Using Hadron resonance gas model to extract freezeout parameters at LHC energies

Tanmay Pani**
4th Year, Integrated Masters in Science,
National Institute of Science Education and Research,
Orissa, Jatani - 752050

Abstract. Hadron resonance gas (HRG) model gives a statistical description of hadrons in the grand canonical ensemble picture. It can be used to obtain the thermodynamic variables like Pressure, Entropy and Energy of the system. We have used one variant of the model called the excluded volume HRG in which the different hadrons are considered as hard spheres that follow the quantum statistics of bosons or fermions. We have used our HRG model to calculate the bulk thermodynamics of a gas of hadrons and compared those calculated from an ideal hadron gas (hadrons are considered as point particles) model and Lattice QCD data. Then we have used the number density of various hadrons calculated using the ideal HRG model, compared it to corresponding measured yields of hadrons in ALICE for Pb-Pb collisions at 2.76 TeV, to obtain the freeze-out volume and temperature.
Keywords: Hadron resonance gas (HRG) model, Freezeout parameters, LHC experiments.

1. THE HADRON RESONANCE GAS MODEL

The basic quantity required to compute the thermodynamical quantities is the partition function $Z(T,V)$ [1]. In the grand canonical (GC) ensemble, the partition function for a particle species i in the limit of large volume takes the following form ($k = \hbar = c = 1$):

$$\ln Z_i^{id.gas} = \frac{g_i V}{2\pi^2}\int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T)) \quad (1)$$

Where, g_i is the degeneracy factor, $E_i = \sqrt{p^2 + m_i^2}$ is the total energy of a particle with mass m_i and μ_i is the chemical potential of the i^{th} species.

This integral can be solved analytically to obtain $\ln Z_i^{id.gas}$ as an infinite sum of bessel functions of second kind.

$$T \ln Z_i^{id.gas} = \frac{g_i m_i^2 V T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{(\pm 1)^{n-1}}{n^2} K_2\Big(\frac{n m_i}{T}\Big)\exp\Big(\frac{n\mu_i}{T}\Big) \quad (2)$$

** tanmay.pani@niser.ac.in

Pressure(P_i), number density(n_i), entropy density(s_i) and energy density(ϵ_i) obtained as:

$$n_i^{id.gas}(T,\mu_i) = \frac{T}{V}\left(\frac{\partial \ln Z_i^{id.gas}}{\partial \mu}\right)_{V,T} \tag{3}$$

$$P_i^{id.gas}(T,\mu_i) = \frac{T}{V}\ln Z_i^{id.gas} \tag{4}$$

$$\epsilon_i^{id.gas}(T,\mu_i) = -\frac{1}{V}\left(\frac{\partial \ln Z_i^{id.gas}}{\partial(1/T)}\right)_{\mu/T} \tag{5}$$

$$s_i^{id.gas}(T,\mu_i) = \frac{1}{V}\left(\frac{\partial \ln Z_i^{id.gas}}{\partial(T)}\right)_{V,\mu} \tag{6}$$

2. INTERACTING HADRON RESONANCE GAS MODEL

The preceding section describes hadrons that donot interact with each other. That is often not the case. Therefore, we also explore the cases where the hadrons interact. We start with adding a repulsive interaction by giving the hadrons dimensions (Excluded volume HRG).

2.1 *Excluded volume HRG*

For deriving the expressions for the thermodynamic variables in the excluded volume HRG, we need to solve the following transcedental equations[2][3]:

$$p^{ex}(T,\mu_1,\mu_2,\ldots,\mu_n) = p(T,\hat{\mu}_1,\hat{\mu}_2,\ldots,\hat{\mu}_n) \tag{7}$$

$$\hat{\mu}_i = \mu_i - v_{0i}p^{ex}(T,\mu_1,\mu_2,\ldots,\mu_n) \tag{8}$$

Where, $v_{0i} = (16\pi/3)R_i^3$, R_i being the radius of the i^{th} hadron.

The other thermodynamic variables can be calculated as,

$$n_i^{ex} = \left(\frac{\partial p^{ex}}{\partial \mu_i}\right)_T = \frac{n_i(T,\hat{\mu}_i)}{1+\sum_k v_{0k}n_k(T,\hat{\mu}_k)} \tag{9}$$

$$s_i^{ex} = \left(\frac{\partial p^{ex}}{\partial T}\right)_{\{\mu_i\}} = \frac{s_i(T,\hat{\mu}_i)}{1+\sum_k v_{0k}n_k(T,\hat{\mu}_k)} \tag{10}$$

$$\epsilon_i^{ex} = Ts - P + \sum_k \mu_i n_i = \frac{\epsilon_i(T,\hat{\mu}_i)}{1+\sum_k v_{0k}n_k(T,\hat{\mu}_k)} \tag{11}$$

One then can numerically calculate and compare the thermodynamic variables P,s and ϵ drawn from Ideal and Excluded volume HRG and lattice QCD calculations[4].

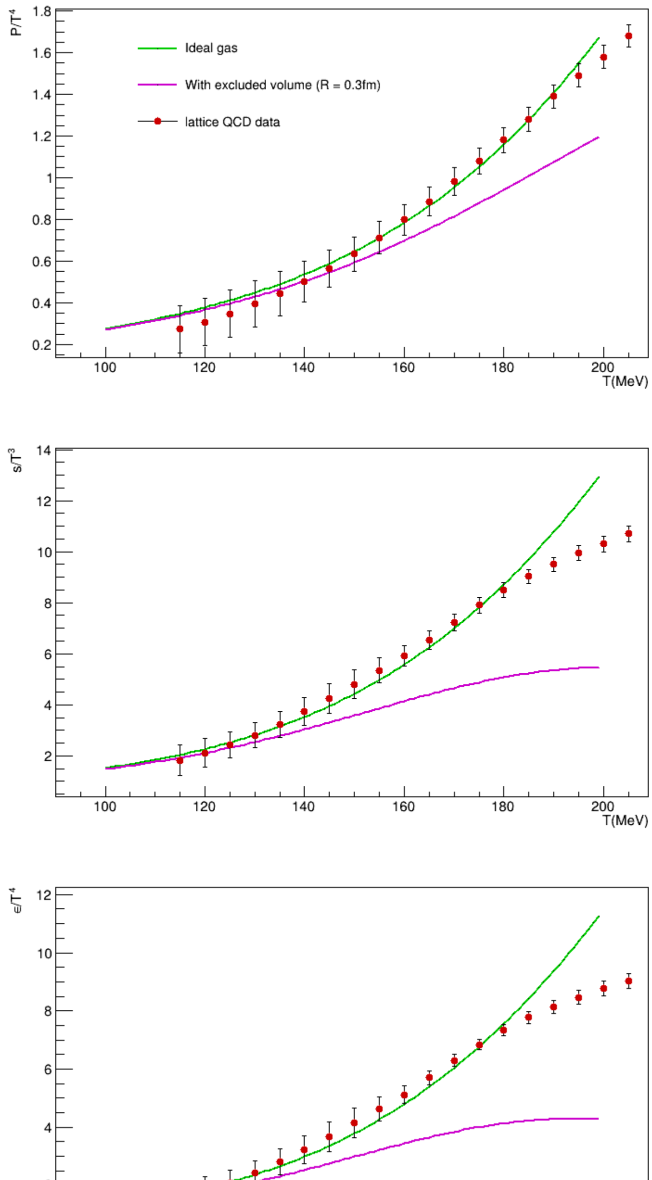

Figure 1. Pressure(top), Entropy density(middle) and Energy density(bottom) with ideal and excluded volume HRG, and comparision with lattice QCD[4].

From Fig. 1, it is apparent that the excluded volume consideration supresses the pressure, entropy density and energy density compared to the ideal case. This supression becomes larger with temperature, as the mean distance between the hadrons keeps getting smaller compared to twice the radius considered in excluded volume HRG. Hadrons upto 2.25 GeV in mass have been considered, which includes 63 mesons and 59 baryons.

3. RESONANCE DECAY AND ESTIMATION OF YIELD

Until now, we were taking all the hadrons to be stable, and not considering their decays. But that is not the case. Most of the hadrons taken are unstable resonances that decay into lower mass hadrons. Considering resonance decay is imperative to estimating the yields of different particles at freezeout[5].

Therefore, we calculate the final multiplicity of a hadron species h as,

$$\langle N_h \rangle = V n_h + V \sum_R \langle n_h \rangle_R n_R \tag{12}$$

Where, V is the volume of the fireball, Vn_h is the primary yield of hadron h, n_R is the primary yield of resonance R and $\langle n_h \rangle_R$ is the average number of particles of species h from a decay of resonance R (also called the branching ratio). In this work, we have compared the yields obtained from the Ideal- HRG model, considering resonance decay to the yields, of $\pi^{\pm}$, $K^{\pm}$, K_0, p, $\bar{p}$, Λ, $\Xi^{\pm}$, $\Omega^{\pm}$ to corresponding measured yeilds in ALICE Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$TeV[6]. The branching ratios are obtained from the PDG[7].

As for LHC energies, the chemical potential tends to zero, we have set μ_B, μ_S and μ_Q to zero. The free parameters are the freezeout temperature (T) and volume (V). We have used the χ^2 minimization method to obtain T and V, where, χ^2 is defined as,

$$\frac{\chi^2}{N_{dof}} = \frac{1}{N_{dof}} \sum_{h=1}^{N} \frac{(\langle N_h^{exp} \rangle - \langle N_h \rangle)^2}{\sigma_h^2} \tag{13}$$

Where, $\langle N_h^{exp} \rangle$ and $\langle N_h \rangle$ are the experimental and theoretical hadron yields respectively, σ_h is the error in experimental yields and N_{dof} is the number of degrees of freedom calculated as the difference between the number of particles considered and the number of free parameters.

We calculated the χ^2 for a range of temperature and volume and then found the T and V for which the value of χ^2 is the lowest (shown in Fig.2).

For our considerations, we get a minimum χ^2/N_{dof} value of 8.08, at a Temperature of (163^{+9}_{-8})MeV and a fireball volume of $V = (3960^{+1240}_{-560}) fm^3$.

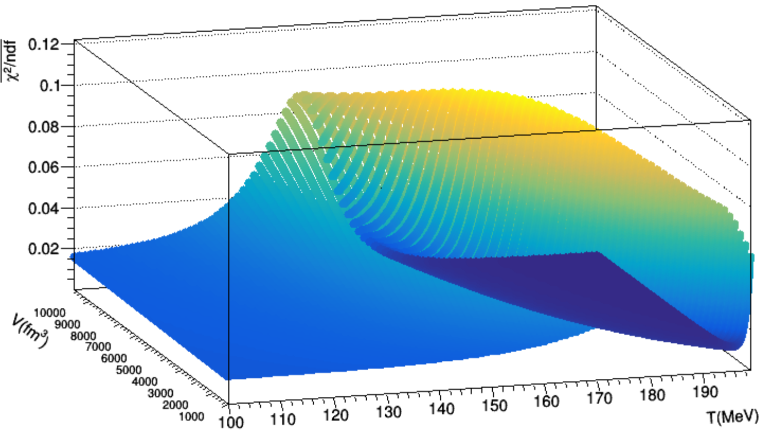

Figure 2. The plot of N_{dof}/χ^2 vs T and V

A comparision of the calculated and experimental yeilds have also been done in both graphically (Fig.3) and tabular manner (Table 1).

From table 1, we observe, that the light hadron yeilds are better determined than their heavier counterparts. This is because the limited number of resonance decays taken into account in our calculations. The heavier hadrons have more significant contributions from the heavy resonance decays than the lighter ones.

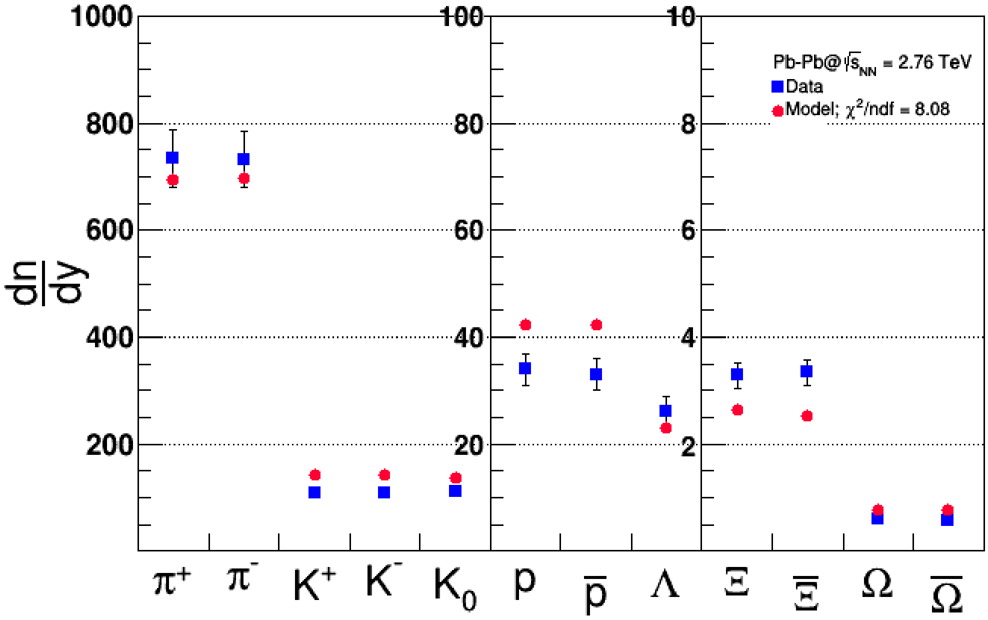

Figure 3. A comparison of experimental data[6] and model calculations for different particles.

Table 1. Experimental and Model Yields.

Particle	$\langle N_h^{exp} \rangle$	σ_h (Experimental error)	$\langle N_h \rangle$
π^+	733	54	693.46
π^-	732	52	695.88
K^+	109	9	143.06
K^-	109	9	143.097
K_0	110	10	138.055
p	34	3	42.17
$\bar{p}$	33	3	42.17
Λ	26	3	22.94
Ξ^+	3.28	0.247	2.64
Ξ^-	3.34	0.238	2.53
Ω^+	0.6	0.103	0.78
Ω^-	0.58	0.098	0.78

4. CONCLUSION

Through χ^2 minimization, we obtain the freezeout temperature and volume as (163^{+9}_{-8})MeV and $(3960^{+1240}_{-560})fm^3$ respectively for heavy-ion collisions at LHC energies.

5. ACKNOWLEDGEMENT

I would like to give my gratitude to Prof. Bedangadas Mohanty, for providing me an opportunity to work on this project. I would also like to thank Dr. Subhasis Samanta for consistently assisting me in the project.

References

[1] R.Vogt, Ultra-relativistic heavy ion collisions, Elsevier (2007)
[2] A. Andronic et al. Phy. Lett. B 718(2012) 80-85
[3] D.H.Rischke, M.I. Gorenstein et al Z. Phys. C - Particles and Fields 51, 485-489 (1991)
[4] S. Borsanyi, et al., JHEP 1011 (2010) 077, arXiv:1007.2580.
[5] P. Alba, V. Vovchenko et al Nuc. Phy. A 974(2018) 22-34
[6] B. Abelev et al. [ALICE Collaboration], Phys. Rev. C 88, 044910 (2013).
[7] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.