String theory at high density

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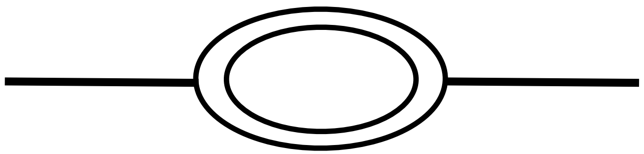
Much of this work is in collaboration with
B. Chowdhury, S. Giusto, O. Lunin, A. Saxena, Y. Srivastava

String theory gives a consistent theory of quantum gravity



Loop divergences cured

What do we want quantum gravity for?

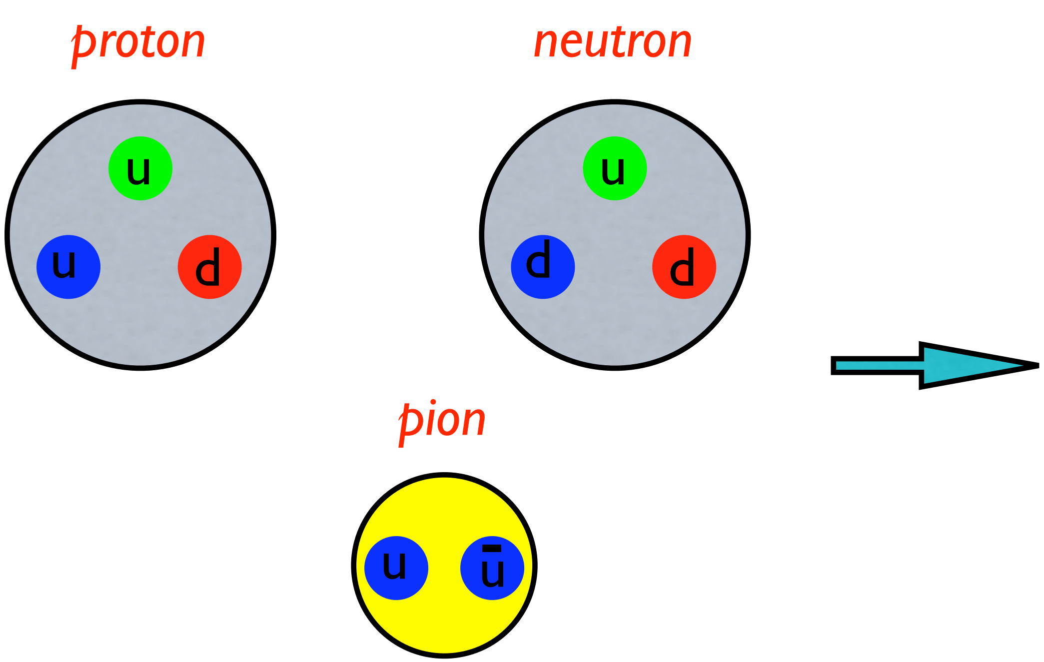
(a) How do we understand black hole entropy?
Where are the states that contribute to this entropy?
How do we resolve the black hole information paradox?
What happens to matter that falls into a black hole?

(b) What was the state of matter in the early Universe?
Can some of this matter be left over as dark matter or dark energy?
What is the solution of the 'horizon problem', flatness problem? (inflation?)

We will discuss some computations that suggest an emerging picture of how matter behaves at high densities.

The computations are themselves rigorous calculations in string theory or supergravity, but the picture we decuce from them will be qualitative.

An analogy is the quark model: From hadron classification and scattering *quarks* were deduced, but QCD came later ....



$$\mathcal{L} = \int dx[-\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu a} + \frac{i}{2}\bar{\psi}\partial\psi + \ldots]$$

Key notions that emerge:

(a) Fractionation: *When different kinds of branes are bound together, they 'fractionate' each other,* so that we get get a large number of objects with very low mass.

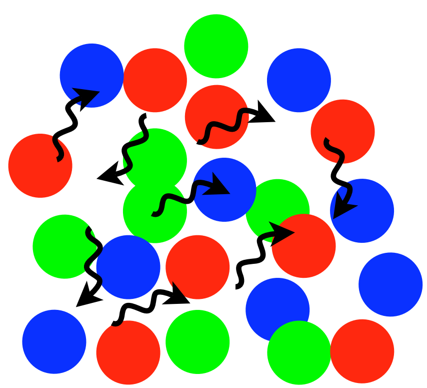
This large number of fractional objects gives the large black hole entropy, and the low mass gives very long distance effects, that stretch upto horizon radius.

Thus we get quantum gravtity effects over macroscopic distances

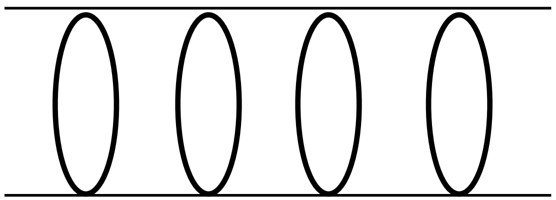
(b) Brane-antibrane pairs: If we have energy but no charge, then we get the maximal entropic state by using the energy to make *brane-antibrane pairs*, which then fractionate as above.

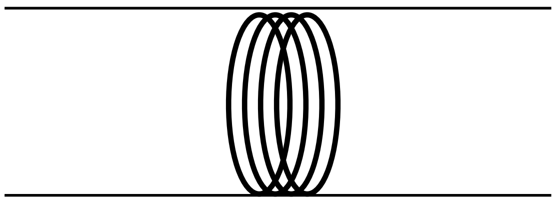
(c) Quasi-free constituents: *These fractional objects seem to be essentially free*, so that we get the total energy, pressure, entropy by just adding the contributions from individual fractional branes.

Analogy: Quark-Gluon plasma: *At high energy density the quarks and gluons are essentially free ...*



Getting entropy: One charge





Degeneracy = 256
(independent of n)

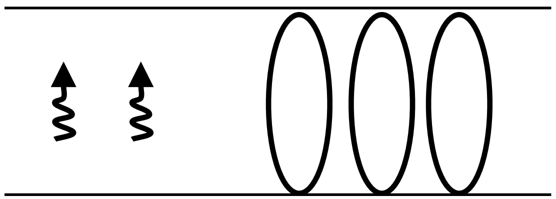
S = ln 256 ~ 0

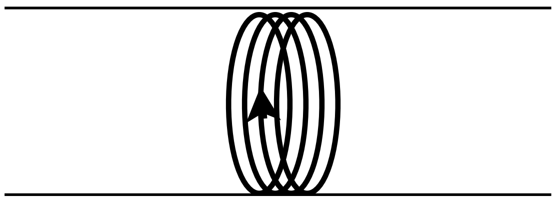
bound state

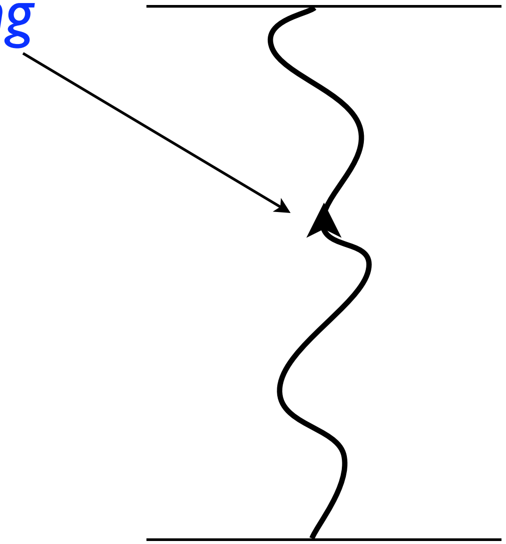
(see however sinha and suryanarayana, ’06)

Two charges

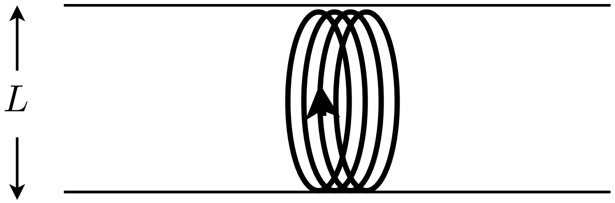
travelling waves on string

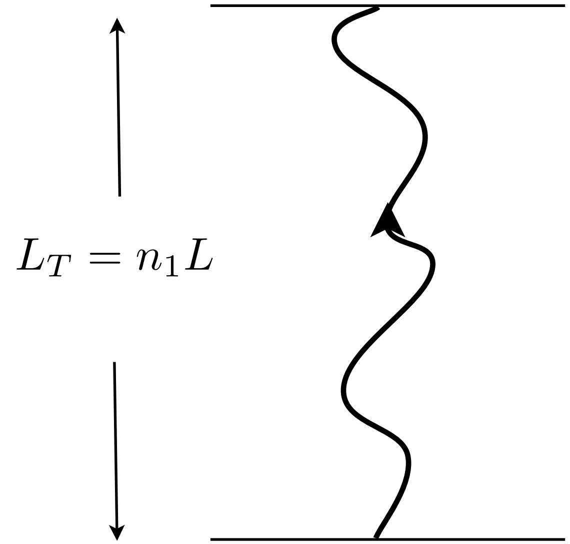






Many ways to partition momentum among different harmonics -- *large entropy*





Total momentum

$$P = \frac{2\pi n_p}{L} = \frac{2\pi (n_1 n_p)}{L_T}$$

Each quantum of harmonic k carries momentum

$$p = \frac{2\pi k}{L_T}$$

So we must have

$$\sum_k k n_k = n_1 n_p$$

So we have to count 'partitions' of $n_1 n_p$

8 bosonic + 8 fermionic degrees of freedom

$$\longrightarrow \quad e^{2\pi\sqrt{2}\sqrt{n_1 n_p}} \quad \text{states}$$

$$\boxed{S = 2\pi\sqrt{2}\sqrt{n_1 n_p}}$$

(Susskind '93, Sen '94)

Three charges

$$M_{9,1} \rightarrow M_{4,1} \times T^4 \times S^1$$

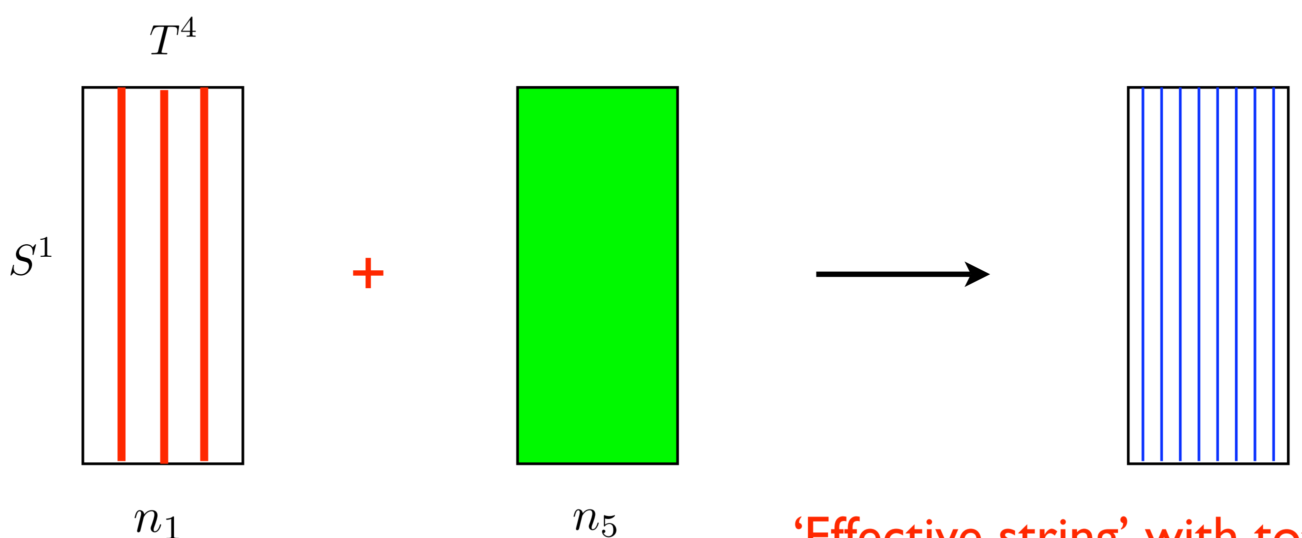
$$\begin{array}{cc} T^4 & S^1 \\ D5 & \\ & D1 \\ & P \end{array}$$

$$\begin{aligned}
\boxed{D1\,D5}\,P\,(IIB) &\xrightarrow{S} NS1\,NS5\,P\,(IIB) \\
&\xrightarrow{T_5} P\,NS5\,NS1\,(IIA) \\
&\xrightarrow{T_6} P\,NS5\,NS1\,(IIB) \\
&\xrightarrow{S} P\,D5\,D1\,(IIB) \\
&\xrightarrow{T_{6789}} P\,D1\,D5\,(IIB) \\
&\xrightarrow{S} \boxed{P\,NS1}\,NS5\,(IIB)
\end{aligned}$$

D1-D5

or

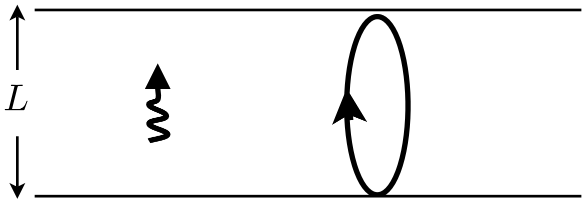
NS1-NS5



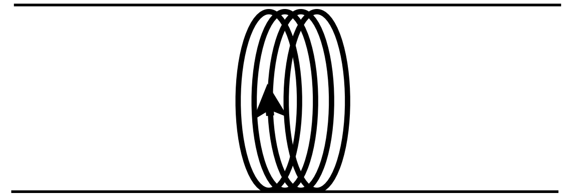
'Effective string' with total winding number

$n_1 n_5$

Fractionation



$\frac{2\pi}{L}$ $\frac{2\pi}{L}$

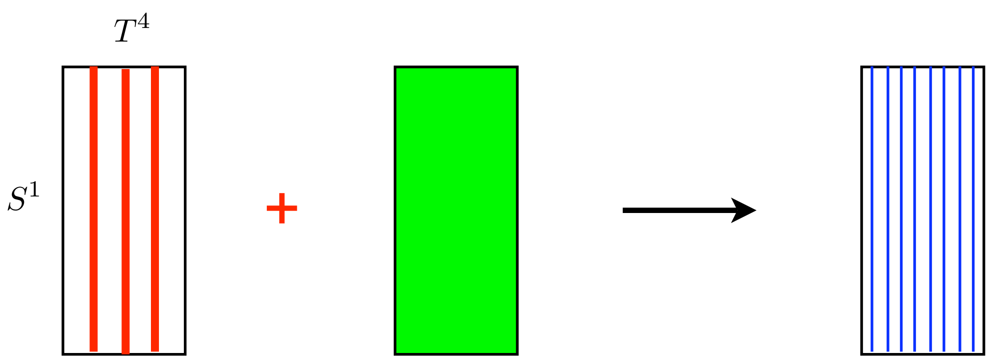


$$\frac{2\pi}{n_1 L} = \frac{2\pi}{L_T}$$

P-NS1

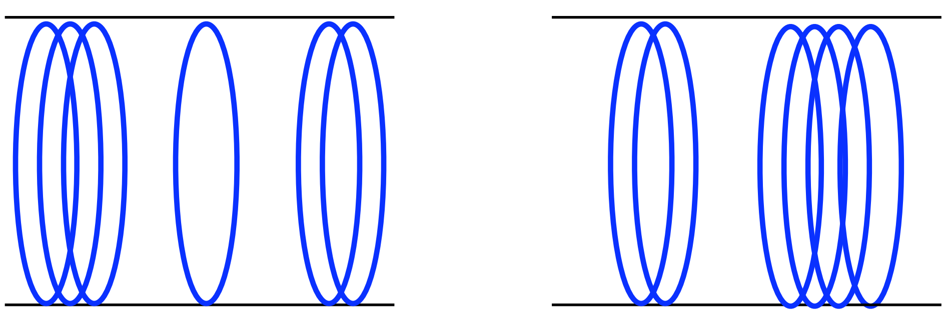
$$\sum_k k n_k = n_1 n_p$$

n_p units of momentum become $n_1 n_p$ *fractional* units of momentum when bound to n_1 strings



n_1 n_5 $n_1 n_5$

D1-D5

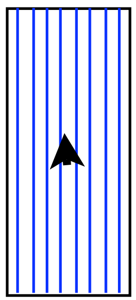


...

$$\sum_k k n_k = n_1 n_5$$

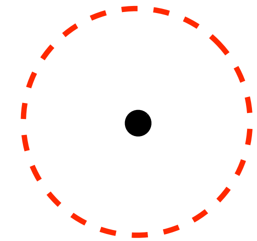
$$\boxed{S = 2\pi\sqrt{2}\sqrt{n_1 n_5}}$$

Three large charges



$n_1 n_5$

4 bosonic + 4 fermionic degrees of freedom



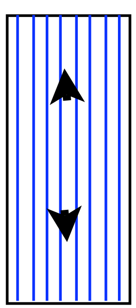
$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$

$$S_{bek} = \frac{A}{4G} = 2\pi\sqrt{n_1 n_5 n_p}$$

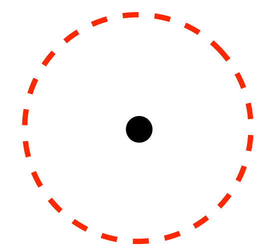
$$\boxed{S_{micro} = S_{bek}}$$

(Strominger + Vafa '96)

Two large charges + nonextremality



$n_1 n_5$



$$\boxed{S_{micro} = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p}) = S_{bek}}$$

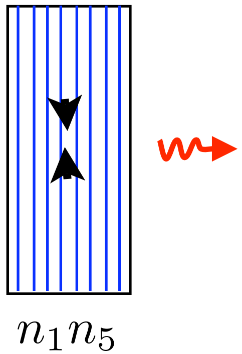
$$n_p - \bar{n}_p = \hat{n}_p$$

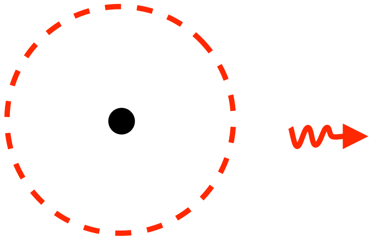
$$E = m_1 n_1 + m_5 n_5 + m_p(n_p + \bar{n}_p)$$

(Callan + Maldacena ' 96)

Thus we see that we reproduce the Bekenstein entropy by assuming that the momentum and anti-momentum excitations do not interact -- the energy is the sum of the two energies and the entropy is the sum of the two entropies

Radiation from near-extremal D1-D5 system





P $\bar{\text{P}}$ excitations collide
and create gravitons

Semiclassical Hawking radiation
from black hole

$$\boxed{\Gamma_{micro} = \Gamma_{hawking}}$$

Exact agreement of radiation rate, spin dependence, grey-body factors

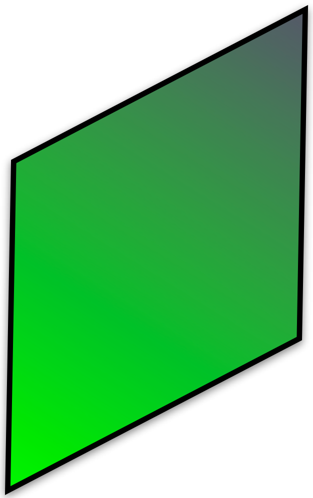
$$D1\ D5\ +\ \Delta E\ \rightarrow\ D1\ D5 + P\bar{P}\ \rightarrow\ \text{radiation}$$

$$NS1\ NS5\ +\ \Delta E\ \rightarrow\ NS1\ NS5 + P\bar{P}\ \rightarrow\ \text{radiation}$$

Callan - Maldacena '96, Dhar, Mandal, Wadia, '96

Das+SDM '96, Strominger+Maldacena '96

One large charge (D5) + nonextremality



$$S_{micro} = 2\pi\sqrt{n_5}(\sqrt{n_1}+\sqrt{\bar{n}_1})(\sqrt{n_p}+\sqrt{\bar{n}_p})$$
$$= S_{bek}$$

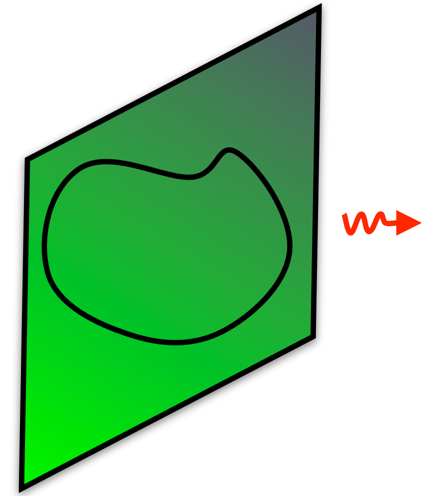
(Maldacena '96)

$$n_1 - \bar{n}_1 = \hat{n}_1$$

$$n_p - \bar{n}_p = \hat{n}_p$$

$$E = m_5 n_5 + m_1(n_1+\bar{n}_1) + m_p(n_p+\bar{n}_p)$$

Maximize the formal expression for S_{micro} subject to these constraints



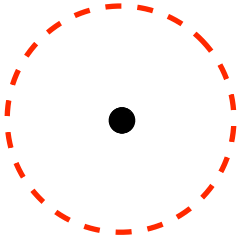
Effective string with fractional tension $\frac{1}{n_5}T_{D1}$

$$\boxed{\Gamma_{micro} \;=\; \Gamma_{hawking}}$$

(Klebanov+SDM '97)

No large charges

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$



Maximize S_{micro} *subject to the constraints*

$$n_5 - \bar{n}_5 = \hat{n}_5$$

$$n_1 - \bar{n}_1 = \hat{n}_1$$

$$n_p - \bar{n}_p = \hat{n}_p$$

$$E = m_5(n_5 + \bar{n}_5) + m_1(n_1 + \bar{n}_1) + m_p(n_p + \bar{n}_p)$$

$$\boxed{S_{micro} = S_{bek}}$$

(Horowitz, Maldacena, Strominger '96)

Take a neutral hole and add charges by boosting + dualities. This relates it to a near extremal hole, and we can find the emission from microscopics:

$$\boxed{\Gamma_{micro} \;=\; \Gamma_{hawking}}$$

(Das, SDM, Ramadevi '98)

Note that boosting in a compact direction is not an exact symmetry, but is presumably a good approximation for large charges (similar to the idea of Matrix theory)

Black holes in 3+1 dimensions

$$M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$$

$D5$

$D1$

P

KK

Nontrivial fiber direction

$$\boxed{\begin{aligned} S_{micro} &= 2\pi(\sqrt{n_5}+\sqrt{\bar{n}_5})(\sqrt{n_1}+\sqrt{\bar{n}_1})(\sqrt{n_p}+\sqrt{\bar{n}_p})(\sqrt{n_{kk}}+\sqrt{\bar{n}_{kk}}) \\ &= S_{bek} \end{aligned}}$$

$$\boxed{\Gamma_{micro} = \Gamma_{hawking}}$$

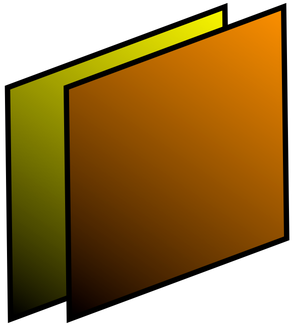
(Horowitz, Lowe, Maldacena '96)

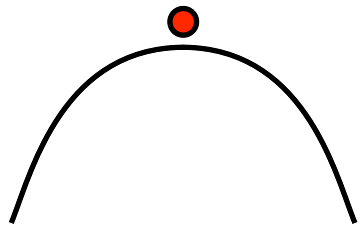
Extremal but not supersymmetric hole: *Emparan + Horowitz '06*

We see that the energy in a black hole goes to creating branes and antibranes; these 'fractionate' each other, and give a large number of degrees of freedom.

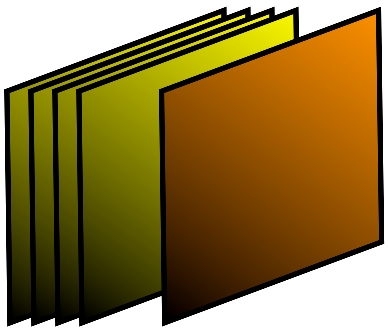
Assuming a noninteracting set of these fractional objects, we get the correct entropy and Hawking radiation for the black hole.

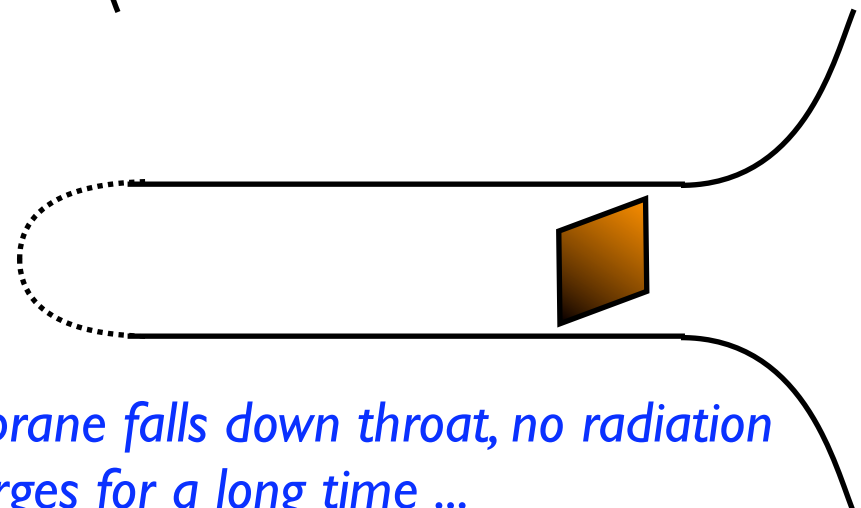
Why don't the branes and antibranes annihilate immediately?





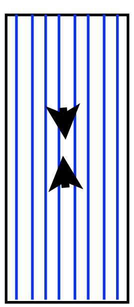
Tachyon at top of potential
(Sen '99)

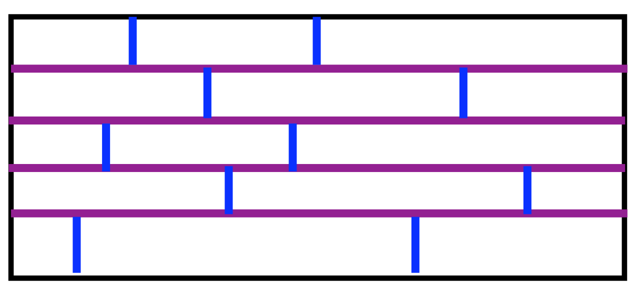




Antibrane falls down throat, no radiation emerges for a long time ...

Dhar, Mandal, Wadia, Yogendran '99
Lunin, SDM, Park, Saxena '03



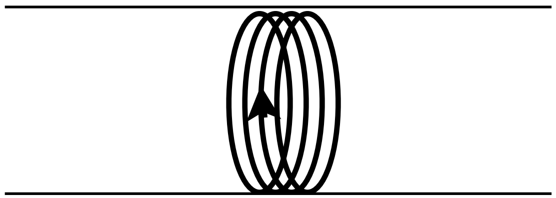


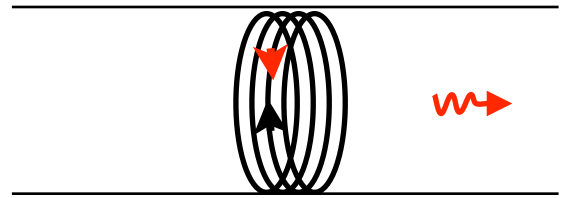
$\frac{2\pi k}{n_1 n_5 L}$ $\frac{2\pi m}{L}$

Fractional branes and antibranes have to find each other before they can annihilate ...

Phase Transitions

$$\boxed{M_{9,1} \;\rightarrow\; M_{4,1} \times T^4 \times S^1}$$



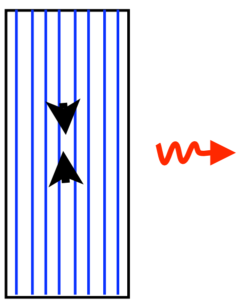


??

Get wrong spins, greybody factors

$NS1\ P$

$$\boxed{NS1\ P + \Delta E \;\rightarrow\; NS1\ P + P\bar{P} \;\rightarrow\; \text{radiation} \quad ??}$$



$n_1 n_5$

$$D1\ D5\ +\ \Delta E\ \rightarrow\ D1\ D5 + P\bar{P}\ \rightarrow\ \text{radiation}$$

$$NS1\ NS5\ +\ \Delta E\ \rightarrow\ NS1\ NS5 + P\bar{P}\ \rightarrow\ \text{radiation}$$

Duality

$$\boxed{P\ NS1\ +\ \Delta E\ \rightarrow\ P\ NS1 + NS5\ \overline{NS5}\ \rightarrow\ \text{radiation}}$$

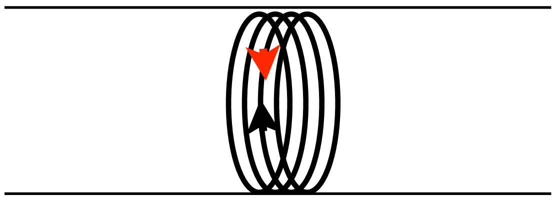
Basic question: Start with NS1-P, and add some excitation energy.

Does this energy go to creating $P\bar{P}$ or $NS5\ \overline{NS5}$?

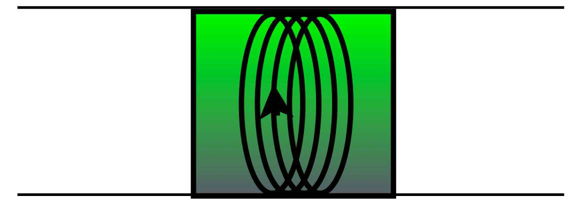
The lighter excitation will give more entropy, so it will be created ...

Which excitation is lighter?

$\boxed{P\bar{P}}$

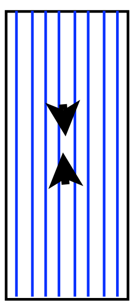


$\boxed{NS5\ \overline{NS5}}$



$$\Delta E = \frac{2\pi}{n_1 L} + \frac{2\pi}{n_1 L} = \frac{4\pi}{n_1 L}$$

$$\Delta E = \frac{2m_5}{n_1 n_p} \sim \frac{LV_4}{g^2 \alpha'^3 n_1 n_p}$$



$n_1 n_5$

$$D1\ D5\ +\ \Delta E\ \rightarrow\ D1\ D5 + P\bar{P}$$

$$\Delta E = \frac{2m_p}{n_1 n_5}$$

The 5-brane pairs are heavy for small g

But they get 'double fractionation', while the momentum modes get 'single fractionation'

So for g infinitesimal the momentum excitations will be lighter, but for a slightly higher g the 5-brane pairs will be lighter

Let us do this more properly ....

Mass of string state $M^2 = (n_1 LT + \frac{2\pi n_p}{L})^2 + 8\pi T N_L = (n_1 LT - \frac{2\pi n_p}{L})^2 + 8\pi T N_R$

Minimum excitation $\delta N_L = \delta N_R = 1$

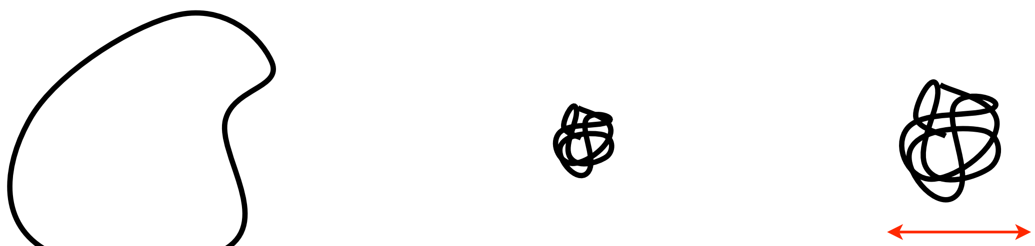
$$2M\delta M \approx 8\pi T = \frac{4\pi}{\alpha'} \quad \Rightarrow \quad \Delta E_{P\bar{P}} = \delta M \approx \frac{2\pi}{\alpha' M}$$

If $$\frac{\Delta E_{5\bar{5}}}{\Delta E_{P\bar{P}}} \sim \frac{LV_4 M}{g^2 \alpha'^2 n_1 n_p} \lesssim 1$$

Then the 5-brane pairs are lighter than string vibrations.

Note that g need not be large for this to happen

States of a string: NS1-P



$\sqrt{\alpha'}$

Generic state contributing to the entropy

$$S = 2\pi\sqrt{2}\sqrt{n_1 n_p}$$

(Sen '94)

Far away, metric will have factors $\approx \quad 1+\frac{Q_1}{r^2}, \quad 1+\frac{Q_5}{r^2}$

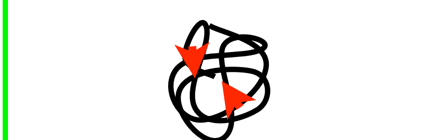
String will strongly feel its own gravity if $\quad (\Delta x)^2 \lesssim Q_1, \; Q_p$

$$\boxed{(\Delta x)^2 \lesssim \frac{Q_1 Q_p}{Q_1 + Q_p}}$$

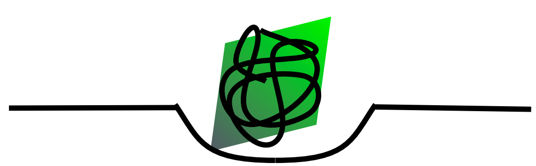
$$\boxed{\alpha' \lesssim \frac{g^2 \alpha'^3 n_1 n_p}{V_4 L M}}$$

(SDM '97)

Phase Transition

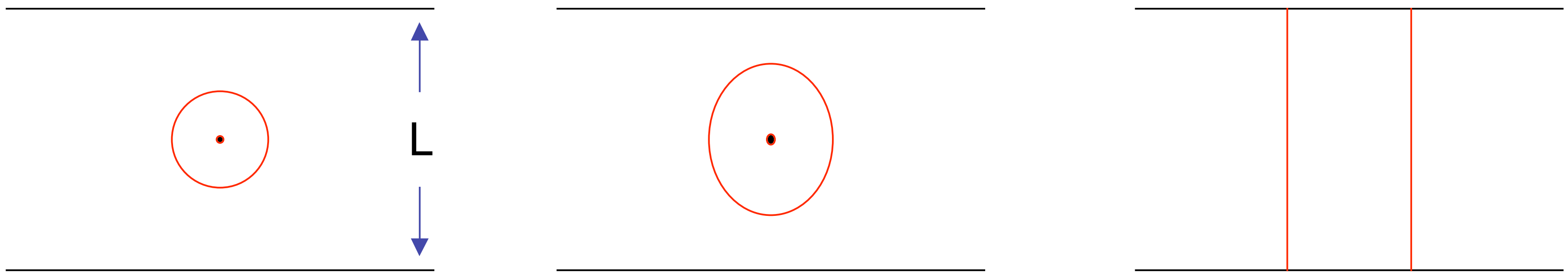


$\boxed{P\bar{P}}$



$\boxed{NS5\ \overline{NS5}}$

The black hole - black string transition



Small mass:
Black hole

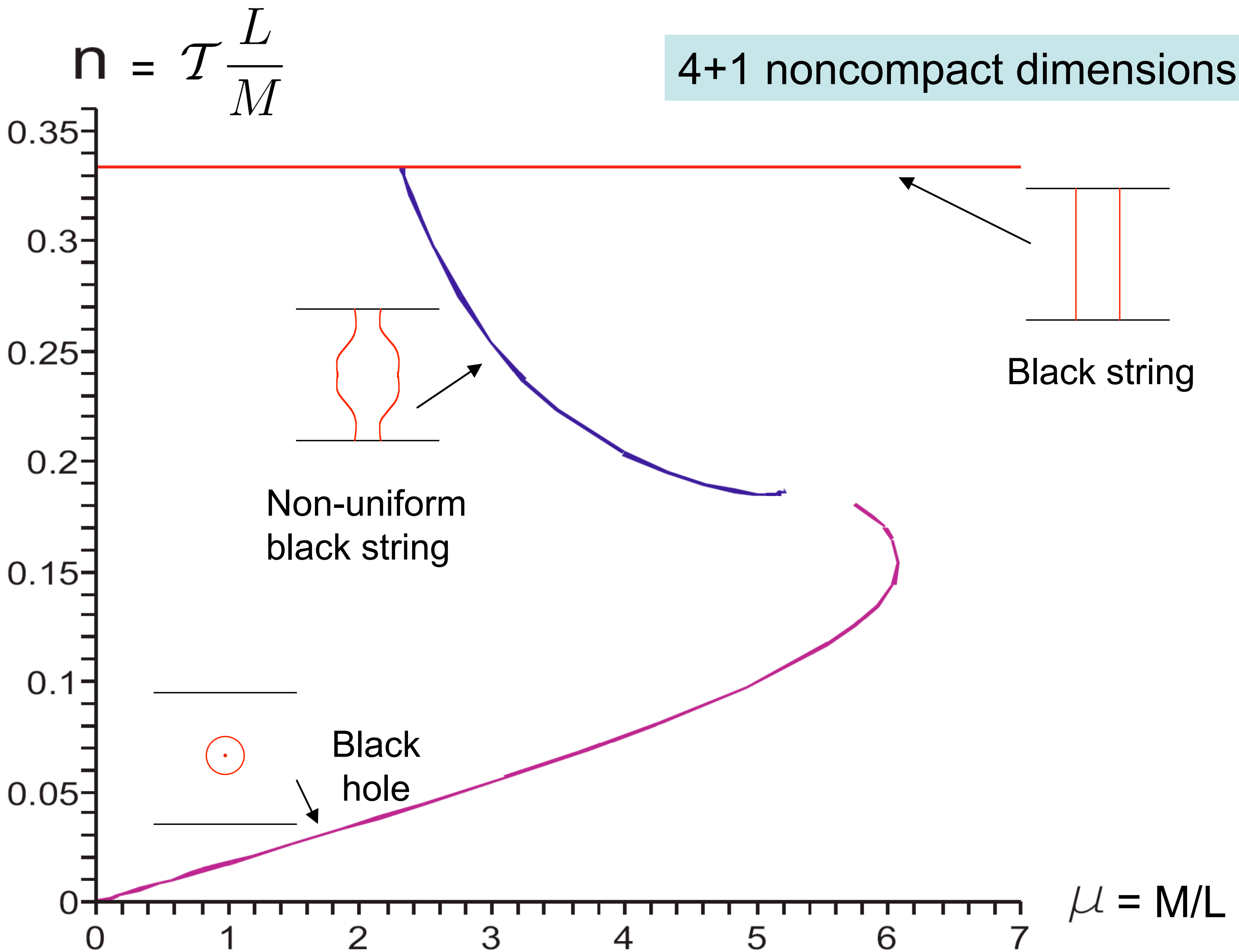
Large mass:
Black string

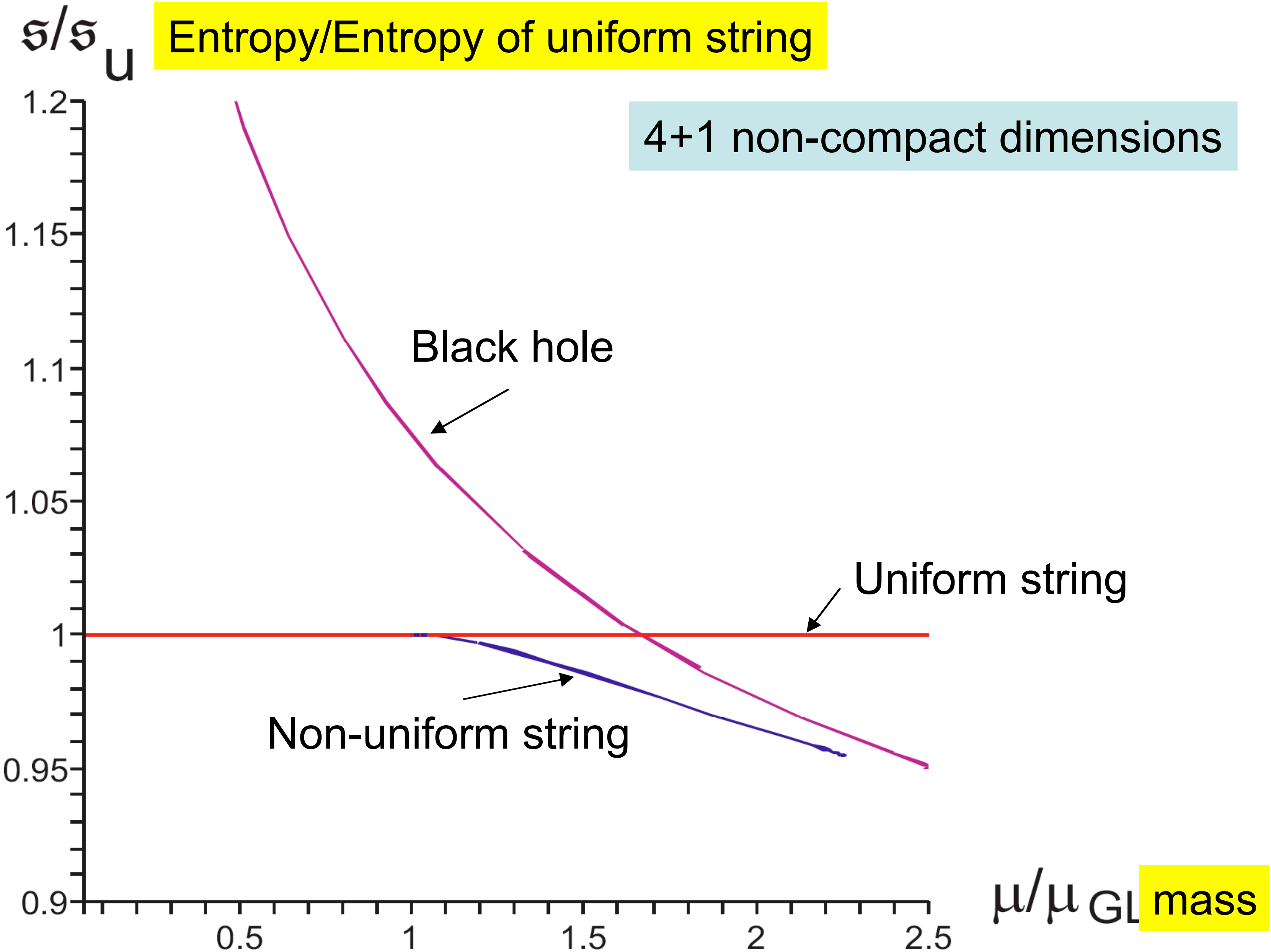
Tension

$$\boxed{\mathcal{T} = -\frac{1}{L}\int T_{zz} = \left(\frac{\partial M}{\partial L}\right)_S}$$

Small black hole $\mathcal{T} \approx 0$

Uniform black string $$\mathcal{T}\frac{L}{M} = const.$$


n = $\mathcal{T}\frac{L}{M}$
4+1 noncompact dimensions
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Black string
Non-uniform
black string
Black
hole
0
1
2
3
4
5
6
7
μ = M/L



s/s u
Entropy/Entropy of uniform string
4+1 non-compact dimensions
Black hole
Uniform string
Non-uniform string
1.2
1.15
1.1
1.05
1
0.95
0.9
0.5
1
1.5
2
2.5
μ/μ GL
mass

Compactify: $M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1$

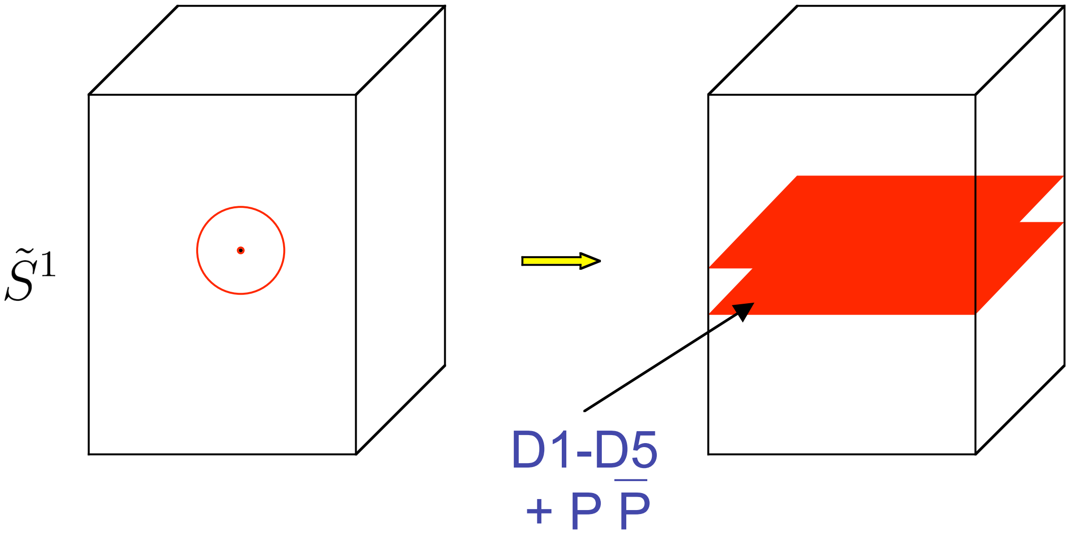
Let $\tilde{S}^1$ be large.

Then we effectively have a black hole in 4+1 non-compact dimensions

(only $T^4 \times S^1$ compact)

Add D1-D5 charges by `boosting+ duality'

→ Near extremal D1-D5



$$S = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})$$
$$= 2\pi\sqrt{N}\ (2\sqrt{\frac{E}{2m_p}})$$

$$N = n_1 n_5$$

$$\boxed{M_{9,1} \to M_{3,1} \times T^4 \times S^1 \times \tilde{S}^1}$$

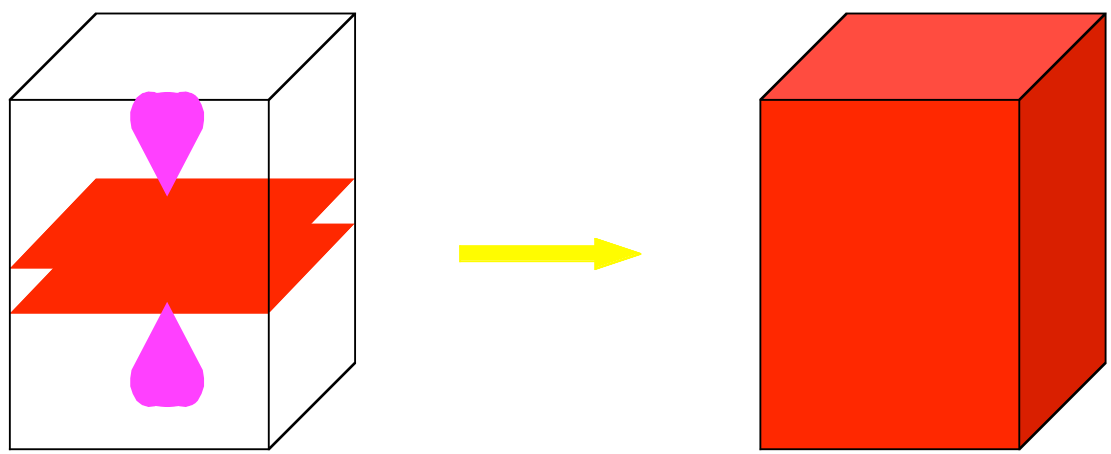
Suppose we could excite all charges appropriate to this compactification

We have 2 charges D1-D5 in 3+1 non-compact dimensions

$$\begin{aligned} S &= 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})(\sqrt{n_{kk}} + \sqrt{\bar{n}_{kk}}) \\ &= 2\pi\sqrt{N}\frac{E}{m_p m_{kk}} \end{aligned}$$

$$\boxed{N = n_1 n_5}$$

D1-D5
\+ P $\bar{\mathrm{P}}$
\+ KK $\overline{\mathrm{KK}}$



Assumption:

A part N_1 of the D1-D5 effective string fractionates the P P charges

The remainder $N - N_1$ fractionates the $\mathrm{P}\,\overline{\mathrm{P}}$ + $\mathrm{KK}\,\overline{\mathrm{KK}}$ charges

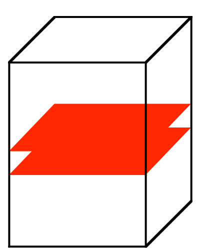
(Suggested by study of supertube excitations,
Giusto + SDM + Srivastava '06)

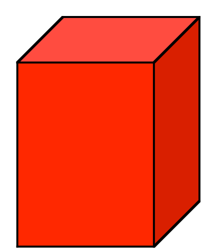
Energy E_1 goes to the $\mathrm{P}\,\overline{\mathrm{P}}$ excitations

Energy $E - E_1$ goes to the $\mathrm{P}\,\overline{\mathrm{P}}$ + $\mathrm{KK}\,\overline{\mathrm{KK}}$ excitations

$$S = 2\pi\sqrt{N_1}\,(2\sqrt{\frac{E_1}{2m_p}}) \quad + \quad 2\pi\sqrt{(N - N_1)}\,\frac{(E - E_1)}{m_p m_{kk}}$$

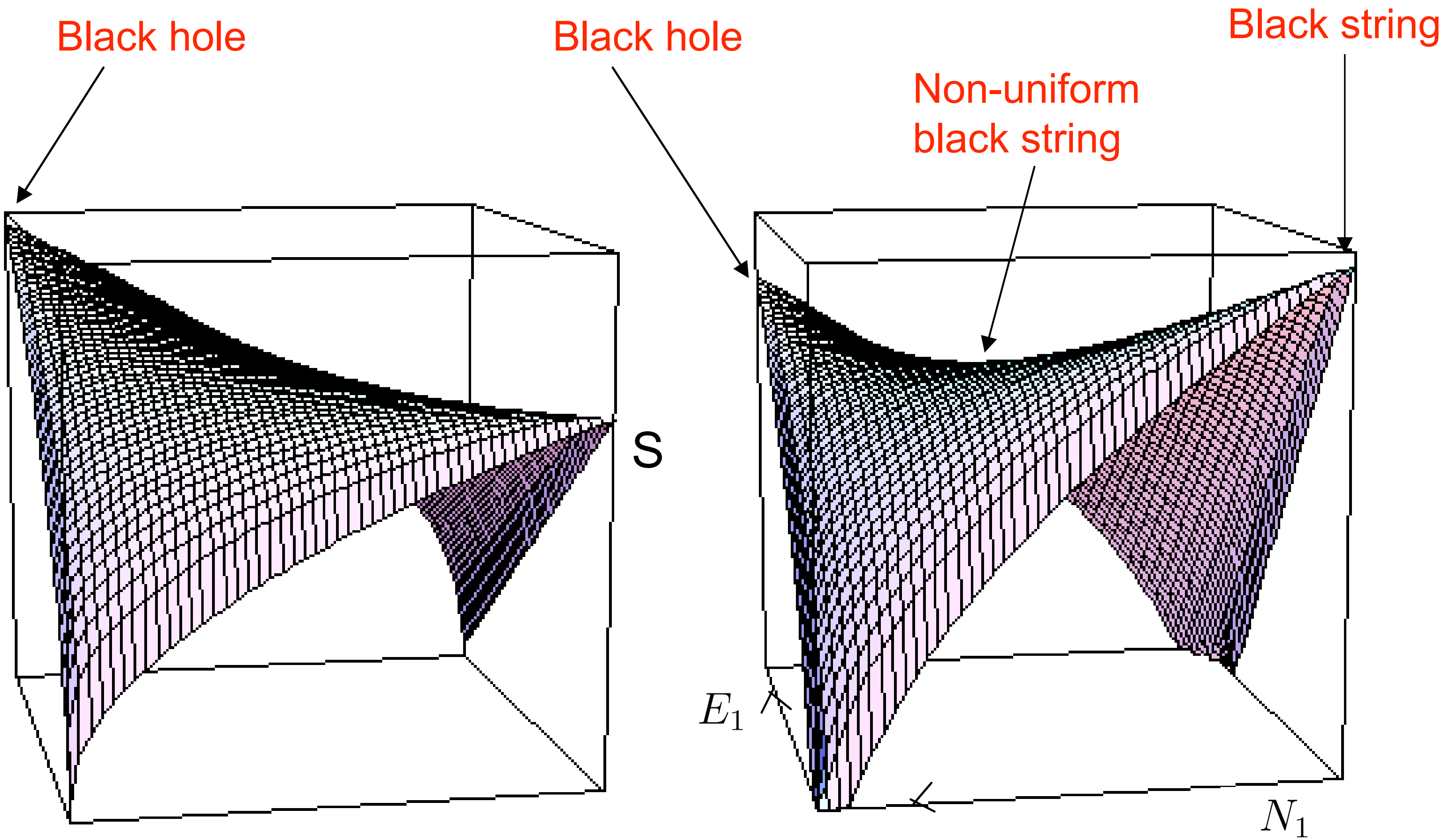
D1-D5
+ $\mathrm{P}\,\overline{\mathrm{P}}$





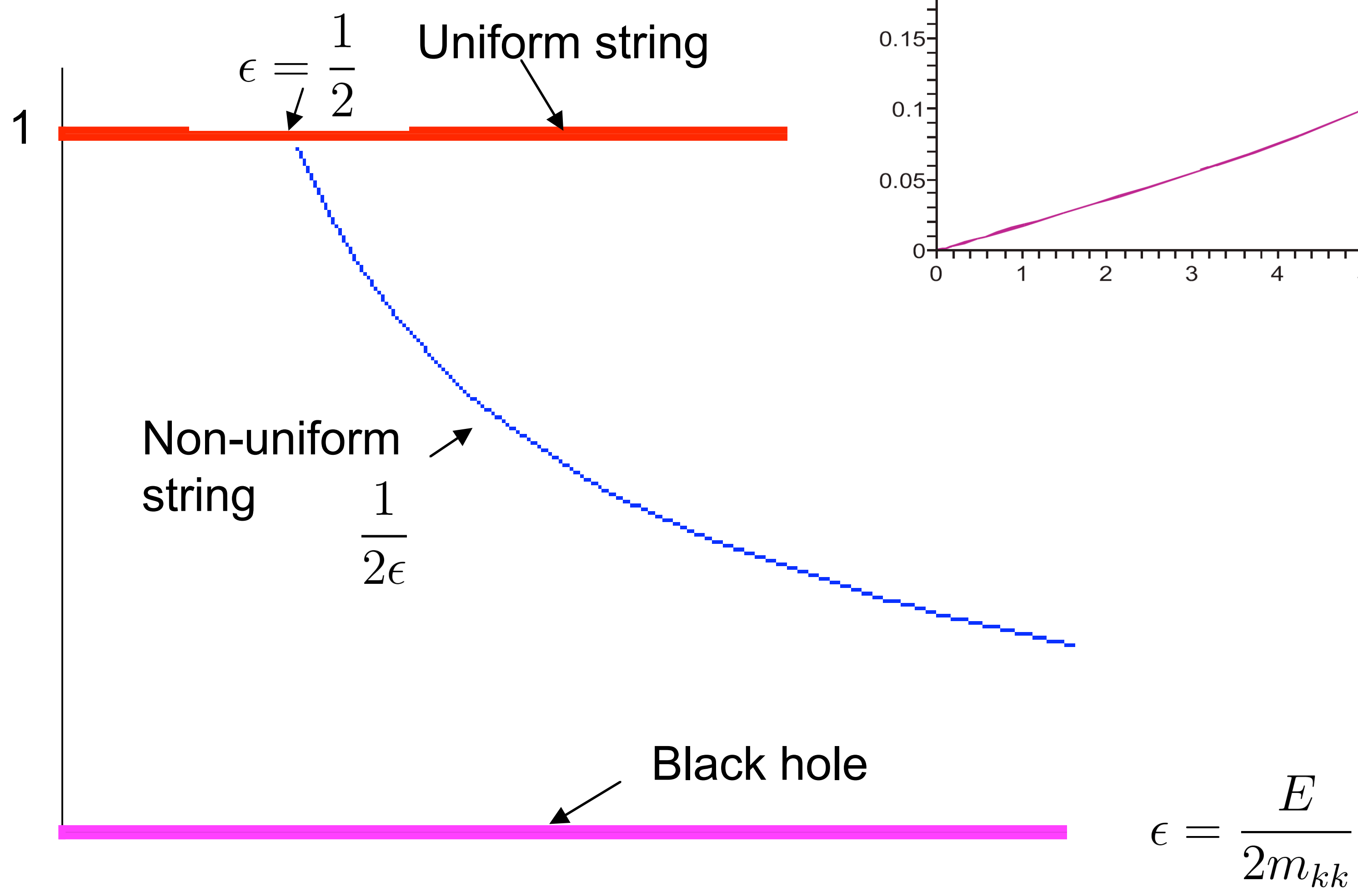
D1-D5
+ $\mathrm{P}\,\overline{\mathrm{P}}$
+ $\mathrm{KK}\,\overline{\mathrm{KK}}$

$$S = 2\pi\sqrt{N_1}\,(2\sqrt{\frac{E_1}{2m_p}}) \quad + \quad 2\pi\sqrt{(N-N_1)}\,\frac{(E-E_1)}{m_p m_{kk}}$$

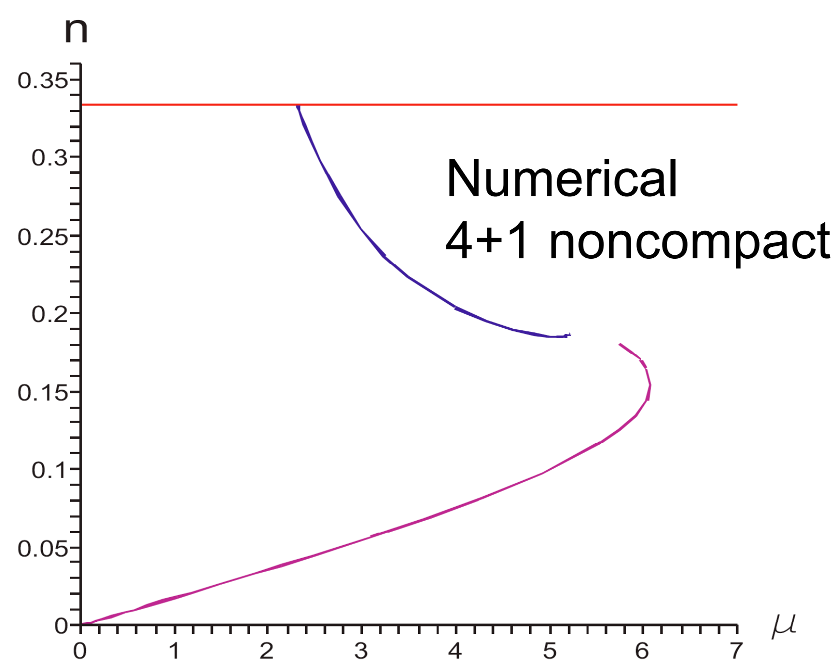


$E/(2m_{kk}) = 0.5$

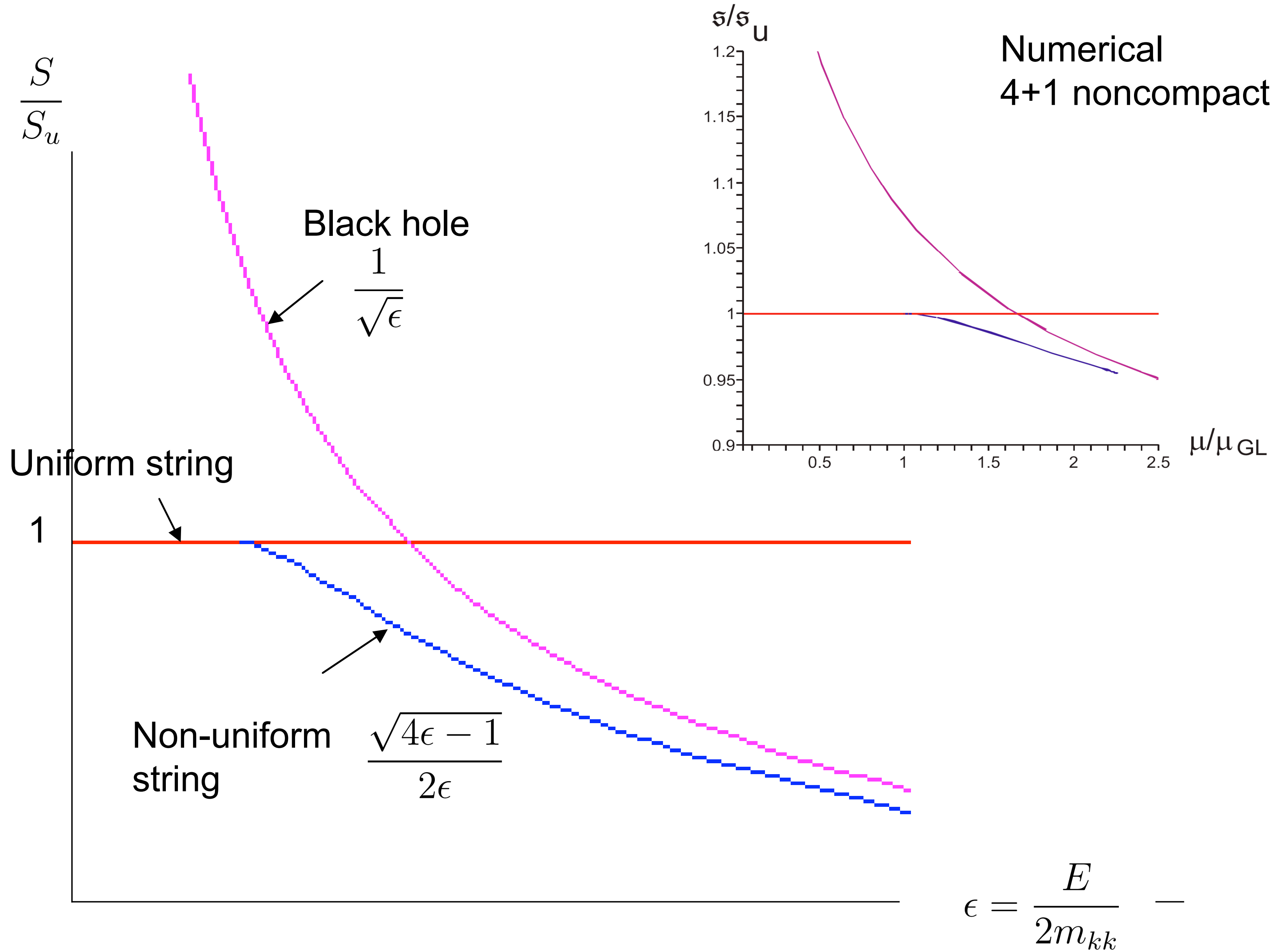
$E/(2m_{kk}) = 1.2$



1
$\epsilon = \frac{1}{2}$
Uniform string
Non-uniform string
$\frac{1}{2\epsilon}$
Black hole
$\epsilon = \frac{E}{2m_{kk}}$

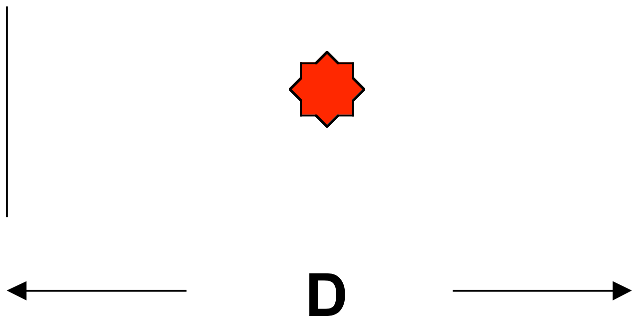


n
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7
μ
Numerical
4+1 noncompact



$\frac{S}{S_u}$
Black hole
$\frac{1}{\sqrt{\epsilon}}$
Uniform string
1
Non-uniform string
$\frac{\sqrt{4\epsilon-1}}{2\epsilon}$
$\epsilon = \frac{E}{2m_{kk}}$
$\mathfrak{s}/\mathfrak{s}_u$
Numerical
4+1 noncompact
μ/μ_{GL}

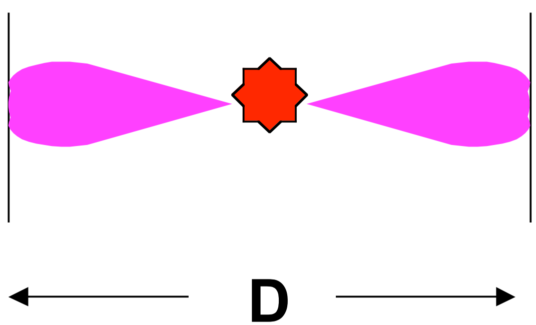
What is the size of a D1-D5-P bound state?



D is big, the bound state does not notice the box

$$S = 2\pi\sqrt{n_1 n_5 n_p}$$

$$\Delta E \sim \frac{1}{D}$$

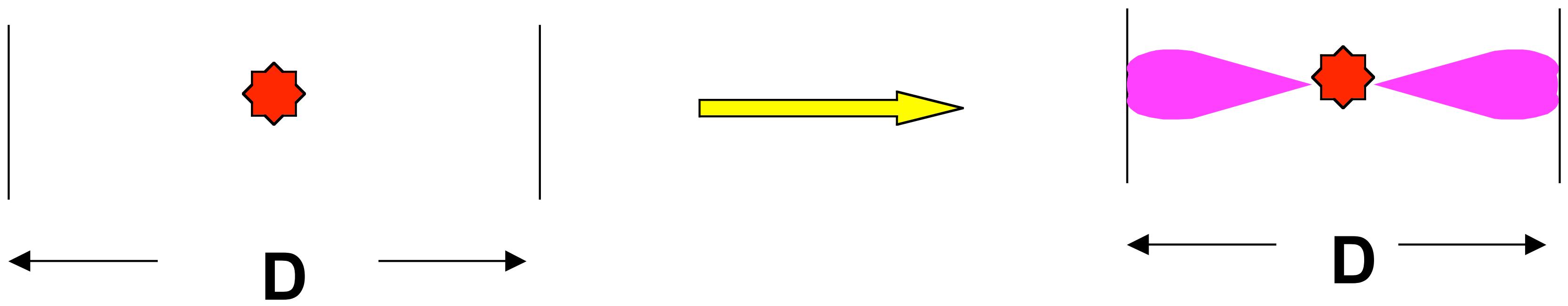


The energy $\Delta E \sim \frac{1}{D}$

is used to create pairs of extended objects that wrap around the circle

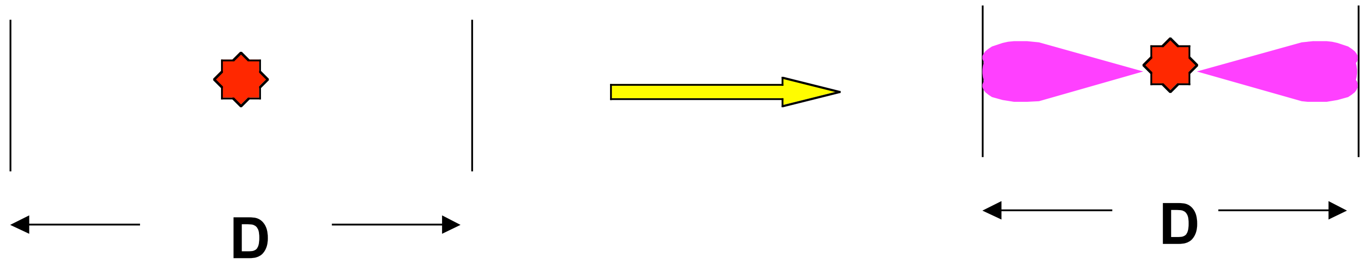
$$S = 2\pi\sqrt{n_1 n_5 n_p} + \Delta S$$

We ask that the creation of pairs be *probable*, not just *possible*



Require $\boxed{\Delta S = 1}$

No pairs, Phase space volume e^{S}	**Pairs form, Phase space volume** $e^{S+\Delta S}$



$$S = 2\pi\sqrt{n_1 n_5 n_p (1-f)} + 2\pi\sqrt{n_1 n_5 n_p f}\left(\sqrt{n_k} + \sqrt{\bar{n}_k}\right)$$

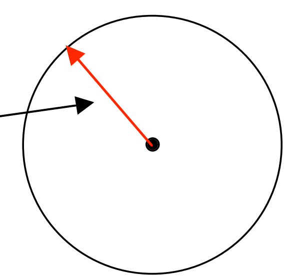
$$n_k = \bar{n}_k = \frac{1}{2}\frac{\Delta E}{m_k} = \frac{1}{2 D m_k}$$

$$m_k \sim \frac{G_5}{G_4^2} \sim \frac{D^2}{G_5}$$

Extremize over f, set

$$\Delta S = S - 2\pi\sqrt{n_1 n_5 n_p} \;=\; 1$$

$$D \sim G_5^{\frac{1}{3}} (n_1 n_5 n_p)^{\frac{1}{6}} \sim R_S$$



(SDM '97)

Make a bound state of a large number of D1,D5 branes.

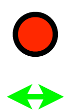
These branes wrap along compact directions, but classically, they are at r=0 in the noncompact space

$$M_{9,1} \rightarrow M_{4,1} \times T^4 \times S^1$$

$D5$

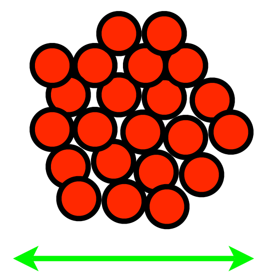
$D1$

$L, \; V_4, \; g$ *held fixed, charges taken large*



$\sim l_s$?

$\sim l_p$?



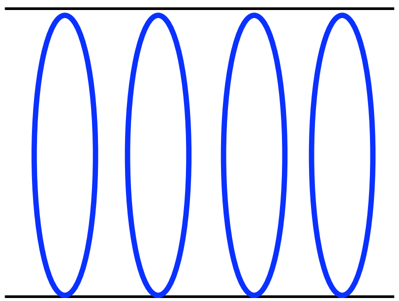
$\sim (n_1 n_5)^{\alpha} l_p$?

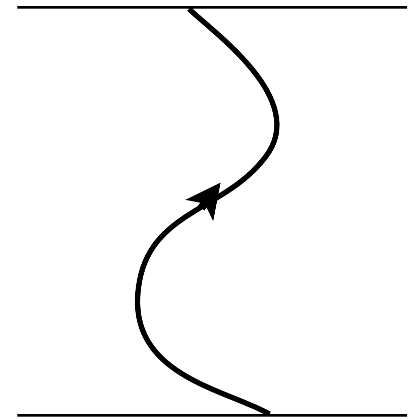
Because of quantum effects, the bound state will a nonzero size.

Is this size string length or planck length ? Or does it grow with the charges ?

S,T dualities

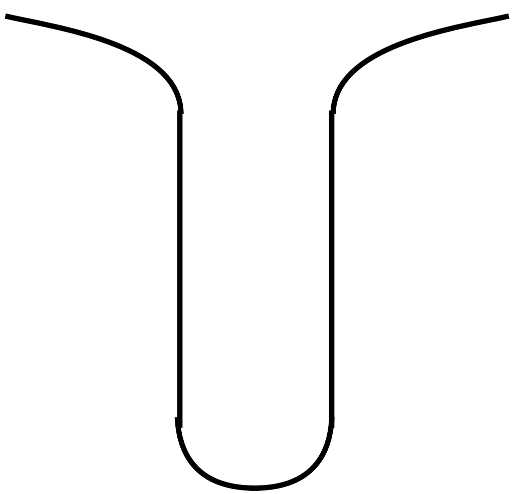
D1D5 | NS1 P

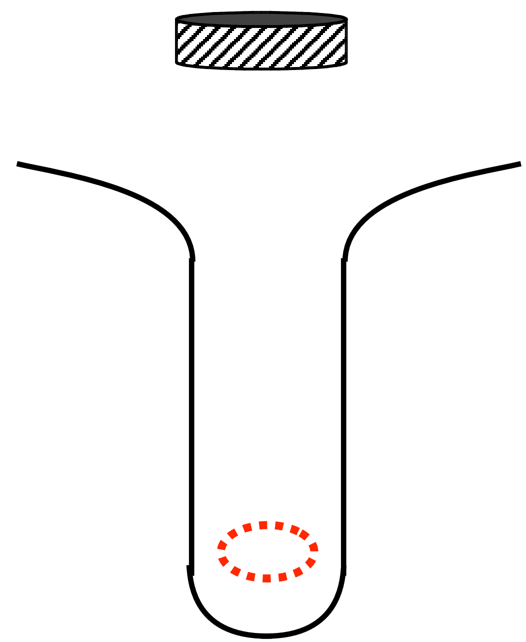




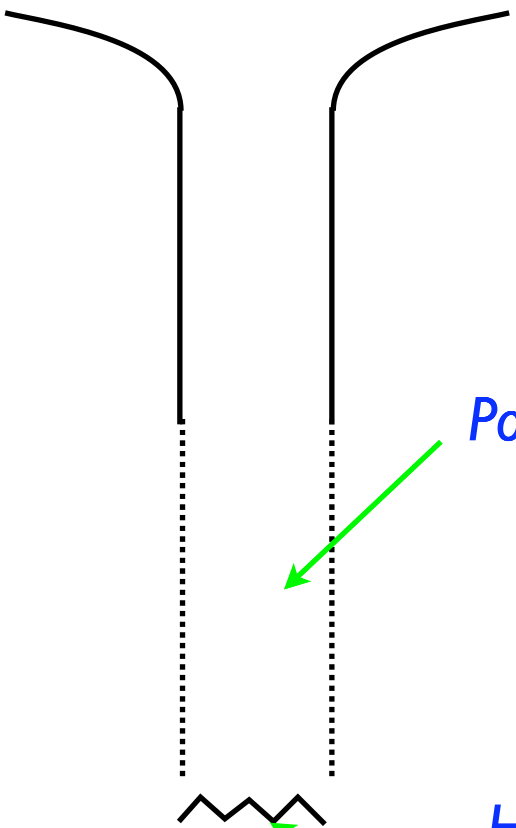
$n_1 n_5$ strands of the 'effective string', each 'singly wound'

$n_1 n_5$ units of momentum, all in the lowest harmonic





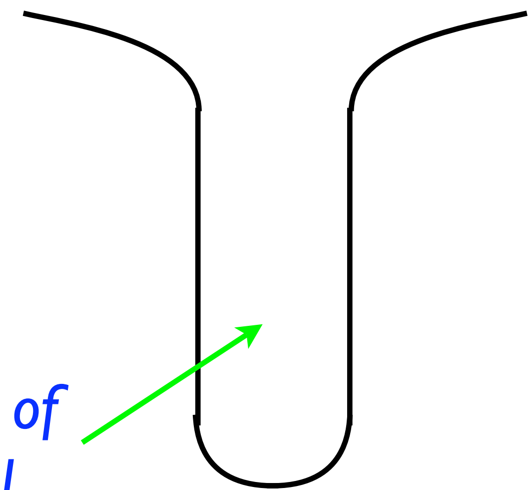
‘Naive’ geometry
of D1 D5



Poincare patch

Horizon coincides with singularity at r=0

Actual geometry
for given microstate

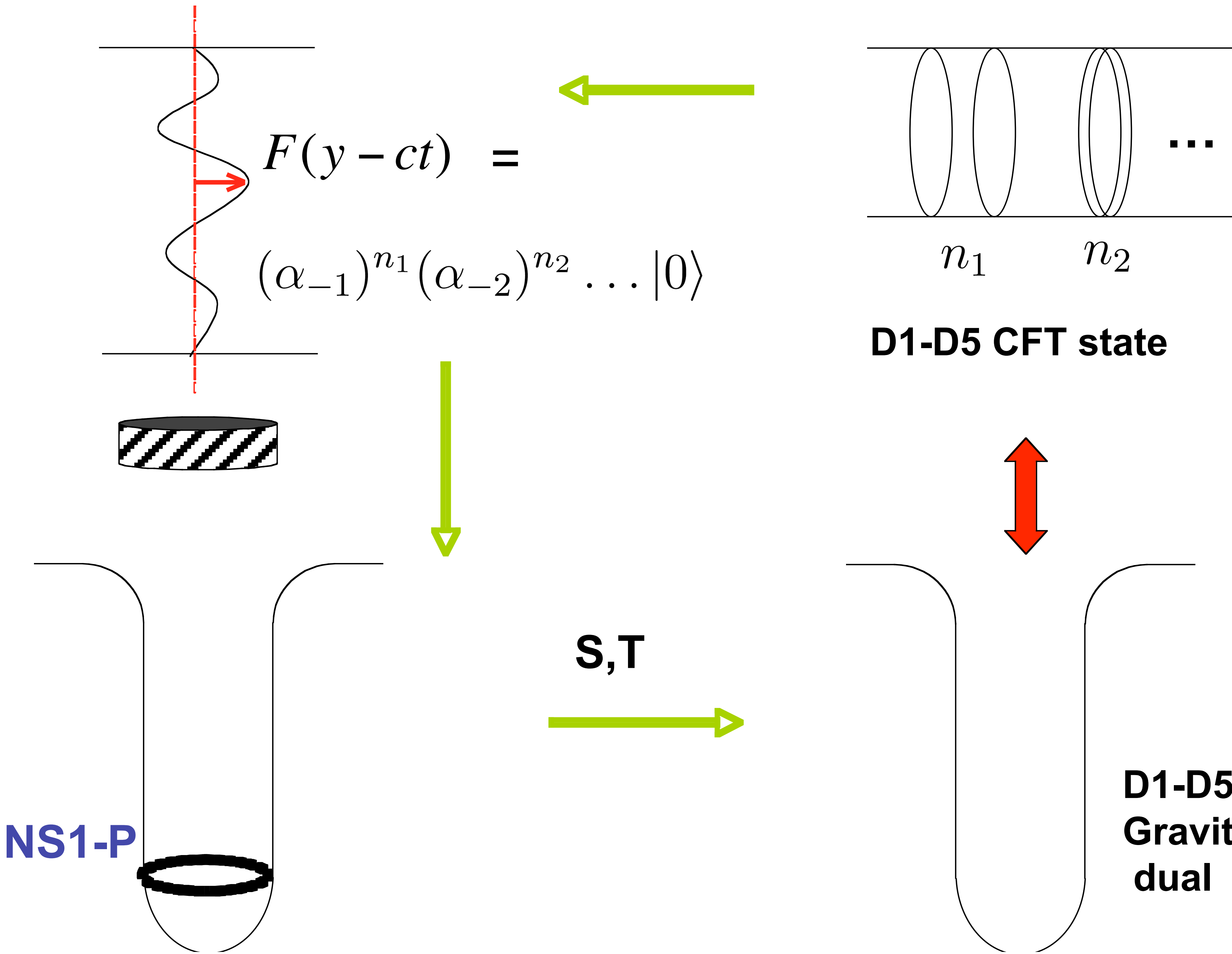


Piece of global

$$AdS_3 \times S^3 \times T^4$$

Cvetic+Youm ’ 95,
Balasubramanian, de Boer, Keski-Vakkuri, Ross ’00,
Maldacena+Maoz ’00

Lunin+SDM ’01



$F(y-ct)$ =
$(\alpha_{-1})^{n_1}(\alpha_{-2})^{n_2}\dots|0\rangle$
n_1
n_2
...
D1-D5 CFT state
S,T
NS1-P
D1-D5
Gravity
dual

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2},$$

$$K = \frac{Q}{L_T}\int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2},$$

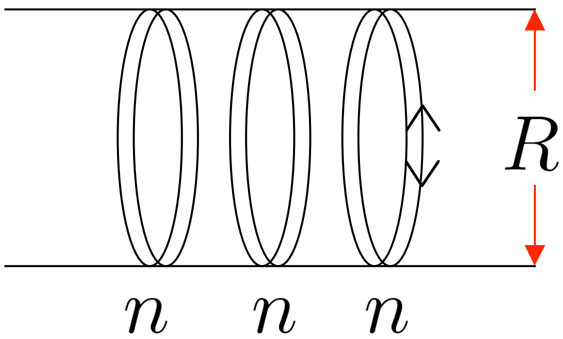
$$A_i = -\frac{Q}{L_T}\int_0^{L_T} \frac{dv\ \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

$$dB = -*_4\, dA$$

General metrics: Lunin+SDM '01

Also,
'Supergravity supertubes' '01
Emparan+Mateos+Townsend

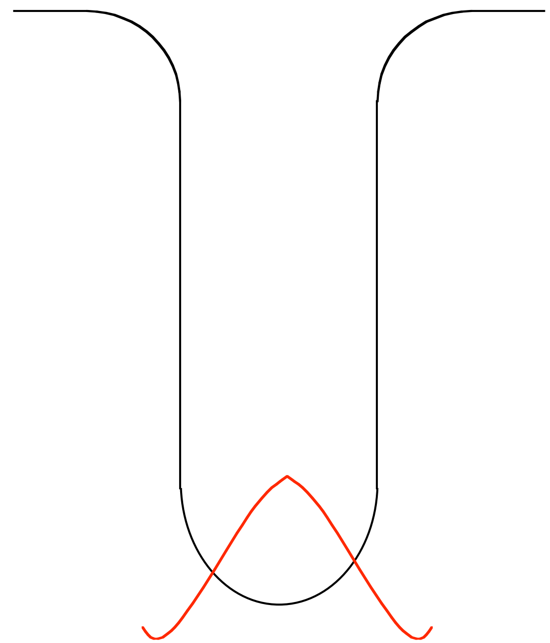
D1-D5 CFT state



$$\Delta E = \frac{1}{nR} + \frac{1}{nR} = \frac{2}{nR}$$

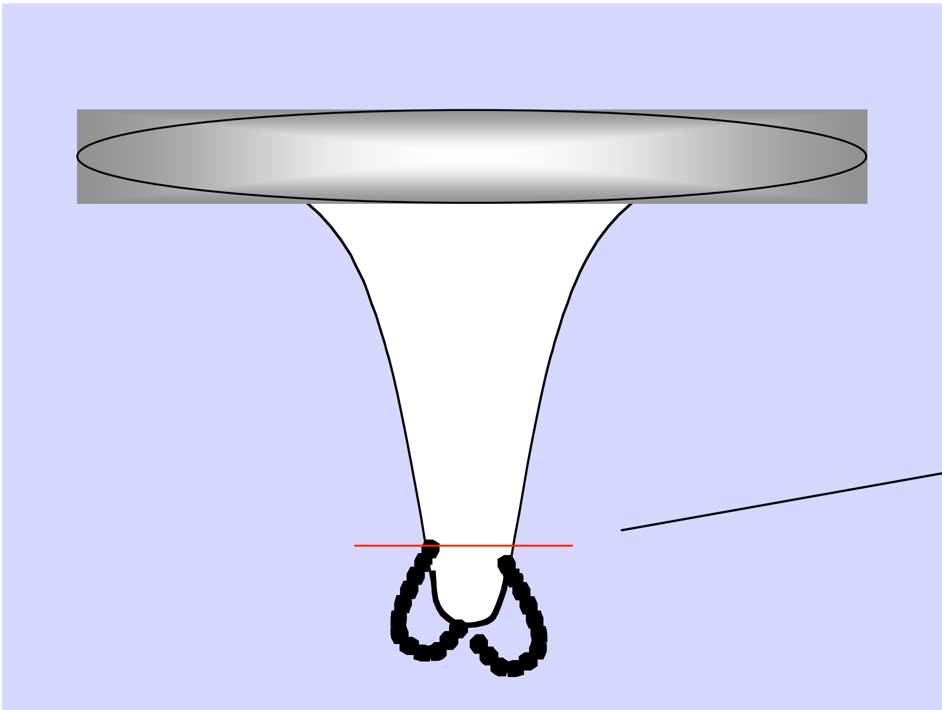
Longer 'component strings' → lower energy

D1-D5 Sugra solution



$$\Delta E = \frac{2}{nR}$$

Deeper throat, more redshift, lower energy

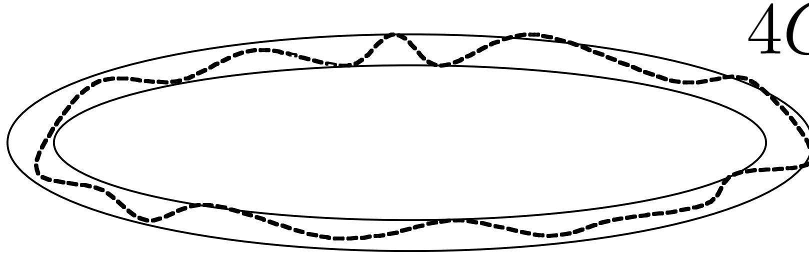


$$\frac{A}{4G} \sim \sqrt{n_1 n_5} \sim S$$

The 'size' of the typical fuzzball is such that the area of its surface yields a Bekenstein type relation

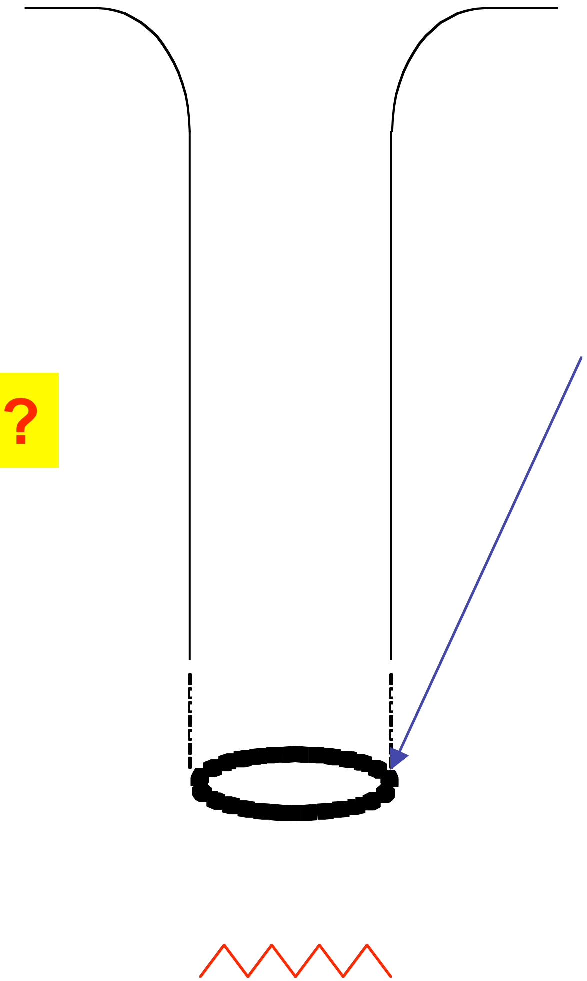
$$\frac{A}{4G} \sim \sqrt{n_1 n_5 - J} \sim S$$

Highly
rotating
2-charge



3-charge holes: NS1-NS5-P (or D1-D5-P)

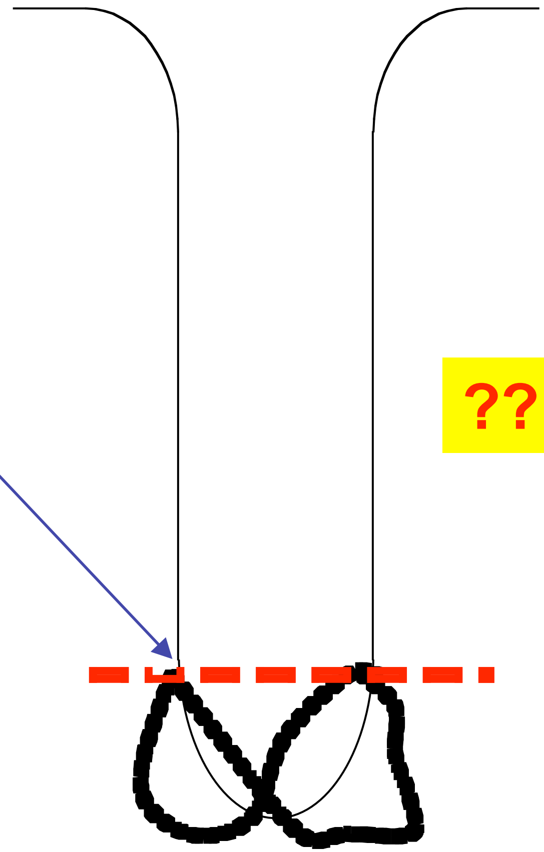
??



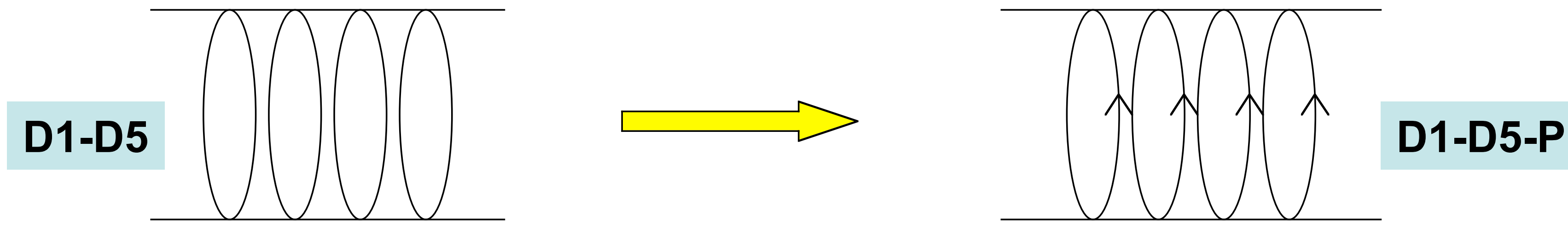
Area of throat saturates

$$\frac{A}{4G} = S$$

$$= 2\pi\sqrt{n_1 n_2 n_3}$$



??



Spectral flow on left movers : **R** → **NS** → **R** (n times)

$$|n\rangle^{total} = (J^{-,total}_{-(2n-2)})^{n_1n_5}(J^{-,total}_{-(2n-4)})^{n_1n_5}\dots(J^{-,total}_{-2})^{n_1n_5}\ |1\rangle^{total}$$

Right movers unchanged

$$h-\bar{h} = n(n+1)n_1n_5$$

P charge

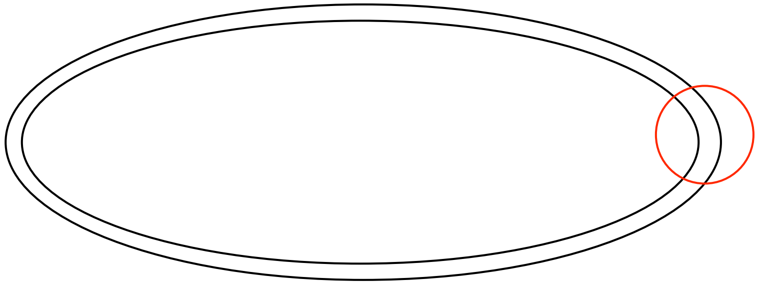
Spectral flow in AdS is a coordinate transformation

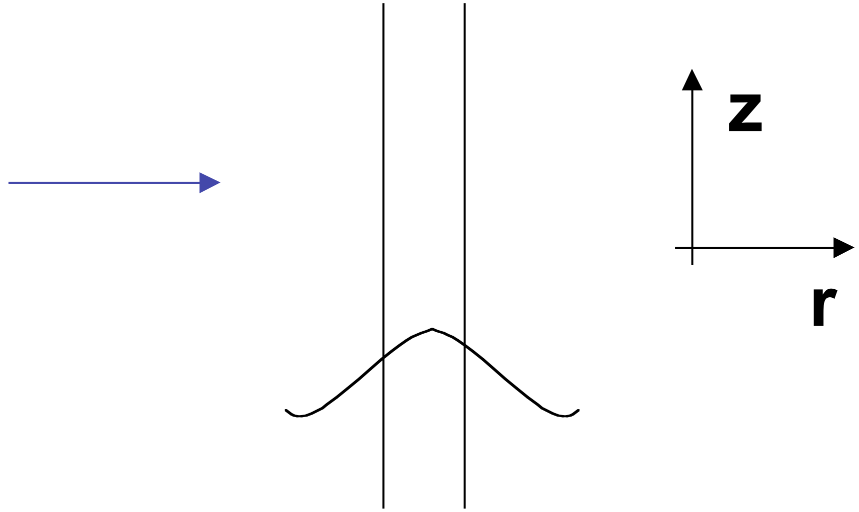
Balasubramanian+De Boer+ Keski-Vakkuri+ Ross '00; Maldacena+Maoz '00
Cvetic-Youm '95

$$
\begin{aligned}
ds^2 &= -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}\,(dt-dy)^2 + hf\left(\frac{dr_N^2}{r_N^2 + a^2\eta} + d\theta^2\right) \\
&+ h\left(r_N^2 - na^2\eta + \frac{(2n+1)a^2\eta Q_1 Q_5 \cos^2\theta}{h^2 f^2}\right)\cos^2\theta d\psi^2 \\
&+ h\left(r_N^2 + (n+1)a^2\eta - \frac{(2n+1)a^2\eta Q_1 Q_5 \sin^2\theta}{h^2 f^2}\right)\sin^2\theta d\phi^2 \\
&+ \frac{a^2\eta^2 Q_p}{hf}\left(\cos^2\theta d\psi + \sin^2\theta d\phi\right)^2 \\
&+ \frac{2a\sqrt{Q_1 Q_5}}{hf}\left[n\cos^2\theta d\psi - (n+1)\sin^2\theta d\phi\right](dt - dy) \\
&- \frac{2a\eta\sqrt{Q_1 Q_5}}{hf}\left[\cos^2\theta d\psi + \sin^2\theta d\phi\right] dy + \sqrt{\frac{H_1}{H_5}}\sum_{i=1}^{4} dz_i^2
\end{aligned}
$$

$$
\begin{aligned}
f &= r_N^2 - a^2\eta\, n\sin^2\theta + a^2\eta\,(n+1)\cos^2\theta \\
h &= \sqrt{H_1 H_5},\;\; H_1 = 1 + \frac{Q_1}{f},\;\; H_5 = 1 + \frac{Q_5}{f}
\end{aligned}
\qquad
\eta \equiv \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}
$$

A microstate for the 3-charge black ring





Smooth D1-D5 geometry

Add p units of P

CFT state $\boxed{|\psi\rangle = J^{-}_{-1}|\psi\rangle_R}$

Wavefunction

$$w = e^{-ip(t+y)-ikz}\,\tilde{w}(r,\theta,\phi)$$

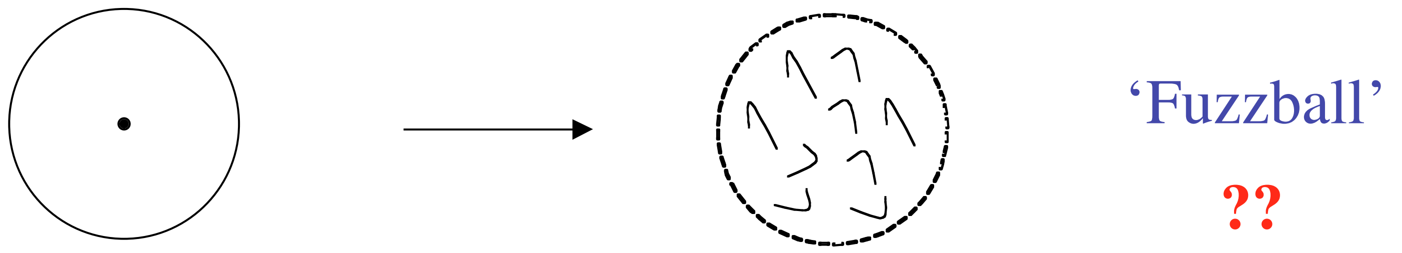
$$B^{(2)}_{MN} = e^{-ip(t+y)-ikz}\,\tilde{B}^{(2)}_{MN}(r,\theta,\phi)$$

$$w = e^{-\frac{1}{2Q}(t+y)}\, e^{i(\phi-kz)}\, \cos\frac{\theta}{2}\, e^{-kr}\, \frac{r^{1/2}}{Q+r}$$

$$\begin{aligned} B^{(2)} = {} & e^{-\frac{1}{2Q}(t+y)}\, e^{i(\phi-kz)}\, e^{-kr}\, r^{1/2} \Big\{ -\frac{1}{2Q}\, \cos\frac{\theta}{2}\, dt\wedge dz \\ & + \frac{r}{2(Q+r)^2}\, \cos\frac{\theta}{2}\, [dy - Q(1+\cos\theta)d\phi] \wedge \Big[dt - \frac{2Q+r}{Q}\, dz\Big] \\ & + ik\, \cos\frac{\theta}{2}\, dr\wedge dz + \frac{1}{2}\sin\frac{\theta}{2}\, dr\wedge [d\theta - i\sin\theta\, d\phi] \\ & - \frac{i}{2}\, r\, \cos\frac{\theta}{2}\, \sin\theta\, d\theta\wedge d\phi \Big\} \end{aligned}$$

(Giusto, SDM, Srivastava '06)

Construction of microstate geometries



2-charge in 4+1 non-compact dimensions: Lunin+SDM

3 charge in 4+1, U(1) X U(1) symmetry: Giusto+SDM+Saxena, Lunin

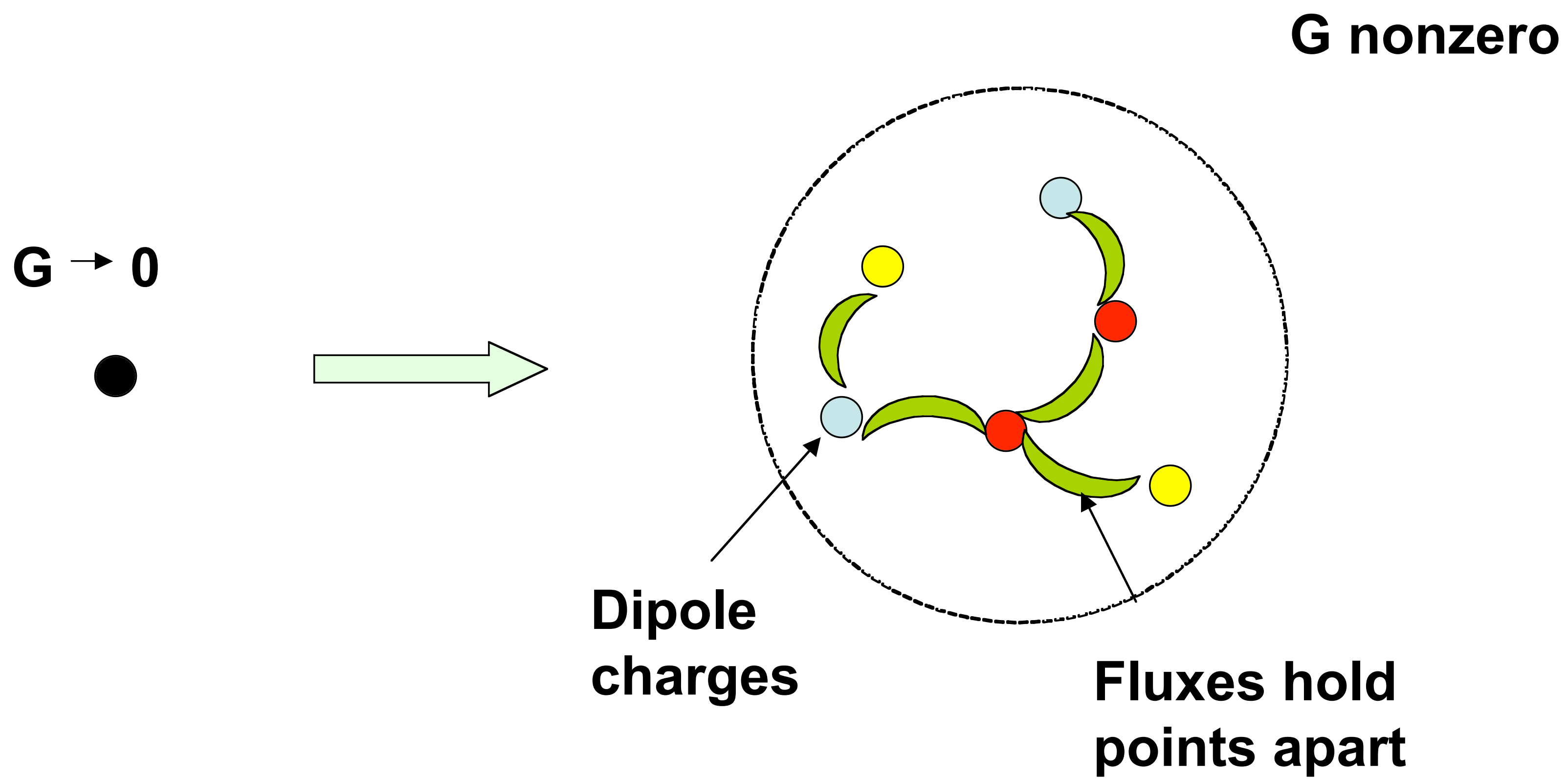
3 charges in 3+1, U(1) X U(1) symmetry: Bena+Kraus

3 charges in 4+1, U(1) symmetry: Bena+Warner, Berglund+Gimon+Levy

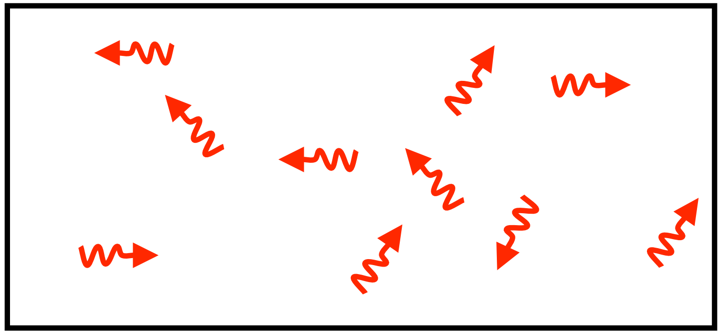
4 charges in 3+1, U(1) X U(1) symmetry: Saxena+Potvin+Giusto+Peet

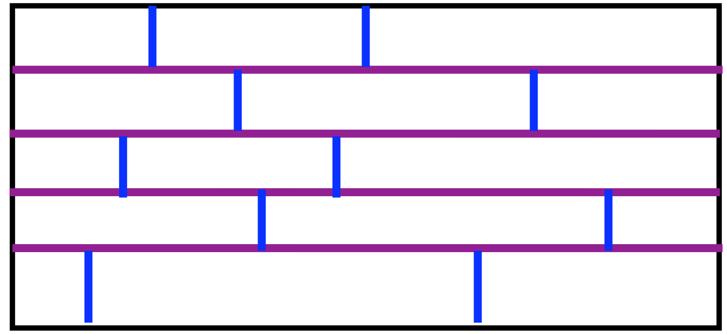
4 charges in 3+1, U(1) symmetry: Balasubramanian+Gimon+Levi

‘Bena-Warner’ equations: As we increase gravitational coupling, a pointlike object splits into dipole charges held apart by integer fluxes



What is the state of matter in the early Universe?





$$\boxed{S \sim E^{\frac{D-1}{D}}}$$

$$\boxed{S = A \prod_{k=1}^{n} (\sqrt{n_k} + \sqrt{\bar{n}_k}) = 2^n A \prod_{k=1}^{n} (\sqrt{n_k} + \sqrt{\bar{n}_k})}$$

So we see that at very high energies the 'fractional brane state' will have more entropy

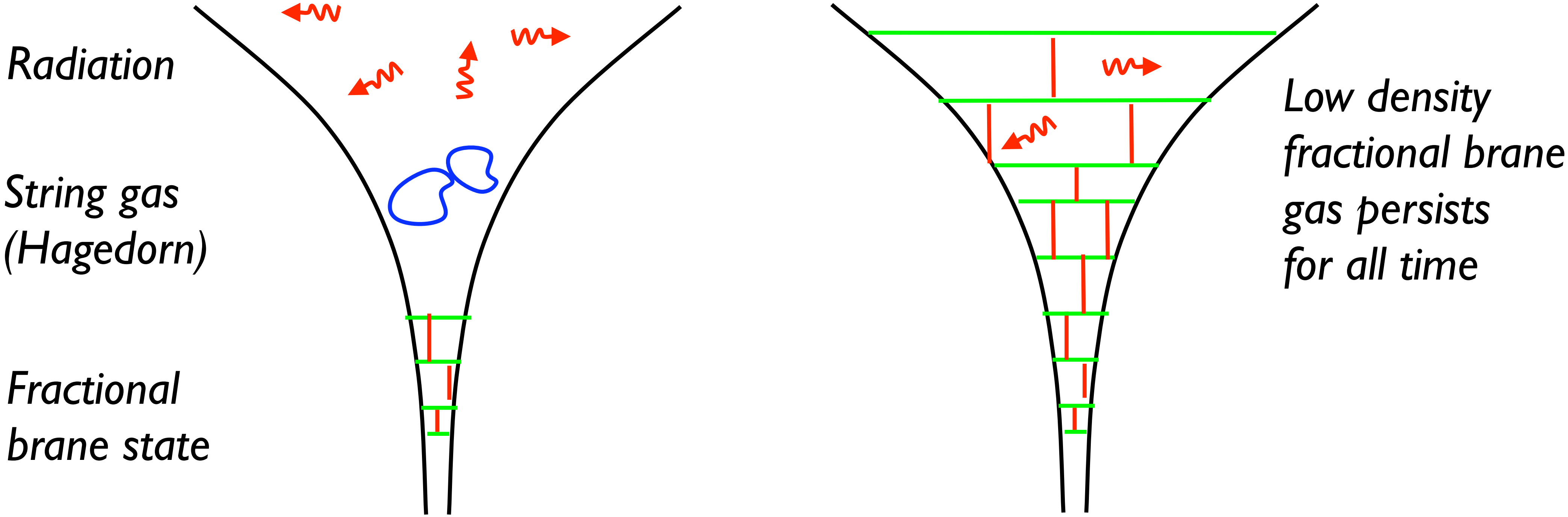
$$n_k \sim E$$

$$\boxed{S \sim E^{\frac{k}{2}}}$$

We find that the equation of state is $p_k = w_k \rho$

The evolution of the geometry can be solved in closed form (hypergeometric functions)

Chowdhury+SDM '06



How many intersecting branes give the maximal entropy state of string theory?

Do the fractional branes persist as a low density fluid for all time?
(Dark matter/dark energy?)

What is the analogue of the macroscopic quantum nonlocality found for fractional branes in the black hole context? (Horizon problem?)

Does the Universe start in a maximal entropy state?

Summary

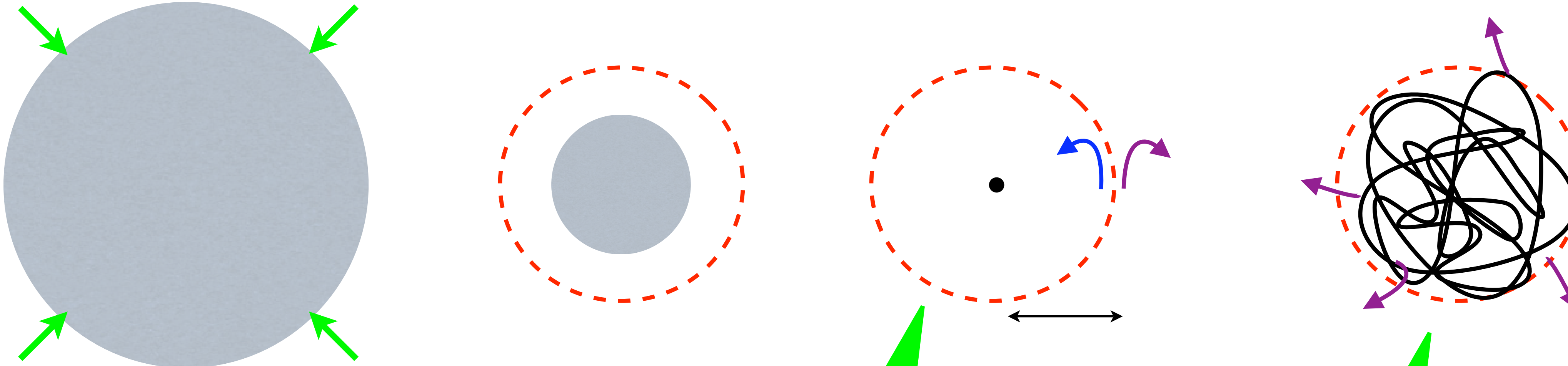
(a) It appears that string theory has very high entropy states where the energy is used to create 'fractional brane-antibrane pairs'.

(b) For time-independent configurations, these states are typically large 'fuzzballs'. Their radius is not string length or planck length; rather it grows with the number if branes in the state and is such that the surface area satisfies a Bekenstein type relation S~A/4G.

This size may be arising for simple 'phase space' reasons. The large entropy implies a large phase space volume, and for time independent configurations this implies a large spatial volume ....

(c) 2-charge extremal holes have been understood, and many states for the 3-charge/4-charge holes have been understood ... these all turn out to be 'fuzzballs' with no horizons.

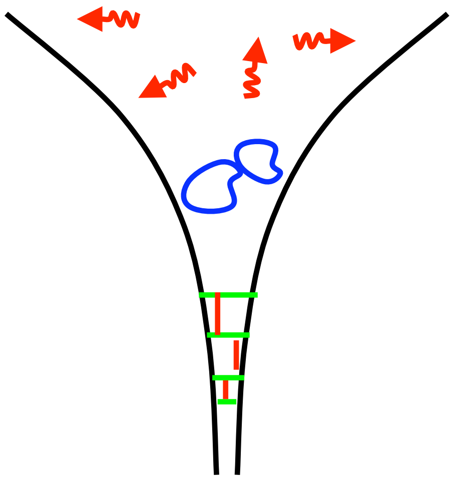
As a corollary, we would resolve the black hole information paradox ....

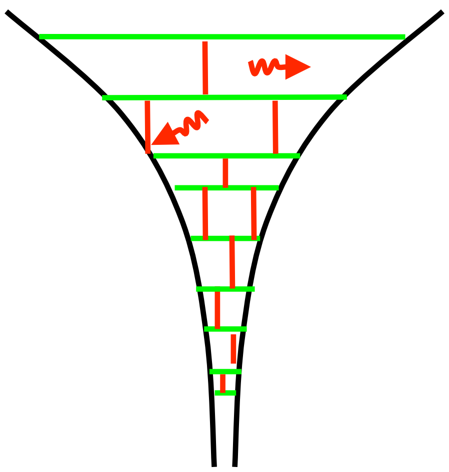


Information disappears into the singularity, but the radiation emerges from the horizon;
This gives information loss

If the state is a horizon sized fuzzball, the radiation leaves from the surface, taking information about the matter in the hole, just like what happens if we burn a peice of coal

(d) These notions suggest a nonconventional resolution to puzzles arising from the early Universe



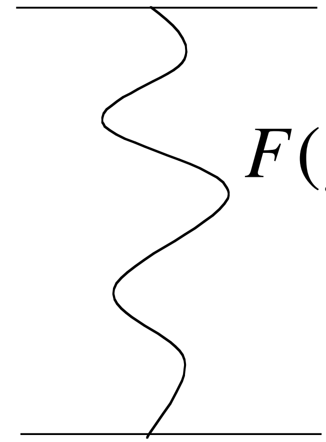


Additional slides for discussion on corrections to geometries

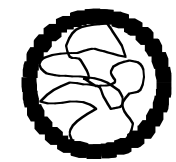
NS1-P state: $(\alpha_{-k_1})^{n_1}(\alpha_{-k_2})^{n_2}\ldots|0\rangle$

Fix total energy

Few modes k, large n : ***Coherent state***

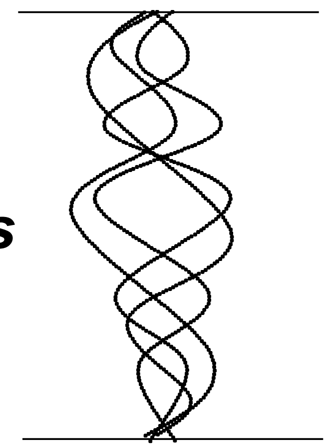


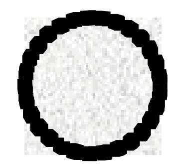
$F(y-ct)$



Many k, All n ~ 1

Quantum energy eigenstate for Harmonic oscillators of each fourier mode





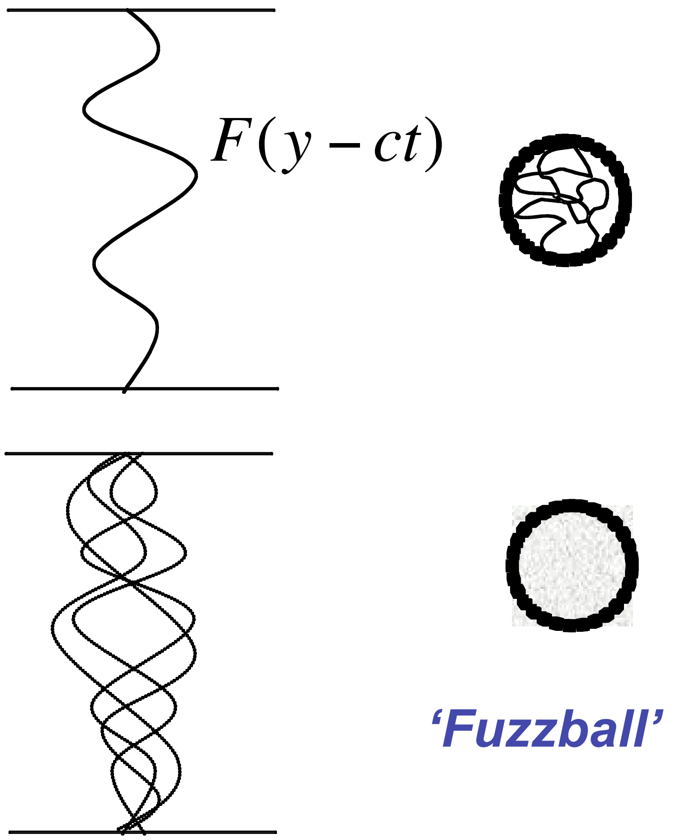
'Fuzzball'

Size for generic state estimated from classical geometries

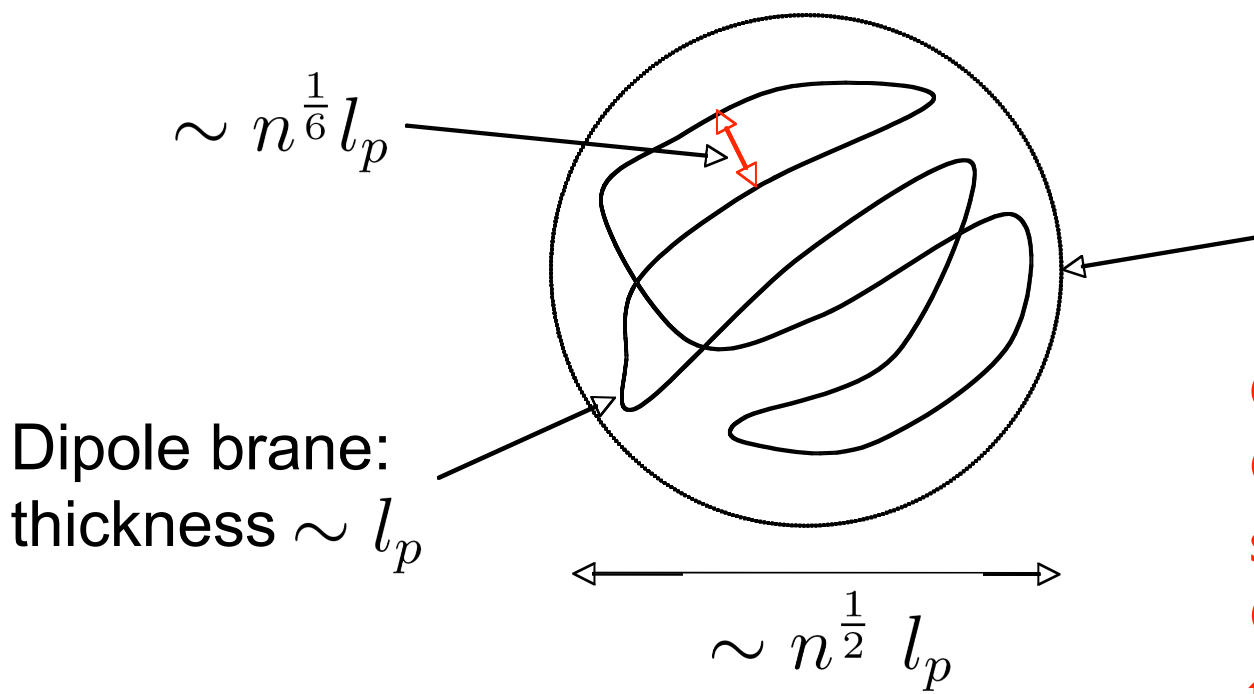
Two sources of corrections:

(a) In a generic state the occupation number of each harmonic is order unity, so the fluctuations are order unity.

There is no essential quantum gravity here -- the same happens for vibrations of any string



(b) R^4 terms: These become significant at the curve where the KK monopole tube has its center. It appears plausible that their effect is to expapnd the radius of this tube from below planck length to planck length, and make no other significant change to the geometry



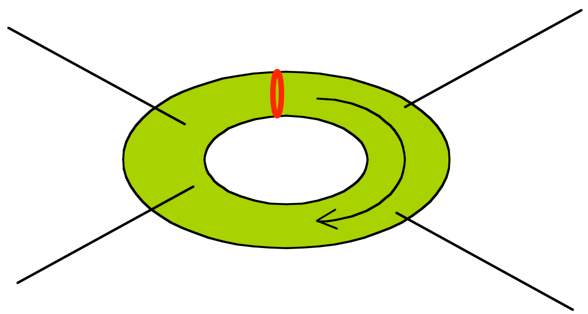
Order of magnitude of Curvature corrections studied in Giusto + SDM '04, need to study their exact effect in particular geometries

$$M_{9,1} \to M_{4,1} \times K3 \times S^1$$

D1

D5

Winding mode of NS1 around S[1]



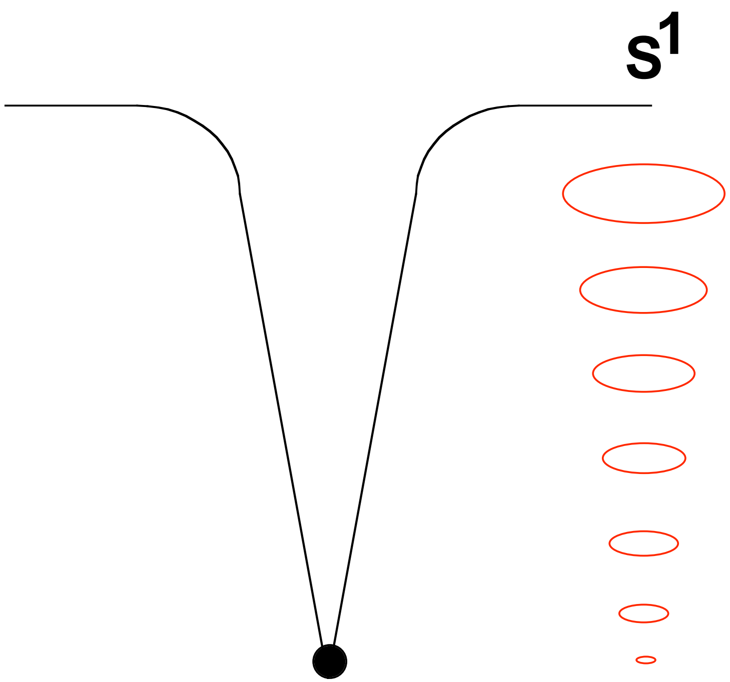
$$\begin{aligned} S_5 &= \frac{2\pi^4 R_0 V_0}{G_{10}} \int dx^5 \sqrt{-g_{(E)}} \Big[R_{(E)} \\ &+ \frac{c_2}{6} \frac{g^2 \alpha'^4}{V_0 R_0^2} \left(\frac{V}{V_0} e^{-2\phi} \right)^{-1/3} \left(\frac{R_0}{R} \right)^{4/3} R^{(E)}_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}_{(E)} \Big] \end{aligned}$$

Radius of S[1]

Cardoso, de Wit, Mohaupt '00,
Dabholkar '04

Naïve geometry:

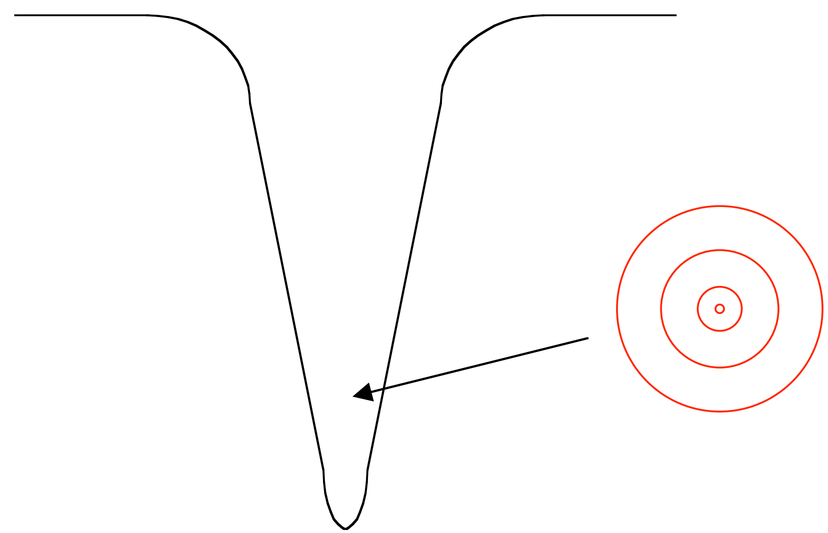
S^1 shrinks to zero size, correction can *diverge*



Actual geometry

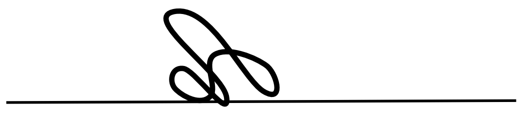
S^1 is nontrivially fibered over the with the angular S^3

Shrinks to zero as the angular circle in a plane, like in the KK monopole



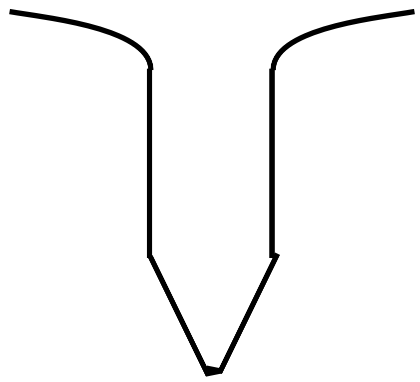
Corrections bounded

Essential question: Can corrections of either type change the fuzzball back to a naive black hole?



This does not appear plausible ....

(i) Note that whatever the corrections, we must still get Exp(S) orthonormal states, so the different states cannot become the 'same' because of quantum corrections ...



(ii) We can follow the BPS state of a 3-charge object as the coupling g is increased. How can a horizon suddenly develop?