PRAYAS

Students' Journal of Physics

Volume 4 Number 1 Jan.-March, 2010

INDIAN ASSOCIATION OF PHYSICS TEACHERS

PRAYAS

Students' Journal of Physics

This is a quarterly journal published by Indian Association of Physics Teachers. It publishes research articles contributed by Under Graduate and Post Graduate students of colleges, universities and similar teaching institutions, as principal authors.

Manuscripts may be submitted in appropriate format to any of the editors. Manuscripts can also be sent directly to prayas@iopb.res.in.

For more information about subscription, membership, renewals, address changes, etc., enquiries should be addressed to the Managing Editor.

Editorial

Revamping of Physics Education

Rote learning and excellence in examination has been the foundation of our Education System in general, and physics education in particular, since its inception in mid 1850s. It has almost remained the same over these intervening years of about one and half century. This system does not foster innovative spirit and creativity, and is ill-equipped to cope up with the demand of present century called 'knowledge century' where 'innovation' has become the buzzword. The present age has become hypercompetitive due to the shrinkage of the world to a global village brought about by globalization and liberalization. India's world ranking on innovation – according to survey amongst 130 countries – is 41 in the innovation index with China and Malaysia at 37 and 25 respectively, and Singapore and Korea within top 10. According to the latest report of UNO-affiliated World Intellectual Property Organization (WIPO), in the list of 15 leading countries India occupies 9th and 12th position in terms of the number of patents applied and granted respectively. The number of patents filed by USA, Japan, China, Korea and India are 4,56,154; 3,96,154; 2,45,161; 1,72,469, and 28,940 and granted are 1,57,954; 1,64,594; 67,948; 1,23,705 and 7,539 respectively. For India, majority of the patents filed and granted are by non-residents. Considering the demographic dimension of our country and its past and ancient laurels, this performance can be termed pathetic if not abysmal.

In view of the above, there is growing concern to remodel the education system in the country for which some soul-searching and commensurate actions are in the process. In the context of physics education it is well recognized that one of the maladies afflicting it, is the considerable degeneration in the standard and quality of laboratory practices called practical in Plus-two, UG, and PG teaching. Due to various reasons, lack of emphasis by the teaching institutions on practical, and growing apathy of the students for the same, has reduced this important component of physics education to the status of ritual to satisfy the statutory requirement of the curricula. The factors which have contributed to this deplorable state are: (i) The entrance test for engineering, medical and IT courses being mostly based on theory,(ii) parallel system of physics education by tutorial homes, (iii) obsolete system of practical examination, and (iv) mushrooming growth of colleges in short span of time without provision of well equipped laboratories.

Physics being an experimental subject, warrants equal emphasis on teaching of both experiment and theory for its holistic understanding and assimilation by the students. No amount of excellence in theory-teaching in class can compensate the deficiency in physics education caused by indifferent and inadequate laboratory practices. The large scale apathy for practical in Plus-two, UG and PG classes has impacted the quality of physics education in a major way, which has cast its shadow on the physics research in the country as a whole. The prospect of attracting bright students to physics by offering liberal pecuniary incentives will not bear desired fruit, until their imaginations are fired and emotions are stirred by proper physics teaching of both theory and practical. How to infuse the enthusiasm and spirit amongst the students, and make the practical classes alluring and attractive, is the major challenge in physics education today. The general panacea often suggested for the above malady is to provide adequate funds for development of laboratories and enhancement of the salary of teachers and laboratory assistants. This will be just polishing and patching the present system which has grown obsolete and infructuous in the current social milieu. To remedy this situation a somewhat radical approach is desirable. In this context the following proposal may be worth considering.

True enthusiasm and spirit for exploration and adventure in physics can be found in the consciousness of young students untainted by commercial aspirations. This valuable resource can be harnessed by our education system if higher class physics students are given the charge of the practical of lower classes. The bright and needy students of Plus-two, B.Sc. M.Sc classes and research scholars may be persuaded with attractive financial assistance to manage the practical classes of High school, Plus-two, B.Sc and M.Sc classes respectively. The benefits flowing out of this system will be immense. It is a well acclaimed experience that, the best way to learn is to teach. The higher class students will develop better understanding and grasp of physics through teaching. The enthusiasm of lower class students for practical will be enhanced many-fold through a playful spirit in company with the higher class students. The atmosphere in the practical classes will be conducive for innovation and exploration, being charged with the fearless display of young spirit. It may be recalled that 40% of university education in western countries is managed by graduate students with meager financial support. They manage the laboratory, teach the practical and offer tutorial lessons to UG students. Why can't India do the same with phenomenal saving on the expenditure on higher education?

It must be realized that the nature of theory-teaching and practical-teaching are diametrically opposite like two poles of a bar magnet; while the former is teacher-centric, the later is student-centric. In theory class, the teacher is the principal actor who is wholly responsible for imparting the education. In the practical class the role is reversed, student has to learn by himself; the teacher being an onlooker or at best a consultant. For such a passive role, the western countries do not afford the luxury of appointing a professor, but assign it to a graduate student called teaching assistant. It is unfortunate that in India, a teacher is being assigned for this job with substantial burden on the exchequer.

L. Satpathy

P R A Y A S
Students' Journal
of Physics

TURNING POINTS

Learning Newton's Laws of Motion

L. Satpathy*
Institute of Physics, Bhubaneswar 751005.

1 INTRODUCTION

On learning and teaching of physics, R.P. Feynman has said:

> I was taught how to pass examinations in Physics
> I also teach the students the same
> I was not taught Physics nor do I teach Physics

Newton's laws of motion herald the advent of modern science, often regarded as the foundation of Physics laid by Newton in seventeenth century. Although science started in many ancient civilizations like Egypt, India and Babylon etc. in pre-christian era more than 2000 years before Newton, still he is accorded a position like that of a founder, and his epoch as the beginning of scientific age. The reason for this unique landmark status in the history of science is that, the quantitative description of nature with ability for prediction of future events became possible for the first time, due to this monumental contribution of Newton in the form of his celebrated three laws of motion along with his discovery of the indispensable mathematical apparatus like Calculus. Newton presented them in his book **"Principia"** published in 1687. The text books in physics generally do not give an account of the historical background and the causes that led to the enunciation of these laws. There is neither derivation of these laws nor any postulate or hypothesis to arrive at them. Mostly these laws are taught in the class rooms like gospels, and also accepted as such. In practice, one often uses them as working rules in solving problems in physics.

For a proper understating of the Newton's laws of motion it is useful to ask:
i) How Newton was led into discovering these laws of motion, and
ii) Why they appeared on the stage of science so late in the 17th century although science started way

*e-mail: satpathy@iopb.res.in

back in the days of Aristotle (384-320 BC), Archimedes (287-212 BC) and Democritus (460-370 BC) et al., and the world had to wait for about 2000 years for the advent of modern science.

Any theory and breakthrough in science can best be understood through the history of its discovery. It has been aptly said:

> "If one knows the past, one can understand the present"

In Section 2, the historical perspective of the birth of the subject Dynamics is given. Galileo's discovery of the first and second law of motion is presented in Section 3. Section 4 describes Newton's role in the establishment of the three laws. In Section 5 the role of Differential Equation in the formulation of physical theories is discussed. Section 6 gives the solution of long standing deep puzzle on the motion of the Earth. Section 7 contains the summary and conclusion.

2 ADVENT OF DYNAMICS: A HISTORICAL PERSPECTIVE

One is amazed to think that, if modern science commencing its journey from seventeenth century could revolutionize human life in a span of three hundred years, what would have been the state of the world today had science made similar rate of progress starting from fourth century BC in Greece and elsewhere. However, practically there was no progress and science remained almost static over these intervening 2000 years because of several reasons, out of which the two most important are:

i. Science-Religion conflict, and
ii. Insufficiency of early science as a discipline.

i. Science - Religion conflict

As is well-known, Galileo (1564-1642) was imprisoned by the State for believing that Earth is not static but revolves round the Sun, in direct conflict with the proclaimed view of the Church. Prior to him, Bruno (1547-1600) was burnt to death for the same reason. The genesis of this antagonism dates back to Aristotle (364-322 BC) who had propagated the geo-centric view that the Sun revolves around the static Earth. Aristotle was a philosopher, scientist, writer and critic, and ranked as the greatest thinker in the-then world. His views on all spheres of knowledge were accepted as the ultimate truth which the succeeding generations were to read and memorize without questioning. Acquisition of knowledge consisted of mastering the works of Aristotle. Roman Catholic Church adopted his views and resolved that Aristotle was above challenge. Thus he exerted overwhelming influence on the course of evolution of science, which continued unabated for about 2000 years until the advent of Copernicus (1473-1543), Galileo and Newton. Andrew Dickson White, the first President and first Professor of History at Cornell University had come to the conclusion in his book "A History of Warfare of Science with Theology" in 1896 that modern science would have developed long before if organized religion would not have suppressed it.

ii. Insufficiency of early science as a discipline.

The methodology of science developed in Greece was insufficient and inadequate to sustain its own evolution. The ancient Greeks had no knowledge of Dynamics. They had excellent understanding on the 'Statics' of the bodies i.e. bodies in equilibrium - all forces acting on the body balanced with no net force and so no motion. However, they did not have good conception about the laws that govern the motion of the bodies. They lacked the knowledge of a good theory of Dynamics which deals with the laws that govern the change of location of a body from one moment to another. How nature actually controls this operation was unknown to them. The primary reason for this was the lack of a device to measure time very accurately i.e. a reliable clock. The velocity is the rate of change of location of the body, and acceleration is the rate of change of the rate of change of location with respect to time. Without having a reliable clock, the ancients would have found difficulty to visualize and compute such quantities. Further the relevant mathematical ideas concerning the 'rate of change' which later on became the central theme of a branch of mathematics known as Calculus was not conceivable at that epoch. In 1583, Galileo happened to observe the oscillation of a chandelier in a church and realized that a pendulum could be used as a reliable means of keeping time. Time could be precisely measured. This had far reaching consequence for the development of science in general. Galileo launched the subject Dynamics 55 years later in 1638 through his publication of the book "Discorsi", four years before his death.

3 GALILEO'S DISCOVERY OF FIRST AND SECOND LAW

Out of the three laws of motion of Newton, the first and second laws are essentially due to Galileo, clearly admitted by Newton in his Principia. By performing innumerable simple experiments and using his deep insight Galileo arrived at these two laws. His revolutionary idea, which should be described as the corner stone of Dynamics, nay the science as a whole, is:

"Force acting on a body determines its acceleration, not velocity"

This is in sharp contrast to Aristotle's theory that force acting on a body determines its velocity which was reigning supreme for about two thousand years. Galileo's new idea released science from this age-old stranglehold and set it on its path of evolution. We recount in the following one of his most important experiment on dynamics.

Consider a large ship at rest in the harbour with some scientists in the main cabin below the deck, from which nothing of the outside is visible. They have with them:
i) some flies, butterflies and some other small flying insects,
ii) a large bowl of water with some living fishes swimming in it, and
iii) a colour-water-filled bottle hanging upside down from a hook, emptying drop by drop to a vessel underneath.

When the ship is standing still, the scientists observe that, the flies, butterflies and the flying insects fly with equal ease to all sides of the cabin; the fishes swim indifferently to all directions, and the drops from the bottle fall into the vessel at the same spot always. Then the ship is set in motion with a constant velocity and the scientists make a new set of observation. They find that their observation is identical to what they had observed when the ship was standing still. The water drops fall into the vessel as before without deflecting towards any side. The flies, butterflies and the insects fly to all sides with equal ease as before. The fishes swim towards all sides of the bowl with no extra effort in any particular direction, which may be opposite to the direction of velocity of the ship. Thus uniform straight line motion (constant velocity) is completely physically indistinguishable from the state of rest. There is no local way of telling if the system is at rest of in motion. In the above experiment, the scientists in the cabin cannot know if the ship is in motion or it is moving with constant velocity. This remarkable fact has been called the ‘Principle of Galilean relativity’. Since, when a body is at rest no net force acts; the above experiment implies that for a body to be in uniform motion in a straight line, no force is necessary. This is counter intuitive, and that is why, humanity took more than two thousand years to discover it, and to come to terms with it. This is the ‘First law of motion’. Stated formally, it reads:

> “Everybody continues in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by a force impressed upon it.”

The second law is derived from the first in a natural way. As implied in the first law, a force will change the state of rest or of uniform motion (velocity). So it will produce acceleration i.e. change of velocity:

$$\begin{aligned}\text{Force} \quad &= \text{change in motion}\\ &= \text{change in (Mass} \times \text{velocity)}\\ &= \text{mass} \times \text{change in velocity}\\ &= \text{mass} \times \text{Acceleration}\end{aligned}$$

This is the second law.

4 NEWTON’S LAWS OF MOTION

Over the superb foundation laid by Galileo, Newton built an edifice of immense grandeur. He adopted the two laws of motion given by Galileo and added the third law purely dictated by his deep insight and intuition:

> *“To every action there is an equal and opposite reaction”*. In clearer terms it implies: ”The force that body A exerts on body B is precisely equal and opposite to the force that body B exerts on body A”.

The question arises: Why these three laws are attributed to Newton when the first two laws which are more profound have been discovered by Galileo? He used them extensively in solving varieties of problems, giving quantitative descriptions of nature for the first time in the history of mankind, most notably founding celestial mechanics with the explanation of the motion of planets and moon. To achieve this, he had to develop requisite mathematical techniques including Differential and Integral calculus. The phenomenal success he achieved laid the foundation of modern science. His treatise ”Principia”, embodying the three laws of motion and their application to the description of natural phenomena, was published in three volumes in July 1687.

What drove Newton to achieve this stupendous task that ushered in the Scientific Revolution in 17th century? He (1642-1727 AD) was born at a time, when the situation was ripe and the world was ready for this breakthrough in science with the necessary ingredients. Preparation for this auspicious moment had started two hundred years before with the advent of Copernicus (1473-1543 AD), who had postulated the heliocentric model of the solar system with the Sun being static at the centre and the planets revolving around it in orbits. Tycho Brahe (1546-1601 AD), with the royal patronage of the King Friedrich II of Denmark, had built the finest observatory Uraniborg of the world at that time, and had made astronomical observations considered most accurate ever made before the invention of telescope. At his death, he had left vast amount of data, which were analyzed by his assistant Kepler (1571-1630 AD), who arrived empirically at the three laws of planetary motion known by his name. They are:

1. Planets move in elliptic orbits with Sun at the focus,
2. Equal areas are swept out in equal times by the radius vector, and
3. Square of the period of revolution is proportional to the cube of the semi major axis.

It was a challenge of the time to explain the origin of these laws at a fundamental level which gave the greatest impetus to Newton. Coupled to this, the two laws of motion and the impressive body of research on Dynamics left by Galileo on his death in 1642 the same year when Newton was born, provided the perfect setting for the latter to carry on his epoch-making research. His derivation of these laws is the core-theme and the most celebrated part of Principia. He truly stood on the shoulders of the giants like Copernicus, Tycho Brahe, Kepler and Galileo.

5 DIFFERENTIAL EQUATION AND PREDICTIVE POWER

When a projectile is thrown up into space, and if its initial velocity and direction of throw are given, then with the help of Newton’s second law of motion it will be possible to predict how high the projectile will rise, and when and where, it will drop on the ground. This predictive ability of the future event of a theory is the very crux, beauty and utility of the modern science inaugurated by Newton. This power is imbued into the theory when it is expressed in terms of Differential Equation. The Differential Equations are written in terms of rate of change of physical variables whose future behaviour are of interest for prediction. The trajectory of projectile in the above example is predictable with the help of Newton’s second law of motion because it is written as a second order differential equation. All important equations of physics like those of Schrodinger, Dirac and Klein-Gordon etc. are differential equation. Newton had to invent calculus to endow physical theories with this life-giving force of predictability. The algebraic equations do not possess this unique property. Modern science, because of this new found ability, made rapid stride with faster pace than ever before.

6 SOLUTION OF A LONG STANDING DEEP PUZZLE

The Earth revolves round the Sun in an orbit, once in a year, at a speed of 18.5 miles per second i.e. one lakh kilometer per hour. This is a very high speed. Why are we not aware of it? About 18 centuries before Copernicus, Aristarchus (310-230 BC) had proposed the picture that the Sun is at rest, the Earth and the other planets which rotate around their own axis, move in orbits round the Sun. People did not accept it with the objection: 'why they do not experience it'. This objection continued even after Copernicus proposed the same picture, so much so that Bruno (1547-1600 AD) was burnt to death for supporting and zealously propagating it. This indeed remained a genuine deep puzzle until Galileo and Newton discovered the laws of motion. Still to many people it remains an enigma and therefore needs an explanation.

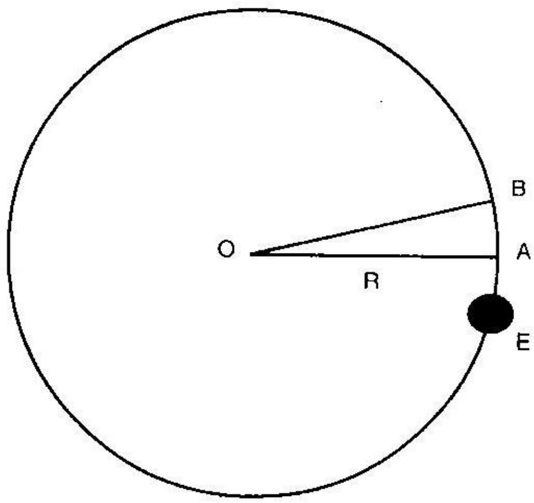


Figure 1. Orbit of the Earth around the Sun.

The Earth is orbiting around the Sun balanced by the gravitational attraction and the centrifugal force. Its motion is not uniform, it is not moving with a constant velocity. It is moving in a circle with an acceleration v^2/R, where v is the speed of the Earth and R is the distance of the Earth from the Sun. This acceleration changes the direction of v but its magnitude remains constant. In Fig. 1 this motion is shown schematically where the Sun is at the Centre O and the Earth E moves from the position A to B in the circular orbit describing the angle AOB equal to $1'$. It will take 24 minutes for the Earth to cover this distance AB which measures 26640 miles. The human mind cannot perceive and angle less than $1'$. If two stars in the sky have an angular separation of less that $1'$, then they will appear as a single star. Thus, an observer on Earth will not experience any change of direction in going from A to B covering a distance of 26640 miles. Thus, this motion will be uniform motion in a straight line with velocity v. According to the principle of Galilean relativity or equivalently Newton's first law of motion, he will not experience any dynamical effect and will feel the Earth is at rest.

The Earth also rotates around its axis which we are not able to feel. Actually here the motion cannot be considered that uniform as in the case of the revolution around the Sun. Due to to this motion, the maximum speed of 1040 miles/hour will be experienced at the equator which will go to zero as one moves to the poles.

A particle moving with the velocity $\mathbf{v}$ in a system rotating with an angular velocity $\mathbf{w}$, experiences a force proportional to $\mathbf{w} \times \mathbf{v}$ called Coriolis force. The direction of this force on a moving particle in the northern hemisphere of Earth is opposite to that in the southern hemisphere. In case of cyclone the wind flows in the counterclockwise direction around its low pressure centre in northern hemisphere while it is clockwise in southern hemisphere. Thus the dynamical effect of the rotation of the Earth is really manifested as observable.

7 CONCLUSION

The importance of the historical perspectives in the understanding and appreciation of the scientific truths has been discussed in the context of the developments of the three laws of motion of Newton. Attempt has been made to explain why this epoch making discovery awaited up to 17th century although science started centuries before Christ. The role of Astronomy in the grand human effort has been recounted. Study of astronomy provided the motive force in this evolutionary breakthrough. Truly astronomy has been called the mother of science. The proper learning/teaching of Newton's laws of motion warrants understanding of all these facets. So, what Feynman has said, is indeed true !

References

[1] Newton's Principia for the common reader by A.S. Chandrasekhar, Clarendon Press, Oxford (1995).

[2] Dialogues concerning two new sciences by G. Galileo.

[3] Emperor's new mind by R. Penrose, Oxford University Press (1999).

P R A Y A S
Students' Journal
of Physics

Simple analytical treatment of Stark resonances using multiple delta potential approximation

Debasis Nayak[1] **and Basudeb Sahu**[2]
[1] 2^{nd} yr. M.Sc. Physics, North Orissa University, Baripada-757003, India
[2] Department of Physics, North Orissa University, Baripada-757003, India

Abstract. With a combined field of an attractive delta potential and a versatile co-ordinate dependent expression for an electric field, we solve the Schrödinger equation using multiple delta potential approximation. The variation of the phase-shift time obtained from the analytical expression of the reflection amplitude with the energy of incidence gives the energy eigenvalues and the lifetime (width) of the resonance. The nature of the resonances produced due to different forms of electric field is analyzed through the estimate of energy position, lifetime and wave function to depict interesting features of resonances in the Stark effect.

Communicated by: L. Satpathy

1. INTRODUCTION

The negative energy (bound state) discrete eigen spectrum of the electron in the hydrogen atom results from the Coulomb interaction potential between the electron and the nucleus. When the atom is placed in a uniform electric field, besides destruction of the bound state spectrum stated above, a discrete spectrum of positive energy quasi-bound or resonance states are generated. The larger the field intensity, the smaller is the lifetime of a given resonance which is a decaying state. If the field strength is small enough, the energy of the resonance state will lie very close to the threshold energy E=0. This physical phenomenon is commonly known as the Stark effect. Some text books on quantum mechanics give an approximate description of the main features of the Stark effect in hydrogen [1]. However, the amount of mathematical work required by the simplest approximate calculation of the resonances is so great that the subject is often avoided in introductory classes in quantum mechanics even though it will be really helpful and fascinating for students [2].

In this paper we shall present the features of the Stark effect in model calculation using a combined potential and calculating the energies and width (lifetime) of resonances generated by the potential by a simple analytical formulation recently developed by us [3,4]. An attractive delta function potential, $-\lambda\delta(x)$ with strength $\lambda > 0$, has been found to be a good one dimensional analog for the Coulomb interaction potential of the electron interacting with the nucleus [2]. The applied electric field is represented by a potential which is expressed as a function of co-ordinate x in a most versatile and general way as

$$V_e(x) = (\frac{Fx_m}{x_m^\alpha})x^\alpha, \tag{1}$$

where F stands for the strength of the field and x_m indicates the maximum range of application of the field. Here, α is a parameter. In figure 1, we plot the potential (1) as a function of x for three different values o

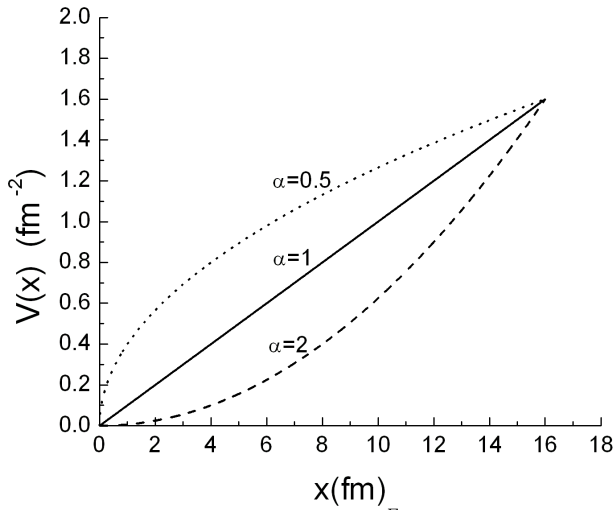


Figure 1. Variation of the potential $V_e(x) = (\frac{Fx_m}{x_m^\alpha})x^\alpha$ given by (1) as function of distance with different values of the parameter α=0.5 (dotted curve), α=1 (straight line) and α=2 (dashed curve).

For α=1, $V_e(x)$=Fx represents a uniform static electric field which is generally used in the literature for the Stark effect [2]. It will be interesting to see the nature of the resonances generated by various forms of the field with different values of α.

The paper is organized as follows. In Section 2, we present the expression for the reflection amplitude derived by solving the Schrödinger equation with the combined potential. The lifetime of resonance is calculated from the phase-shift time using the derivative of the phase-shift of the reflection amplitude with respect to energy or the wave number. In Section 3, the variation of reflection coefficient or the phase-shift time as a function of energy is studied for different sets of potential to demonstrate the nature of resonances. Section 4 contains summary and conclusions.

2. FORMULATION

With the combined potential

$$V(x) = (\frac{Fx_m}{x_m^\alpha})x^\alpha - \lambda\delta(x), \tag{2}$$

we solve the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x). \tag{3}$$

In a recent paper [4], we have solved this equation for any co-ordinate dependent potential V(x) by transforming this potential to a chain of delta function potentials.

We make a simple transformation which reduces the equation of motion for a potential $V(x)$ of arbitrary shape to an equation with a chain of delta function potentials V_d(x)=$g\delta$(x). A potential $V(x)$ as a function of the coordinate x can be considered as a chain of 'n' number of rectangular potentials, each of which has an arbitrarily small width w making 'n' infinitely large. Having simulated the potential up-to a maximum range of x=D, we have D=$\sum_i^n w_i$, where $w_i = w$ is the spatial width of the i^{th} rectangle. Let in the j^{th} region $\sum_{i=1}^{j} w_i < x \leq \sum_{i=1}^{j+1} w_i$, equivalently $x_j < x \leq x_{j+1}$, the magnitude of the potential is denoted by V_j and the width is given by $w_j = x_{j+1} - x_j = w$. The potential V_j (j=0, 1, 2, ...n) is replaced by a delta potential V_{dj}=$g_j\delta(x - (\frac{x_j+x_{j+1}}{2}))$, where the strength parameter g_j=$(\frac{2m}{\hbar^2})wV_j$ with m indicating the mass of the particle moving across the potential. The wave functions on either side of this potential centered at $x = (\frac{x_j+x_{j+1}}{2}))$ at a given incident energy E with wave number $k = \sqrt{2mE}/\hbar$ are represented by simple plane waves

$$\psi_j = a_j\, e^{ikx} + b_j\, e^{-ikx}, \tag{4}$$

$$\psi_{j+1} = a_{j+1}\, e^{ikx} + b_{j+1}\, e^{-ikx}, \tag{5}$$

with arbitrary coefficients a_j, b_j, a_{j+1} and b_{j+1}. Establishing the continuity of these wave functions around the center of delta potential using the rule $\psi_j = \psi_{j+1}$ and $\frac{d\psi_{j+1}}{dx} = \frac{d\psi_j}{dx} + g_j\psi_j$ calculated at $x = (\frac{x_j+x_{j+1}}{2}))$, we express the coefficients a_j, b_j etc. in terms of potential parameters g_j and w. The reflection amplitude is expressed as

$$r = r_{1234.....n} = \frac{b_0}{a_0} = \frac{(ik_0 - ik + g_1) + (ik_0 + ik + g_1)\, r_{234.....n}\, e^{2ikw}}{(ik_0 + ik - g_1) + (ik_0 - ik - g_1)\, r_{234.....n}\, e^{2ikw}}, \tag{6}$$

$$r_{234.....n} = \frac{g_2 + (2ik + g_2)\, r_{345.....n}\, e^{2ikw}}{(2ik - g_2) - g_2\, r_{345.....n}\, e^{2ikw}}, \tag{7}$$

$$r_{345.....n} = \frac{g_3 + (2ik + g_3)\, r_{456.....n}\, e^{2ikw}}{(2ik - g_3) - g_3\, r_{456.....n}\, e^{2ikw}}, \tag{8}$$

.

.

$$r_n = \frac{ik - ik_n + g_n}{ik + ik_n - g_n}. \tag{9}$$

Here, $k_0 = \sqrt{2m(E-V_0)}/\hbar$ and $k_n = \sqrt{2m(E-V_n)}/\hbar$ indicate the wave number at the beginning and the end points of the potential chain, respectively. The reflection coefficient R_c as a function of incident energy E across the potential $V(x)$ subject to delta potential approximation (DPA) is obtained by taking the square of the modulus of the reflection amplitude $r = r_{1234.....n}$ given by (6) as

$$R_c = \mid r \mid^2 = \mid r_{1234.....n} \mid^2 . \tag{10}$$

Expressing complex reflection amplitude r=e$^{i\beta}$, where β stands for the phase-shift, the presence of a resonance can clearly be manifested as a positive peak in the variation of $\frac{d\beta}{dk}$ with k or $\frac{d\beta}{dE}$ with E. The position of this peak will indicate the resonance energy E=$\frac{\hbar^2}{2m}k^2$ and the value of the height of this peak will stand for the width

$$\Gamma = 4\frac{\hbar^2 k}{2m}(\frac{d\beta}{dk})^{-1} = 2(\frac{d\beta}{dE})^{-1}. \tag{11}$$

The lifetime of resonance is given by

$$\tau = \frac{\hbar}{\Gamma} = (\frac{\hbar}{4})(\frac{2m}{\hbar^2})\frac{1}{k}\frac{d\beta}{dk} = (\frac{197}{4})(\frac{2m}{\hbar^2})\frac{1}{k}\frac{d\beta}{dk} \quad fm/c, \tag{12}$$

where 1fm/c=(1/3)10$^{(-23)}$sec. As r=e$^{i\beta}$, $\frac{d\beta}{dk}$=$\frac{1}{ir}\frac{dr}{dk}$. Hence, (12) is written as

$$\tau == (\frac{197}{4})(\frac{2m}{\hbar^2})\frac{1}{irk}\frac{dr}{dk} \quad fm/c, \tag{13}$$

It is straight forward to differentiate expressions (6), (7), (8), . . (9) with respect to k and compute the results of $\frac{dr}{dk}$ along with r given by (6) to give finally the results of τ as function of energy E using (13).

Further, the analysis of wavefunction given by (4) at a resonance energy clearly exhibits the nature of the resonance. The modulus of the wavefunction shows prominent peak structure within the interaction region and the number of peaks is exactly equal to the total number of resonance states present at and below the energy at which the wavefunction is analyzed.

In the next section we apply this formulation to calculate the energy, width or lifetime and the wavefunction of the resonances produced by the combined potential given by (2).

3. RESULTS

Using the variation of phase-shift time τ given by (13) as function of energy, we calculate the results of energy eigenvalue, width and the lifetime of the resonances generated by the combined potential given by (2) for different shapes of the applied potential (1). These results are tabulated in table 1. Using 2m=$\hbar^2$=1, the energy (E) and width (Γ) are expressed in fm^{-2} and the lifetime (τ) is in 10^{-23}s where the maximum distance (x$_m$) of the application of the field is in fm. For all cases, the

Table 1. Results of resonance energy (E), width (Γ) and lifetime (τ) of resonance states generated by the potential given by (2) with three different values of α. Other values of the potential parameters λ=1, F=0.1 are common for all three cases.

$\alpha = 1$			$\alpha = 0.5$			$\alpha = 2$		
E (fm^{-2})	Γ (fm^{-2})	τ (10^{-23}s)	E (fm^{-2})	Γ (fm^{-2})	τ (10^{-23}s)	E (fm^{-2})	Γ (fm^{-2})	τ (10^{-23}s)
0.568	0.0094	6949	0.945	0.0106	6191	0.278	0.0049	13200
0.937	0.0095	6899	1.270	0.0081	8117	0.602	0.0077	8517
1.241	0.0093	7022	1.500	0.0067	9272	0.924	0.0094	6951
1.506	0.0089	7390				1.242	0.01066	6167
						1.545	0.01068	6157

strength of the attractive delta potential is λ=1 and it is located closed to the starting point x=0 of the applied field where the end point indicating the maximum range is x_m=16 fm. For the Stark resonances, α=1 and the strength of the electric field is F=0.1. The results of τ are plotted as a function of energy E in figure 2(a). The positions of the peaks in this plot give four resonances with energies 0.568 fm^{-2}, 0.937 fm^{-2}, 1.241 fm^{-2} and 1.506 fm^{-2}. The maximum values of these peaks at different peak positions account for the lifetime of the resonances. The results of energies, widths and lifetimes for these resonances are presented in first three columns of table 1 under α=1. It is seen that the lifetime τ_1=6949 x10^{-23}s of the lowest resonance (E_1=0.568 fm^{-2}) is found to be smaller than that τ_4=7390 x10^{-23}s of the highest resonance (E_4=1.506 fm^{-2}). Thus, the high-lying resonance states are found to be long-lived as compared to the low-lying resonances in this field. It will be interesting to see whether this systematic of resonances occurs amongst the resonances produced by other shapes and sizes of the applied electric field.

With the same strength F=0.1 and range x_m=16 fm, the applied potential with α=0.5 is shown in figure 1 as a dotted curve showing a convex shape. In this case, the results of τ as a function of E shown in figure 2(b) give only three resonances with energies 0.945 fm^{-2}, 1.270 fm^{-2} and 1.500 fm^{-2}. These energies along with corresponding widths and lifetimes are recorded in the 4th, 5th and 6th columns of table 1 under α=0.5. As in the case with α=1 for the Stark resonances, the higher states are long-lived and the lowest state is short-lived.

Coming to the case with α=2, the results of resonance energies and the corresponding widths and lifetimes are given in the 7th, 8th and 9th columns of table 1 and the variation of τ with energy is shown in figure 2(c). As seen here, there are five resonances altogether and the low-lying states are having larger lifetimes than the higher states. This situation is just opposite to the case with α=0.5. Further, the number of resonances in the case of α=2 is more than that in the case with α=0.5. As we see in figure 1, for α=2 the shape of the potential (dashed curve) is concave whereas it is convex (dotted curve) for α=0.5. This difference in the nature of the potentials is clearly depicted in the generation of the resonances having different nature and systematics amongst the resonances.

For the case case with α=0.5, the square modulus of the wavefunction ($|\ \psi\ |^2$) at the respective resonance energies are plotted as a function of distance in figure 3. A study of these plots reveals the following systematics about the resonances produced in this case. (i) At each resonance, the ampli-

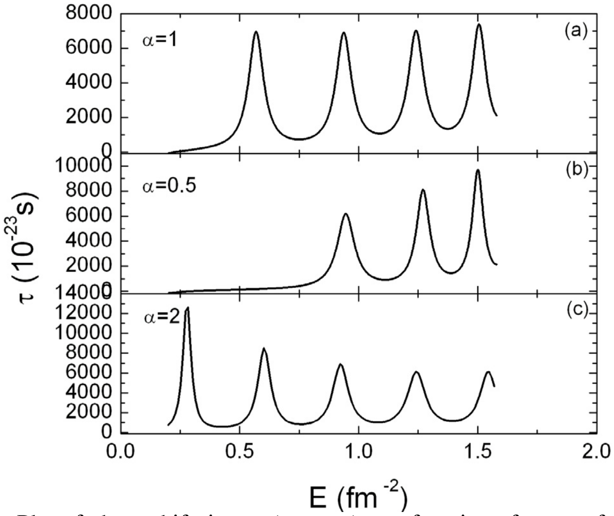


Figure 2. Plot of phase-shift time τ (see text) as a function of energy for three cases of potential given by (2) with (a) α=1, (b) α=0.5 and (c) α=2. Values of other common parameters are F=0.1 and λ=1.

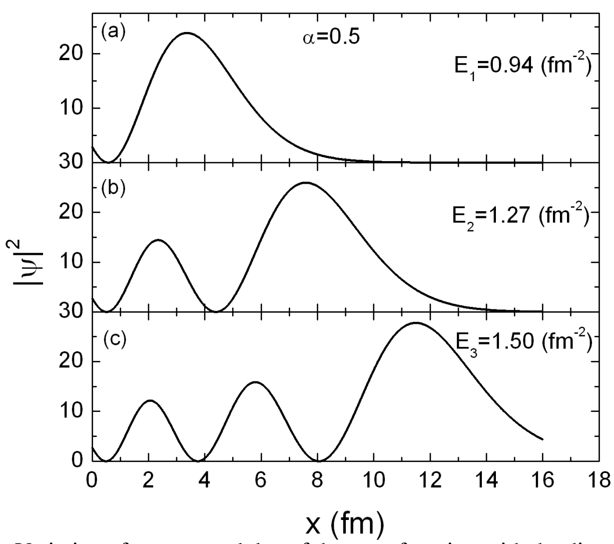


Figure 3. Variation of square modulus of the wavefunction with the distance x at three resonance energies (see table 1) corresponding to the potential (2) with α=0.5, F=0.1 and λ=1.

tude of the wavefunction is large within the interaction region which clearly depicts the confinement property of a resonance state.

(ii) The number of peaks for the first (E_1=0.945 fm^{-2}), second (E_2=1.270 fm^{-2}) and third (E_3=1.500 fm^{-2}) resonances is one, two and three, respectively. Thus, the lowest resonance (E_1=0.945 fm^{-2}) with only one peak denotes the ground state resonance. The number of peaks at the third resonance being three clearly indicates that there exists two more resonance states below this resonance energy. This is the simple manifestation of the established fact in quantal potential scattering where one can predict the number of resonances present below a given resonance by counting the nodes or the peaks in the spatial variation of the wavefunction.

4. SUMMARY AND CONCLUSIONS

In summary, we may mention that one can very easily identify the presence of resonances and also give correct estimate of the energy and lifetime of the discrete resonances generated by a combined potential representing an attractive Coulomb field and a repulsive electric field through the present formulation using variation of phase-shift time with energy. The expected nature of resonances observed in Stark effect is clearly demonstrated and understood. The different nature and features of the resonances generated by applied fields in various shapes are analyzed. Using the present formulation, further analysis can be carried out to find and understand hitherto unknown features of resonances produced by delta potential with different strengths and oscillatory electric field. It is believed that the simplicity of application of the present formulation in the detailed study of resonances generated by composite potentials which may not be analytically soluble will help the students analyze and understand decaying quantum mechanical states in any complicated applied fields.

References

[1] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, (Addison-Wesley, Reading, MA, 1958)

[2] F. M. Fernandez and E. A. Castro, *Am. J. Phys.* **53**, 757 (1985)

[3] Swetarekha Ram and Basudeb Sahu, *Prayas* **3**, 149 (2008)

[4] Basudeb Sahu and Bidhubhusan Sahu, *Phys. Lett. A* **373**, 4033 (2009)

P R A Y A S
Students' Journal
of Physics

Rolling down solution in a simple mechanical model

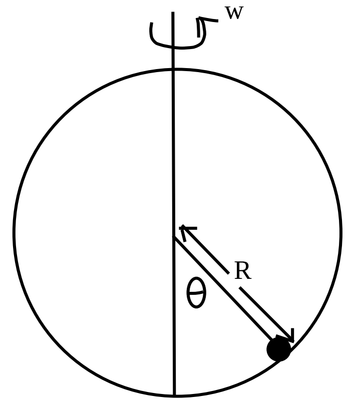
Swagat Saurav Mishra
IInd Year, School of Physical Sciences, NISER, Bhubaneswar 751 005

Abstract. We explicitly construct a *time-dependent* rolling down solution from symmetry preserving phase to the symmetry broken one within a simple mechanical system. It consists of a particle which is free to move on a loop. The loop, attached at the top to a support, rotates about the vertical axis passing through its center. As one tunes the frequency, this model provides a toy example of spontaneous symmetry breaking and continuous phase transition.

Communicated by: L. Satpathy

1. INTRODUCTION

Often simple mechanical systems provide instructive insights into complicated physical processes that occur in our nature. Consider, for example, the Alben model [1]. Very simple and exactly solvable, yet this model captures various features of phase transition. Another system, that is of our interest in this work, is discussed in [2]. It considers motion of a particle of mass m constrained to move on a frictionless circular loop of radius R. The loop is rotated about a vertical axis passing through its center as in the figure below. The analysis of [2] shows that when the loop rotates with a small angular frequency ω, the particle sits at the bottom of the loop. This is the symmetry preserving phase of the model. However, as one increases the frequency beyond a critical value, which we call ω_c, the ball settles at a non-zero value of θ. As soon as this happens, the particle breaks $\theta \to -\theta$ symmetry, and consequently, a symmetry broken phase is reached. If we increase ω further, θ increases continuously to a limiting value of $\pi/2$. This model provides a close analogy with the second order phase transition where θ plays the role of the order parameter and ω the temperature. Indeed, it is possible to construct a Landau potential in powers of the order parameter [3].



The generalization of the above model to include first order transition was discussed in [3]. Behaviour of this model in presence of friction was partially analysed in [4]. The purpose of this work is to understand the transition from the symmetric phase to the symmetry broken phase in more detail. In particular, we explicitly construct the *time-dependent* solution for θ which shows how the ball goes from the unstable position $\theta = 0$ to the stable $\theta \neq 0$ one when $\omega > \omega_c$.

In passing, we note that our analysis might provide an analogy with the inflationary phase of our universe. Inflation is triggered by rolling down of a scalar field called inflaton. As inflaton rolls down from unstable phase to its true minimum, it releases energy. This energy, in turn, causes exponential growth of the universe. In the model of [2], θ mimics the inflaton field and the time dependent solution that we construct is the analogue of rolling down solution of the inflaton [5].

In paper is structured as follows. In the next section, we review the model within the Lagrangian framework [3]. Then we solve the classical equation of motion with required boundary condition. This leads to a rolling down solution from unstable to the stable phase. Finally, in the last section of this paper, we summerise our results.

2. THE LAGRANGIAN AND THE EQUATION OF MOTION

As discussed in [3], the model has an effective Lagrangian description. Let us assume that at any instant of time the mass is at a position $\theta(t)$ The Lagrangian then reads [3]

$$L = \text{kinetic energy} - \text{potential energy}. \tag{1}$$

While the kinetic energy is given by

$$KE = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta, \tag{2}$$

the potential energy is

$$PE = -mgR\cos\theta. \tag{3}$$

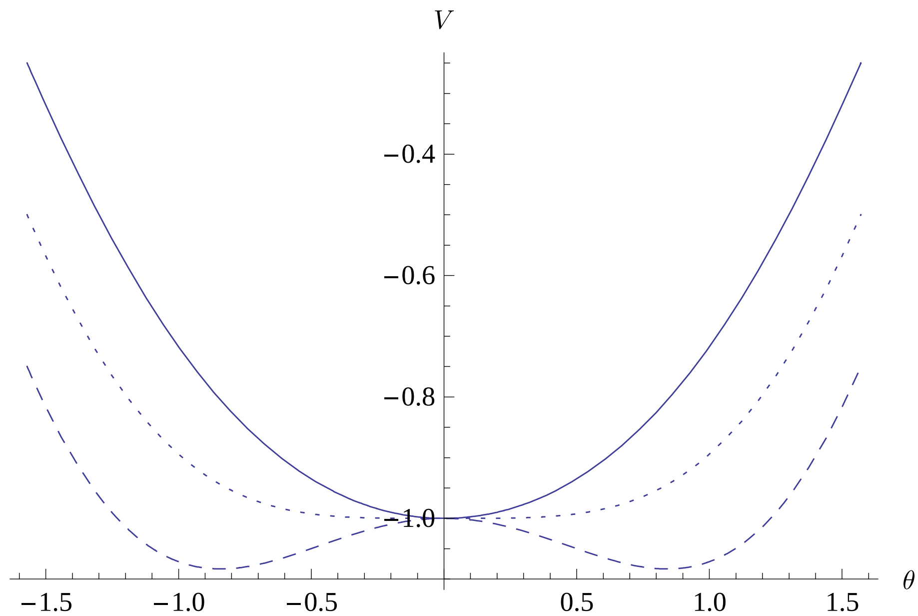


Figure 1. Effective potential for various values of ω. We have taken $g/R = 1$ for this plot. The solid, dotted and dashed curves are for $\omega = .5, 1$ and 1.5 respectively.

Therefore the total Lagrangian is

$$L = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta + mgR\cos\theta. \tag{4}$$

This can be re-written as

$$L = \frac{1}{2}mR^2\dot{\theta}^2 - (-(\frac{1}{2}mR^2\omega^2\sin^2\theta + mgR\cos\theta)) \tag{5}$$

This allows us to have a description of the system in terms of an *effective* potential

$$V = -(\frac{1}{2}mR^2\omega^2\sin^2\theta + mgR\cos\theta) \tag{6}$$

Note that V is symmetric under $\theta \to -\theta$. The nature of the effective potential is shown in figure (1). The stable positions correspond to the extrema of this effective potential

$$\frac{dV}{d\theta} = 0. \tag{7}$$

This gives

$$\sin\theta(g/R - \omega^2\cos\theta) = 0. \tag{8}$$

The solutions are

$$\theta = 0, \ \ \text{or}, \theta = \cos^{-1}\frac{g}{R\omega^2}. \tag{9}$$

Since $\cos\theta \leq 1$, the second condition holds only when $\omega > \omega_c$ where we have defined

$$\omega_c = \sqrt{\frac{g}{R}}. \tag{10}$$

Consequently, for $\omega < \omega_c$, the particle remains at $\theta = 0$. However, as soon as we increase the frequency beyond ω_c, the second solution in (9) becomes the minimum and therefore, the mass settles at $\theta = \cos^{-1}\frac{g}{R\omega^2}$. This, in turn, breaks $\theta \rightarrow -\theta$ symmetry spontaneously and we reach a symmetry broken phase. Our aim is now to explicitly find the time dependent solution for $\theta(t)$ which represents a rolling down solution from $\theta = 0$ to some non-zero stable value for $\omega > \omega_c$. To proceed, we first write down the Euler-Lagrange equation for the mass m. This follows from

$$\frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}}) - \frac{\partial L}{\partial\theta} = 0, \tag{11}$$

giving

$$mR^2\ddot{\theta} - \omega^2\sin\theta\cos\theta + {\omega_c}^2\sin\theta = 0. \tag{12}$$

This equation can be rewritten as

$$\frac{d}{dt}(\frac{1}{2}\dot{\theta}^2 + \frac{1}{4}\omega^2\cos 2\theta - {\omega_c}^2\cos\theta) = 0. \tag{13}$$

This is noting but the energy conservation equation. Upon integrating, we get

$$\frac{1}{2}\dot{\theta}^2 + \frac{1}{4}\omega^2\cos 2\theta - {\omega_c}^2\cos\theta) = c, \tag{14}$$

where c is a constant. Now c can be fixed by appropriate boundary condition. In particular, we use $\dot{\theta} = 0$ at $\theta = 0$. This gives

$$c = \frac{1}{4}\omega^2 - {\omega_c}^2. \tag{15}$$

Substituting c in (14), we get

$$\frac{1}{2}\dot{\theta}^2 + \frac{1}{4}\omega^2\cos 2\theta - {\omega_c}^2\cos\theta) = \frac{1}{4}\omega^2 - {\omega_c}^2. \tag{16}$$

Further simplifying, we reach at

$$\dot{\theta}^2 = \omega^2(1-\cos\theta)(1+\cos\theta - \frac{2}{\omega^2}{\omega_c}^2). \tag{17}$$

Defining $\frac{2}{\omega^2}{\omega_c}^2 = a$, the above equation becomes

$$\dot{\theta}^2 = \omega^2(1-\cos\theta)(1+\cos\theta - a). \tag{18}$$

Equation (18) can now be easily solved or can be read off from a similar equation appeared in [6] in a different context. The solution is

$$\theta = \pm 2\tan^{-1}\Big(\sqrt{\frac{2-a}{a}}\mathrm{sech}\{\sqrt{\omega(\omega - \frac{a\omega}{2})}(t-t_0)\}\Big). \tag{19}$$

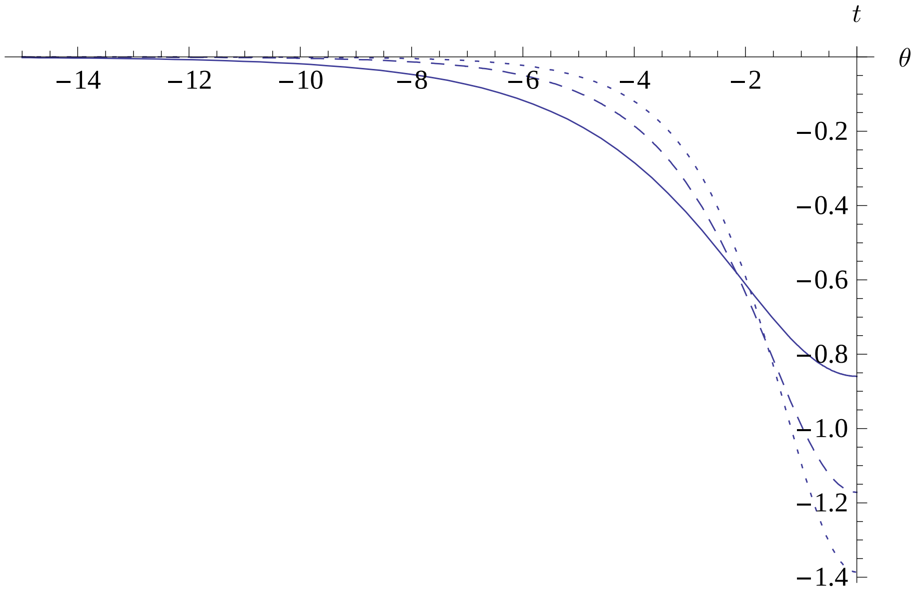


Figure 2. This is plot of $\theta(t)$ with time for $\omega > \omega_c$. The mass rolls down from unstable $\theta = 0$ position to some non-zero (negative) θ in time. The solid, dashed and dotted lines are for $\omega = 1.1, 1.2$ and 1.3 respectively. ω_c has been set to 1. We see that the rate of rolling down increases as we increase $\omega - \omega_c$.

Substituting the value of a, we get

$$\theta = \pm 2 \tan^{-1}\left(\sqrt{\frac{\omega^2 - {\omega_c}^2}{{\omega_c}^2}} \operatorname{sech}\{(\sqrt{\omega^2 - {\omega_c}^2})(t - t_0)\}\right). \qquad (20)$$

In the above equation, t_0 is an arbitrary constant. This appears due to the time translational invariance of the differential equation (18). To fix the integration constant, we have used the boundary condition that at $t = -\infty$ the particle is at $\theta = 0$. Note that equation (20) is only real for $\omega > \omega_c$. This is what we expect. For $\omega < \omega_c$, only $\theta = 0$ is a stable minimum of the effective potential. Furthermore, we expect that as we increase ω beyond ω_c, the rate at which the particle rolls down would be more. This is indeed the case as can be seen from figure (2) where we have plotted the negative θ part of the solutions (20). We also notice that, it requires infinite time to reach finite θ value from zero. Consequently, it would never reach the symmetry preserving phase again. Note that the non-zero stable theta value in the figure is $\theta = -\cos^{-1}(\frac{\omega_c^2}{\omega^2})$. It increases as we increase ω.

3. SUMMARY

To conclude, in this paper, we have constructed a time-dependent rolling down solution from symmetry preserving phase to the symmetry broken one within the context of the simple mechanical model of [2]. This model, though very simple, captures various features of spontaneous symmetry breaking and second order phase transition. We hope our results will be useful to understand more complicated scenarios including the models of infation of our universe.

ACKNOWLEDGMENTS

I would like to thank Sudipta Mukherji, Institute Of Physics, Bhubaneswar for supervising this work. I also thank Somendra M. Bhattacharjee for his insightful comments and also for suggesting improvements.

References

[1] R. Alben, *An exactly solvable model exhibiting a Landau phase transition*, Am. J. Phys 40, 3 (1972).

[2] J. Sivardiere, *A simple mechanical model exhibiting a spontaneous symmetry breaking*, Am. J. Phys. 51, 1016 (1983).

[3] G. Fletcher, *Mechanical analog of first and second order phase transition*, Am. J. Phys. 65, 1 (1997).

[4] R. Banerjea, *Analogy between mechanical systems, phase transitions and possible experimental implementation*, Prayas: Students' Journal of Physics, 3, 140 (2008).

[5] S. Weinberg, *Cosmology*, Oxford University Press.

[6] S. Kar and A. Khare, *Classical and quantum mechanics of a particle in a rotating loop*, Am. J. Phys. 68, 1128 (2000).

P R A Y A S
Students' Journal
of Physics

Neutrino Oscillation Phenomenology*
(Solving the mystery of the missing solar neutrinos)

Ipsita Satpathy†
IIIrd Yr., School of Physics, IISER Kolkata, HC-Block, Sec-3, Salt Lake, Kolkata-700 106.

Abstract. This is a brief review of the theory and experiments of solar neutrino oscillation. I start with a brief description of neutrino oscillation formalism and solar matter effect. Then, I describe the solar neutrino experiments, alongwith the KamLAND reactor neutrino experiment, in terms of neutrino oscillation.

Keywords. solar neutrino, neutrino mass, neutrino oscillation

Communicated by: D.P. Roy

1 NEUTRINO MIXING AND OSCILLATIONS

Neutrinos are the Standard Model fermions that interact only via the weak interaction. They travel at speeds nearly equal to the speed of light. Over the last 50 years since their discovery, neutrinos have changed from being a physics oddity into one of the most powerful tools of experimental physics. The strong evidence found in favour of the neutrino oscillation phenomenology, through pioneering experiments like Super Kamiokande (SK), Solar Neutrino Observatory (SNO) and KamLAND (KL), has opened up an entirely new direction to particle physics. The last few years, in particular, are important, since they have seen a major anomaly put to rest.

As per the Standard Model, the basic constituents of matter [1] are a set of spin $\frac{1}{2}$ particles (called fermions) alongwith their antiparticles, as shown in Table 1. It clearly shows the three flavours of neutrinos.

---

*A Review

†e-mail: ranisatpathy@gmail.com

Table 1. The basic constituents of matter alongwith their charges

Leptons	ν_e	ν_μ	ν_τ	0
	e	μ	τ	-1
Quarks	u	c	t	$2/3$
	d	s	b	$-1/3$

2 NEUTRINO MIXING AND OSCILLATIONS

In 1968, Pontecorvo observed that if neutrinos have non-zero mass, mixing between flavour and mass eigenstates will be possible, leading to the neutrino oscillations.

It turns out that one can get a good approximation to three neutrino mixing by simply considering mixing between two neutrino flavours, i.e.

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$$

and

$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta. \tag{1}$$

Writing out (1) in the form of a mixing matrix, with ν_1 and ν_2 as mass eigenstates with eigenvalues m_1 and m_2, we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{2}$$

The ν_1 and ν_2 components in any state ν_e produced will thus, in general, propagate with different velocities, since they have different masses, so that their relative sizes do not remain constant over the distance. This means transformation of ν_e into ν_μ.

As per the uncertainty principle, a neutrino state with definite energy-momentum does not travel as a point particle but as a plain wave, described by the wave-function,

$$\psi = e^{-i(Et-pl)} \tag{3}$$

where E, p, t and l represent the energy, momentum, time and distance respectively. We know that E is on the order of MeV and hence much larger than the neutrino mass which is less than eV, $E >> m$. Therefore, these neutrinos are relativistic particles. Hence, we can write,

$$t \simeq \frac{l}{c} \simeq l, \text{in natural units (c} = 1 \text{ and } \hbar = 1),$$

$$E \simeq p + \frac{m^2}{2p} \simeq p + \frac{m^2}{2E} \tag{4}$$

Substituting (4) in (3), we get

$$\psi = e^{-i\left(\frac{m^2 l}{2E}\right)} \tag{5}$$

Thus the neutrino mass eigenstate propagates with a phase of $e^{-i\left(\frac{m^2 l}{2E}\right)}$. Now, after traveling a distance l the $\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$ admixture becomes

$$\nu_e \rightarrow \nu_1 \cos\theta e^{-i\left(\frac{m_1^2 l}{2E}\right)} + \nu_2 \sin\theta e^{-i\left(\frac{m_2^2 l}{2E}\right)} \tag{6}$$

Upon inverse transformation of ν_1 and ν_2, using (2), the ν_μ coefficient does not cancel out because the phases are different. This means that after a distance l some component of ν_μ appears. Probability of $\nu_e \rightarrow \nu_\mu$ oscillation is the square of this component:

$$\begin{aligned} P_{\nu_e \rightarrow \nu_\mu}(l) &= \left| \sin\theta\cos\theta \left(-e^{-i\frac{m_1^2 l}{2E}} + e^{-i\frac{m_2^2 l}{2E}} \right) \right|^2 \\ &= \left| \sin(2\theta) e^{\frac{(m_1^2+m_2^2)}{4E} il} \sin\left(\frac{(m_1^2 - m_2^2)}{4E} il \right) \right|^2 \\ &= \left| \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 l}{4E} \right) \right| \end{aligned} \tag{7}$$

where the first factor represents the amplitude, the second the phase of oscillation, with $\Delta m^2 = m_1^2 - m_2^2$, and l is the source-detector distance, and E for the neutrino energy. Thus, the probability of the neutrino flavour change has an oscillatory behaviour with the amplitude $\sin^2 2\theta$ and the phase $\sin^2\left(\frac{\Delta m^2 l}{4E}\right)$. Writing out convenient units for $\Delta m^2 (eV^2), l(m)$ and $E(MeV)$, we have,

$$P_{\nu_e \rightarrow \nu_\mu}(l) \; = | \sin^2(2\theta) \sin^2\left(\frac{1.3 \Delta m^2 l}{E} \right) | \tag{8}$$

We observe that if the mixing angle is zero or the neutrinos have equal masses, then such neutrino oscillations do not occur. Particularly, the oscillation would be impossible if both of them had zero masses.

We note from (8) that the oscillation wavelength is,

$$\lambda = (\pi/1.3).(E/\Delta m^2) \simeq 2.4E/\Delta m^2. \tag{9}$$

Table 2. Change in oscillation probability with increase in source-detector distance for large mixing angle ($\sin^2 2\theta \simeq 1$)

	$l \ll \lambda$	$l = \lambda/2$	$l \gg \lambda$
$P_{\nu_e \rightarrow \nu_\mu}$	0	$\sin^2 2\theta$	$1/2 \sin^2 2\theta$
$P_{\nu_e \rightarrow \nu_e}$	1	$1 - \sin^2 2\theta$	$1 - 1/2 \sin^2 2\theta > 1/2$

Thus, we have the pattern of neutrino oscillation illustrated in Table 2. In each case of Table 2, the survival probability is given by,

$$P_{ee} = P_{\nu_e \rightarrow \nu_e} = 1 - P_{\nu_e \rightarrow \nu_\mu} \tag{10}$$

The typical energy of solar and reactor neutrinos, is of the order of 1 MeV. The distance l between the source and the detector for the solar neutrino experiment is around $10^{11}m$. Thus, Δm^2 can remarkably be measured upto $10^{-11}eV^2$. For a long baseline experiment like KamLAND, l is around $10^5 m$. So, it can probe Δm^2 upto $10^{-5}eV^2$. Thus, these oscillation experiments provide a precise tool to measure very tiny neutrino masses, much beyond the capacity of any other method.

21 CC and NC interactions

There are two types of interactions. The neutral current (NC) interaction is the one occurring through exchange of the Z^0 boson. The charged current (CC) interaction occurs through the charged $W^{+/-}$ boson exchange. The ν_e can interact both via CC and NC interactions.

22 Solar Neutrino Source

The vast amount of energy generated inside stars like our sun comes mainly from the thermonuclear pp chains of reactions (Fig 1). Production of energy inside the sun is accompanied by the emission of electron neutrinos in a huge flux detectable at the surface of the earth. From the figure, we find

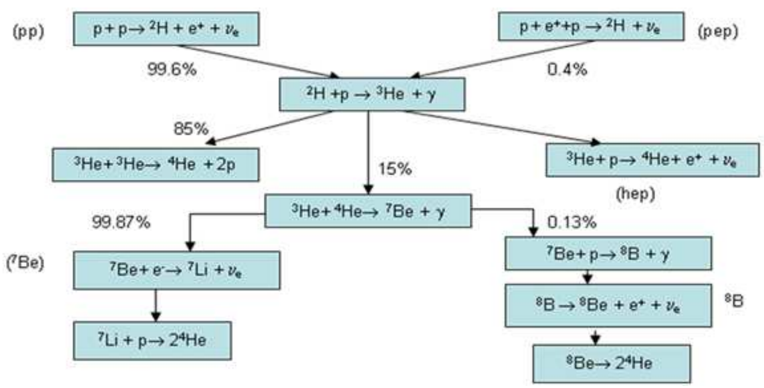


Figure 1. The pp cycle.

that the pp, ^7Be, and ^8B neutrinos are important. The low energy pp neutrino has the highest flux. The ^7Be neutrinos have intermediate energy, whose flux is suppressed by one order of magnitude compared to the pp neutrino. Finally, the flux of the high energy ^8B neutrinos is further suppressed by three orders of magnitude. Fig 2 shows the Standard Solar Model (SSM) predictions [2] for the above neutrino fluxes. It also shows the neutrino energy ranges covered by the different solar neutrino experiments discussed below.

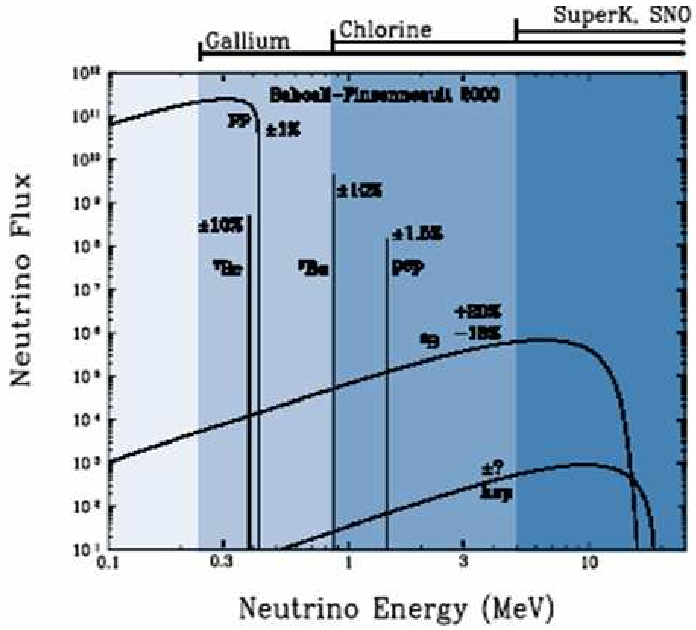


Figure 2. The SSM prediction for the solar neutrino fluxes and the neutrino energy ranges probed by the solar neutrino experiments.

3 SOLAR NEUTRINO EXPERIMENTS

The Homestake Chlorine Experiment pioneered by Raymond Davis Jr. et al, in the late 60s, basically consists of a detector tank of volume 6×10^5 litres filled with 400,000 litres of percholoroethylene (C_2Cl_4). It is a radiochemical experiment, in which through the following CC interaction,

$$\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar, \quad (11)$$

radioactive ${}^{37}Ar$ is produced from ${}^{37}Cl$. The produced argon is extracted after every few weeks by purging the detector with ${}^{4}He$. By measuring the number of radioactive decays through a GM counter, one can estimate the number of produced argon atoms. Then, the neutrino fluxes for the exposition period can be estimated and compared with those predicted by SSM.

In the radiochemical gallium experiments, GALLEX [5] and SAGE, ν_e could be detected through the following CC interaction,

$$\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge. \quad (12)$$

Again, every few weeks, the amount of Ge atoms produced was extracted and measured by radiochemical method.

The Super-Kamiokande detector [6] is a large cylindrical tank containing 50,000 metric tons of pure water. It is situated deep underneath the Kamioka mine in Japan, so that the cosmic ray muons would be shielded by the rocks. It is a real-time experiment which uses photomultiplier tubes to detect the neutrino interaction via Cherenkov light produced by the electron. The following elastic scattering takes place:

$$ve^- \to \nu e^- (\text{threshold energy} \ = \ 5 \ \text{MeV}). \tag{13}$$

Though principally dominated by the CC interaction, it also has some sensitivity for the corresponding NC interaction.

$$\nu_{e,\mu} e^- \to \nu_{e,\mu} e^- \tag{14}$$

Hence the detection rate,

$$R_{el} = P_{ee} + (1 - P_{ee})\sigma^{NC}/\sigma^{NC+CC} \simeq P_{ee} + (1/6)(1 - P_{ee}). \tag{15}$$

Unlike the earlier experiments, it can measure both the energy and direction of the incident neutrino from those of the outgoing electrons.

The SNO collaboration [7] is a one-kiloton heavy-water detector. A geodesic array of photomultiplier tubes detects the Cherenkov radiation. It could detect both NC and CC events on account of the low binding energy of the deuteron (2.2 MeV):

$$\begin{aligned} &\nu_e + d \underrightarrow{CC} p + p + e^- \\ &\nu_{e,\mu} + d \underrightarrow{NC} \nu_{e,\mu} + p + n. \end{aligned} \tag{16}$$

For the first phase of the experiment, the NC events were detected through neutron capture on deuteron [7], $n + d \ \to \ t + \gamma$. In the second phase, NaCl was added, to enhance the NC event detection efficiency, using neutron capture on chlorine via $n + {}^{35}Cl \ \to \ {}^{36}Cl + \gamma$. Recently, in the last phase, helium filled gas counters were used to detect the NC events through the following reaction: $n + {}^3He \to p + t$.

Table 3. Neutrino flux composition and energy threshold for various solar neutrino experiments

Experiment	GALLEX & SAGE	Chlorine Homestake Experiment	SK	SNO - 1
Neutrino beam composition	$pp(55\%)$, ${}^7Be(25\%)$, ${}^8B(10\%)$	${}^7Be(15\%)$, ${}^8B(75\%)$	${}^8B(100\%)$	${}^8B(100\%)$
Energy threshold (MeV)	0.2	0.8	5	5
R	0.55 $\pm$ 0.03	0.33 $\pm$ 0.03	0.465 $\pm$ 0.015 (0.36 $\pm$ 0.015)	0.35 $\pm$ 0.03

In Table 3, the ν_e survival probability P_{ee} measured by the charged current event rate R relative to the SSM prediction is indicated. The values shown in parenthesis in the last row indicate the former for SK after taking into account the NC corrections. We observe that at low energies, as is the case of the radiochemical gallium experiments, the survival probability is a little above $1/2$. But, at higher energies, it falls to $1/3$. The effect of solar matter on the neutrino oscillation is considered, to explain the magnitude and the energy dependence of this survival probability.

4 SOLAR MSW EFFECT

As the neutrinos traverse the interior of the sun from the core it gets an induced mass. The phenomenon affects neutrino oscillations, which is known as the MSW effect (after Mekheyev, Smirnov and Wolfenstein) [8] or the "matter enhancement". It arises because of the following CC interaction:

$$\nu_e e^- \xrightarrow{CC} e^- \nu_e$$

while the NC interaction has no overall effect because it is common to all flavours.

The interaction potential energy V depends upon the solar electron density N_e:

$$V = \sqrt{2} G_F N_e, \tag{17}$$

where G_F and N_e respectively denote Fermi coupling and the solar electron density.

We have to introduce the potential energy term in the free-particle wave equation for the neutrino,

$$i \frac{d\psi_{1,2}}{dt} = (p + m^2_{1,2}/2E)\psi_{1,2}. \tag{18}$$

At this stage, we rewrite the equation (18) using the rotation matrix of the equation (2)

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = (p + M^2/2E) \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \tag{19}$$

where,

$$M^2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{20}$$

Now, introducing the potential energy term on the RHS is equivalent to replacing M^2 by:

$$\begin{aligned} M'^2 &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} 2\sqrt{2}G_F N_e E & 0 \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta^2 m_1^2 + \sin^2\theta m_2^2 + 2\sqrt{2}G_F N_e E & -\sin\theta\cos\theta\Delta m^2 \\ -\sin\theta\cos\theta\Delta m^2 & \cos^2\theta m_2^2 + \sin^2\theta m_1^2 \end{pmatrix} \end{aligned} \tag{21}$$

This is called the effective mass or energy. Now, all that we have to do is find the eigenvalues and eigenstates to the above matrix. For simplicity, we assume that the mixing angle θ is small, so that the off-diagonal elements are small. In this situation, the diagonal elements roughly correspond to the eigenvalues and their eigenstates to the flavour eigenstates.

Both the eigenvalues are plotted against the solar electron density (Fig. 3) upon the assumption that $m_1 < m_2$. We find that at the surface of the sun $(N_e = 0)$, λ_1 is smaller than λ_2. However, with increase in N_e, λ_1 increases steadily and finally becomes larger than λ_2 at the solar core.

We find that the eigenvalues of M come close to each other at a critical density N_e^C

$$N_e^C = \frac{\Delta m^2}{2\sqrt{2} G_F E} \cos 2\theta \Rightarrow M_{11}'^2 = M_{22}'^2. \tag{22}$$

and cross-over occurs.

However, the two eigenvalues actually never cross. The gap between them always has a minimum value equal to the value of the non-diagonal elemment:

$$\Gamma = \Delta m^2 \sin 2\theta. \tag{23}$$

Therefore, it is clear from the above figure, that the ν_e produced in the solar core will come out as

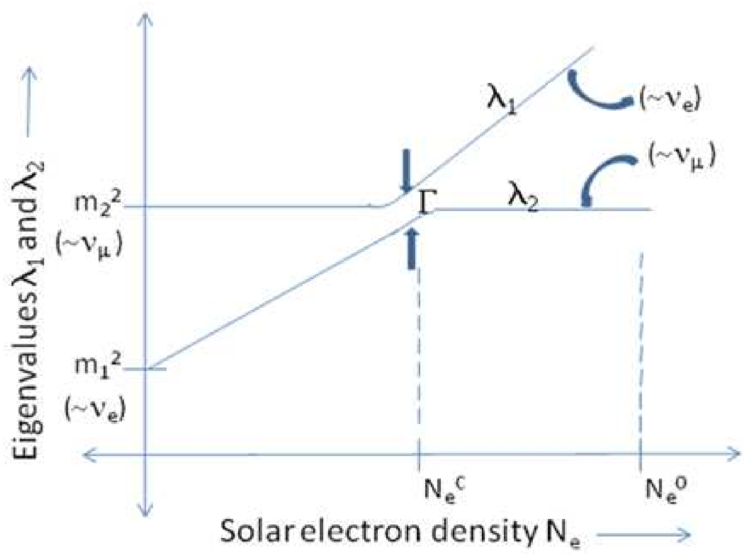


Figure 3. The two eigenvalues as functions of the solar electron density assuming $m_1 < m_2$.

ν_2, provided there is not much transition between the energy levels. Further, this remarkable result does not depend on the small angle assumption as we see now. The mixing angle in matter is the rotation angle which diagonalises the matrix (21), i.e.

$$\tan 2\theta_M = \frac{2M_{12}'^2}{\mid M_{22}'^2 - M_{12}'^2 \mid} = \frac{\sin 2\theta}{\mid \cos 2\theta - 2\sqrt{2} G_F N_e / \Delta m^2 \mid}, \tag{24}$$

Since, at the solar core $N_e^0 >> N_e^C$, the second term in the denominator is much larger than the first term. It implies that $\theta_M << 1$ at the solar core for any vacuum mixing angle θ. Thus, the

ν_e produced in the solar core travels first as ν_1 uptill the critical density. From equation (22), we can see that at the critical density, the denominator of equation (24) becomes zero. Here, maximal mixing occurs ($\theta_M = \pi/4$), again independent of θ. Hence, it is called resonant oscillation. Now, the neutrino comes out as ν_2 with a ν_e survival probability,

$$P_{ee} = \sin^2\theta \tag{25}$$

Again, this occurs provided there is no appreciable transition between the two energy levels. Particularly, the critical density region is most important for this transition, because the gap is the smallest. The transition probability in this region is given by the Landau- Zenner formula:

$$\begin{aligned} T &= e^{-\pi/2\gamma} \\ \gamma &= \frac{\lambda_C}{\Gamma}(d\lambda_l/dl)_c \propto \frac{\lambda_C}{N_e^C}(dN_e/dl)_c, \end{aligned} \tag{26}$$

where λ_c gives the oscillation wavelength in the critical region. If $(dN_e/dl)_c$ is small, then $\gamma << 1$, and the transition rate goes down exponentially. This is known as the adiabatic condition. Summing up the two conditions, we have:

$$\frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}G_F N_e^0} < E < \frac{\Delta m^2 \sin^2 2\theta}{2\cos 2\theta (dN_e/dlN_e)_C} \tag{27}$$

The first condition simply ensures that,$N_e^0 > N_e^c$, whereas the second one represents the adiabatic condition.

5 THE MSW TRIANGLE

In the $\Delta m^2 - \sin^2 2\theta$ plot (Fig 4.), the region satisfying equation (27) for typical neutrino energy E = 1 MeV, forms a triangle, known as the MSW triangle. The horizontal side follows from the first inequality because $\cos 2\theta \simeq 1$. The lower bound on $\sin^2 2\theta$ is given by the second inequality. Since this inequality applies to the product $\Delta m^2 \sin^2 2\theta$, it corresponds to the diagonal side on the log-log plot. The vertical side is given by the physical limit for maximal mixing. The survival probability inside the triangle is given by $\sin^2\theta$ (equation 25), while that outside the triangle is given by $1 - (1/2)\sin^2\theta$ as we see from Table 2. Therefore, the survival probability inside the triangle $< 1/2$, whereas it is $> 1/2$ outside it.

The best solution to the observed energy dependence of the survival probability P_{ee} (Table 3), alongwith its energy independence above 5 MeV, as observed by the SK and SNO experiments [6,7], comes from the so-called Large Mixing Angle (LMA) region of Fig 4, with $\Delta m^2 \sim 10^{-5} eV^2$. We see from this figure that the low energy Gallium experiment (E << 1 MeV) falls above the MSW triangle in $\Delta m^2/E$, corresponding to a $P_{ee} > 0.5$. On the other hand, the Chlorine, SK and SNO experiments (E >> 1 MeV) fall inside the triangle in $\Delta m^2/E$, corresponding to $P_{ee} = \sin^2\theta < 1/2$.

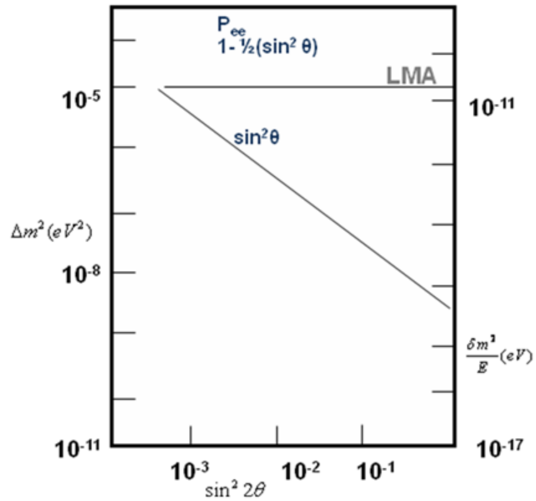


Figure 4. The MSW triangle at E = 1 MeV. Since Δm^2 scales with E in the formulae for the survival probability, the P_{ee} values shown against the ratio on the right hold at all energies.

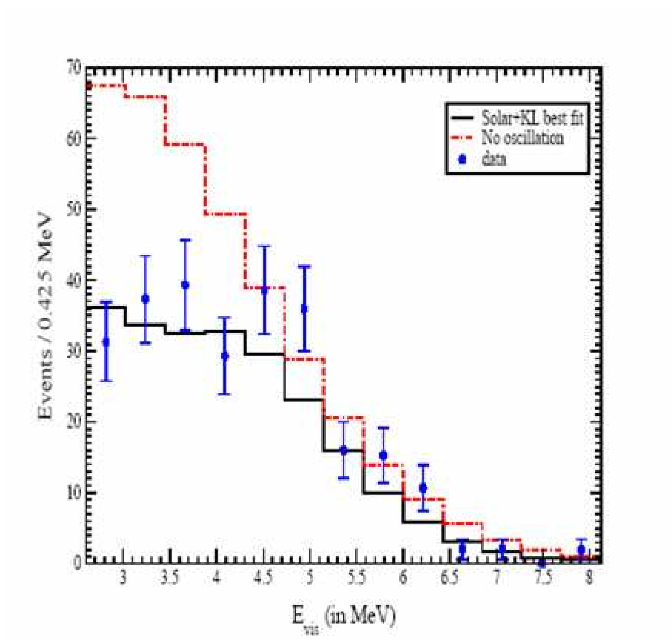


Figure 5. Comparing the 766 ty KL spectrum compared with the no-oscillation and the best oscillation fits.

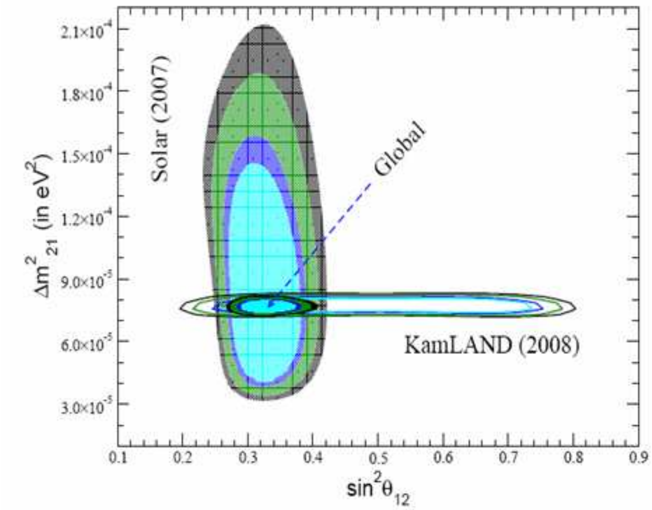


Figure 6. The best fit values of the solar neutrino mass and mixing parameters, from a global fit to the solar neutrino data and the very recently released 2.8kty KL data.

51 Zeroing in on the LMA by KAMLAND

It was only after the reactor antineutrino data from KamLAND came out that the LMA solution could be confirmed independently [9]. It is a one kiloton liquid scintillator neutrino detector. Using the reaction, $\bar{\nu}_e + p \rightarrow e^+ + n$, it detects $\bar{\nu}_e$ coming from the Japanese nuclear reactors situated at distances between 80 to 800 kms, with a mean baseline distance of ~ 180 km, i.e. $\sim 10^5$m. Thus, it is sensitive to the region $\Delta m^2 \geq 10^{-5}eV^2$, as explained below equation (9). Hence, it is designed specifically to probe the LMA region. The incident $\bar{\nu}_e$ energy is measured through the visible scintillation energy produced by the positron and its annihilation with a target electron, i.e.

$$E_{vis} = E + m_e + m_p - m_n = E - 0.8MeV \tag{28}$$

The first KL result (162 ton-year data) showed a $P_{ee} \sim 0.6$, thus validating the LMA prediction and ruling out the alternative ones. Considered together with the global solar neutrino data, it split the allowed LMA region into low and high LMA bands [10], corresponding to the first and second oscillation minima, with the latter having lower statistical significance. Following this, the 766 ty (ton year) KL data zoroed in on the low-LMA band [11]. Comparing the 766 ty KL spectrum with the no-oscillation and the best oscillation fits (fig 5), it is found that the two fits come closest at $E_{vis} \approx 5MeV (E \approx 6MeV)$, corresponding to the first oscillation node ($\langle l \rangle = \lambda = 180$ km). Plugging into equation (9) gives $\Delta m^2 = 8 \times 10^{-5}eV^2$.

Fig 6 shows the best fit values of the solar neutrino mass and mixing parameters, from a global fit to the solar neutrino data and the very recently released 2.8kty KL data[12]. The best fit values are:

$$\Delta m^2_{sol} = \Delta m^2_{21} = 7.7 \times 10^{-5}eV^2, \sin^2\theta_{sol} = \sin^2\theta_{12} = 0.33 \tag{29}$$

Acknowledgements

This project is done as part of NIUS programme of HBCSE, TIFR, under the guidance of Prof. D.P.Roy. I am grateful to him, the person who first took me by the hand and opened up the mystery for me. It is a pleasure to acknowledge his warm encouragement and kind guidance throughout this work.

References

[1] D.P.Roy, Basic Constituents of Matter and their interactions, hep-ph/9912523.

[2] Bahcall, Pinsonneault, Basu, Astrophys J. 555 (2001) 990; http://www.sns.ias.edu/ jnb.

[3] D.P.Roy, Solar Neutrino Oscillations, hep-ph/0409336 (2004).

[4] B.T.Cleveland et al, Astro Phys. J. 496 (1998) 505.

[5] SAGE: J.N. Abdurashitov et al, JETP 95 (2002) 181. GALLEX: W.Hampel et al, Phys. Lett. B447 (1999) 127.

[6] SK Collaboration: S. Fukuda et al, Phys. Lett. B539 (2002) 179

[7] SNO Collaboration : Q. R. Ahmad et al., Phys. Rev. Lett. 89 (2002) 011301.

[8] L.Wolfenstein, Phys. Rev. D17(1978) 2369.

[9] KamLAND: K.Eguchi et al, Phys. Rev. Lett. 90 (2003) 021802.

[10] A. Bandopadhyay, S.Choubey, R.Gandhi, S.Goswami, and D.P.Roy, J.Phys.Lett. B559 (2003) 121.

[11] KL Collaboration : T. Araki et al, [hep-ex/0406035].

[12] KL Collaboration: S. Abe et al., arXiv:0801.4589[hep-ex].

PROBLEMS IN PHYSICS

Readers are invited to submit the solutions of the problems in this section within two months. Correct solutions, along with the names of the senders, will be published in the alternate issues. Solutions should be sent to: H.S. Mani, c/o A.M. Srivastava, Institute of Physics, Bhubaneswar, 751005; e-mail: ajit@iopb.res.in

Problems set by H.S. Mani

In this issue we discuss three quantum mechanics problem and their relation to the macroscopic world. The first problem is found in many textbooks in Quantum Mechanics(Schiff's book for example). For such problems the solution lies in the use of uncertainty principle after ensuring the problem gets properly posed in terms of macroscopic quantities. We will provide the solutions in the next issue.

1. Consider a pencil (length L = .2m, mass 0.05Kg.) standing vertically on its tip, which can be considered as a point.Using the uncertainty principle estimate the time it can stay up without falling. Assume the tip is fixed during the fall.

2. Consider a one dimensional motion of a particle along the x-axis under the action of a potential $V(x) = V_0 > 0$ for $x \leq 0$ and $V(x) = 0$ for $x > 0$.If the particle moves to the right from $x < 0$, with an energy $2V_0$, standard quantum mechanics gives for the reflection coefficient at $x = 0$ of the order $0(1)$ (The exact number is $(\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)})^{1/2}$ and the result is independent of the mass of the particle.

   Now consider a car travelling with a speed v towards a cliff (height H). From the previous calculation the probability of reflection at $x = 0$ should be $0(1)$ (Assume the kinetic energy of the car to be of the same order as mgH where m is the mass of the car). This is an absurd result. Do a correct modeling for the cliff and obtain a physically reasonable result.

3. A person is dropping stones at a mark on the floor from a height H. Show that the minimum spread of the stones would be

$$\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}$$

where m is the stone's mass and g is the acceleration due to gravity. Calculate the spread for a Cesium atom ($At.wt = 133$) dropping a height of .2m.

Solutions to the problems given in Vol.3 No.5

Vivek Singh
Student - M.Sc. Physics, Banaras Hindu University, Varanasi - 221005.
(Mr. Vivek Singh has provided solutions for problems 1 and 3 and these are correct solutions. Solutions to all the 3 problems are given below.)

Communicated by: H.S. Mani

Problem 1: Consider a rigid sphere of radius R rolling without slipping at the points of contact with two rough planes $x-y(z=0)$ and $y-z(x=0)$ at P_1, P_2 respectively. The centre of the sphere at $t=0$ is at ($x=R, y=0, z=R$) and the velocity of the centre is $v\hat{j}$. Find the velocity of the point $P_3(x=R, y=0, z=2R)$ and $P_4(x=R, y=R, z=R)$.

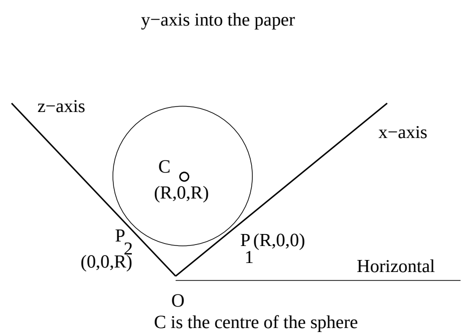


Figure 1. Figure for Problem 1.

Solution to Problem 1:
We know that the points $P_1(0,0,R)$ and $P_2(R,0,0)$ are at rest at $t=0$ and hence the instantaneous axis of rotation must pass through these points. Thus the axis is along $\vec{P_1P_2}$. As $\vec{OP_1} = R\hat{i}$ and $\vec{OP_2} = R\hat{k}$, the $\hat{\omega}$, the unit vector along the axis of rotation is

$$\hat{\omega} = \frac{1}{\sqrt{2}}(\hat{k} - \hat{i}) \tag{1}$$

If C is the centre then $\vec{OC} = R(\hat{i} + \hat{k})$ and hence $\vec{P_1C} = \vec{OC} - \vec{OP_1} = R\hat{k}$. Using $\vec{v} = \vec{\omega} \times \vec{P_1C}$, we have

$$v\hat{j} = \omega\frac{1}{\sqrt{2}}(\hat{k} - \hat{i}) \times R\hat{k} \quad (2)$$

giving

$$\omega = \sqrt{2}v \quad (3)$$

. or

$$\vec{\omega} = \frac{v}{R}(\hat{k} - \hat{i}) \quad (4)$$

We can calculate the velocity of any point at t=0. For P_3, we have $P_1\vec{P}3 = 2R\hat{k}$ which gives

$$v(\vec{P}_3) = \vec{\omega} \times P_1\vec{P}_3 = 2v\hat{j} \quad (5)$$

and $P_1\vec{P}_4 = R(\hat{j} + \hat{k})$ which leads to

$$v(\vec{P}_4) = \frac{v}{R}(\hat{k} - \hat{i}) \times R(\hat{j} + \hat{k}) = v(-\hat{i} + \hat{j} - \hat{k}) \quad (6)$$

Problem 2: Consider two cylinders of radius r and mass m on a rough belt which is moving with acceleration A. There is a massless plank of wood on top of which are two similar cylinders (See figure 2). Find the acceleration of the centres of the bottom cylinders. Assume all the cylinders are rolling without slipping.

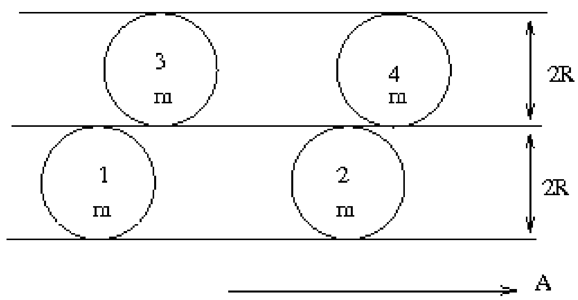


Figure 2. Figure for Problem 2.

Solution to Problem 2:
The various forces on the cylinders are shown in the figure 3.

The various equations are for the bottom cylinders (a_1, a_2 and α_1, α_2 are the accelerations and the angular accelerations of the cylinders 1 and2 respectively)

$$F_1 + F_3 = ma_1; F_2 + F_4 = ma_2 \quad (7)$$

$$(F_1 - F_3)R = \frac{mR^2}{2}\alpha_1; (F_2 - F_4)R = \frac{mR^2}{2}\alpha_2 \quad (8)$$

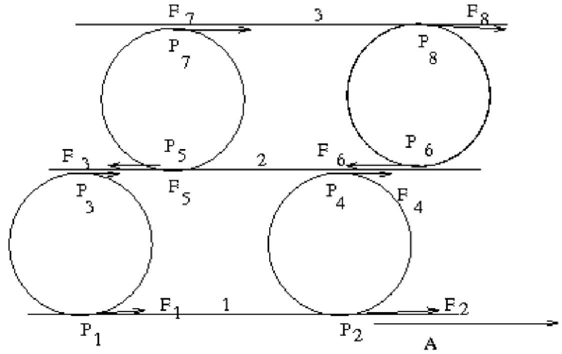


Figure 3. Figure for the solution of Problem 2.

The acceleration of P_1 is $a_1+R\alpha_1 = a_2+R\alpha_2 = A$. Acceleration of P_3 is $a_1-R\alpha_1 = a_2-R\alpha_2 = A'$, where A' is the acceleration of the plank 2. These two imply

$$a_1 = a_2; \alpha_1 = \alpha_2 \tag{9}$$

Similar conclusions hold for the cylinders 3 and 4. This shows we need to analyse only cylinders 1 and 3. Using the fact that plank 2 is massless we get $F_3 = F_5$ (in magnitude). A similar argument for plank 3 gives $F_7 = 0$. We write all the relevant equations below

$$F_5 = -F_3 = ma_3; R(-F_5) = RF_3 = \frac{mR^2}{2}\alpha_3 \tag{10}$$

The acceleration of P_3 and P_5 must be the same. This gives

$$a_1 - R\alpha_1 = a_3 - R\alpha_3 \tag{11}$$

We have six unknowns: $F_1, F_3, a_1, \alpha_1, a_3, \alpha_3$ and we have six equations :Eq.(7),Eq.(8),the two in Eq.(10),Eq.(11) and the fact that acceleration of P_1 is A, which gives

$$a_1 + R\alpha_1 = A \tag{12}$$

these equations can be solved to give

$$a(P_1) = \frac{17}{7}A; \alpha_1 = \frac{10A}{17R} \tag{13}$$

(This problem is a simplified version of the problem posted in http://www.physics.harvard.edu/academics/undergrad/problems.html (problem 85)- you are encouraged to browse through these excellent problems)

Problem 3: Consider a sphere of radius r rolling on a horizontal surface without slipping with its centre having a velocity $\vec{v}$ and an angular velocity $\omega = v/r$ (see figure 4). No frictional force is acting as there is no slipping and there is no torque about the centre as the normal reaction passes through the centre. However in actual practice the sphere does stop, which means an opposing force (opposite to the direction of $\vec{v}$ has to act between on the ball and this gives a torque in the direction of the angular velocity, which means the angular velocity will increase. Then how come as we know from experience the sphere comes to rest with zero angular velocity. Make appropriate physical assumptions and find the opposing torque.

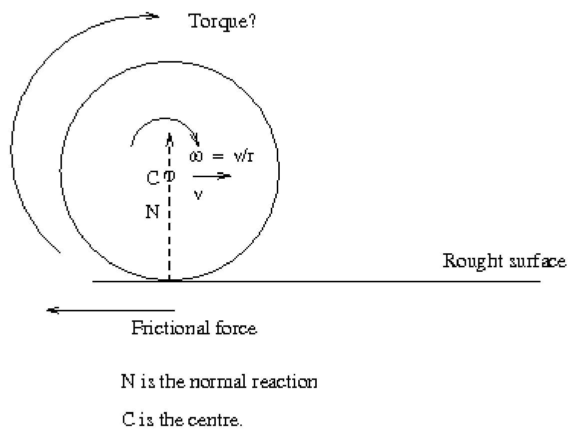


Figure 4. Figure for problem 3.

Solution to Problem 3:
Figure 5 explains the state of affairs.

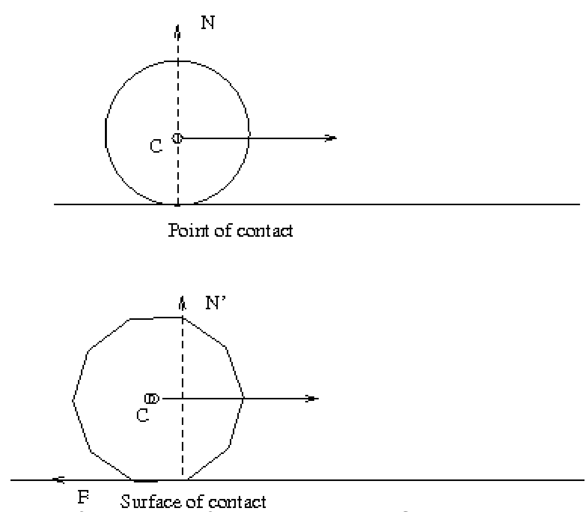


Figure 5. Figure for solution of problem 3. Lower figure is exaggerated to clarify the presence of the opposing torque.

The sphere has some elasticity and hence the contact with the plane horizontal surface is not a point but a surface (shown in the lower figure). Now the normal reaction N from the ground need not pass through the centre but as the sphere moves forward it is shifted as shown . This force produces a retarding torque (overcoming the torque from the frictional force) and the frictional force F the retarding acceleration. Thus both the velocity and the angular velocity decrease eventually the sphere comes to a halt.

One can also see why the normal force N is to the right of centre C in the following way- As the body moves the left side of the surface in contact moves upward whereas the right side ”presses” the ground - Further the right part which touches the ground is brought to a halt at the ground which needs a larger force. Thus there is a greater normal reaction on the right side, leading to the total normal reaction to be right of centre.

....

INSTRUCTION TO AUTHORS

Authors are advised to visit the PRAYAS homepage (http://www.iopb.res.in/~prayas/prayas.html), where instruction for the preparation of the manuscript is provided. They are advised to follow exactly the format of the Templates.

ANNOUNCEMENT

Commencing from 2010, PRAYAS will be published quarterly.

PRAYAS

Quarterly Prayas will now be available by subscripton.

Annual (4 issues)	Rs. 150/-
Single Issue	Rs. 40/-

Payment by crossed bank draft in favour of "Indian Association of Physics Teachers" payable at Chandigarh, may be sent to Dr. C.N. Kumar, Managing Editor, Prayas, Deptt. of Physics, Panjab University, Chandigarh - 160014.

PRAYAS

Students' Journal of Physics

IAPT

Volume 4 Number 1 Jan.-March, 2010

Contents

Published on behalf of Indian Association of Physics Teachers,
128/2, J.P. Naik Road, Kothurd, Pune - 411 038, Phone : 020 25455504
Printed at Shovan, 106 Acharya Vihar, Bhubaneswar - 751 013