Mirage in geometrical optics and the horizontal ray

Dhiman Biswas[1] [*], **Simran Chourasia**[2] [†], **Rathindra Nath Das**[1] [‡], **Rajesh B. Khaparde**[3] [§], **Ajit M. Srivastava**[4] [¶]

1. B.Sc. final year, Ramakrishna Mission Residential College Narendrapur, Kolkata - 700103
2. M.Sc. 3rd year, National Institute of Science Education and Research, Bhubaneswar
3. Homi Bhabha Centre for Science Education, TIFR, Mankhurd, MUMBAI – 400088
4. Institute of Physics, Bhubaneswar 751005

Abstract. Mirage forms when light rays traversing a medium with spatially varying refractive index bend and undergo total internal reflection. Typical example is when downward going light rays bend upward when heated air in contact with hot earth surface leads to vertical gradient of refractive index with refractive index increasing in upward direction. A conceptual issue arises when considering the part of the light trajectory where light ray becomes horizontal. With refractive index varying only in the vertical direction, one will expect from symmetry considerations that a horizontal ray should not bend, contrary to the observed phenomenon of mirage. This issue has been discussed in literature, e.g. by Raman and Pancharatnam and by Berry. We discuss their arguments and argue that there are subtle conceptual issues in understanding mirage strictly in the framework of geometrical optics. In particular, we consider a horizontally moving light ray and argue that within geometrical optics, such a ray should continue to move horizontally. Bending of such a ray, as required by the mirage phenomenon, must require considerations of wave nature of light.

Keywords: Mirage, geometrical optics, wave optics, Fermat's principle, horizontal ray

1. INTRODUCTION

Nature is full of beautiful optical phenomena and mirage is undoubtedly one of the most interesting one of those. Mirage has been studied for a long time from the aspect of wave and ray optics but still there remain confusions about the bending of light while traveling in a medium with gradient in refractive index. This problem has been addressed by many scientists and a good details of the literature is available in Berry's paper [1] where he has analyzed subtleties of this issue. The popular explanation of mirage in the context of geometric ray considers medium as composed of thin layers with refractive index fixed in each layer and gradually varying with height. We consider a ray making a certain angle of incidence with the plane of stratification and then the ray continues to proceed through the layers from denser to lighter medium and hence changing the angle of incidence.

* e-mail:biswasdhiman44@gmail.com
† simran.chourasia@niser.ac.in
‡ rdasph@gmail.com
§ rajesh@hbcse.tifr.res.in
¶ ajit@iopb.res.in

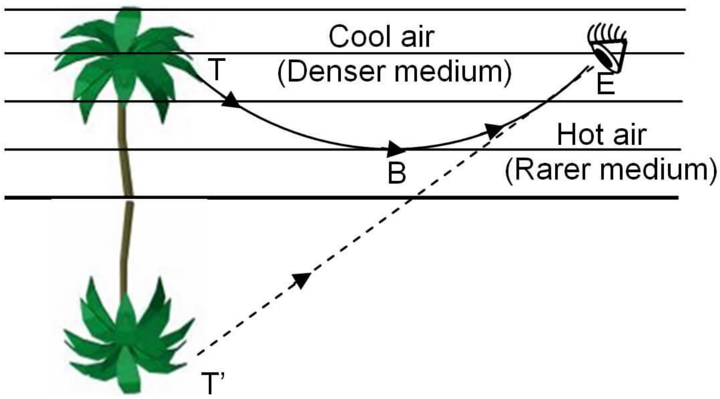

Figure 1. Trajectory of light in mirage

Finally the angle of incidence crosses the critical angle between two successive layers and at that point the light ray undergoes total internal reflection which results in the bending of light towards the denser medium, thereby forming the mirage.

However, this simple sounding explanation ignores underlying questions and subtleties. There are certain parts of this argument which lead to conceptual problems. As the refractive index changes continuously in the vertical direction, the change of incident angle in each (hypothetical) layer will be infinitesimally small. As the ray bends upward continuously, there must be a point where the light ray becomes horizontal. The importance of this *horizontal ray* was discussed by Raman and Pancharatnam [2] and later by Raman [3]. They argued that while continuously bending upward, the light ray should become horizontal (see Figure 1) after successive refractions and then it would not face any change in refractive index in the medium and it should move in a straight line. Basically, the argument relies on symmetry consideration. In geometrical optics, a single light ray is a thin light beam (of thickness approaching zero). Such a ray traversing a medium horizontally where refractive index is constant in horizontal plane should have no reason to change its direction. Note that this argument of symmetry fails if we consider light as a wave since a wave of finite wavelength will necessarily sense vertical gradient of refractive index thereby violating considerations solely relying on horizontal symmetry.

We will discuss the standard approach of explaining mirage in geometrical optics using the laws of reflection and refraction which can be derived, e.g. using Fermat's principle. A light ray traveling from a denser to a lighter medium, with critical incident angle leads to a horizontal (parallel to the interface between the two media) refracted ray. As Fermat's principle is simply a statement of extremal time taken by a light ray between two points, it must respect time reversal symmetry. It then implies that a light ray traveling horizontally (parallel to the interface) will bend towards denser medium, thereby resolving the confusion about the fate of horizontal ray as needed for the mirage phenomenon. However, it should be noted that geometrical optics (hence the use of Fermat's principle) only has the information about direction of reflected and refracted rays, and any information about the relative intensities of these rays requires the treatment of wave nature of light (e.g. using Fresnel's coefficients with the information about the polarization of the light etc.). For example, a complete treatment of reflection and refraction at a *sharp* interface between two media using Maxwell's equations shows that when the incident angle of light ray traveling from a denser to a lighter medium approaches the critical value (with the refracted light ray becoming parallel to the interface), the intensity of this refracted ray approaches zero [4]. This is the situation of total internal reflection when the entire intensity of the incident light is contained in the reflected component alone. With this knowledge, one can see that there is no sense in which one can use time reversal argument to conclude that a horizontal light ray should bend towards denser medium, as the horizontal ray which is being time reversed carries no energy. Indeed, Maxwell equations directly show that a horizontal light ray (traveling parallel to a sharp interface) leads to zero intensity for refracted ray, with the entire energy carried by the reflected ray (which continues in the horizontal direction).

We will also discuss arguments of Berry who has addressed the problem of this horizontal ray [1]. It is argued in [1] that the argument in ref.[2] about horizontal ray is not correct due to a peculiar singularity in the equation governing the bending of light ray in a medium with vertically varying refractive index. We will re-visit these discussions and argue that, while the issue of the singularity is indeed non-trivial, the conclusions reached in Berry's paper are also not very clear. We will argue that the standard approach of focusing on refracted ray cannot lead to correct understanding of mirage in the geometrical optics framework. What one needs is that at certain point during the bending trajectory of light, one needs to switch attention to the reflected light (which carries the entire intensity of incident light in that part of the medium), while any refracted light continues with zero intensity. This reflected light then leads to a refracted component which continues bending upward, finally leading the the standard mirage. (It then becomes an interesting problem to check what differences arise in the actual trajectory of light ray, especially near the horizontal segment, when one switches attention to the reflected ray compared to continuing follow up of the transmitted ray via Fermat's principle.)

To have our discussion well defined (and not subject to the non-trivial issue of singularity in the bending equation as discussed in [1]) we will focus our discussion on the light ray which is initially traveling horizontally (parallel to the plane of symmetry of refractive index, which is assumed to vary only in vertical direction). We will then argue that within geometrical optics, this light ray should not bend. This is what one expects from the symmetry of the problem. A ray which is horizontal and is moving in a medium (of thin layer) where the refractive index is constant should not *know* about the change of refractive index in the layers which are above and below of it. We will also recall discussion from [5] where bending of light in medium for the general case with refractive index gradient has been discussed. We will see that the direction of principle normal of a straight line which is used to derive the bending of light ray can not be defined for a horizontal light ray and hence that approach does not work. In fact, if we take a special case of horizontal ray and follow the same argument then we obtain the result that it should go in a straight line. We will also briefly discuss the derivation by Raman and Pancharatnam [2] using wave nature of light which gives correct explanation of mirage.

A convenient realization of mirage phenomenon generally uses sugar solution in a jar with vertically varying sugar concentration (with higher concentration, hence higher refractive index being in the downward direction). We have used such a setup and performed measurements to check various aspects of the arguments used above. An important measurement for this purpose is the value of $\frac{dn}{dz}$ for which we have followed a method used in literature by Barnard and Ahlborn [6].

The paper is organized in the following manner. In Sect.2, we discuss about the consideration of intensity for the geometric ray approach of light bending in a medium with vertically varying refractive index. We use equations for the intensity of refracted ray obtained from Maxwell equations for the case of a sharp interface separating two media of different refractive indices. We then argue why the application of Fermat's principle for discussing bending of a horizontal ray gives misleading conclusions. In Sect.3 we describe our experiment which we carried out to confirm these conclusions about intensity measurements of refracted ray. In Sect.4, we first present the argument of Raman and Pancharatnam [2] about the issue of bending of horizontal ray in geometrical optics where they conclude that within geometrical optics a horizontal ray should not bend. We then present arguments of Berry [1] who pointed out a peculiar singularity in the equation of bending of light in mirage within geometrical optics thereby concluding that the argument in [2] does not hold. We will discuss the analysis in ref.[1] and argue that the treatment of singularity is not satisfactory. In Sect.5 we start by considering all these different arguments, including the calculations in [5] about bending of light. As we are not able to reach any definite conclusions about the issue of singularity in the general bending equation (as discussed in refs.[1, 5]) for a medium with continuously varying refractive index, we will then focus on a special case of initially horizontal ray. We will analyze the propagation of such a ray in the medium and draw conclusions from all these different approaches. In Sect.6 we give the results of our experiments which we have carried out to measure $\frac{dn}{dz}$ using the method of ref.[6]. In Sect.7, we conclude by referring to the satisfactory wave optics explanation of mirage given by Raman and Pancharatnam [2, 3], and present our conclusions about the problems with the geometrical optics approach. We also point out future experiments/analysis which can be carried out for further understanding this very intriguing issue of the beautiful phenomenon of mirage.

2. CONSIDERATION OF INTENSITY IN GEOMETRICAL OPTICS

When light beam is incident on a sharp interface of two media with different refractive indices, Maxwell's equations give the following expression for reflection coefficient of the incident light (for s-polarized light) [4]:

$$R = |\frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}|^2 \tag{1}$$

Consider the case when R = 1, i.e. intensity of transmitted light is zero.

$$R = 1$$
$$\implies (n_1 \cos\theta_i - n_2 \cos\theta_t)^2 = (n_1 \cos\theta_i + n_2 \cos\theta_t)^2$$
$$\implies \cos\theta_i \cos\theta_t = 0$$
$$\implies \text{either } \cos\theta_i = 0 \text{ or } \cos\theta_t = 0$$
$$\implies \text{either } \theta_i = \pi/2 \text{ or } \theta_t = \pi/2$$

(This result also holds for the p-polarized light as in that case the only change in Eq.(1) is that n_1 and n_2 are exchanged.)

Case 1: $\theta_t = \pi/2$

Snell's law gives

$$\theta_i = sin^{-1}(\frac{n_2}{n_1}) \tag{2}$$

Since domain of sin^{-1} is [-1,1], we have $n_2 \leq n_1$ (dense to rare medium)

$$\therefore \theta_i = sin^{-1}(\frac{n_<}{n_>}) = \theta_c = \text{ critical angle (Figure 2)}$$

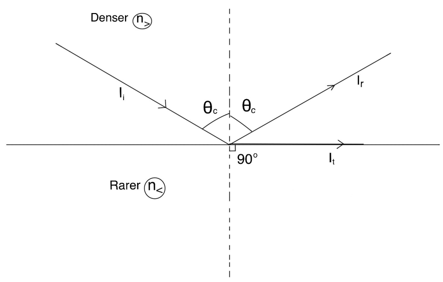

Figure 2. Ray diagram for $\theta_t = \pi/2$

Case 2: $\theta_i = \pi/2$

From Snell's law, $n_1 \sin\theta_i = n_2 \sin\theta_t$

$$\implies \theta_t = sin^{-1}(\frac{n_1}{n_2})$$

Again, with the domain of sin^{-1} is [-1,1], we have $n_1 \leq n_2$ (rare to dense medium)

$$\therefore \theta_t = sin^{-1}(\frac{n_<}{n_>}) = \theta_c = \text{ critical angle (Figure 3)}$$

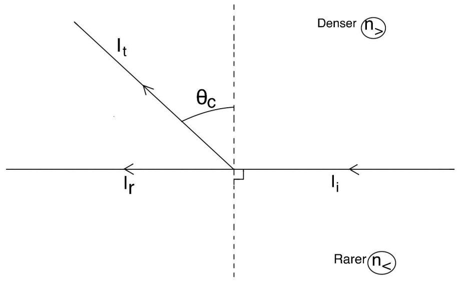

Figure 3. Ray diagram for $\theta_i = \pi/2$

Therefore, horizontal incident light ray with zero intensity (because R=1) bends toward denser medium at angle θ_c. And complete intensity gets reflected at angle 90^o (see Figure 3).

Thus we see that in discrete layer picture of mirage, the bending of light towards the denser layer after the ray becomes horizontal (i.e. tangential to the interface of two layers, with the incident angle $\theta_i = \pi/2$) can be understood in geometrical optics. However, this conclusion holds if we only consider the directions of different rays without any consideration of their respective intensities.

Let us now re-visit the argument about bending of horizontal ray in the framework of geometrical optics. The basic principle of geometrical optics is *Fermat's principle*. It says that to go from one point to another point, any light ray will follow the path in which optical path will be extremum. Using this principle we can derive the basic laws of geometrical optics like *The law of reflection* and *The law of refraction (Snell's law)*. Here our interest lies in the *Law of refraction* which says

$$n_i \sin\theta_i = \text{constant} \tag{3}$$

Here n_i is the refractive index of ith medium and θ_i is the angle of refraction in ith medium.

Now if a light ray is traveling from one medium (say medium 1) to anther medium (say medium 2) the Snell's law reduces to $n_1 \sin\theta_1 = n_2 \sin\theta_2$. But Snell's law only gives us the information of the direction of light ray. It has no information about relative intensities of different rays. This becomes of crucial importance when critical angle is involved. From Snell's law we can see that at critical angle of incidence, the transmitted ray becomes horizontal. But we saw above that Maxwell's equations show that in this case, the transmitted light has zero intensity, i.e. it will not carry any energy. In this condition the total light will be reflected at critical angle which is the case of total internal reflection. Let us continue with this use of Fermat's principle (neglecting considerations of intensity) that a light ray incident at critical angle θ_c $(= sin^{-1}(n_</n_>))$ on a sharp interface from denser to rarer medium gets refracted at angle 90^o. Time reversal symmetry of Fermat's principle implies that a ray incident at 90^o on a sharp interface from rarer to denser medium will get refracted to denser medium at an angle equal to θ_c. This is what underlies the standard explanation of mirage in geometrical optics using Fermat's principle. So at the lowest point of trajectory in mirage, the ray becomes horizontal (90^o incidence) and gets refracted to the upper denser layer at an angle θ_c corresponding to the concerned two layers.

Now we bring in the consideration of intensities of different rays (Eq.(1)). Case 2 above (for ray propagating from rarer medium to the denser medium with incidence angle $\theta_i = 90^o$) shows that although the ray bends towards in the denser medium at θ_c, it's intensity is zero and the total intensity of light continues in a straight line. Main conclusion one can draw from this discussion is that to have a correct explanation of mirage in discrete layer picture of the medium, one cannot just focus on the transmitted light with the equation $n_i sin\theta_i =$ constant for each successive layer. At some point, when the incident angle equals critical angle between two consecutive layers, the attention must be shifted to the reflected ray with the equation $\theta_i \equiv \theta_{incidence} = \theta_{reflected}$. This takes care of the main puzzle of horizontal ray, in the sense that in discrete layer picture there will simply be no horizontal ray ever having a non-zero intensity. The horizontal transmitted ray will carry zero intensity at the point when one shifts attention to the reflected ray. Subsequently, one can just follow that ray, again focusing on the transmitted ray all along, finally leading to the phenomenon of mirage. Of course, all this holds in the discrete layer picture of the medium and these arguments need to be made rigorous for a medium with continuous variation of refractive index.

In the next section we will describe our experiment which we have carried out to verify the above conclusions about intensities of refracted and reflected rays using Eq.(1).

3. EXPERIMENTAL STUDY OF TRANSMISSION OF LIGHT AT AN INCIDENT ANGLE OF 90^O

In this experiment, relative intensity of reflected light for different angles of incidence is measured for air-water interface and for air-acrylic interface using the apparatus in Figure 4. The experiments were done using solid state red laser whose angle of incidence was measured. Intensity of reflected light was measured using a CCD detector. The heights of the laser source and the detectors could be changed depending on the angle on incidence.

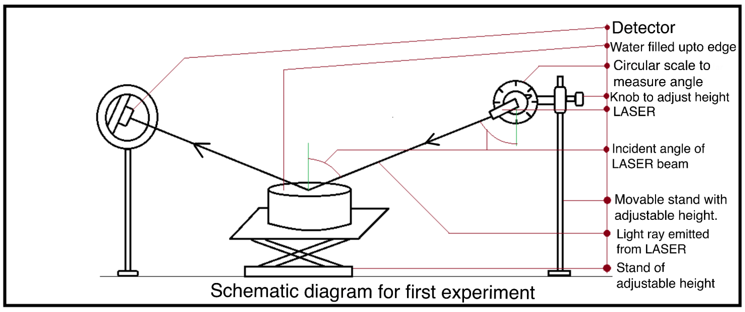

Figure 4. Apparatus for measuring reflectivity at different angles of incidence

We first give detailed measurements for the air-acrylic interface. Table 1 gives the data where reflected intensity is measured in terms of the current in the CCD detector with proper calibration. Dark current was properly accounted for. Refracted intensity was inferred by subtracting the reflected intensity. Direct measurement of refracted intensity could not be done due to various difficulties (such as strong scattering of light in the medium, we will discuss these issues in the last section). Figure 5 gives the data for reflected intensity as a function of incident angle. Our experiment was carried out without controlling the polarization of light, hence we give for comparison theoretical values (from Eq.(1)) for the two different polarizations. As we see, our results lie in between the two curves for the two polarizations. In Figure 6 we re-plot reflected intensity along with the refracted intensity (obtained by just subtracting the reflected intensity form the incident value). Though there is no new information in the figure, it is instructive to see that the intensity of refracted light drops to zero as the incident angle approaches 90^o, and the entire intensity is reflected back. This confirms that between two horizontal layers of different refractive indices, if a light ray is travelling nearly horizontal, i.e. tangential to layers then most of the light continues in the same medium and negligible proportion of intensity (approaching zero as incident angle approaches 90^o) is transmitted in the other medium. Figure 7 and Figure 8 show the observations for air-water interface. We do not give the corresponding table in this case as the required information is contained in these plots.

It is evident that, for 90^o incidence, the refracted light ray at angle θ_c predicted by Fermat's principle will carry zero intensity. As we discussed above, since time reversal symmetry of Fermat's principle does not take intensity into consideration, arguments based on Fermat's principle cannot explain bending of light after it becomes horizontal in a mirage.

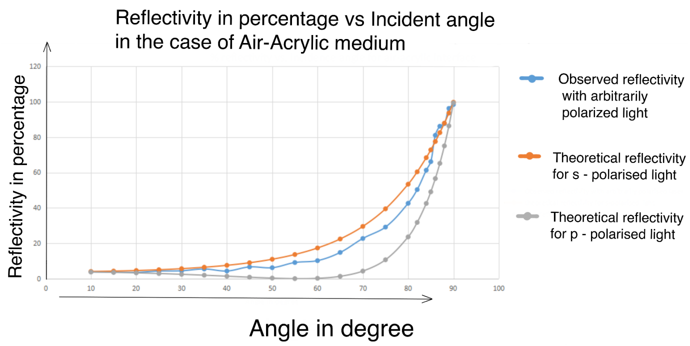

Figure 5. % Reflectance vs. incident angle for air-acrylic interface

4. THE ISSUE OF BENDING OF HORIZONTAL RAY IN GEOMETRICAL OPTICS

The crucial issue of the bending of the horizontal light in mirage was discussed by Raman and Pancharatnam [2, 3]. Here it was argued that the phenomenon of mirage cannot be described using geometrical optics. We reproduce here a summary of their arguments for this conclusion. They

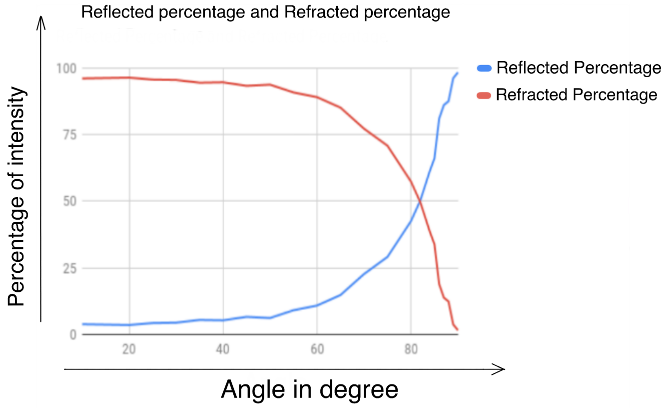

Figure 6. % Reflectance and % transmittance vs. incident angle for air-acrylic interface

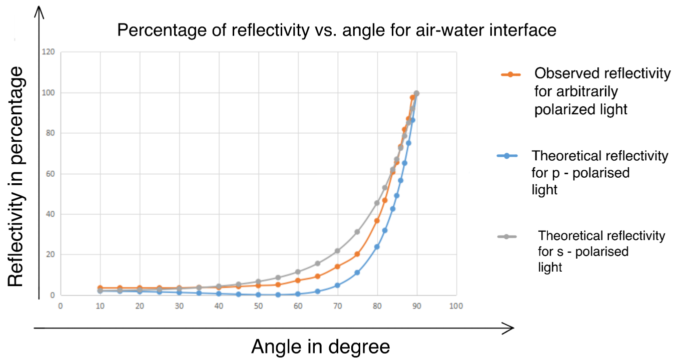

Figure 7. % Reflectance vs. incident angle for air-water interface

consider the medium as composed of several discrete thin layers, each of constant refractive index, which varies in the vertical direction. It is argued in [2, 3] that if an incident ray is making an angle (ϕ_1) with the plane of stratification then as the ray will continue through different mediums the angle (ϕ) will keep changing and after some time it will hit a limiting layer where μ_l=$\mu_1(\cos(\phi_1))$ (where μ_l is the refractive index of limiting layer and μ_1 is the refractive index of the first layer), according to Fermat's principle thence the ray should continue in a straight path. In other words, the total energy of the light will be confined in that infinitesimally thin layer and the intensity of the beam will be zero just below and above of that layer. It is shown in [2, 3] that proper explanation of the bending of horizontal ray (hence the phenomenon of mirage) can be obtained using wave optics.

We have argued above that application of Fermat's principle to determine the bending of transmitted light fails at critical angle. From the proper treatment of Maxwell's equations we saw that at critical angle the intensity of transmitted ray is zero and the total energy will be carried by the

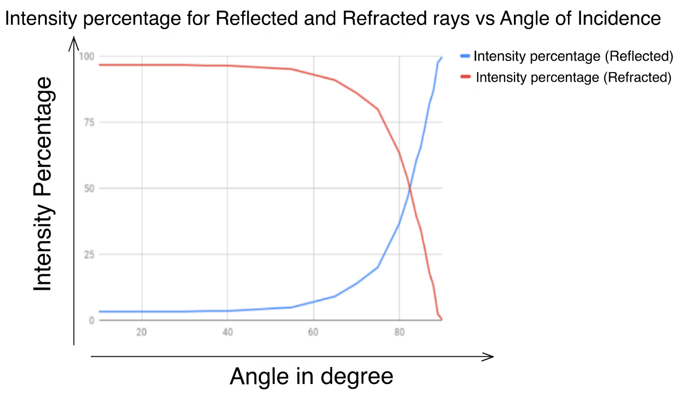

Figure 8. % Reflectance and % transmittance vs. incident angle for air-water interface

reflected ray. In other words there will be no horizontal ray for critical angle of incidence. In this context we note that the main source of the confusion about the role of horizontal ray originates from the conventional definition of the critical angle of incidence which is defined as *An angle of incidence for which the the angle of refraction is* $90°$. In view of our discussion above pointing out that the transmitted ray in this case has zero intensity (actually existing only as an evanescent wave), this definition does not look very appropriate. A more appropriate (equivalent) definition for the critical angle would be *The minimum angle of incidence for which the intensity of the reflected ray is* 100% *of the incident ray* or in other words *The minimum angle for which light undergoes total internal reflection*.

Geometrical optics analysis of [2, 3] was questioned by Berry [1], basically arguing that in the case of mirage the ray actually never becomes horizontal, it just has a horizontal tangent at one point which is a very different situation than the ray being absolutely horizontal. Starting with the standard argument that a continuous refractive-index gradient curves a ray in a way that is analogous to the way the gravity bends the trajectory of a massive particle, the analysis in [1] considers the curved rays in a smoothly varying medium by caricaturing the refractive index profile as a stack of horizontal layers of thickness Δ. The refractive index μ is taken to be constant in each layer and limiting process $\Delta \to 0$ is examined using one of several possible discretizations. Here $\mu_{disc}(z) = \mu(z_n) = \mu(n\Delta) = \mu_n$ where n is an integer and $z_{n-1} < z \leq z_n$. So here the ray has also been divided into segments wth the horizontal location at the end of *m*th segment (Figure 9, see ref.[1] for details).

$$x_{N+m} = x_c + \sum_{l=1}^{m} X_l, \quad where\ X_l = \frac{K}{\sqrt{l-\gamma}} \quad and \quad K = \sqrt{\frac{\mu_c \Delta}{2\mu_c'}} \tag{4}$$

It is assumed that the limiting layer lies between the layers N and N+1, so the the critical layer is denoted as N+γ where $0 < \gamma \leq 1$. Now, one replaces the summation by integration and shows that

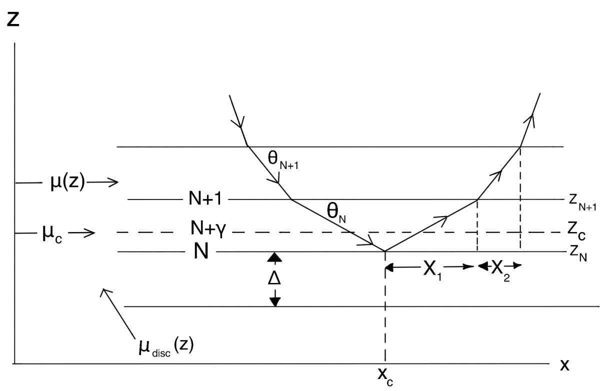

Figure 9. Trajectory of ray in discrete layered picture of mirage

the equation of the path of the ray will be

$$x(z) \approx x_c + 2k\sqrt{m} = x_c + 2k\sqrt{\frac{z - z_c}{\Delta}} = x_c + \sqrt{\frac{2\mu_c(z - z_c)}{\mu_c'}} \tag{5}$$

Subsequent discussion in [1] discusses the limiting case for which $\Delta \to 0$ and $\gamma \to 1$ which is actually the situation envisaged by Raman. In that case in order to calculate the summation in Eq.(4), the l=1 term is separated due to problem when $\gamma \to 1$, and the summation is replaced by integration for other values of l. It is then argued that *For any fixed γ, however small, the term l=1 in the above summation vanishes as $\Delta \to 0$.* With that it is then shown that the ray will follow parabolic path.

It is certainly of crucial importance to understand the behavior of the $l = 1$ term for understanding the fate of the *horizontal* ray. It appears to us that the argument used in [1] for deciding the fate of horizontal ray using the limit $\Delta \to 0$ is not conclusive. We discuss this issue in the next section where we present our overall understanding about the issue of the horizontal ray.

5. THE HORIZONTAL INCIDENT RAY

As mentioned above, ref.[1] points out the issue of an important singular term in the bending equation of light. This issue remains unclear to us for the most general case of continuous bending of light. To have a well defined discussion, we consider a special situation, that of a light ray which is initially completely horizontal, propagating through a medium in which refractive index has only vertical gradient (say, increasing in downward direction for concreteness). For the ray optics approach of this problem we will consider Berry's argument and also the approach in [5].

First, we consider the equation of the location of *m*th segment in [1].

$$x_{N+m} = x_c + K \sum_{l=1}^{l=m} \frac{1}{\sqrt{l-\gamma}} \qquad where\ K = \sqrt{\frac{\mu_c \Delta}{2\mu_c'}} \tag{6}$$

It is also mentioned in [1] that the lowest ray for which l=1, can be arbitrarily long, we see that it tends to infinity as γ approaches 1 for the case of layered medium. This means that in the limit considered in [1], the horizontal location of the end of the *m*th segment which is given by

$$X_l = K \frac{1}{\sqrt{l-\gamma}} \tag{7}$$

should be tending to infinity for l=1 and γ tending to 1 for layered medium. For continuous medium however as Δ also tends to zero the limit becomes a $\frac{0}{0}$ condition and hence undefined. This suggests that although the ray will become horizontal and should not bend in this limiting condition for layered medium, this conclusion cannot be drawn for a continuous medium (with limit $\Delta \to 0$). However, for an initially horizontal ray, in layered picture $\gamma = 1$, exactly implying that the horizontal ray will not bend for any non-zero value of Δ.

We now consider the equation of bending of light as discussed in [5] where one starts with the differential equation for light ray given by $n\frac{d\vec{r}}{ds} = \vec{\nabla} S$. (We refer to [5] for details, here we only present explanation of terms relevant for our discussion.) The differentiation with respect to arc length s is taken on both of the sides leading to the equation

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \vec{\nabla} n \tag{8}$$

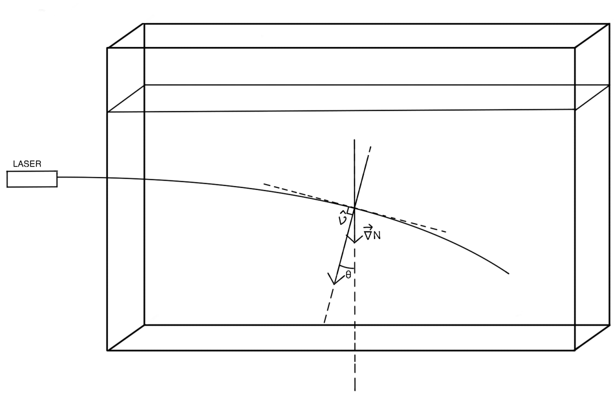

Figure 10. Light entering horizontal to the surface of layers

The curvature vector is defined to be $\vec{K} = \frac{1}{\rho}\vec{\nu}$ (see Figure 10) . Here $\vec{\nu}$ is the unit principle normal at that particular point. Then using Eqn.(8) and this definition of curvature, the general formula for the ray traveling in a medium with refractive index function n was derived. The differential equation of light rays after the inclusion of the curvature and taking the modulus value becomes

$$|K| = \frac{1}{\rho} = \vec{\nu}.\vec{\nabla}(ln\, n) \tag{9}$$

This equation shows that the refractive index increases as we move towards the principle normal which implies the bending of light ray towards the denser medium. The interesting thing here happens when we try to see this bending for a horizontal incident ray. For that case the direction of principle normal of a straight line is not defined. So we cannot use Eq.(9) to show the bending as the scalar product on the right side isn't defined. We can anyway use Eq.(8) for ray approximation. For that case $\vec{\nabla}n$ is zero. As the ray moves through just one medium in a plane of constant refractive index and is unable to *feel* the change of refractive index along the vertical direction. So using this we get

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = 0 \tag{10}$$

This equation gives a straight line solution which implies that the ray will not bend for this case. So we can see that the bending of a horizontal ray in a layered medium cannot be explained by these arguments. Here Raman's argument of wave optics is useful and can successfully show the bending of light. The symmetry in the case of ray approach for horizontal light in layered medium results in the straight line solution of those equations. Raman's wave argument has been discussed later. There are also conditions for the validity of geometric ray approach to this problem. As mentioned in [1–3] and also discussed in [5], the validity of ray optics requires that $\frac{\Delta n}{n} << 1$ for the length scale of wavelength of light in the medium. This brings us to the consideration of the gradient of the refractive index, which we address in the next section.

6. EXPERIMENTAL STUDY OF GRADIENT OF REFRACTIVE INDEX

It is standard to use a sugar solution with vertically varying concentration to realize the mirage situation in laboratory conditions. We used such a set up for investigating bending of light ray, especially focusing on the ray which is initially horizontal. A transparent cuboidal container is taken and saturated sugar solution is poured in its bottom. Now a layer of water is poured on top of this sugar solution and the interface of the two layers is stirred gently. A medium with gradient refractive index is obtained on letting this solution settle for some time. The method mentioned in [6] is adopted to determine the refractive index profile of the sugar solution prepared as we describe below.

A laser beam is passed through a glass rod (diameter around 1mm). The emerging fan of rays is then passed through the sugar solution and its image is obtained on a screen. The experimental arrangement is as shown in Figure 11.

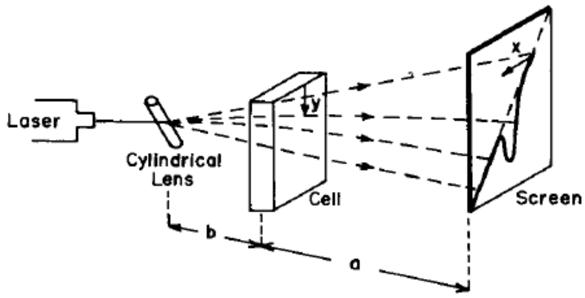

Figure 11. Schematic of the experimental arrangement

If the refractive index of the sugar solution was constant throughout its height, the image on the screen would have been a straight line. But the solution had gradient of refractive index which causes bending of light, and therefore the obtained image deviates from straight line. By measuring this deviation (Figure 12), rate of change of refractive index with height, dn/dy, can be calculated using the following formula.

$$\frac{dn}{dy} = \frac{z}{at} \tag{11}$$

where n is the refractive index, y is the height, z is the deflection on the screen with respect to straight line, t is the thickness of the cell or container in which the sample solution is kept, and a is the distance of the screen from the cell. Approximation involved in the derivation of equation 11 is that the thickness t is small [6] (so that relative deflections remain small). We have used thickness t $\simeq$ 18 mm for the experiment. Fig.13 shows the image obtained on the screen (for the blue laser) for the sugar solution used in our set up. Analyzing Figure 13 with equation 11, we obtain the profile of the refractive index gradient dn/dy as shown in Figure 14 for the solution used for the study.

As we discussed in the previous section, the crucial issue is to understand the bending of a ray which is initially horizontal. As expected, we observe that when a horizontal beam of light enters the sugar solution, it bends towards the direction of increasing refractive index (Figure 15). Trajectory of the light beam was also studied for the case when light enters the sugar solution at an angle (Figure 16). In all these cases, bending of light is more where dn/dy is large.

We also see that the sugar solution introduces magnification of the light beam as light passes through it due to the gradient of refractive index. Different dn/dy gave different magnification of the beam. Figure 17 is the image obtained on the screen after light passes through the maximum dn/dy region. The main purpose for measuring dn/dy was to probe the condition for the validity

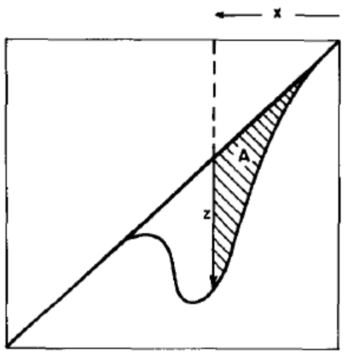

Figure 12. The initial and deflected beam trace on the screen

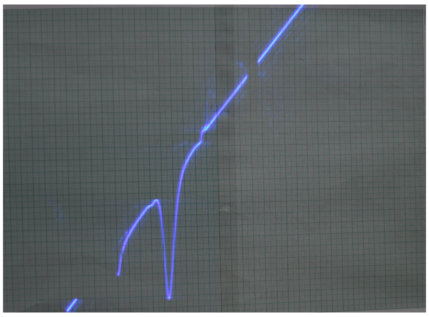

Figure 13. Image obtained on the screen which is then used for determining refractive index gradient throughout the height of solution

of geometrical optics which, as we mentioned above, requires that $\frac{\Delta n}{n} << 1$ for the length scale of wavelength of light in the medium. We expect that the intensity of the transmitted light (the ray which is bending, starting with horizontal direction) should show qualitative changes as one crosses this condition. For this purpose we measured dn/dy for different wavelengths. Unfortunately, it was not possible to measure the intensity of the transmitted light due to strong scattering of light from the medium as well as a very significant spread of intensity in the vertical direction. This is seen in the left two pictures in Figure 17 where one can see the spread of the beam profile in vertical direction. Because of this we were unable to complete the investigation of the intensity of transmitted light depending on dn/dy and the wavelength of light. We still provide the results of our investigations to make the reader familiar with the very useful technique of [6] for measuring refractive index gradients. Also, hopefully our study provides a platform for conducting further experimental investigation of the bending of horizontal light in the regime of refractive index gradient which crosses the limit for the validity of geometrical optics.

As we mentioned, the relevant quantities for our investigations are dn/dy and the wavelength

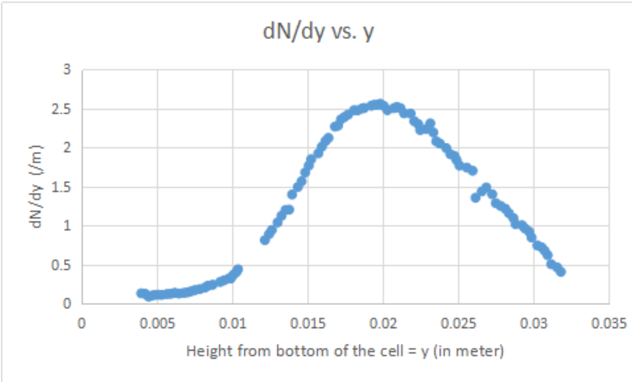

Figure 14. Refractive index gradient vs. height of liquid from bottom of container

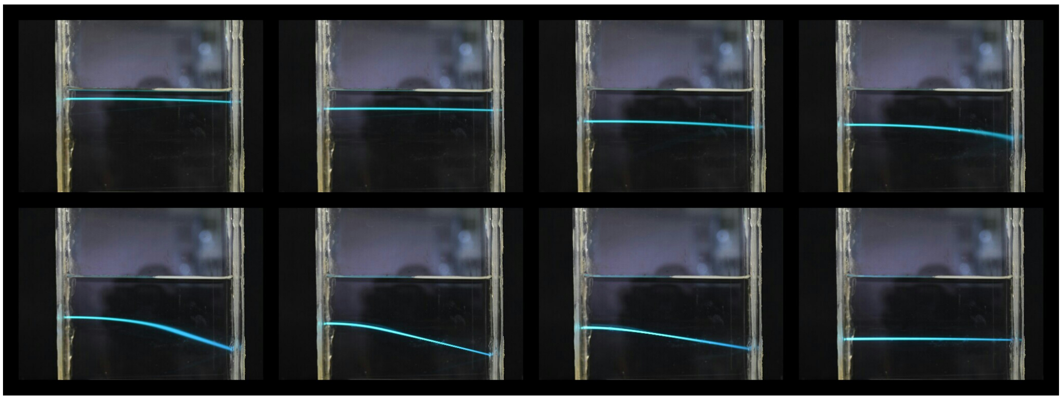

Figure 15. Trajectory of light entering horizontally in sugar solution (light beam is entering from left)

of light. To that purpose, we have repeated the experiment for measurement of dn/dy with different color lasers. Two laser beams of different colors were overlapped using a beam-splitter which was then passed through a glass rod to generate a fan of rays. We observed that the smaller wavelength light bent more. And this was observed only around the region where dn/dy was maximum. Third picture from left in Figure 17 shows that bending of blue light is more than that of red light when passing through the layer of maximum dn/dy, while the right most picture in Figure 17 shows the separation of constituent colors of white light when it is passed through the layer of maximum dn/dy. Here also blue light bends more. Since bending (i.e. deflection) is directly proportional to dn/dy, the observation implies that dn/dy for blue is more than that for red in the layer of maximum dn/dy as one will expect.

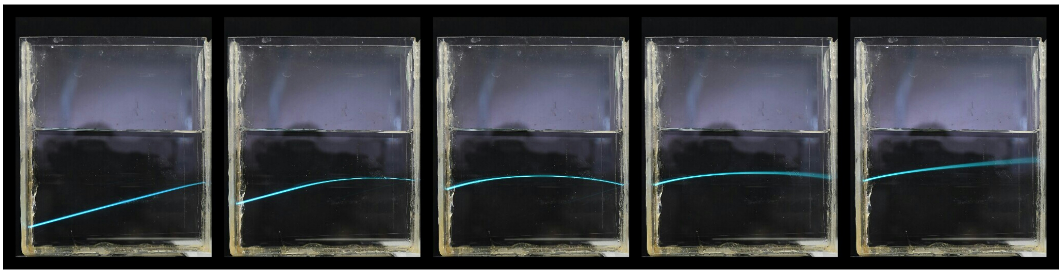

Figure 16. Trajectory of a light beam entering obliquely in sugar solution (light beam is entering from left)

7. SUMMARY AND CONCLUSIONS

We have argued above that for the case of horizontal incident ray the ray optics approach fails to show the bending of light. A satisfactory explanation of this bending was discussed by Raman and Pancharatnam [2, 3]. They considered the light to be a wave which propagates in the x direction and whose amplitude changes along z direction. The grading of refractive index is in the z direction. So the wave was described by the function $\psi = e^{ikpx}u(z)$. Here u(z) is the z dependent amplitude. Then by using general wave equation solution the intensity distribution of light as a function of distance from the limiting layer was derived. Using wave optics Raman and Pancharatnam explained all the phenomena observed experimentally. We refer to ref.[2, 3] for details of this approach.

For the conventional explanation of mirage within geometrical optics approach we have argued that the treatment of the horizontal section of the light ray requires crucial inputs from wave optics. We have argued that the standard approach of focusing on refracted ray cannot lead to correct understanding of mirage in the geometrical optics framework. What one needs is that at certain point during the bending trajectory of light, one needs to switch attention to the reflected light (which carries the entire intensity of incident light in that part of the medium, while any refracted light continues with zero intensity). This reflected light then propagates upwards undergoing refraction through successive layers. This refracted component continues bending upward, finally leading the the standard mirage. The standard derivations uses the bending of only transmitted light obtained from Eq.(3) with the application of Fermat's principle. Hence, one will expect that there must be some difference in the light trajectory if we require that one switches attention to the reflected ray instead of continuing to follow up of the transmitted ray via Fermat's principle, especially near the horizontal segment. It will be an interesting problem to find any such differences and try to carry out experimental observation of that. We hope to carry out such an investigation in future.

To have well defined discussion, especially in view of the singularity in the equation for bending of light as pointed out in [1], we have focused on a light ray which is initially horizontal. We have analyzed this situation within the geometrical optics framework as discussed in [1, 5] and have discussed the difficulties in understanding the bending of such a ray. Both the cases of mirage and horizontal incident beam can be well explained by wave theory of light. If the refractive index varies

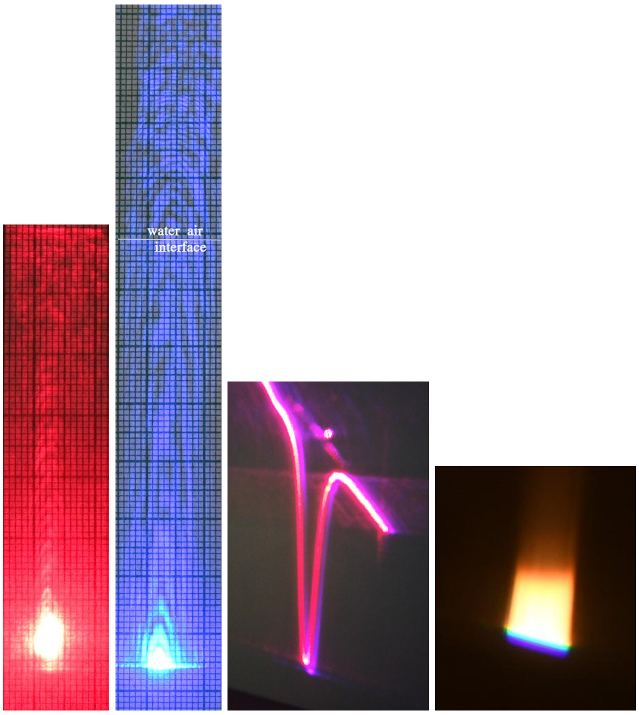

Figure 17. Left two pictures show the vertical spread of intensity after light beam passes through the sugar solution around the region of maximum dn/dy (with red and blue LASER respectively). Third picture from left shows that bending of blue light is more than that of red light when passing through the layer of maximum dn/dy, while the right most picture shows the separation of constituent colors of white light when it is passed through the layer of maximum dn/dy. Here also blue light bends more.

significantly in the range of the wavelength of light then ray treatment will be strictly invalid. Several factors may be important for this discussion, for example the behavior of intensity of the horizontal ray resulting from a ray incident at critical angle from denser to rarer medium (with sharp interface) needs to be studied in the presence of several layers, within the range of the wavelength of light. This will more closely represent the situation of a continuously varying refractive index medium. The evanescent wave nature of this horizontal transmitted ray may be important in such a situation. One needs to carry out controlled experimental study of the changes in the intensity of transmitted light along the bending curve when dn/dy is increased crossing the validity of geometrical optics regime.

ACKNOWLEDGEMENT

We are very grateful to Rajaram Nityananda for very insightful discussions on this topic and for making us aware of refs.[1–3]. We are also grateful to the National Initiative on Undergraduate Science (NIUS) at the Homi Bhabha Centre for Science Education, TIFR, Mumbai, India, for providing the required facilities, staff and funding for the study. The authors are thankful to the NIUS (Physics) laboratory staff for all their help. We are also thankful to IOP, Bhubaneswar for providing facilities during a visit of some of the authors for this project.

References

[1] M. V. Berry, ”Raman and the mirage revisited: confusions and a rediscovery”, Eur. J. Phys. 34, 1423 (2013)

[2] C.V. Raman and S. Pancharatnam, ”The optics of mirages” Proc. Ind. Acad. Sci. A49, 251 (1959)

[3] C.V. Raman, ”The optics of mirages”, Curr. Sci. 29, 309 (1959)

[4] John David Jackson, ”Classical Electrodynamics”, 3rd edition, Page 305-306

[5] Max Born and Emil Wolf, ”Principles of optics”, Cambridge University Press, 7th edition

[6] A. J. Barnard, and B. Ahlborn, ”Measurement of refractive index gradients by deflection of a laser beam”, American Journal of Physics 43, 573 (1975)

Data of reflected and refracted light intensity by air-acrylic surface						Theoretical % reflectivity for	
Angle	Dark current (A)	Current (A) (Reflected)	Net current(A)	Percentage of reflected current	Percentage of refracted current	s- polarised	p- polarised
10	0.07	2.7	2.63	3.845	96.155	4.051	3.729
15	0.07	2.6	2.53	3.698	96.302	4.263	3.530
20	0.07	2.5	2.43	3.552	96.448	4.581	3.249
25	0.07	3	2.93	4.283	95.717	5.025	2.887
30	0.07	3.1	3.03	4.429	95.571	5.629	2.448
35	0.07	3.8	3.73	5.453	94.547	6.441	1.939
40	0.07	3.7	3.63	4.307	95.693	7.528	1.380
45	0.07	4.6	4.53	6.622	93.378	8.987	0.812
50	0.07	4.3	4.23	6.184	93.816	10.958	0.309
55	0.07	6.3	6.23	9.108	90.892	13.642	0.014
60	0.07	7.5	7.43	10.162	89.838	17.327	0.189
65	0.07	10.2	10.13	14.809	85.191	22.427	1.312
70	0.07	15.6	15.53	22.704	77.296	29.530	4.259
75	0.07	20	19.93	29.137	70.863	39.465	10.657
80	0.07	29.2	29.13	42.587	57.413	53.376	23.587
82	0.07	34.5	34.43	50.336	49.664	60.382	31.780
84	0.07	42	41.93	61.301	38.699	68.383	42.510
85	0.07	45.3	45.23	66.125	33.875	72.796	49.064
86	0.07	55.5	55.43	81.038	18.962	77.508	56.571
87	0.07	59	58.93	86.154	13.846	82.536	65.172
88	0.07	60	59.93	87.616	12.384	87.899	75.035
89	0.07	65.9	65.83	96.242	3.758	93.618	86.355
90	0.07	67.4	67.33	98.435	1.565	99.712	99.362
					Max. current = 0.0684 mA		

Table 1. Observations of Exp. 1 for air-acrylic surface