NULL SINGULARITIES AND THEIR VARIOUS HOLOGRAPHIC DUALS

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I INTRO

- SPACELIKE SINGULARITIES ARE DIFFICULT TO UNDERSTAND — IT IS HARD TO MAKE SENSE OF TIME WHICH BEGINS OR ENDS SOMEWHERE
- SOMEWHAT RELATED — NULL SINGULARITIES
- THIS TALK: SOME ATTEMPTS TO ADDRESS ISSUES IN TOY MODELS WHERE THERE IS A GOOD GUESS FOR HOLOGRAPHIC DUALS OF BACKGROUNDS WITH NULL SINGULARITIES
- HOPE: HOLOGRAPHIC DUAL IS MORE FUNDAMENTAL — DYNAMICAL SPACE-TIME EMERGES IN AN APPROXIMATE FASHION

  NEAR "SINGULARITIES" THE HOLOGRAPHIC DUALS MAKE SENSE

THE SETUPS

(1) MATRIX BIG BANGS

BULK : 10 D SPACE-TIMES WITH NULL SINGULARITIES

- FLAT + DILATON
- PP + DILATON

HOLOGRAM : BFSS TYPE MATRIX THEORIES

- Craps, Sethi, Verlinde
- S.R.D & J. Michelson

(2) AdS DEFORMATIONS

BULK : DEFORMATIONS OF $AdS_5 \times S^5$ WITH NULL SINGULARITIES

HOLOGRAM : 3+1 DIM GAUGE THEORY WITH "TIME" DEP BACKGROUND AND COUPLINGS

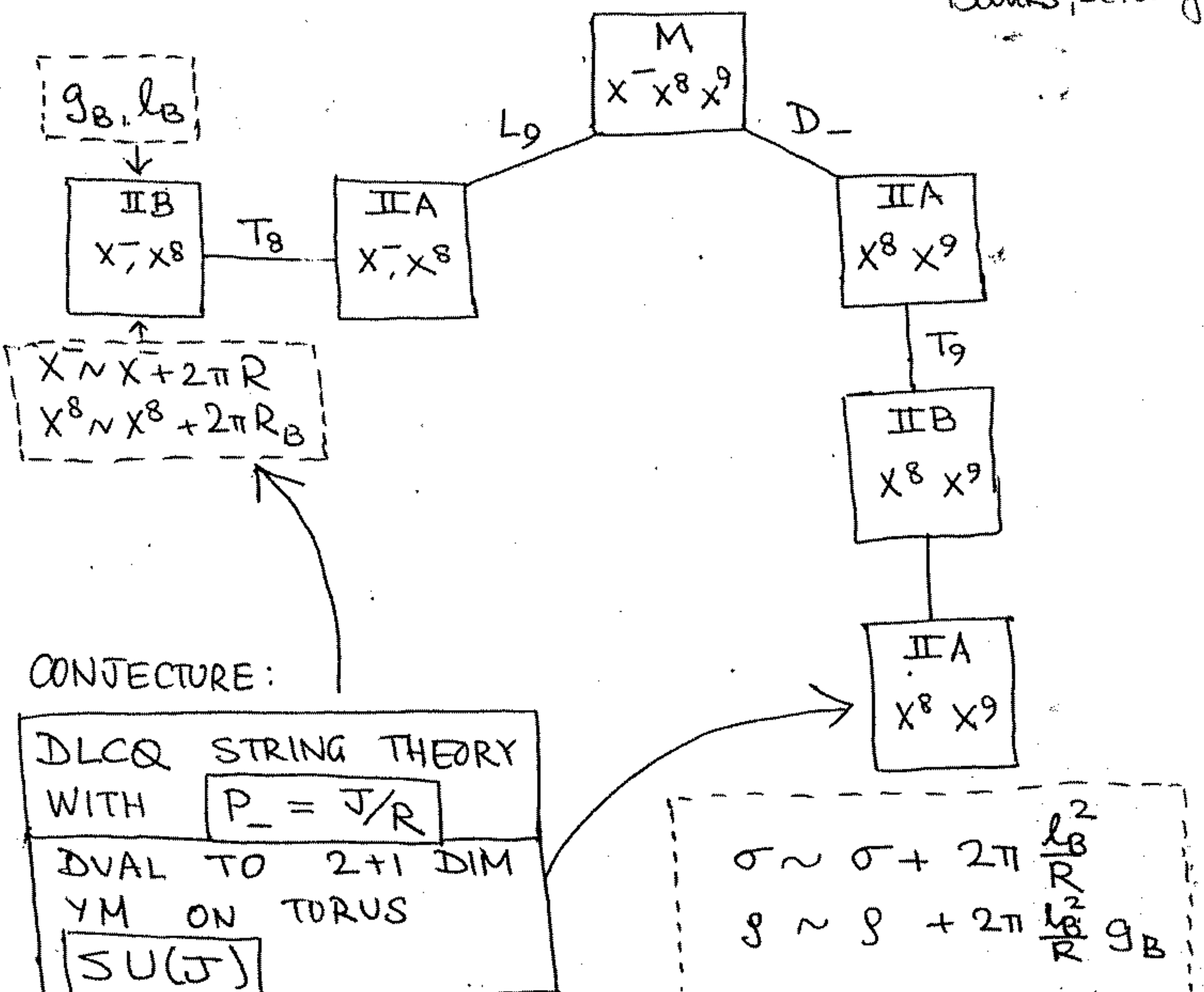
- S.R.D., J. Michelson, K. Narayan & S. Trivedi

$\boxed{\text{(2) WILL BE MOSTLY DEALT WITH BY NARAYAN}}$

MATRIX MEMBRANE THEORY

START WITH IIB STRING IN SOME BACKGROUND WITH A COMPACT NULL DIRECTION + A COMPACT SPACE DIRECTION

Motl
Dijgraf, Verlinde^2
Banks, Seiberg



- THERE ARE A VERY LIMITED NUMBER OF KNOWN BACKGROUNDS
  - FLAT SPACE
  - PP - WAVES

- LUCKILY ALSO POSSIBLE FOR BCKGNDS WITH NULL LINEAR DILATON

$$\boxed{\begin{aligned} ds^2 = \; & 2dx^+dx^- - 4\mu^2(x_1^2+\cdots+x_6^2)(dx^+)^2 \\ & -8\mu\, x^7 dx^8 dx^+ + d\vec{x}^2 \end{aligned}}$$

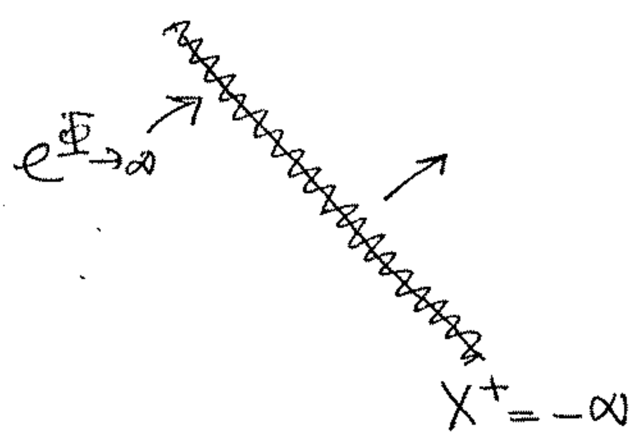
$$\boxed{F_{+1234} = F_{+5678} = \mu e^{Qx^+}}$$

$$\boxed{\Phi = -Qx^+}$$ (S.R.D & J. Michelson)

Note: Related to usual Brinkmann coordinates by a coordinate transformation.

1/2 SUSY - $\boxed{\Gamma^+ \varepsilon = 0}$

- THIS HAS A NULL SINGULARITY AT $X^+ = -\infty$ FOR $Q > 0$
  $X^+ = +\infty$ FOR $Q < 0$



- $X^+ = -\infty$ IS REACHED IN FINITE AFFINE PARAMETER ALONG NULL GEODESICS
- TIDAL FORCES DIVERGE HERE
- HOWEVER CURVATURE INVARIANTS FINITE.

$e^{\Phi} \to \infty$ AT THIS NULL SINGULARITY.

- FOR $Q=0$ (USUAL ~~PP~~ PP) THERE ARE TWO POSSIBILITIES FOR DUAL THEORY

  (a) A 3+1 DIMENSIONAL GAUGE THEORY (QUIVER) — AdS/CFT TYPE
  — Mukhi, Rangamani, Verlinde

  (b) A 2+1 DIM GAUGE THEORY — MATRIX MEMBRANE —
  Following the logic of BMN in M Theory context

- FOR $Q \neq 0$ — WE AS YET DO NOT KNOW (a)., BUT WE KNOW HOW TO CONSTRUCT (b).

BOSONIC FIELDS

$$X^i(\rho,\sigma) \qquad i = 1\dots 7$$
$$F_{ab} \qquad a,b = 0,1,2$$

$$\boxed{\begin{aligned}\mathcal{L} = \frac{1}{2}\mathrm{Tr}\Big\{&\big[(D_\tau X)^2 - (D_\sigma X)^2 - e^{2Q\tau}(D_\rho X)^2\big] \\ &+ \frac{1}{G_{YM}^2 e^{2Q\tau}}\big[F_{\tau\sigma}^2 + e^{2Q\tau}(F_{\rho\tau}^2 - F_{\rho\sigma}^2)\big] \\ &- 4\mu^2(X_1^2 + \dots + X_6^2) \\ &+ \frac{1}{2} G_{YM}^2 e^{2Q\tau}\,[X^i, X^j]^2 \\ &- \frac{8\mu}{G_{YM}e^{Q\tau}}\, e^{Q\tau} X^7 F_{\rho\sigma} \\ &- 8\mu i\, G_{YM} e^{Q\tau} X^7 [X^5, X^6]\Big\}\end{aligned}}$$

$$\boxed{\begin{aligned} 0 \le \sigma &\le 2\pi \frac{l_B^2}{R} \\ 0 \le \rho &\le 2\pi \frac{l_B^2}{R} g_B \\ G_{YM}^2 &= \frac{R R_B^2}{g_B l_B^4}\end{aligned}}$$

TO STRINGS

- FOR $Q=0$ (TIME-INDEPENDENT BACKGRNDS) THIS REDUCES TO STRING THEORY WHEN $g_B \lll 1$

- $R_\rho / R_\sigma = g_B \lll 1 \Rightarrow \rho$ KK MODES DECOUPLE AND WE HAVE 1+1 DIM THEORY

- IN THE IR, POTENTIAL TERM

$$G_{YM}^2 \, [X^i, X^j]^2$$

RESTRICT TO DIAGONAL X^i IN A SUITABLE GAUGE $X^i = \mathrm{diag}\,(X^i_1 \cdots X^i_J)$

- THERE COULD BE NON-TRIVIAL BOUNDARY CONDITIONS IN σ DIRECTION

e.g.

$$\begin{aligned} X^i_1(\sigma+2\pi) &= X^i_2(\sigma) \\ X^i_2(\sigma+2\pi) &= X^i_3(\sigma) \\ &\cdots \\ X^i_J(\sigma+2\pi) &= X^i_1(\sigma) \end{aligned} \quad \rightarrow \quad \text{Effective length } 2\pi J R_\sigma$$

GENERALLY l FIELDS FOR EACH i ON CIRCLES OF LENGTHS λ_a

$$\sum_{a=1}^{\sigma} \lambda_a = 2\pi J R_\sigma = 2\pi J \frac{l_B^2}{R}$$

$$= 2\pi l_B^2 P_-$$

- THE GAUGE FIELD MAY BE DUALIZED TO A SCALAR

$$F_{ab} \sim \varepsilon_{abc} \partial_c \phi$$

- MAKE A ROTATION

$$\begin{pmatrix} X^7 \\ G_{YM}\phi \end{pmatrix} = \begin{pmatrix} \cos 2\mu\tau & \sin 2\mu\tau \\ -\sin 2\mu\tau & \cos 2\mu\tau \end{pmatrix} \begin{pmatrix} Y^7 \\ Y^8 \end{pmatrix}$$

- THEN Y^α, $\begin{pmatrix} Y^\alpha = X^\alpha & \alpha = 1..6 \\ Y^7, Y^8 & \end{pmatrix}$

  BECOME THE 8 SCALARS ON THE GS WORLDSHEET WHOSE σ SIZE IS $2\pi l_B^2 P_-$ as required

- Gopakumar : IRRELEVANT OPERATORS OBTAINED BY INTEGRATING OUT THE KK MODES LEAD TO THE CORRECT EFFECTIVE STRING COUPLING

$$g_{eff} \sim \frac{\mu}{M_{KK}} = g_B \left(2\pi l_B^2 P_-\right)$$

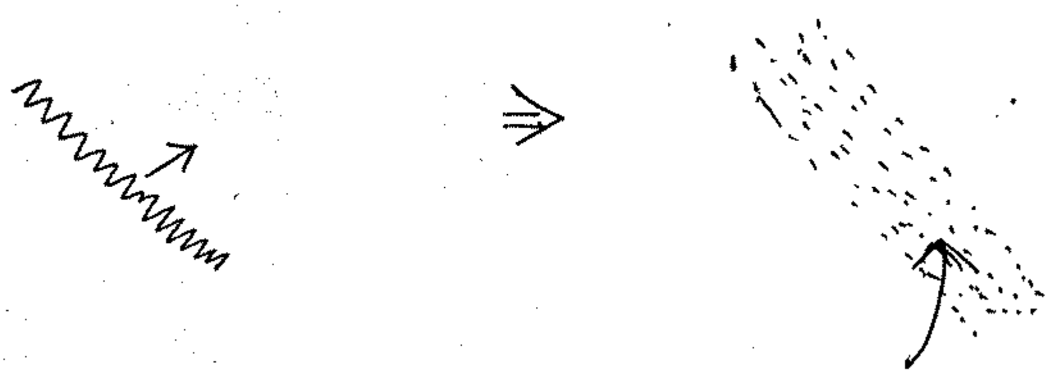
- FOR $Q \neq 0$ COUPLING IS WEAK AT $\tau = -\infty$

$\Rightarrow$ NON-ABELIAN STRUCTURE OF THEORY BECOMES SIGNIFICANT

$\Rightarrow$ 2+1 DIM NATURE RATHER THAN 1+1 DIM

$\Rightarrow$ NO INTERPRETATION IN TERMS OF STRINGS MOVING IN 10 DIMENSIONS

BUT THIS IS PRECISELY WHERE THE GRAVITY DESCRIPTION BECOMES <u>SINGULAR</u>



- AT LATE TIMES $\tau \to +\infty$
  - G_{YM} BECOMES LARGE
  - R_ρ / R_σ BECOMES SMALL IF $g_B \ll 1$

EXPECT USUAL PERTURBATIVE STRING THEORY IN PP-WAVE

— LATE TIME EMERGENCE OF 10 D SPACE-TIME

- BUT THERE IS A LITTLE MORE TO THE STORY

FUZZY ELLIPSOIDS

- BY RESCALING FIELDS

$$S(G_{YM},\mu) = \left(\mu / G_{YM}^2\right)^2 S(1,1)$$

- WHEN $\mu \gg G_{YM}^2$ IT MAKES SENSE TO LOOK AT CLASSICAL SOLUTIONS

- A PARTICULARLY INTERESTING CLASS
  - FUZZY ELLIPSOIDS
  - D3 BRANES IN ORIGINAL IIB

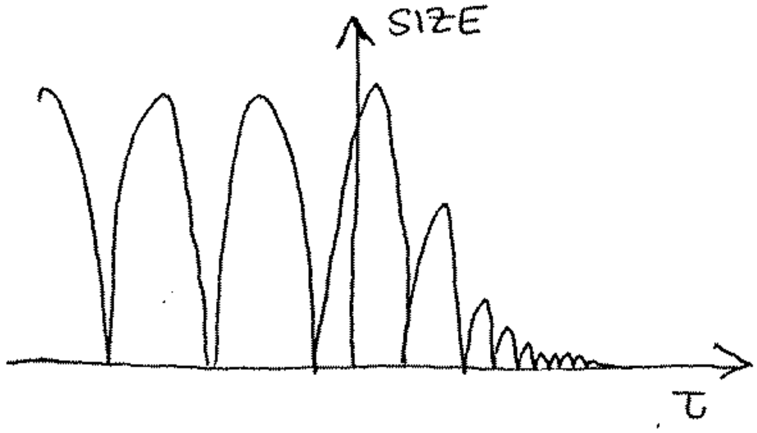
$$X^5(\tau) = R_1(\tau) J^1 \qquad X^7(\tau) = R_2(\tau) J^3$$
$$X^6(\tau) = R_1(\tau) J^2$$
$$[J^1, J^2] = i J^3 \quad \text{etc.}$$

J^a : J-DIMENSIONAL REP. OF SU(2)

- THESE ARE OF COURSE VERY NON-ABELIAN CONFIGURATIONS

- QUESTION: IF WE START WITH SOME TYPICAL SUCH OBJECT AT $\tau = -\infty$, WHAT HAPPENS TO IT?

- AT CLASSICAL LEVEL



GENERIC EVOLUTION.

- INDICATES THAT ANY STATE OF SUCH OBJECT EVOLVES INTO A TRIVIAL ZERO SIZE OBJECTS

  — EFFECTIVELY THE X^i BECOME DIAGONAL

(P,q) STRINGS

- FOR US TO OBTAIN PERTURBATIVE STRINGS AT LATE TIMES THE 2+1 DIM THEORY MUST BECOME A 1+1 DIM THEORY

- KINETIC TERMS

$$(D_\tau X)^2 - (D_\sigma X)^2 - e^{2Q\tau}(D_\rho X)^2$$

IN TERMS OF MOMENTUM MODES IN ρ DIRECTION

$$X(\tau,\rho,\sigma) = \sum_m X_m(\tau,\sigma)\, e^{\frac{2\pi i m}{R_\rho}\rho}$$

$$M_{KK}(\tau) = \left(\frac{m}{R_\rho}\right) e^{Q\tau}$$

MULTIPLE WINDING $\ni$ $\frac{m}{JR_\rho} e^{Q\tau}$

$M_{KK}(\tau)$ GROWS

DOES THIS MEAN WE CAN SURELY FORGET ABOUT THESE ?

- WE HAVE A 1+1 DIMENSIONAL THEORY WITH TIME DEP MASSES & COUPLINGS

QUESTION TO ASK : HOW DOES A $|in\rangle$ VACUUM EVOLVE

OR IN REVERSE :

IF WE HAVE $|out\rangle$ VACUUM IN THE FUTURE — WHAT IS THE $|in\rangle$ STATE

$|0\rangle_{OUT}$ ———————— $\tau = +\infty$

$|0\rangle_{IN}$ ———————— $\tau = -\infty$

$$|0\rangle_{OUT} \neq |0\rangle_{IN}$$

- AT THE FREE FIELD LEVEL

$$\boxed{\varphi^{(in)}_{m,n} \sim e^{i\left(\frac{mR}{l_B^2}\sigma + \frac{nR}{g_B l_B^2}\rho\right)} J_{-i\frac{\omega_m}{Q}}\left(k_n e^{Q\tau}\right)}$$

$$\omega_m^2 = 4\mu^2 + \frac{m^2R^2}{l_B^4} \qquad k_n = \frac{nR}{Q g_B l_B^2}$$

m : KK mode number

$$\boxed{\varphi^{(OUT)}_{m,n} \sim e^{i\left(\frac{mR}{l_B^2}\sigma + \frac{nR}{g_B l_B^2}\rho\right)} H^{(2)}_{-i\frac{\omega_m}{Q}}\left(k_n e^{Q\tau}\right)}$$

$$\boxed{|0\rangle_{out} = \left\{\prod_{\substack{n\neq 0\\ m}} \left[(1-|\gamma_m|^2)^{1/4}\, e^{\frac{1}{2}\gamma_m^* a^{\dagger(in)}_{m,n} a^{\dagger(in)}_{-m,-n}}\right\}|0\rangle_{in}}$$

IF WE DEMAND THAT THERE ARE NO KK MODES IN THE FUTURE — HAVE TO START WITH THIS SQUEEZED STATE

- WHAT ARE THE KK MODES?

RECALL $X^i(\tau,\sigma)$ — INDEPENDENT OF ρ ARE THE FUNDAMENTAL STRING OPERATORS

$$\ldots a^{+}_{m',0}\, a^{+}_{m,0} |0\rangle \quad : \quad \text{SINGLE STRING STATE}$$

(Consider eg. the single string, ie. longest cycle)

- USING THE CHAIN OF DUALITIES

$$a^{+}_{0,n} \quad : \quad \text{D-STRING OSCILLATOR}$$

$$\boxed{a^{+}_{m,n} \quad : \quad (p,q) \text{ STRING OSCILLATOR}}$$

Note: NET MOMENTUM ALONG ρ (OR σ) $\Rightarrow$ NET WINDING ALONG X^-

$(\partial_\sigma X^- = L_0 - \bar{L}_0)$. IN LIGHT CONE GAUGE.

$\Rightarrow$ $\boxed{\text{SQUEEZED STATE IS A HIGHLY EXCITED STATE OF A SINGLE (p,q) STRING}}$

SUMMARY SO FAR

- MATRIX STRING THEORY / MEMBRANE THEORY BECOMES STRING THEORY ONLY WHEN DYNAMICS RESTRICTS MATRICES TO BE COMMUTING
  — THIS IS HOW SPACE EMERGES

- AT THE (NULL) TIME WHERE THE SPACE-TIME DESCRIPTION APPEARS SINGULAR WE FIND THAT THERE IS INDEED NO SUCH SPACE-TIME DESCRIPTION
  — NEW DEGREES OF FREEDOM BECOME "MASSLESS"

- TYPICALLY PERTURBATIVE STRING THEORY EMERGES WHEN ONE STARTS OUT WITH A SUITABLE STATE IN THIS YM THEORY

AdS/CFT EXAMPLES

- SINCE THE IIB PP-WAVE HAS A AdS/CFT TYPE OF DUAL DESCRIPTION ONE MIGHT WONDER WHAT WOULD THE MATRIX BIG BANG LOOK LIKE IN THE 3+1 DIM GAUGE THEORY

- THIS HOWEVER REQUIRES THAT WE IDENTIFY SOME DEFORMATIONS OF $AdS_5 \times S^5$ WHOSE PENROSE LIMIT IS OUR PP-WAVE WITH LINEAR DILATON

- WE DONT KNOW THIS YET

- HOWEVER WE KNOW AN INTERESTING RELATED CLASS OF AdS DEFORMATIONS WITH NULL "SINGULARITIES" WHICH HAVE GAUGE THEORY DUALS IN THE AdS/CFT SENSE

- THE SUPERGRAVITY SOLUTION

$$\boxed{\begin{aligned} ds^2 &= \left(\frac{r}{R}\right)^2 \tilde{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + dr\left(\frac{R}{r}\right)^2 + R^2 d\Omega_5^2 \\ \Phi &= \Phi(x). \\ F_5 &= R^4(\omega_5 + *\omega_5) \end{aligned}}$$

$$\boxed{\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi}$$

- OUR CONJECTURE :

> THE DUAL IS THE USUAL $N=4$ YM IN 3+1 DIM LIVING ON METRIC $\tilde{g}_{\mu\nu}(x)$ AND A COUPLING $e^{\Phi(x)/2}$

- A PARTICULARLY TRACTABLE CASE

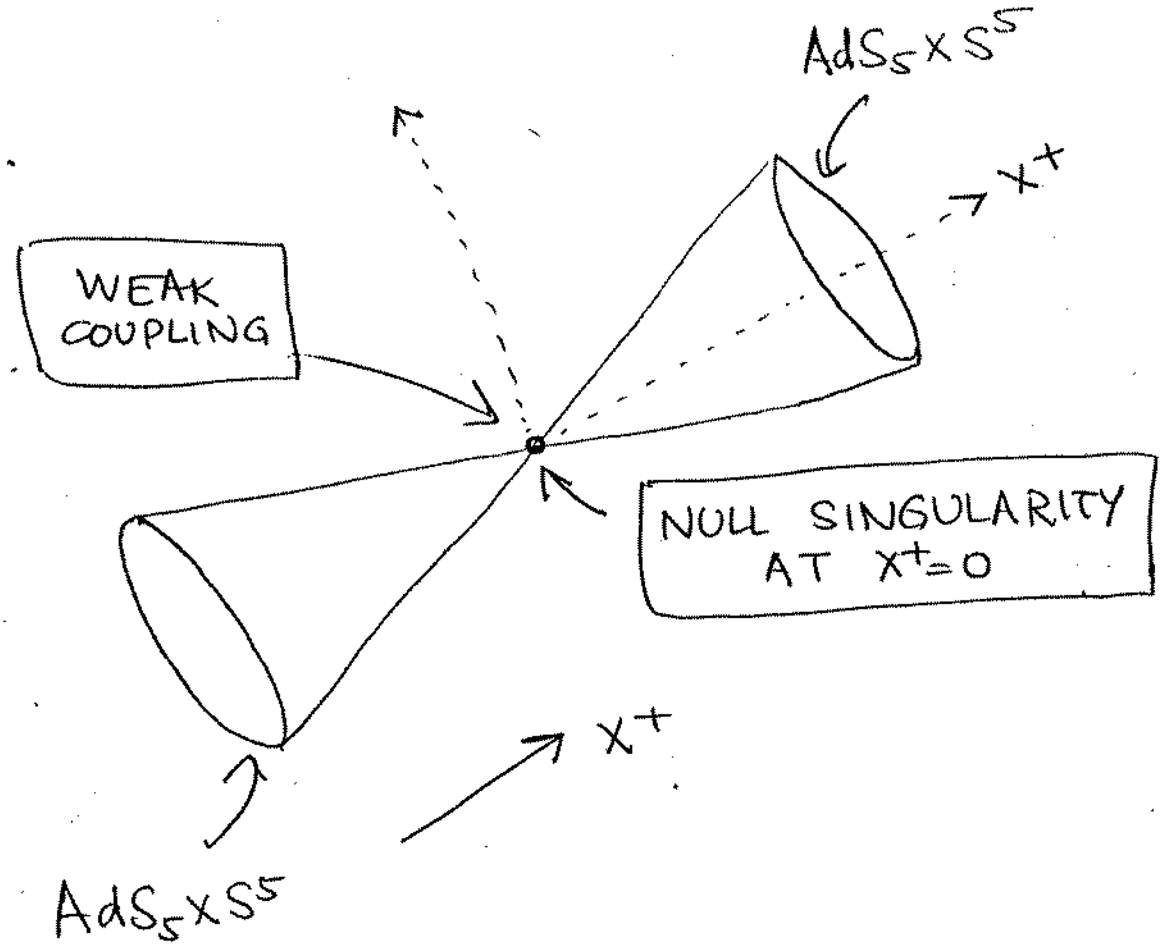
$$\tilde{g}_{\mu\nu}(x) = e^{f(x^+)}\eta_{\mu\nu}\, dx^\mu dx^\nu$$

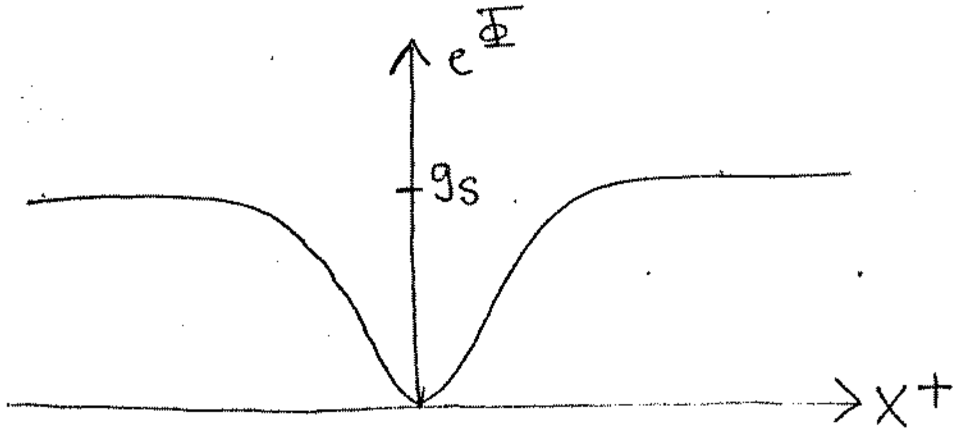
x^+ : Null direction in $\{x^\mu\}$ SPACE

$$\Phi = \Phi(x^+)$$

e.g

$$\boxed{\begin{aligned} e^{f(x^+)} &= \tanh^2 x^+ \\ e^{\Phi(x^+)} &= g_s \left|\tanh\frac{x^+}{2}\right|^{\sqrt{8}} \end{aligned}}$$


$AdS_5 \times S^5$
x^+
WEAK COUPLING
NULL SINGULARITY AT $x^+=0$
x^+
$AdS_5 \times S^5$


e^{Φ}
g_s
x^+

- PRETTY MUCH LIKE THE MATRIX EXAMPLE —

  HOLOGRAPHIC DUAL IS <u>WEAKLY COUPLED</u> AT THE PLACE WHERE CLOSED STRING DESCRIPTION BECOMES SINGULAR

- <u>NARAYAN'S TALK</u> : ARGUMENTS INDICATING WHY GAUGE THEORY IS WELL BEHAVED NEAR $X^+ = 0$

- SINCE THE DUAL GAUGE THEORY IS <u>WEAK</u> NEAR $x^+=0$ — ONE MIGHT EXPECT THAT STRINGY CORRECTIONS ARE LARGE HERE

- INDEED THIS IS EVIDENT IN THE GS WORLDSHEET ACTION IN LIGHT CONE GAUGE (eg. Metsaev & Tseytlin
Polchinki & Susskind)

- IN THE FORM OF THE STRING FRAME METRIC

$$ds^2 = e^{\Phi/2}\left[\frac{e^{-f(x^+)}}{Y^2}\left[2dx^+dx^- + d\vec{x}^2\right] + \frac{1}{Y^2}d\vec{Y}^2\right]$$

$Y^i \quad i = 1..6$

WORLDSHEET ACTION IN $\boxed{x^+=\tau}$ GAUGE

$$\boxed{S = \int d\sigma d\tau\left[(\partial_\tau\vec{x})^2 + e^{-f(\tau)}(\partial_\tau\vec{Y})^2 - \frac{1}{Y^4}e^{2f(\tau)}e^{\Phi(\tau)}(\partial_\sigma\vec{x})^2 - \frac{1}{Y^4}e^{f(\tau)}e^{\Phi(\tau)}(\partial_\sigma\vec{Y})^2\right]}$$

- GRADIENT TERMS

$$e^{2f(\tau)} e^{\Phi(\tau)} (\partial_\sigma \vec{X})^2 \quad \text{etc.}$$

BECOME SMALL AT $\tau = 0$

$\Rightarrow$ STRINGY MODES GET EASILY EXCITED.

- WE DO NOT KNOW WHETHER PERTURBATIVE STRING THEORY MAKES SENSE IN THIS BACKGROUND

- MAYBE THE PENROSE LIMIT IS MORE TRACTABLE.

Penrose Limit

- STARTING WITH OUR METRIC

$$\boxed{ds_E^2 = \left(\frac{r}{R}\right)^2 e^{f(x^+)}\left[2dx^+dx^- + d\vec{x}^2\right] + dr^2\left(\frac{R}{r}\right)^2 + d\psi^2 + \sin^2\psi\, d\Omega_4^2}$$

PERFORM A PENROSE LIMIT ALONG A NULL GEODESIC IN (r,ψ,t) PLANE.

$$\Downarrow$$

$$\boxed{ds_E^2 = 2dU\,dV + d\vec{X}^2 + d\vec{Y}^2 - \left[H(U)\,\vec{X}^2 + \vec{Y}^2\right](dU)^2}$$

THE ORIGINAL SINGULARITY NOW APPEARS AT $U = \pi/2$

$H(U)$ MAY BE OBTAINED FROM $f(x^+)$, $\Phi(x^+)$ NEAR SINGULARITY

$$H(U) \sim \frac{1}{(U-\pi/2)^2}$$

$$e^{\Phi(U)} \sim (U-\pi/2)^{\sqrt{8}/3}$$

- SIMILAR PP WAVES HAVE BEEN STUDIED BY

  Papadopolous, Russo & Tsyetlin

  GS ACTION SOLVABLE. — STRINGS CAN PASS THROUGH

- OUR BACKGROUNDS ARE SLIGHTLY DIFFERENT — BUT COULD BE SOLVABLE AS WELL

  — WORK IN PROGRESS.

Matrix Membranes

- ONE CAN — FOLLOWING THE WORK DESCRIBED IN THE FIRST PART OF THIS TALK — WRITE DOWN MATRIX THEORY IN DLCQ APPROACH IN THESE pp WAVES

- THIS MATRIX MEMBRANE IS SIMILAR TO OUR PREVIOUS EXAMPLE HOWEVER
  - G_{YM} IS TIME INDEPENDENT
  - BOTH ∂_σ and ∂_ρ HAVE TIME DEPENDENT FACTORS
  - MASSES ARE TIME DEP.

$\Rightarrow$ IN APPROPRIATE LIMIT THE GS WORLDSHEET ACTION REPRODUCED.

$\Rightarrow$ JUST AS IN OUR NULL LINEAR DILATON EXAMPLE

HIGHER MODES OF (P,q) STRINGS ARE PRODUCED

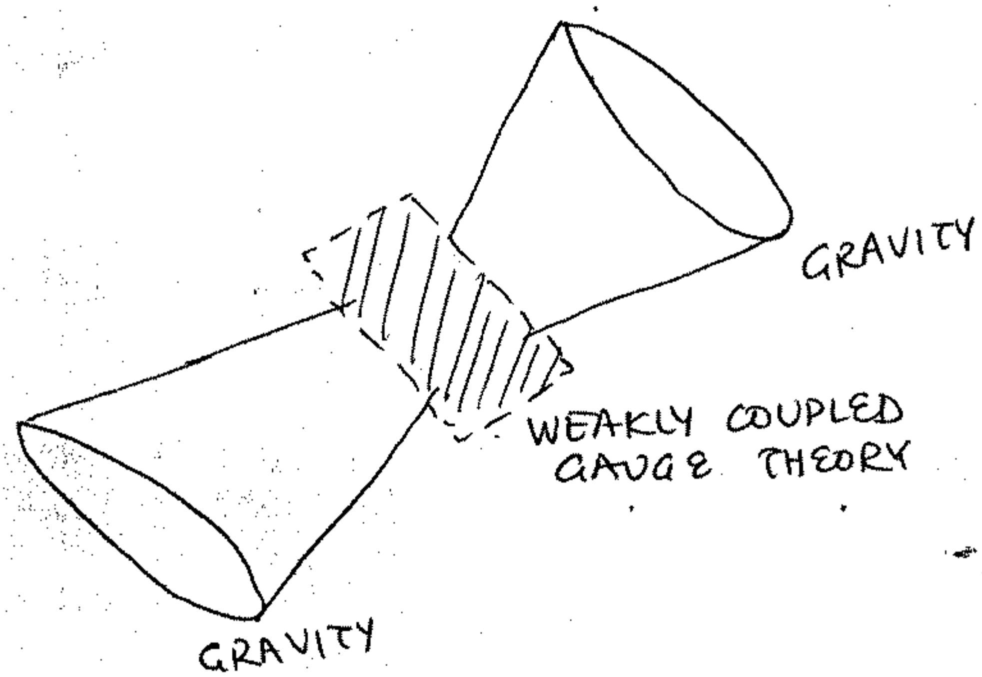
CONCLUSIONS

- WE HAVE DESCRIBED NULL SINGULARITIES USING
  (a) MATRIX THEORY
  (b) AdS/CFT
  IDEAS.

- IN THE MATRIX THEORY STORY, NEW DEGREES OF FREEDOM — THE NON-ABELIAN MODES IN THE MODEL — ARE EXCITED COPIOUSLY NEAR THE (LIGHT CONE) TIME WHERE THE GRAVITY DESCRIPTION BECOMES SINGULAR. THIS MEANS WE SHOULD NOT TALK ABOUT CONVENTIONAL SPACE-TIME HERE — RATHER, THE MATRIX THEORY DUAL IN ITS FULL FORM.

- IN THE AdS|CFT STORY THE DUAL THEORY AGAIN BECOMES WEAKLY COUPLED NEAR THE "SINGULARITY".
- THIS IS ONE OF THE REASONS WHY THE THEORY APPEARS TO MAKE SENSE (Narayan's Talk)

- THERE ARE NO NEW DEGREES OF FREEDOM — RATHER BECAUSE OF WEAK GAUGE THEORY COUPLING THE GRAVITY DESCRIPTION BREAKS DOWN



- FINALLY – IN BOTH OUR EXAMPLES, SOME SPACE DIMENSIONS ARE MANUFACTURED – TIME IS ALWAYS PRESENT

- IT WOULD BE INTERESTING TO LOOK AT TIME OR NULL DEPENDENT BACKGROUNDS IN DUAL FORMULATIONS WHERE EVEN TIME IS MANUFACTURED (eg IKKT)