TYPICALITY & UNIVERSALITY IN BLACK HOLE PHYSICS

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INTRODUCTION

- BLACK HOLE PUZZLES:
  - STATISTICAL ORIGIN OF $S = \frac{A}{4G\hbar}$
  - UNITARITY OF Q.M. IN PRESENCE OF BLACK HOLE $\approx$ IDENTIFYING THE STATE OF A BLACK HOLE
  - RECONCILING INFALLING & ASYMPTOTIC OBSERVERS

- STRINGY PICTURES OF ENTROPY

D-BRANE BOUND STATES LOCALIZED AT A POINT (STROM.-VAFA ...)

SPATIALLY EXTENDED SPACETIME FOAM (Mathur et al ; Topological String)

Where Is The Horizon?

- TODAY:
  (1) STATISTICAL ORIGIN OF ENTROPY GUARANTEES UNITARITY
  (2) SEMICLASSICAL OBSERVERS EFFECTIVELY SEE CONVENTIONAL BLACK HOLE WITH INFORMATION LOSS
  (3) RECONCILING D-BRANE BOUND STATES (AT A POINT) WITH SPACETIME FOAM (SPATIALLY EXTENDED)

INFORMATION RECOVERY FROM BLACK HOLES

VB
MAROLF
ROSALI

- ASSUME: BH ENTROPY ARISES FROM UNDERLYING DISCRETE MICROSTATES

- IN QM THE SPECTRUM OF THE HAMILTONIAN IS GENERICALLY NON-DEGENERATE, UP TO SYMMETRIES
  $\Longrightarrow$ EACH BH MICROSTATE HAS A UNIQUE MASS (UP TO SYMMETRIES)

- IN GRAVITY MASS IS MEASURED AT ∞ $\Longrightarrow$ PRECISE MASS MEASUREMENT ALLOWS *Asymptotic* OBSERVERS TO MEASURE BH MICROSTATE!

- TO MAKE THIS CAREFUL MUST PUT BLACK HOLE IN A BOX (AdS SPACE) AND CONSIDER BLACK HOLES IN EQUILIBRIUM WITH RADIATION (LARGE AdS Bholes)

How PRECISE?

(i) $N(E)\,\Delta E \sim e^{S(E)}$ STATES BET. E & $E+\Delta E$

(ii) $\Delta E \sim M_p c$

(iii) $\delta M \sim$ level spacing $\sim M_p e^{-S}$ Super-Planckian Precision

TO MEASURE WITH SUCH PRECISION:

$$\Delta t \, \Delta E \gtrsim \hbar \Longrightarrow \boxed{\Delta t \sim \frac{\hbar}{M_p} e^{S}}$$

ALSO: $S \sim \frac{A}{4G\hbar} + \cdots \Rightarrow$ AT FIXED $A, G, \hbar \to 0 \Rightarrow \Delta t \to \infty$

ALL INFO. ABOUT STATE IS PRESENT AT INFINITY, BUT SEMICLASSICAL OBSERVER CAN'T ACCES IS $\Longrightarrow$ INFO. LOSS AS $\hbar \to 0$

$$\Updownarrow$$

CAUSAL DISCONNECTION OF BH INTERIOR () IS AN ARTIFACT OF REPLACING $e^{1/\hbar}$ BY ∞

- N.B. $e^{S/\hbar} \sim$ POINCARÉ RECURRENCE TIME

IDENTIFYING SUPERPOSITIONS

- PREPARE UNIVERSE REPEATEDLY IN

$$|\psi\rangle = a_1 |E_1\rangle + a_2 |E_2\rangle$$

- LET $[B,H] \neq 0$

$$|B_1\rangle = \sum b_{1n} |E_n\rangle$$
$$|B_2\rangle = \sum b_{2n} |E_n\rangle$$

- REPEATED MEASUREMENT OF H GIVES $|a_1|, |a_2|$

- REPEATED MEASUREMENT OF B GIVES RELATIVE PHASES

$$|\langle B_1|\psi\rangle|^2 = |a_1 b_{11} + a_2 b_{12}|^2$$
$$|\langle B_2|\psi\rangle|^2 = |a_2 b_{21} + a_1 b_{22}|^2$$

TYPICALITY & UNIVERSALITY

- BTZ $M=0$ BLACK HOLE ($2+1$ d ; $\Lambda<0$)

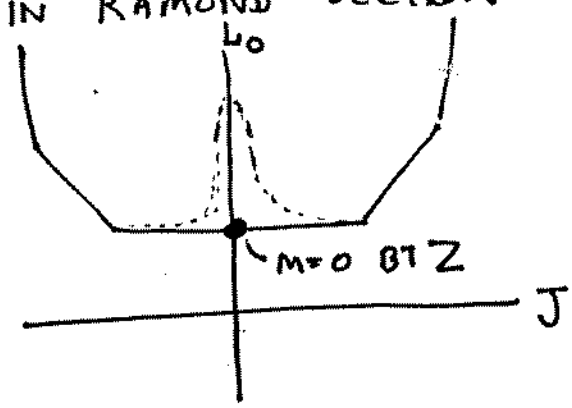
$$ds^2 = -\frac{r^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2}dr^2 + r^2 d\phi^2$$

- WHAT ARE THE UNDERLYING MICROSTATES?

EMBED IN STRING THEORY ON $AdS_3 \times S^3 \times T^4$ (AdS Scale ℓ) $\Longleftrightarrow$ (4,4) SUSY σ-MODEL ON $(T^4)^N/S_N$ (FREE LIMIT) ($\ell \sim N^{1/4}$)

$M=0$ BTZ $\Longleftrightarrow$ $L_0 = \bar{L}_0 = 0$; $J = \bar{J} = 0$ IN RAMOND SECTOR

- SMOOTH, HORIZON-FREE CLASSICAL MODULI SPACE OF ~~STATES~~ GEOMETRIES CORRESPONDING TO MICROSTATES (MUST QUANTIZE) $\Longleftrightarrow$



↳ TOPOLOGICALLY COMPLEX, HORIZON FREE NOTHING LIKE BTZ

$\boxed{\text{CLAIM: TYPICAL PROBES OF TYPICAL QUANTIZED MICROSTATES GIVE A UNIVERSAL MEASUREMENT, IDENTICAL TO BTZ} \implies \text{BTZ = CORRECT EFFECTIVE SEMICLASSICAL DESCRIPTION}}$

- RAMOND GROUND STATES ARE PRODUCTS OF TWIST OPERATORS: $\{\underset{\text{BOSONIC}}{\sigma^\mu_n}, \underset{\text{FERMIONIC}}{\tau^\mu_n}\}$ $\begin{smallmatrix} n=1\cdots N \\ \mu=1\cdots 8 \end{smallmatrix}$

- OPERATORS CREATING $M=0$ BTZ MICROSTATES

$$\boxed{\begin{aligned} \sigma &= \prod_{n,\mu} (\sigma^\mu_n)^{N_{n\mu}} (\tau^\mu_n)^{N'_{n\mu}} \\ &\sum_{n\mu} n (N_{n\mu} + N'_{n\mu}) = N \end{aligned}}$$

- TYPICAL MICROSTATE: RANDOM DISTRIBUTION OF $\sigma, \tau \Longrightarrow$ RANDOM BOSONIC, FERMIONIC PARTITION OF N

- ALMOST ALL SUCH RANDOM PARTITIONS ARE SMALL FLUCTUATIONS AROUND A "TYPICAL PARTITION":

$$\boxed{\underset{\text{B.E.}}{N_{n\mu} = \frac{1}{e^{\beta n}-1}} \; ; \; \underset{\text{F.D.}}{N'_{n\mu} = \frac{1}{e^{\beta n}+1}} \; ; \; N_n = \sum_\mu N_{n\mu} + N'_{n\mu} = \frac{8}{\sinh \beta n}}$$

HERE β, A LAGRANGE MULTIPLIER, IS FIXED BY $\sum_n n N_n = N$. $\frac{1}{\beta}$ IS <u>NOT</u> A PHYSICAL TEMP.

- TOTAL NUMBER OF SUCH PARTITIONS $\sim e^{\sqrt{N}}$

$$\Longrightarrow \boxed{S \sim \sqrt{N}}$$

- HOW DOES A PROBE REACT TO THE TYPICAL STATE? (ALMOST ALL STATES WILL BE SIMILAR)

- CONSIDER A NON-TWIST PROBE: $\mathcal{A} = \frac{1}{\sqrt{N}} \sum_{A=1}^{N} \mathcal{A}_A$ — NON-TWIST OP. IN A^{th} T^4

  e.g. $\mathcal{A}_A = \partial X_A^a \, \partial X_A^b$ — DUAL TO METRIC FLUCTUATION ON T^4

- WE WANT TO COMPUTE

  $\langle \sigma | \mathcal{A}^\dagger \mathcal{A} | \sigma \rangle$; $|\sigma\rangle = \sigma |0\rangle$; σ = TYPICAL STATE OP.

EXPLICIT COMPUTATION (IN ORBIFOLD LIMIT)

- $$G_T = \langle \sigma | \mathcal{A}^\dagger \mathcal{A} | \sigma \rangle$$

$$= \frac{1}{N} \sum_n n \, N_n \sum_{k=0}^{n-1} \frac{c}{\left[2n \sin\left(\frac{w - 2\pi k}{2n}\right)\right]^2 \left[2n \sin\left(\frac{\bar{w} - 2\pi k}{2n}\right)\right]^2}$$

HERE: $\quad w = \phi - t \qquad \bar{w} = \phi + t \qquad N_n = \frac{8}{\sinh \beta n}$

- COMPARE WITH BTZ $M = 0$

  $\mathcal{A}_{CFT}$ $\qquad$ ϕ (BULK FIELD)

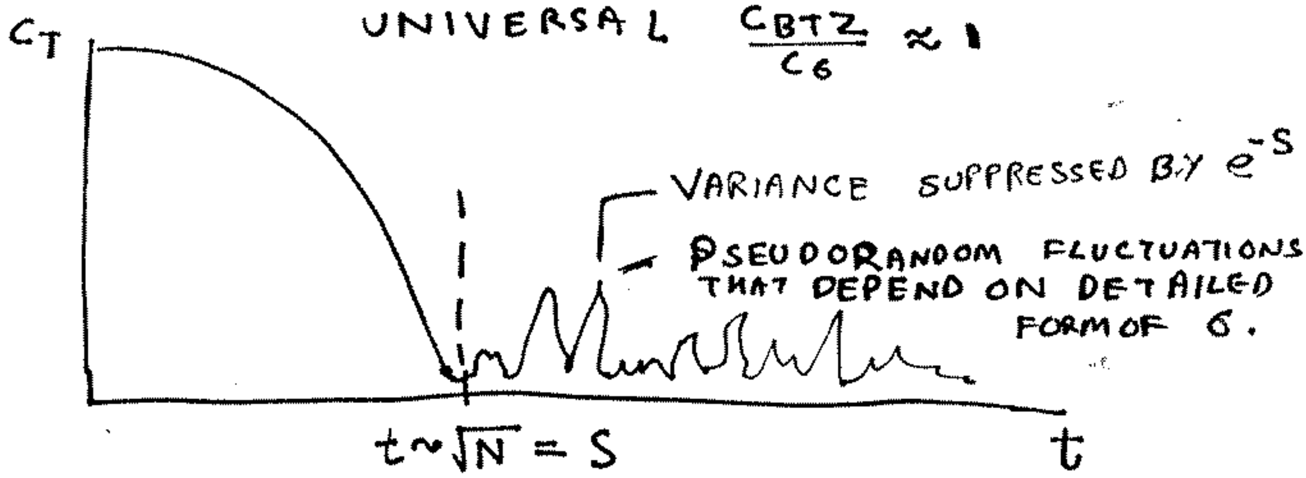
$$C_{BTZ} = \langle BTZ | \mathcal{A}^\dagger \mathcal{A} | BTZ \rangle$$

$$= \lim_{z_1, z_2 \to \infty} G_F(z_1, \bar{\omega} \, ; \, z_2 \, \omega) \Big|_{BTZ \; M=0}$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{(\omega - 2\pi k)^2 (\bar{\omega} - 2\pi k)^2}$$

$$C_T = \frac{1}{N} \sum_n N_n n \sum_{k=0}^{n-1} \frac{c}{\left[2n \sin\left(\frac{\omega - 2\pi k}{2n}\right)\right]^2 \left[2n \sin\left(\frac{\bar{\omega} - 2\pi k}{2n}\right)\right]^2}$$

$$C_{BTZ} = \sum_{k=-\infty}^{\infty} \frac{1}{(\omega - 2\pi k)^2 \, (\bar{\omega} - 2\pi k)^2}$$



- CAN SHOW ANALYTICALLY THAT

  $C_T \approx C_{BTZ}$ FOR $t \lesssim \sqrt{N}$

- SEMICLASSICAL LIMIT $\hbar \to 0$, $N \to \infty$

  $\Longrightarrow$ UNIVERSAL RESULT

LESSONS: ① INFORMATION ABOUT MICROSTATE IS AVAILABLE AT INFINITY

② BTZ $M=0$ IS THE CORRECT ~~SE~~ EFFECTIVE CLASSICAL GEOMETRY

③ SEMICLASSICAL OBSERVERS PERFORMING IMPRECISE MEASUREMENTS LOSE INFORMATION

THE SIZE OF THE MICROSTATE

- SO FAR:
  - USUAL ~~C~~ SINGULAR GEOMETRIES ARE THE CORRECT DESCRIPTION OF BLACK HOLES IN SEMICLASSICAL GRAVITY
  - BUT INFO. LOSS IN AN ARTIFACT OF SEMICLASSICAL LIMIT

- WHAT DOES AN INFALLING OBSERVER SEE? NEED TO STUDY FINITE AREA BLACK HOLES.

- M-THEORY ON T^7 OR IIA ON T^6 $\Longrightarrow \mathcal{N}=8$ SUGRA IN 4D

  (i) SPECTRUM OF BLACK HOLES CHARACTERIZED BY CHARGE VECTOR.

  (ii) FINITE AREA $\Longrightarrow$ 1/8 BPS, AT LEAST 4 CHARGES

e.g.

12	Q_1
34	Q_2
56	Q_3
123456	Q_6

$$S \sim \sqrt{J_4(\vec{Q})} = \sqrt{Q_1 Q_2 Q_3 Q_6}$$

- STATES HAVE BEEN COUNTED BY D-BRANE METHODS (VB, LARSEN, KLEBANOV, TSEYTLIN, STROM., MALD., WITTEN)

  (1) MODULI SPACE OF NON-SINGULAR CLASSICAL GEOMETRIES ("FOAM" WITH SPATIAL EXTENT)

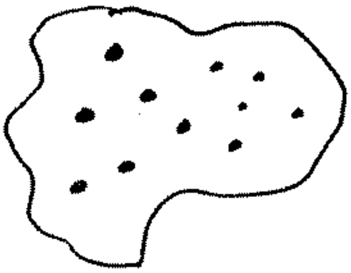
  (2) RELATION BETWEEN D-BRANE COUNTING & FOAM?

- LARGE CLASS OF SOLUTIONS WITH NECESSARY CHARGES BY COMPACTIFYING 5d SOLNS. $\binom{BGL}{BW}$ ON TAUB-NUT

$$\Rightarrow \quad ds^2_{IIA} = -I_4^{-1/2}\,(dt + R_a\, dx^a)^2 + I_4^{1/2}\left(ds^2_{R^3} + \sum_{i=1}^{3} (-Z_i M_0)^{-1} ds^2_{T_i}\right)$$

$$I_4 = \frac{L^2}{4}\left[(Z_1 Z_2 Z_3) H - k_0^2 H^2\right]$$

Z_i, H, k_0 etc. ARE FUNCTIONS OF $\mathbb{R}^3$ eg. $H = \sum_{p=1}^{N} \frac{n_p}{|\vec{x} - \vec{y}_p|}$ etc



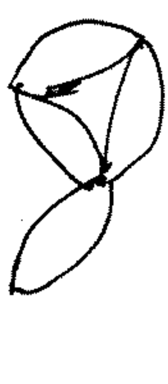
N centers, each with charge vector $\vec{Q}_p$

Total 1/8 BPS $\quad \vec{Q} = \sum_p \vec{Q}_p$

$$\vec{Q}_p = (\underbrace{Q_6}_{n_p}, \underbrace{Q_i^2}_{\frac{S_{ijk} d^j d^k}{n_p}}, \underbrace{Q_4^i}_{d_p^i}, \underbrace{Q^0}_{S_{ijk} ddd})_p \quad \Rightarrow \quad \text{1/2 BPS CHARGE (6-BRANE WITH FLUXES}$$

<u>PROPOSAL</u>:	EVERY 1/8 BPS FINITE AREA BLACK HOLE ~~HO~~ OF $\mathcal{N}=8$ SUGRA CAN BE SPLIT INTO A "GAS" OF 1/2-BPS "ATOMS"

- LIFT BACK TO M-THEORY BY OPENING UP x^{11}



- Foam of blown up 2-cycles
- Completely non-singular, Topologically complex

CONSTRAINTS

- CENTERS SATISFY TRIANGLE INEQUALITIES
- NO CLOSED TIMELIKE CURVES
- $$\Gamma_{pq} = \langle \vec{Q}_p, \vec{Q}_q \rangle$$
$$\sim \frac{1}{2}\left(Q_p^0 Q_6^q - Q_q^0 Q_6^p + \vec{Q}_4^p \cdot \vec{Q}_2^q - \vec{Q}_q^4 \cdot \vec{Q}_p^2 \right)$$

$$r_{pq} = |\vec{X}_p - \vec{X}_q|$$

$$\theta_p = \mathrm{Im}(e^{-i\alpha} Z_p) \qquad \alpha = \mathrm{Im}(Z = \Sigma_p Z_p)$$

$$\boxed{\sum_q \frac{\Gamma_{pq}}{r_{pq}} = \theta_p}$$

BASICALLY RIGID (UP TO ROTATIONS), BOUND BY MUTUAL NONLOCALITY OF CHARGES.

(1) HOW MANY WAYS OF SPLITTING $Q \to \Sigma Q_p$?

(2) HOW MANY SOLUTIONS TO CONSTRAINTS FOR EACH SPLITTING.

CONJECTURE

TYPICAL STATE IS A DENSELY PACKED FOAM OF SUCH STRUCTURES.

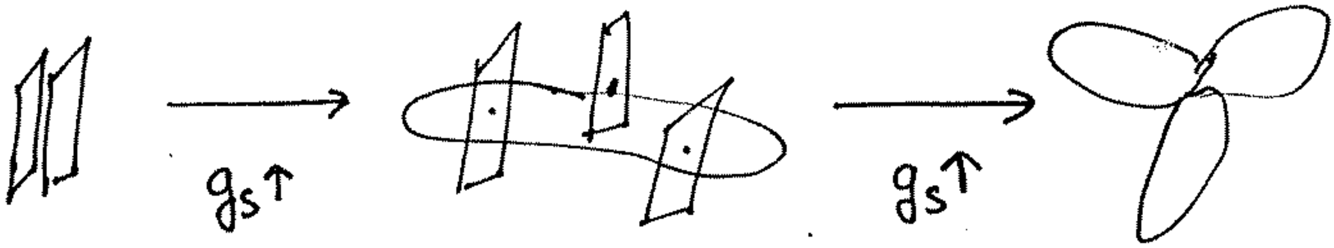
ACCOUNTS FOR BLACK HOLE ENTROPY?

SCALING RELATION

$$g_s \to \beta g_s \Rightarrow \sum_q \frac{\Gamma_{pq}}{r_{pq}} = \mathcal{V}_p \longrightarrow \beta^3 \sum \frac{\Gamma_{pq}}{r_{pq}} = \beta Z_p$$

$\Rightarrow r_{pq} \longrightarrow \beta^2 r_{pq}$ IS A SOLUTION

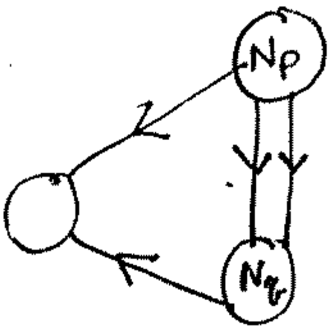
$\Rightarrow$ D6 CENTERS FLOW TOGETHER AT WEAK COUPLING.



- WHEN $r_{pq} < \ell_s \rightarrow$ COULOMB BRANCH OF D-BRANE GAUGE THEORY

$\Rightarrow$ INTERSECTING 6-BRANES OF A TORUS $\Rightarrow$ COMPACTIFY TO GET QUIVER Q.M.

[TO ONLY KEEP LIGHTEST MODES NEED BRANES TO HAVE SUFFICIENTLY ALIGNED CHARGE VECTORS ETC...]



$N_p = \gcd$ [Charge Vector]

Γ_{pq} = # OF CHIRAL BIFUNDAMENTALS

FIELDS: Vector Mult $(A, \vec{X}, F)_p$ — $\vec{X}$: 3 real scalars

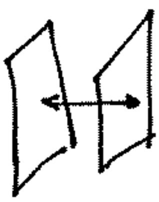
Chiral Bifund. $(\phi_{pq}, D\ldots)_p$

Adjoints (Moduli on Torus) — MOSTLY A SPECTATOR ROLE

- FOR SIMPLICITY $N_p = 1$

$\downarrow$ FOLLOWING PAPERS OF DENEF

COULOMB BRANCH



- $\langle \vec{X} \rangle \neq 0 \implies \phi$ ARE MASSIVE

$$m^2_{\phi_{pq}} = |\vec{X}_p - \vec{X}_q|^2 + (\theta_p - \theta_q)$$

- INTEGRATE ϕ OUT
- TO FIND D-TERM EQUATION WE CAN DO THIS AT ONE LOOP (NON-REN) AND IGNORE ANY SUPERPOTENTIAL

D-TERM EQN: $$\boxed{\sum_q \frac{\Gamma_{pq}}{|\vec{X}_p - \vec{X}_q|} = \theta_p}$$

$\implies$ IDENTICAL TO SUGRA!

- $g_s \to 0 \implies |\vec{X}_p - \vec{X}_q| \to 0$ AS BEFORE

$$m^2_{\phi_{pq}} \sim |\vec{x}_p - \vec{x}_q|^2 + (\theta_p - \theta_p)$$

BOUND STATE: $\theta_p - \theta_q < 0$ FOR AT LEAST ONE q FOR EACH p

$\Longrightarrow$ FOR SMALL $|\vec{x}|$, ϕ_{pq} IS MASSLESS, THEN TACHYONIC

$\Rrightarrow$ CONDENSATION ONTO HIGGS BRANCH

HIGGS BRANCH

- $\langle \phi \rangle \neq 0$ INTEG. OUT $\vec{x}$
- IF QUIVER HAS CLOSED LOOPS, THERE CAN BE A SUPERPOTENTIAL $W(\phi)$

— WE DONT HAVE A GENERAL FORMULA FOR THIS

— SO, FOR NOW, LOOK AT D-TERMS EQUATION

$$\sum_q (-1)^{\text{sign}(\Gamma_{pq})} |\phi_{pq}|^2 = \theta_p \qquad \left(\begin{array}{l}\text{SOME INTERSECTION} \\ \text{OF } \mathbb{CP}_k\text{'s}\end{array}\right)$$

DIAGONAL ANSATZ: $|\phi_{pq}| = \frac{1}{g_{pq}} \Longrightarrow \sum_q \frac{\Gamma_{pq}}{r_{pq}} = \theta_p$

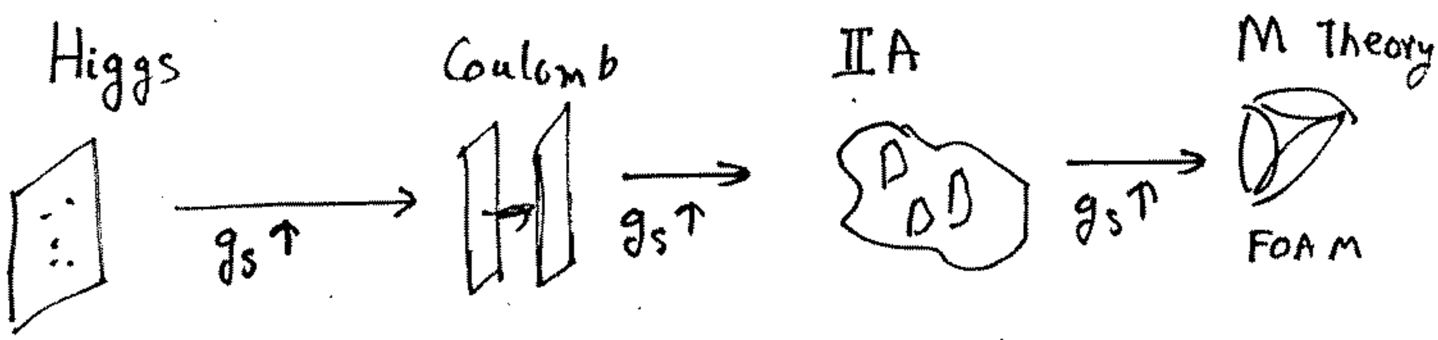
$\Longrightarrow$ MATCHES COULOMB AND SUGRA.

QUANTUM MECHANICAL MATCHING?

OF HIGGS BRANCH STATES = Σ OF BETTI NUMBERS OF VACUUM MANIFOLD $\overset{?}{=}$ # OF GROUND STATES OF PARTICLE-MONOPLE MECHANICS IN COULOMB BRANCH.

- CHECKED FOR SOME QUIVERS WITHOUT CLOSED LOOPS (Denef)

$$\Downarrow$$



$$\Downarrow$$

CONJECTURE

D-BRANE BOUND STATES GROW A TRANSVERSE SIZE AS COUPLING INCREASES.

THIS IS THE RELATION BETWEEN D-BRANE & FOAM PICTURES OF MICROSTATES & THE ORIGIN OF THE HORIZONS SEEN BY SEMICLASSICAL OBSERVERS

CONCLUSION

(1) STATISTICAL ORIGIN OF ENTROPY GUARANTEES UNITARITY

(2) SEMICLASSICAL OBSERVERS EFFECTIVELY SEE CONVENTIONAL BLACK HOLE WITH INFORMATION LOSS

~~RECONCILING D-BRANE BOUND STATES~~

(3) D-BRANE BOUND STATES CAN GROW A TRANSVERSE SIZE AT FINITE COUPLING, TURNING INTO A SPACETIME FOAM.