_{\mathcal{M}_{sup}} = \frac{d^k}{d\theta_1^k} \frac{d^k}{d\theta_2^k} \left[\langle C_2(\theta_1, \theta_2) \rangle_{\mathcal{M}_{sup}} \right]_{\theta_1=\theta_2=0}$$

- The quantity of interest to us is the relative r.m.s deviation:

$$\frac{\sigma[c^k]_{\mathcal{M}_{sup}}}{\langle c^k \rangle_{\mathcal{M}_{sup}}} = \frac{\sqrt{\mathrm{Var}[\mathrm{c}^k]_{\mathcal{M}_{\mathrm{sup}}}}}{\langle c^k \rangle_{\mathcal{M}_{sup}}}$$

Back to variances