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PRAYAS

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Prayas Office : Institute of Physics, Bhubaneswar - 751 005. Phone : (0674) 2306452

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Guest Editorial

Ways of Advancing Physics

In January 1974 I had the opportunity to listen to an evening talk by Dirac during a Coral Gables Conference in Florida, USA. He looked tall, frail and a little bent down with age. As a speaker, however, he was remarkably focussed and articulate. He was describing two complementary ways of advancing physics based on his own experience. He had been a contemporary and close friend of Heisenberg. He said that he was so enamoured by the mathematical beauty and completeness of Bohr's theory of the atom that he could not bring himself to abandoning it even though the fuzziness of the atomic spectra was staring them at the face. On the other hand, guided by the fuzziness of this experimental spectra Heisenberg abandoned Bohr's theory for a heuristic formulation of uncertainty principle, leading to quantum mechanics. So he lost this game, which was won by Heisenberg. Of course Dirac soon reinforced Heisenberg's formulation of quantum mechanics with his own, which became the standard textbook formulation of quantum mechanics. He then described his theory of the electron, which was entirely guided by the priciples of consistency and mathematical beauty. Of course to be accepted as a theory of physics, he said, it had to make physical predictions, to be verified by experiments. But experimental data was not the guiding principle in this case. He then went on to further elaborate on these two complementary ways of advancing physics.

More recently, I read an article by Nambu in the December 2008 Bulletin of the AAPPS (Association of Asian and Pacific Physical Societies). Here he has described three different ways of advancing the frontier of physics. He calls the first way Hisenberg's way, which is heuristic, bottom-up and inductive. Apart from Heisenberg's uncertainty principle, he describes Yukawa's meson theory in this category. Guided by the experimental measurement of the range of the nuclear force to be about 1 fermi (or femto-meter) he used the uncertainty principle to predict that the carrier particle of this force must have a mass (i.e. rest mass energy) of about 100 MeV (Million electron volt). This particle was subsequently discovered in cosmic ray experiments, and is called the pi meson. Incidentally the name meson was given by Bhabha, as its mass was midway between those of electron and proton (or neutron).

The second way is Einstein's way, which is axiomatic, top-down and deductive. The physical results of the special theory of relativity were largely known before Einstein. But Einstein raised the thory to a new level by invoking the geometrical concept of symmetry as a guiding principle of physics. This new paradigm has guided the advancement of physics througout the twentieth century. First came the general theory of relativity by Einstein himself. This was followed by the gauge theory of quatum electrodynamics and its nonabelian extensions to strong and weak interactions, the use of spontaneous symmetry breaking in describing phase transition phenomena in condensed matter physics like ferromagnetism, superfluidity and superconductivity and its extension to strong and weak interaction theories to describe particle masses, and a host of others.

It is worth briefly describing here the confrontation between the ways of Heisenberg and Einstein. The quantum mechanics developed by Heisenberg was reinforced by Dirac, Pauli, Fermi and above all by Bohr himself. However, Einstein felt the theory was incomplete as it violated what he called the principle of local reality. He and his followers tried to develop a more complete theory, which will respect local reality, while preserving the experimental predictions of quantum mechanics. Finally the basic conflict between quantum mechanics and local reality was brought out by John Bell through the so called Bell's inequalities, which can be experimentally tested to determine which theory is correct. In recent years the two photon experiment of Alain Aspect has ruled out local reality in favour of quatum mechanics using this test. Moreover, this victory of quantum mechanics over local reality underlies the current development of quantum computing, which promises an exponential increase in the computing power compared to the present classical computers. Of course the antagonists of quantum mechanics have played a very positive role in this development by sharpening the conflict of quantum mechanics with local reality to a point, which made it possible to experimentally rule out one in favour of the other.

According to Nambu the third way is Dirac's way, which is abstract, aesthetic and revolutionary. The concept of anticommuting variables existed in mathematics. But Dirac took the revolutionary step of importing them into physics as an alternative to commuting variables in developing his theory of the electron. This led to the prediction of half-integral spin of electron, the existence of its antiparticle (positron), the Fermi-Dirac statistics, Pauli exclusion principle, all of which were experimentally verified soon. This in turn led to the development of much of quantum physics of that time (i.e. quatum electrodynamics) and practically all of quantum chemistry. It is hard to emphasize adequately the importance of Dirac's theory of electron to the development of science.

Nambu then puts string theory in this third category and expresses his hope that it will be able to solve the longstanding problems of gravitation and cosmology. While agreeing that string theory is abstract, aesthetic and revolutionary, I do not share his optimism about its physical relevance to gravitation and cosmology. Let me promptly add here that I have no involvement in this thory. However, I was involved in its earlier version as a theory of hadrons e.g. the Veneziano model and the Nambu-Gotto string theory. It had many attractive features like duality between resonances and Regge poles, along with the right analyticity, crossing symmetry and asymptotic behavior of the scattering amplitudes. Moreover, the Veneziano model was phenomenologically successful in describing hadron scattering data to a fair degree of accuracy, say within 10-20%. However, the efforts to develop a quantitatively predictive string theory of hadrons from this model using a variety of perturbative techniques did not succeed in closing this 10-20% gap. Finally the interest in this approach waned after the emergence of quantum chromodynamics as a viable theory of hadrons in the mid-seventies. About 5-10 years later the string theory reappered as a potential theory for solving the problems of quantum gravity. It has again shown many promising features and made a lot of mathematical breakthroughs over the last thirty years. But there is little in terms of its phenomenological predictions with which one can compare the considerable ammount of cosmological data at our disposal to rule in or rule out the theory. Arguably the last frontier of physics for our present epoch is the quantization of gravity. So the advancement of this frontier is of utmost importance to physics. As an outsider to this field the two broad questions to me here are firstly, whether the right quantum theory of gravity will emerge from within or outside the string theory framework and secondly, how long it will take. I cannot honestly hazard a guess on either.

D.P. Roy

P R A Y A S
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TURNING POINTS

Thinking new thoughts: creativity in science

Sunil Mukhi
Tata Institute of Fundamental Research, Colaba, Mumbai 400 005, India

Communicated by D.P. Roy

1. INTRODUCTION

What really is creativity? We tend to believe that it's a very rare quality, that little can be achieved by discussing it and that it "just happens" to be present in a gifted few people. My purpose in putting down some historical facts about this topic is to help us develop a better sense of what it means. The facts suggest creativity is not quite so rare, that it can be a collective — rather than purely individual — trait, and that it can be encouraged or discouraged by society around us, as well as by the attitudes we adopt.

In dictionaries, creativity is defined as "the quality or ability to create or invent something; originality." All of us are creative at some time or other and there is no single formula for scientific creativity. Great leaps in understanding are often just extensions of ordinary creativity during an era of intellectual ferment or crisis, or are brought about by individuals with an uncommon drive to understand.

While creativity occurs in all fields of human endeavour, this article will focus on the scientific (rather than technological or artistic) variety. We will start by trying to extract the essence of creative thought by looking at discoveries and innovations from the ancient past. The idea is to see where and how different types of scientific progress came about and what drove them. In the second part of the article we will survey some creative steps taken in less ancient times, spanning the 17th to the early 20th century. Finally I will analyse in some detail the jagged path that led, in the late 20th century, to the currently accepted theory of the strong nuclear interactions.

In this article I will consistently use the abbreviations "BC" (Before the Christian Era) and "CE" (Christian Era) to label dates. This usage carries the same meaning, in terms of dates, as the earlier "BC" and "AD" respectively, but is considered more scientific.

2. CREATIVITY IN ANCIENT HISTORY

2.1. Town Planning

The age was 3000-2000 BCE and the location, the Indus Valley Civilisation familiar to us from archaeological remains at places like Mohen-jo-daro, Harappa and Lothal. Detailed historical knowledge of this era is somewhat limited and occasionally controversial, but the artefacts and sites that survive contain a great deal of information.

This civilisation had urban conglomerations similar to modern cities, with large numbers of people staying in close proximity to each other. This raised an issue that was not so critical in the spread-out lifestyle of a village: how can one supply water and drain away waste from homes? Evidently the

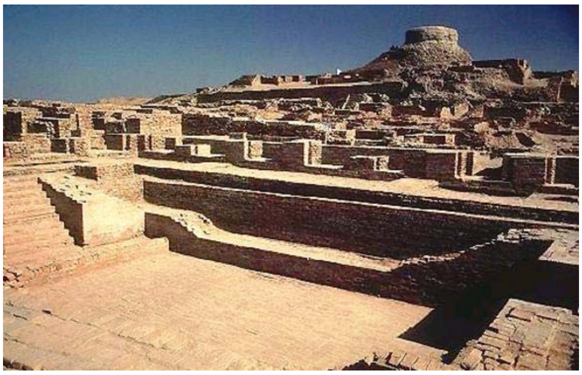

Figure 1. View of an Indus valley excavation (used under Creative Commons license)

inhabitants put their minds to this problem and discovered methods that today impress us as being highly creative for their time. For example, to carry away waste they made drainpipes of brick and, crucially, they angled them so that the waste would be drawn in the desired direction. In this process they harnessed the force of gravity! Additionally they installed filters to separate solid from liquid waste as this would make disposal easier. Surviving artefacts from this period include rulers and compasses for measurement, without which the engineering marvel of their towns would not have been realisable.

Unfortunatly little was gained by future generations from these remarkable discoveries. The Indus Valley civilisation was destroyed by floods and its brilliance lost to posterity. Concepts like rulers, compasses and drainage channels had to be rediscovered many centuries later. Other ancient societies were luckier and passed on their learning through trade and travel. The knowledge possessed by human beings began to be cumulative, each successive society building upon and improving the discoveries of the previous one.

2.2. Geography

What is the shape of the earth? The earliest major progress on this question took place in ancient Greece, during the period 500-200 BCE. A question like this would not have occurred to a civilisation localised over a very small area, for on scales of a few kilometres it matters little what the shape of the earth is. But the Greeks were seafarers and the farther they travelled, the more they were led to ask where will it all end. Indeed, Greek sailors provided early experimental evidence that the earth is not flat. Ships in the distance were observed not to fade from view uniformly, instead their lower ends became invisible first and they steadily sank below the horizon.

Remarkably the proposal that the earth is spherical (or approximately spherical, as we know today) was initially a guess! Some thinkers guessed alternate shapes like a capped cylinder, which could also partially explain the way ships disappeared at sea. Now observations of the sky played a crucial role in testing the different proposals. The creative act consisted in realising that a distant system (like the shadow on the moon during an eclipse) could provide independent experimental evidence for a terrestrial question. But the culmination of this creative study, which provided a definitive answer and even a value for the earths radius, was enabled by geometry. The angle of elevation of the sun, and distances between nearby cities, were famously used by Eratosthenes to estimate the earths circumference and his proposed figure (which is ambiguous because of the units he used) was within about 15% of the correct answer.

The studies by the ancient Greeks had a lasting impact on Western civilisation. Latitude and longitude were also invented at this time. But due to the lack of communication among different cultures, distant civilisations such as China are said to have believed in a flat earth right until the 17th century CE.

2.3. Medicine

While disease has always been a fact of nature, its causes were not always clear. This question obviously had a more direct and urgent bearing on human life than say the shape of the earth, but for some reason little progress was made on it until over ten centuries after the ancient Greeks . Evidently it took centuries to properly formulate the question. Religious beliefs presumably intervened, as life and death were ascribed to mystical causes.

The 10th century CE was an intellectually stagnant period in Europe, and the focus of intellectual activity had shifted to the Muslim world. In this period a break through on the problem of disease was made in Bukhara, Persia. The methods used by the Persian scholars reveal an important aspect of creativity. Instead of using abstract or mathematical reasoning, they hit upon a simple (and to us today, obvious) methodology: systematic observation and quantification. To understand what causes disease, they carefully studied people who acquired the disease. What were they doing when they got affected? What activities were they engaged in and who were they with? What was the connection between one person and another who were both stricken by the same disease?

Figure 2. From cover of Ibn Sina's book

The results of this study were dramatic and are described in the monograph *Kitab Al-Qanun fi al-Toubb* or "The Canon of Medicine", one of the most famous "modern" books on medicine, by Abu Ali ibn Sina. This book explains that certain diseases are contagious and are transmitted through contact with an infected person. With this understanding, the way to limit the spread of a disease became clear – keep infected persons separate from the healthy populace as long as they are displaying symptoms. This gave rise to the practice, still in use today, of quarantine.

2.4. Mathematics

Problems in the areas of arithmetic, algebra and geometry appear rather abstract even today to the lay person, who is unlikely to understand the questions let alone their answers. But ancient Indian scholars successfully grappled with these subjects and obtained new and important results. One set of questions involves the solutions to a quadratic equation in a single unknown. This problem had been around since 2000 BC and progress had been made in stages across different cultures.

The developments of interest to us took place in ancient India due to the mathematician Brahmagupta and his colleagues in the 7th century CE, and to Bhaskara II in the 11th century CE. The methodology had been around for a while: precise statements and logical step-by-step proofs of mathematical theorems. But it was the Indian mathematicians, notably Brahmagupta, who discovered and explained the number zero. His successors built on this knowledge and developed theorems showing for example that a positive number always has two square roots. They found the general formula for the solution of a quadratic equation that is taught in high schools across the world today. The books *Lilavati*, *Bija Ganita* and *Siddhanta Shiromani*, by Bhaskara II, were the most famous mathematics texts of his time.

Here we encounter another intriguing aspect of creativity. The concept of zero was conceptually difficult to grasp. Apparently the ancient Greeks grappled with it but were baffled by the philosophical implications of "nothing" and gave up. However the Indian mathematicians accepted it and named it "shunya", borrowing the existing Sanskrit word for "nothingness" or the "void". This word and its significance in Hindu philosophy seem to have been determining factors in the acceptance of the idea of zero.

3. CREATIVITY IN THE "MODERN" AGE

In the last three or four centuries, the scientific method flowered as never before and its epicentre was undoubtedly Europe. Here too we can identify some of the socio-cultural factors that made discoveries possible. As befits a more mature stage of science, the discoveries made in this period were more complex, subtle and technical than the ones we have discussed above.

3.1. Laws of motion

The problem was to understand what laws govern the motion of bodies. This gave rise to the science of mechanics. Key developments took place in 17th century Europe. Careful observation played a role, for example in Galileo's famous experiments involving objects falling under gravity. By noticing that they accelerate equally regardless of their weight, he overturned a mistaken view held for centuries, that heavier objects fall faster. Apparently no one had bothered to test such a "self-evident" notion!

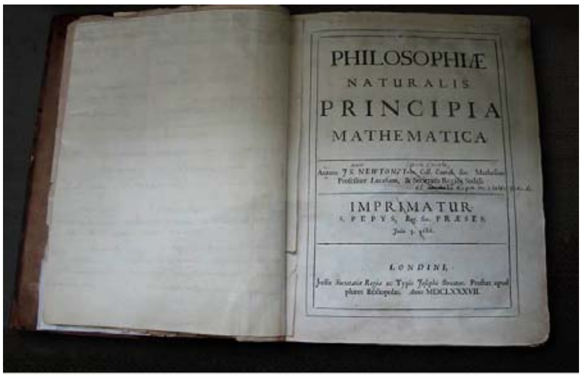

Figure 3. Newton's "Principia Mathematica"

But careful observation alone was not sufficient. Progress in technology, such as precision grinding of lenses, notably in colder countries like the Netherlands, gave rise to tools such as powerful telescopes. These in turn enabled new experiments. Additionally, thanks to Leibnitz and Newton, a new mathematical tool – calculus – came into being and played an invaluable role in building a solid theoretical foundation for the experimental observations. These developments gave rise to the first detailed theories of gravitation, optics and planetary orbits. They also presaged the birth of modern, quantitative astronomy.

3.2. Periodic table

From ancient times, attempts have been made to reduce all substances in nature to a few basic constituents. Early attempts amounted to some sort of guesswork (e.g. "earth", "air", "fire" and "water") that was not very well-defined and could not be substantiated. Then chemistry developed to a point where definite "elements" could be identified as the constituents of matter. It was at this time that Mendeleev, in 19th-century Russia, had the inspiration that led to a major breakthrough.

He started by arranging elements in order of their atomic mass. Soon, periodically recurring properties began to be evident. For example, both Sodium and Potassium are extremely reactive and soft metals though they differ in atomic mass. This suggested that the list of elements be drawn up as a table, with elements having similar chemical properties being placed one below the other.

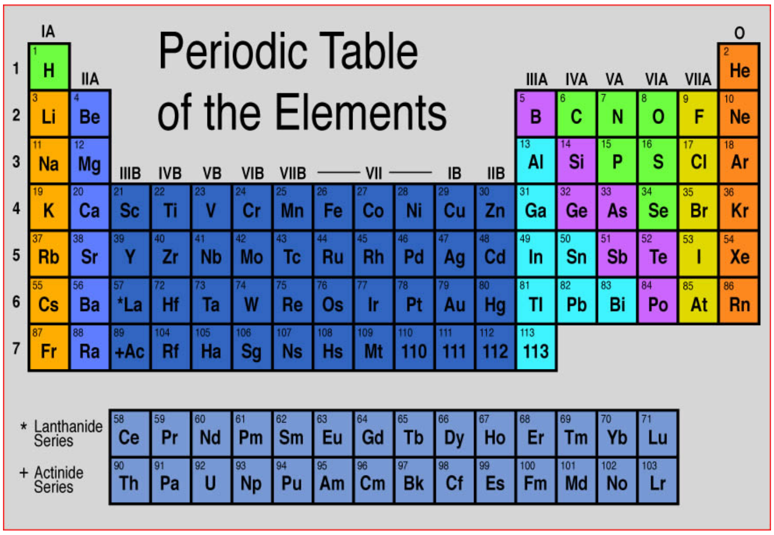

Figure 4. The Periodic Table of the elements

The table unambiguously suggested that atoms are made of more elementary particles. These eventually came to be known as protons, neutrons and electrons. The result was a revolution in chemistry, medicine and materials. We see that systematic careful and detailed study, sometimes mistakenly thought to be remote from the domain of insight and genius, was at the root of this creative breakthrough.

3.3. Natural selection

Just as the profusion of materials in nature demanded an explanation, so too did the profusion of species of plants and animals. How did so many distinct and diverse species of living creatures originate? Many people, believing in some myth of creation, thought this was not a valid question at all and doubted that it could possibly have an answer.

Charles Darwin thought differently. It was chance that took him on his voyages in the 19th century, but once he started to experience the diversity of life on the planet, he was driven to understand it. Like so many brilliant minds before him, he started off by performing experiments and taking careful notes during his travels. Eventually he came to hypothesize that species originated by natural selection. His novel idea, combined with earlier studies of heredity, provided a detailed understanding of diversity and adaptation.

3.4. Special relativity

The propagation of light had long been a puzzle. It was widely believed that waves needed a medium in which to propagate. Outer space appeared to be empty, but perhaps it was actually filled with a medium in which light propagated but which was itself very subtle and hard to detect. This hypothetical medium was called "ether". If this picture was correct then motion relative to the ether would change the velocity of light and this effect could then be observed. The puzzle deepened when experiments failed to detect any such change.

Despite these experiments, the scientific community was reluctant to accept the absence of ether. Early in the 20th century, Einstein provided a simple and brilliant solution to the puzzle. The first step was to accept what the experiments were telling us: there is no ether. He further suggested there is no preferred inertial reference frame. Applying additional physical insights through thought-experiments (relative motion, simultaneity) and performing some fairly simple calculations, Einstein arrived at his Special Theory of Relativity, changing forever our notions about mechanics and motion.

3.5. Quantum mechanics

New paradoxes emerged in the early 20th century from the experimental study of very small distance scales. Atoms are made of electrons orbiting around a nucleus, but such charged orbiting particles should lose energy and fall into the nucleus,which is not observed to happen. Also it was puzzling that some atoms are found to emit electrons when light shines on them, and in very specific ways.

It was natural to try and explain atomic behaviour using the same laws of physics that operate at familiar distance scales. But in early 20th-century Europe, many scientists started to realise that at small scales the world is not a mere copy of what we know, but differs in essential ways. In this field there was no individual scientist who proposed a complete and correct theory – instead the subject grew through a group effort involving several communities of scientists working independently and influencing each other's ideas.

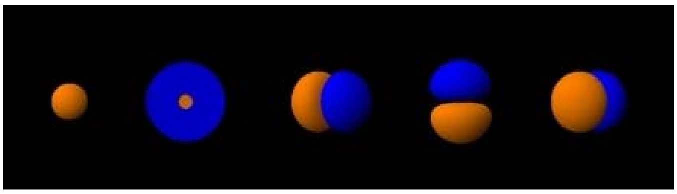

Figure 5. Atomic orbits as visualised in quantum mechanics (used under Creative Commons license)

As with Special Relativity, here too many precise and novel experiments guided theorists in their search. They in turn responded by proposing a radical new theory. It incorporated familiar notions of both mathematics and physics but in unusual ways. The two best-known examples are the wave equation due to Schrodinger, and the matrix mechanics proposal of Heisenberg. All this culminated in the theory of Quantum Mechanics, changing forever our notions about the subatomic world. The name "quantum" comes from the fact that in many systems – including atoms – energies turn out to be discretely spaced or "quantised" rather than being able to continuously vary.

4. IMPACTS ON CREATIVITY

Having surveyed a large number of examples, one can start to discern some features of the creative act. Here I would like to consider what features of society can influence people to be more, or less, creative.

4.1. Creativity can be encouraged.

"Discovery consists of looking at the same thing as everyone else and thinking something different." said Albert Szent-Gyorgyi, Nobel prize winner in medicine, 1937. A statement like this encourages each individual to believe she or he can really "make a difference" in the world of science. Creative discovery comes about not from some kind of genius that one is simply born with (or without) or a mountain of knowledge one has acquired, but rather it arises primarily from the desire – and capacity – to see differently.

4.2. Creativity can be stimulated.

"The best way to get a good idea is to get a lot of ideas." said Linus Pauling, winner of Nobel prizes in 1954 and 1962. To get two separate Nobels, one imagines he must have applied his own observation very seriously! The essential content of this quote is that one can consciously stimulate one's own creativity (again this contrasts with a commonly held notion that creativity is inherently unpredictable and cannot be influenced by one's behaviour patterns).

4.3. Creative ideas can also be discouraged.

"Heavier-than-air flying machines are impossible." said Lord Kelvin, then president of the Royal Society, 1895. It's fortunate that the Wright brothers did not pay attention to him. Sadly, history abounds with examples where extremely gifted and creative people end up giving discouraging advice to others. This suggests that young and ambitious scientists should take whatever advice they get from seniors with a pinch of salt!

5. QUANTUM CHROMODYNAMICS: DRAMA OF A CREATIVE SUCCESS

We now discuss one example of creativity in modern physics in some detail. This concerns the understanding sub-nuclear interactions. It has been believed since the early 20th century that the protons and neutrons inside an atomic nucleus are bound to each other through a powerful force. For lack of a better word, this force was dubbed the "strong interaction". Several other particles observed in nature also experience this force and they are collectively called "hadrons".

5.1. Quark Model (1964)

The properties of strongly interacting particles suggested they fit into tables analogous to the periodic table of elements. Following this analogy, Gell-Mann and Zweig proposed that hadrons are made up of smaller particles called quarks. Since hadrons have electric charge, so do quarks. Since hadrons are found to be spinning on their axis, quarks too must have spin. In quantum mechanics, all particles are allowed to possess a quantised spin. It was evident that, if they exist, quarks must have a $\frac{1}{2}$ unit of spin in units of Planck's constant. Thus quarks belong to the family of "fermions", particles with a half-integer value for their spin. Fermions in turn obey the "Pauli exclusion principle", according to which no two of them can be in an identical quantum state.

To account for the hadrons known in the 1960's, three species, or "flavours", of quarks were needed. They were labelled "up", "down" and "strange", these being merely convenient labels for certain abstract mathematical attributes. Later, as more hadrons came to be discovered, three more quark flavours were needed. These were called "charm", "top" and "bottom".

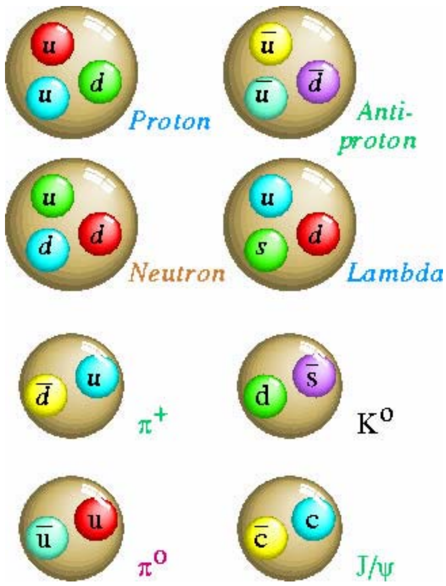

Figure 6. Various hadrons and their quark constituents

A puzzle soon arose for the quark model. A particular hadron, the Δ^{++} particle, can only be explained in terms of three up-quarks. Moreover their spins must be aligned, putting them in the same quantum state. But this contradicts the Pauli exclusion principle.

5.2. Colour (1965)

To circumvent this problem, Greenberg, Han and Nambu proposed that each quark species comes in 3 types labelled by an abstract "colour". Then the exclusion principle can be obeyed by putting them in different colour states even as their spins remain aligned. In this way the Δ^{++} particle could be explained without any evident theoretical contradiction.

Despite much criticism within the community, these scientists (along with the original proposers of the quark hypothesis and a few others) refused to get discouraged about the quark hypothesis or give up on it. Even after colour was proposed, questions persisted over whether quarks are real and some skeptics labelled them a mere "mathematical construction". It was not at all clear how this story was going to turn out.

5.3. Deep Inelastic Scattering (1968-70)

To probe nuclear sub-structure, experiments were perfomed where protons were "hit" by energetic electrons. The way in which the electrons were scattered suggested they were bouncing off point-like objects inside the proton. This strongly supported the quark model and the tangible physical reality of these quarks. These experiments earned J. Friedman, H. Kendall and R. Taylor the Nobel prize for physics in 1990.

However the same experiments – and subsequent ones – produced no evidence of free quarks. This was a new puzzle. When a powerful beam hits an atom, it knocks out electrons. But hitting protons completely failed to knock a quark out! So it seemed they were very tightly bound. At the same time, details of the scattering process suggested that each quark in a proton or neutron behaved largely independent of the others. This could only be the case if the different quarks were loosely bound to each other.

The situation was now absurdly contradictory: how could quarks inside protons be both tightly bound and loosely bound? Remarkably it turned out that both statements were true!

5.4. Asymptotic Freedom (1973)

The puzzle was eventually resolved, at least conceptually, through calculations by David Gross and Frank Wilczek and independently by David Politzer. They shared the 1999 Nobel prize for this work, in which they studied certain types of quantum theories where the force between quarks is mediated by unit-spin particles called "gluon". In these theories the force can grow stronger with increasing distance if a phenomenon called "anti-screening" holds.

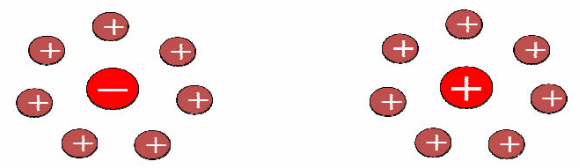

Figure 7. Screening and Anti-screening

In conventional screening, which takes place in conducting media, the force between charged objects decreases with distance faster than Coulomb's law would predict. This happens because mobile charge carriers crowd around a fixed charge centre and screen it from its neighbours. In anti-screening, mobile charges can instead enhance the charge of the fixed centre and cause it to effectively increase with distance. The relevance of anti-screening for the problem of quarks was clear. This would make it possible for nearby quarks to be weakly interacting, consistent with the theoretical interpretation of the scattering process, but cause the force between quarks to grow so large as their separation increases that liberating a quark would become impossible.

The above authors reduced the question of screening versus anti-screening to the sign of a mathematical quantity called the "beta-function", denoted β. If the sign of β is positive then screening holds, while if it is negative then anti-screening results. They then did a computation which showed that, if N denotes the number of quark "colours" and M denotes the number of "flavours", then:

$$\beta = -\frac{1}{16\pi^2}\left(\frac{11}{3}N - \frac{2}{3}M\right)$$

Now according to the quark hypothesis, $N = 3$ (three colours were needed to solve the paradox about Δ^{++}) and $M = 6$ (recall there were six flavours, from "up" all the way to "bottom"). Therefore β is negative, and anti-screening (now renamed "Asymptotic Freedom") does hold in the theory! Being a quantum theory for the dynamics of "coloured" quarks, it came to be known as Quantum Chromodynamics (QCD) because "Chromo" is the Greek root word for "colour".

Today, QCD is considered a highly successful theory of quarks and also gluons. It is solidly established and has passed repeated experimental tests. Nevertheless some questions within QCD still remain open. Despite anti-screening providing a clear physical mechanism, no one has rigorously proved that quarks are permanently confined. It is also believed that confinement holds only at low temperatures and normal matter densities. Scientists today believe that at high temperatures and densities, hadrons instead dissolve into a soup called "quark-gluon plasma". In this soup, quarks could actually be de-confined. Experiments at the Relativistic Heavy-Ion Collider (RHIC) have been probing this plasma and the Large Hadron Collider will teach us yet more about it.

One might think that quark-gluon plasma is a new and exotic state of matter. But it's not so new! This extremely hot and dense substance is what we believe the universe was like, microseconds after the Big Bang when the universe originated.

6. NATURE OF CREATIVE PEOPLE

The above episodes from the ancient and recent history of science give us many pointers to the nature of the creative process and thereby the nature of the people involved. These pointers can guide us, both young scientists beginning their careers as well as senior scientists who formulate policy, to create the conditions most favourable for creativity in our country's scientific establishments.

Curiosity: Creative people are driven by the urge to understand. They pursue their question with passion.
Belief in oneself: they are confident and willing to create new knowledge. They do not suffer from excessive modesty or diffidence or fear.
Enjoyment: Research is fun. Creative people are seen to be enjoying themselves in the pursuit of their science.
Precision: Meticulous, careful and precise observations are a hallmark of creativity.
Independence as well as collaboration: People who can work by themselves when necessary, but also work with others as part of a team and a community when that is called for, are most able to harness creative forces from around themselves.
Being informed: It's essential to carefully study what is already known, and to talk to people and find out what they are thinking. At the same time one should not be too easily guided/misguided by what others say.

But most of all, creativity is about being oneself. It is indeed the highest form of self-expression. And creative people are guaranteed a bright future because, no matter what we know today, there is always that much more remaining to be discovered and understood.

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Physics of Neutrino Mass, Mixing and Oscillation

P. Sruthi
2^{nd} yr. Dual degree BS + MS,
Indian Institute of Science Education and Research, Pune, India
NIUS Physics (Batch 6)

Abstract. In the Standard Model of Particle Physics, neutrinos are massless. However, recent breakthroughs in the field of neutrino physics has shown that neutrinos are indeed particles with small but non-zero mass and that they show leptonic mixing and oscillation. In this review, we discuss the model built in order to explain neutrino masses and also discuss the phenomenon of neutrino oscillations in vacuum.

Communicated by L. Satpathy

1. NEUTRINO MASS

In the Standard Model of particle physics, the matter fermions are spin half particles which include the quarks and leptons. These fermions occur either in right-handed or left-handed helicity states. If the direction of spin of the particle is along the direction of momentum, then it is a right-handed fermion and if the two directions are opposite each other, then it is a left-handed fermion. These fermions participate in one or more of the three interactions - strong, electromagnetic and weak. The strong and electromagnetic interactions are invariant under parity and charge conjugation transformation. Weak interactions break parity (**P**)maximally. This means that a left handed particle and a right handed particle do not show the same interactions. It implies that only the left-handed fermions carry isospin (I = 1/2), which is the gauge charge for weak interaction, whereas the right handed neutrino is an isospin singlet and hence doesn't interact weakly. Weak interactions also break charge conjugation (**C**) symmetry maximally. Under the operation of charge conjugation, a particle is turned into its anti particle. However, the *combined CP symmetry is conserved*. Therefore, *only the left-handed fermions and the right-handed anti fermions carry iso-spin 1/2* and participate in weak interactions whereas the *right-handed fermions and left-handed anti fermions carry isospin zero*.

Therefore, in the Standard model, we have three isospin doublets of left-handed leptons and quarks. The leptons are the electrons, muons and tau particles and their corresponding neutrinos. The neutrinos do not participate in strong or electromagnetic interactions. Only the left handed neutrinos (and right-handed anti neutrinos)have non-zero isospin and participate in weak interactions.

Since there is no interaction that the right handed neutrino and the left handed anti neutrino can participate in, there is no need for them in the Standard Model and hence the SM does not contain any.

Now, we discuss the general mechanism by which fermions acquire mass, according to the Standard Model.

The wave functions of fermion and anti-fermion are written as a superposition of that of left-handed and right handed fermion and anti-fermions respectively.

$$\psi = \psi_L + \psi_R \qquad \bar{\psi} = \bar{\psi_L} + \bar{\psi_R} \tag{1}$$

The mass term in the Lagrangian ($m\bar{\psi}\psi$) can be expanded as,

$$m_f\bar{\psi}\psi = m_f\bar{\psi_L}\psi_L + m_f\bar{\psi_R}\psi_L + m_f\bar{\psi_L}\psi_R + m_f\bar{\psi_R}\psi_R \tag{2}$$

In the more rigorous language of Quantum Field Theory, $\bar{\psi_R}$ and ψ_L represent the field operators for the absorbing of left-handed anti-fermion and left-handed fermion, or equivalently creating of right-handed fermion and right-handed anti fermion respectively (since the absorption of a left handed antifermion is equivalent to emission of its antiparticle, i.e. a right handed fermion). The terms in the above Lagrangian can be represented as follows:

$$\bar{f}_{L(R)} \qquad \stackrel{\Leftarrow(\Rightarrow)}{\longrightarrow} \qquad \stackrel{\Rightarrow(\Leftarrow)}{\longleftarrow} \qquad f_{L(R)} \tag{3}$$

where f represents a fermion, $\bar{f}$ represents an anti fermion and the subscripts L and R mean left handed and right handed respectively. The two long arrows give the direction of momentum in the center of mass frame(that is, direction of motion). The arrows on top of these two represent the orientation of spin. Orbital angular momentum has no component in the direction of motion since it is **r** x **p**. Non-zero spin in the direction of motion means non-zero total angular momentum. We can see that only the LL and RR terms in the above illustration are allowed. The LR and RL terms are not allowed since such terms have a non-zero total angular momentum. Non-zero spin term in the Lagrangian means it has a preferred direction and hence not rotationally invariant in ordinary space. And it is the rotational invariance that results in the conservation of angular momentum. Hence for the conservation of angular momentum to hold, only the second and third terms in the Lagrangian given in (2) are allowed.

However, a mass term like this carries a total isospin of $1/2$ since $f_R(\bar{f}_L)$ has isospin 0 while $f_L(\bar{f}_R)$ has isospin $1/2$. But isospin conservation requires Lagrangian to have zero total isospin. A non-zero isospin means that the Lagrangian has a preferred direction in isospin space and hence not invariant under rotation in this space, just like how a non-zero spin term in the Lagrangian means that it has a preferred direction and hence not invariant under rotation in ordinary space. Therefore, the above mass term breaks the isospin gauge invariance of the Lagrangian.

So, fermions cannot possess a bare mass and their mass arises from their Yukawa coupling to the isospin doublet of Higgs boson, which acquires a vacuum expectation value by spontaneous breaking of isospin gauge symmetry.

The scalar field (ϕ) can be written as the sum of the physical Higgs field, h (Higgs boson) and the background Higgs field present in the vacuum, $< \phi >$. $\phi = h + < \phi >$. It is the coupling to this background Higgs field that gives mass to the particles. In case of fermions, it is through Yukawa coupling given as below:

$$y_f \phi f_L \bar{f}_R = \underbrace{y_f < \phi >}_{m_f} f_L \bar{f}_R + y_f h f_L \bar{f}_R \tag{4}$$

where m_f is the mass of the fermion and y_f is the yukawa coupling.

The first term is the mass term and the second one represents the coupling of fermion to the physical Higgs boson. The mass thus aquired by the Higgs mechanism is called the Dirac Mass for quarks and leptons.

Similarly, the mass of the gauge boson (W) and its coupling to the physical Higgs boson follow from the gauge interaction term in the Lagrangian:

$$g^2 \phi \phi W^+ W^- \Rightarrow \underbrace{g^2 < \phi >^2}_{M_W^2} W^+ W^- + g^2 < \phi > h W^+ W^- \tag{5}$$

where g is the gauge coupling constant. From the above equation, we have

$$M_W = < \phi > g \tag{6}$$

From equations (4) and (6), we see that the heavier fermions and gauge bosons couple more strongly to the physical Higgs boson. From the experimentalvalue of the gauge boson mass ($M_W = 80GeV$) and the gauge coupling ($g \sim 0.6$), the vacuum expectation value of Higgs field $< \phi > \sim 100GeV$. Thus the perturbative limit on the Yukawa coupling ($y_f =< 1$) implies that all the fermion masses are less than or in the range of 100 GeV.

A short note here about the dimensions of the fermion and boson fields is in order. The Lagrangian density has mass dimensions of Energy/Volume. In natural units, the mass dimension is 1 for energy and -1 for length. Therefore, each term of the Lagrangian, including the mass term has a mass dimension of 4. For a fermion, the mass term is $m_f \bar{\psi} \psi$, so that the fermion field has dimension 3/2. On the other hand a boson mass term is $m^2 \bar{\phi} \phi$; so that the boson field has dimension 1. It is a revolutionary hypothesis of Dirac to ascribe half integral dimension to spin 1/2 fields, which leads to antisymmetric wave function, anti-commutation relations, Fermi-Dirac statistics, Pauli exclusion principle. This principle underlies not only physics but practically the whole of chemistry.

Finally, the physical Higgs boson mass comes from the quartic self coupling term of the Higgs field in the Lagrangian:

$$\lambda \phi \phi \phi \phi \Rightarrow \underbrace{\lambda < \phi >^2}_{M_h^2} h^2 \tag{7}$$

Now, from equation (6) and (7), we get

$$M_h^2 = \lambda M_W^2 / g^2 \tag{8}$$

We can observe that the mass of the Higgs boson is just the mass of the gauge boson times the ratio of the self coupling to the gauge coupling. By plugging in the value of the gauge coupling (g), mass of the W boson (M_W), along with a perturbative limit on λ ($\lambda \leq 1$)we get an upper limit of a few hundred GeV for the mass of the Higgs boson. This Higgs boson is expected to be detected in the LHC experiment.

This way, particles acquire mass through their interaction with the background Higgs field present in the vacuum. This interaction of a fermion/gauge boson passing through a background Higgs field can be easily understood by taking an analogy of what happens in a Oscar ceremony. In such a ceremony, there will be many people gathered and spread around the entire hall. This congregation can be thought of as the background Higgs field that is 'all-prevalent'. Suppose a Oscar Award winner passes through this congregation, then many people will gather around her. As she moves forward in the room, new set of people start accumulating around her but nonetheless, she is always surrounded by a big group of people. This cluster of people slows her down. Equivalently, we can say, she has thus aquired 'mass'. A smaller actor will have fewer people around him. In the same way, a particle passing through the Higgs field will be 'clustered around' due to its interaction with the Higgs field. This restricts the movement of the particle or in other words, there is inertia i.e., mass is acquired. Now, how much mass is aquired depends on how strongly the particle interacts with the background field.

Since in the standard model right handed neutrinos and left handed anti neutrinos don't exist at all, there can be no neutrino mass. However, we can extend the Standard model to include an isospin singlet (right handed neutrino and a left handed anti neutrino). By doing this, we are not breaking any basic symmetry of the model. Since this particle has no gauge charge, the only way it can be distinguished from its anti particle is through their helicity and lepton number. Such particles are called Majorana Particles. We can add a mass term of the form $M\psi_R\psi_R$ (representing the absorption of two right handed neutrinosor transformation of right handed neutrino into left handed anti neutrino) and it won't break any gauge symmetry. Of course a term like this will have a total lepton number of 2. Lepton number is not conserved in such a process. However, lepton number is not a gauge quantum number and hence need not be conserved. So, such particles attain a mass called Majorana mass (M). Now that we have included the right handed neutrino into the model, the neutrinos can also have a Dirac mass like any other fermion. Unlike the Dirac Mass, the majorana mass can be much larger than 100 GeV since it does not break any gauge quantum number. Note here that a left handed neutrino and its anti neutrino has an isospin $1/2$ therefore, we cannot give an analogous mass term for the left handed neutrino (since that would mean that the term has total isospin of 1). Thus in the basis of the singlet and doublet neutrinos, we have a 2x2 mass matrix

$$\begin{bmatrix} 0 & m \\ m & M \end{bmatrix} \tag{9}$$

in which the first term is the mass associated with the LL term in the Lagrangian, the second and third (m) are for the LR terms and the last one (M) is that for the RR term in the (mass term of) Lagrangian.

The eigenvalues of this matrix (which form the diagonal elements of the matrix when it is diagonalised) are:

$$\frac{M}{2}(1 \pm \sqrt{1 + \frac{4m^2}{M^2}}) \tag{10}$$

Assuming that $M \gg m$, the diagonal elements will be,

$$a \approx \frac{M}{2}(1 - (1 + \frac{1}{2}\frac{4m^2}{M^2})) = -\frac{m^2}{M} \tag{11}$$

and

$$b \approx \frac{M}{2}(1 + (1 + \frac{1}{2}\frac{4m^2}{M^2})) = M(1 + \frac{m^2}{M^2}) \approx M. \tag{12}$$

So we can now write the mass matrix as

$$\begin{bmatrix} \frac{m^2}{M} & 0 \\ 0 & M \end{bmatrix}. \tag{13}$$

Thus we obtained the masses of these two mass eigenstates. The first of the two diagonal elements of the diagonalised mass matrix is: $a = m^2/M$ and under the assumption that $m \ll M$, this turns out to be a very small number. Thus, the standard doublet neutrino has aquired a tiny mass.

The mechanism discussed above is called the **See-saw mechanism**. The larger the majorana mass, the smaller the mass of the doublet. (One particle becomes light as the other becomes heavy see saw effect).

We know that a Dirac fermion mass is $m \leq 10^2$GeV. Experimentally, we know that the mass of a neutrino, i.e., $\frac{m^2}{M} \approx 10^{-2}eV = 10^{-11}GeV$. This implies,

$$M \approx 10^{15}GeV. \tag{14}$$

Majorana mass term is that which results from Lepton number non-conservation. In an extension to the Standard model, called Grand Unified Theory (in which, at high energies, all the three interactions weak, strong and electromagnetic will have the same strength and the 'merged' interaction is characterised by a single gauge symmetry), lepton number is taken as a gauge quantum number. Then, Majorana mass term will represent the symmetry breaking scale of GUT just as the Dirac mass represents the symmetry breaking at the Standard model scale.

2. NEUTRINO MIXING

Since neutrinos have mass, their flavor eigenstates need not coincide with mass eigenstates. Instead, there will be mixing. That is, a flavor eigenstate will be a 'mixture' of more than one mass eigenstate and vice versa. So, at any time, a neutrino of a given flavor will be made up of components of all the masses. This mixing is characterised by three mixing angles and a phase angle. However, to a good aproximation it is enough to consider mixing between two neutrino states for which we require only one mixing angle. That is,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{15}$$

where, ν_1 and ν_2 are the mass eigenstates with masses m_1 and m_2. The phenomenon of mixing is a purely quantum mechanical phenomenon. A mixed state

$$\nu_e = \cos\theta\nu_1 + \sin\theta\nu_2 \tag{16}$$

violates energy conservation because of the following reason. Suppose we look from the rest frame of the first mass eigenstate (which we assume to be of the lower mass), then the second mass eigenstate obviously has a greater rest mass energy. However, in quantum mechanics, this problem is taken care of by the Uncertainty principle according to which you can take up additional energy as long as you give it back within a certain period of time. In case of neutrinos, the mass difference involved is so small that a quantum mechanical coherent admixture of unequal mass eigenstates can be maintained over long time scales. This mixing leads to the phenomenon of neutrino oscillation.

3. NEUTRINO OSCILLATION (IN VACUUM)

Consider a ν_e produced in a nuclear beta decay. It will be composed of two components - one of ν_1 and the other of ν_2. Now, since both of these states have different masses, they will travel with different speeds. Their relative sizes changes with the distance which means transformation of ν_e into a ν_μ. This is the phenomenon of Neutrino Oscillation. We shall delve into the mathematical and other details in the paragraphs to follow.

Neutrinos of definite mass, m and momentum, p do not travel as point particles due to uncertainty principle. They travel as a plane wave with the wave function (expressed in natural units where $c = \hbar = 1$) :

$$\psi = e^{px-Et}. \tag{17}$$

Moreover, neutrinos are extremely relativistic particles. This can be seen because their energies ($\sim MeV$) are much much greater than their masses ($< 1eV$). Hence they travel with very high speed (almost equal to the speed of light) Therefore, we can say,

$$t \approx x \tag{18}$$

Also, we know that $E = \sqrt{p^2 + m^2}$
$\Rightarrow E = p\sqrt{(1 + \frac{m^2}{p^2})} \approx p(1 + \frac{m^2}{2E^2})$. So,

$$E = p + \frac{m^2}{2p} \approx p + \frac{m^2}{2E}. \tag{19}$$

Put equations (18) and (19) in (17). The neutrino mass eigenstates propagate with a phase of $e^{-i\frac{m^2x}{2E}}$. So, after travelling a distance x, the wavefunction will become:

$$\nu_e \longrightarrow \nu_1 \cos\theta e^{-i\frac{m_1^2 x}{2E}} + \nu_2 \sin\theta e^{-i\frac{m_2^2 x}{2E}} \tag{20}$$

Now, writing ν_1 and ν_2 in terms of ν_e and ν_μ, using

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{21}$$

we see that there will be a non-zero coefficient of ν_μ in the wavefunction. The coefficient of ν_μ is $\cos\theta\sin\theta(-e^{-i\frac{m_1^2 x}{2E}} + e^{-i\frac{m_2^2 x}{2E}})$, so that the probability of a ν_e converting into a ν_μ is

$$P_{e\mu}(x) =\mid \cos\theta\sin\theta(-e^{-i\frac{m_1^2 x}{2E}} + e^{-i\frac{m_2^2 x}{2E}}) \mid^2 \tag{22}$$

Take $\theta_1 = m_1^2 x/2E \qquad \theta_2 = m_2^2 x/2E$

Now, $\mid -e^{-i\theta_1} + -e^{-i\theta_2} \mid^2 =\mid (\cos\theta_2 - \cos\theta_1) + i(\sin\theta_1 - \sin\theta_2) \mid^2 = 4\sin^2(\frac{\theta_2-\theta_1}{2})$

Therefore,

$$P_{e\mu}(x) = \sin^2 2\theta \ \sin^2(\Delta m^2 x/4E) \tag{23}$$

where, $\Delta m^2 = m_2^2 - m_1^2$. Converting to more convinient units where Δm^2 is in eV^2, x is in meters, E is in MeV, (use the conversion factor $1m = 5.07 \times 10^{15} GeV^{-1}$) we have,

$$P_{e\mu}(x) = \sin^2 2\theta \sin^2(\frac{1.3\Delta m^2}{E}x) = \sin^2 2\theta \sin^2(\frac{\pi x}{\lambda}). \tag{24}$$

The above equation describes neutrino oscillation where, the first factor gives the amplitude of oscillation and the second term gives the phase. From this phase factor, we can get the wavelength of neutrino oscillation, which is

$$\lambda = \pi/(\frac{1.3\Delta m^2}{E}) = 2.4E/\Delta m^2. \tag{25}$$

The surviving probability of the ν_e is,

$$P_{\nu_e \to \nu_e} = 1 - P_{\nu_e \to \nu_\mu}. \tag{26}$$

The following inferences can be made from the above:

For large mixing angles, $\sin^2 2\theta \approx 1$. If $x \ll \lambda$, then from equation (24), $\sin(\frac{\pi x}{\lambda}) \approx 0$ and $P_{e\mu}$ is zero implies survival probability is 1. If $x \sim \frac{\lambda}{2}$, $P_{e\mu} \sim \sin^2 2\theta \sin\frac{\pi}{2}$ and therefore,$P_{e\mu} \sim \sin^2 2\theta \approx 1$. Finally, if $x \gg \lambda$, we get $P_{e\mu} = (1/2)\sin^2 2\theta \approx 1/2$, where the factor of 1/2 comes from averaging over $\sin^2(\frac{\pi x}{\lambda})$. Therefore, when $x \geq \lambda/2$, we can find out the values of the mixing angles and Δm^2 from neutrino oscillation experiments.

Various solar and atmospheric neutrino oscillations and reactor neutrino experiments have been carried out over the past few years and yielded values of the mixing angle θ, wavelength of oscillation and mass squared gaps(Δm^2). For solar or reactor neutrino experiments, the source of ν_e is nuclear reaction and the energy of the outcoming neutrinos is of the order of 1 MeV. For solar neutrino experiments, the value of x is the distance between earth and sun ($x \sim 10^{11}m$). Therefore, it can probe values of Δm^2 down to $10^{-11}eV^2$. The distance between the source and detector for a long baseline experiment like KamLAND is about 10^5m and hence it can probe neutrino masses down to $\Delta m^2 = 10^{-5}eV^2$.

The typical energies of accelerator and atmospheric neutrinos is of the order of 1GeV. We can use equation (24) again and this time measuring x in km. For a long baseline accelerator neutrino experiment like MINOS ($x \sim 10^3km$), we can probe values of Δm^2 down to $10^{-3}eV^2$. For atmospheric neutrinos traversing the earth, the value of x is given by the diameter of the earth (so, $x \sim 10^4km$). Therefore, we can probe neutrino masses down to $\Delta m^2 = 10^{-4}eV^2$.

All the above mentioned experiments show definitive evidence for neutrino masses of the order of a few percent of an eV. The tiny neutrino mass can give new insights into some entirely new physics. As mentioned earlier, the neutrino and Majorana mass help in probing energies at the GUT scale. Therefore, the small neutrino mass is indicative of physics at the high energies of $M \geq 10^{10}GeV$. Also, if neutrinos turn out to be Majorana particles, then that would mean entirely new physics beyond the Standard Model.

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P R A Y A S
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The Doppler Effect in a Circular Motion

Rohit Gupta [1] and Parijat Sarkar [2]
III yr. Int. M.S. Indian Institute of Science Education and Research (IISER) - Kolkata, Mohanpur Campus, PO: BCKV Campus Main Office, Mohanpur - 741252, Nadia, West Bengal, India
[1] Major Chemical Sciences, [2] Major Biological Sciences

Abstract. The equation for the Doppler effect for a sound source moving in a circular motion was derived and verified by a simple experimental setup. The observer was kept at rest and the velocity and the radius of circular path of sound source were also kept constant. The frequency of the sound waves produced by the source was studied using the spectrogram software. The data obtained by analysis of sound waves by the spectrogram software was in perfect agreement with the theoretical data.

Keywords. Doppler effect, frequency, circular motion

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1. INTRODUCTION

The speed at which waves travel in a medium is usually independent of the velocity of the source. Thus, if a pebble is thrown into a pond with a horizontal velocity, the resulting water waves will travel radially outwards from the centre of disturbance in the form of concentric circles, with a speed which is independent of the velocity of the pebble that caused them. When we have a moving source, producing sound waves continuously as it moves, the velocity of the waves is often unchanged, but the wavelength and frequency, as noted by a stationary observer, may be altered. However, the velocity is changed, when there is dispersion. This phenomenon of the change of frequency, when a source is moving is known as the Doppler effect. The effect is equally well observed, if the observer is moving instead of the source, or if both are moving.

2. THEORY

Let the receding/approaching velocity of the sound source be V_D. It will be positive if the sound source is receding and negative if it is approaching the observer.If the observer is at rest, the relationship between f (the apparent frequency due to motion of source) and f_0 (frequency of the source, while there is no relative velocity between the observer and the source) will be given by the well-known Doppler effect equation: [1–3].

$$f = f_0(\frac{v_s}{v_s + V_D}) \tag{1}$$

where v_s is the velocity of the sound in air.

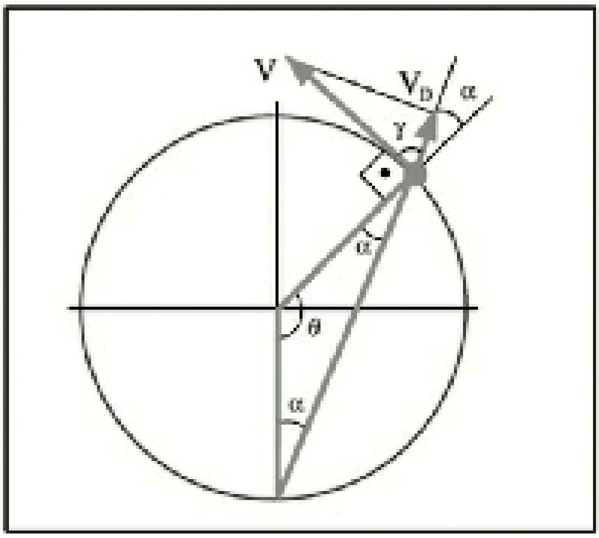

Figure 1. Schematic geometry to calculate the approaching and receding speeed.

In order to find V_D as a function of time we need to make some geometrical consideration on the basis of figure 1,

$$V_D = V\, cos\, \gamma \tag{2}$$

where V the tangential velocity of a buzzer, see figure 2 and γ is the angle between V and V_D . If T is the time period of rotation of the buzzer and R the radius of the circle, then

$$V = \frac{2\pi R}{T} \tag{3}$$

which yields

$$V_D = \frac{2\pi R\, cos\gamma}{T} \tag{4}$$

Observing the angles in figure 1 we find that $\theta + 2\alpha = \pi$, $\alpha + \gamma = \frac{\pi}{2}$ and $\theta = \frac{\gamma}{2}$. For a constant , $\theta = \frac{2\pi t}{T}$, which makes $\gamma = \frac{\pi t}{T}$. Direct substitution into Eq. (2) yields:

$$V = \frac{2\pi R}{T} cos(\frac{\pi t}{T}) \tag{5}$$

and one finally obtains,

$$f = f_0 \frac{v_s}{v_s + \frac{2\pi R}{T} cos(\frac{\pi t}{T})} ....Eq.(3) \tag{6}$$

3. EXPERIMENTAL SET UP

In order to observe the Doppler effect for a circularly moving sound source, the apparatus shown in figure 2 was set up.

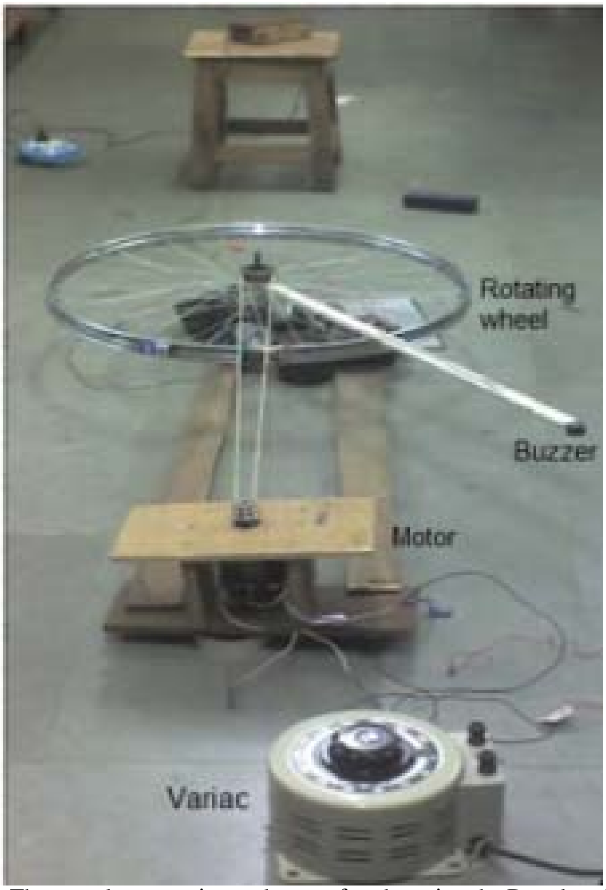

Figure 2. The complete experimental set up for observing the Doppler effect in circular motion

A motor controlled by a variable ac power supply (variac) spins a cycle wheel. On this cycle wheel another plastic bar was attached to produce a larger diameter for the circular motion. At the end of the bar, one buzzer was attached. The power supply to the buzzer was provided by a constant dc source (9V). Initially two 9 V batteries were used for the power source for the buzzer, but it was found that the power of the batteries were decreasing and as a result the frequency of the buzzer was also decreasing, which would have led to wrong conclusions. So we switched to constant ac power supply but providing a constant ac source to a rotating wheel was somewhat difficult. It was implemented by attaching one wire to the portion on the wheel which was free from rotation, another wire was attached to a copper plate which was made to have contact with the wheel via a non conducting tape. The bar along with the cycle wheel was set to spin at a constant angular speed by the help of the regulator and the sound was recorded by an mp3 recorder. The mp3 recorder was placed beneath the buzzer. The sound of the buzzer at rest was also recorded for reference. The sound produced by the moving source was captured as a .wav file and was then analyzed in the spectrogram software program. The spectrogram software used in our study displays the audio signal as a frequency versus time plot with signal amplitude at each frequency represented by intensity (or colour).

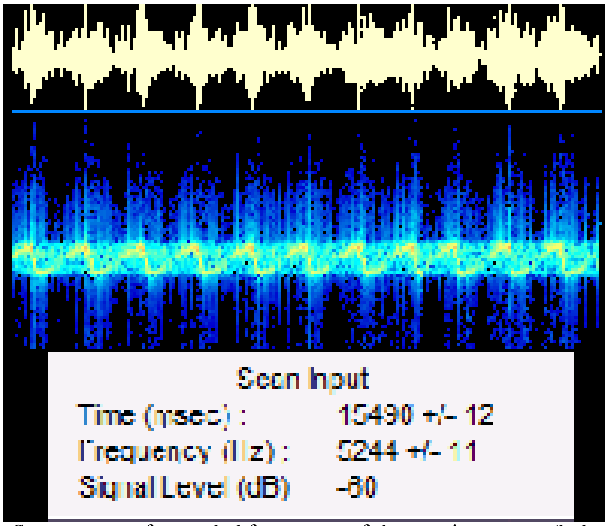

Figure 3. Spectogram of recorded frequency of the moving source(below) and source intensity (above)

Also, a continuous readout of time (ms), frequency (Hz), and signal level (dB) at the position of the mouse pointer (cursor) allows an easy sampling of the frequencies with maximum signal level. Figure 3 shows a typical display given by the spectrogram software [4]. The recorded frequency of the buzzer increases to its maximum value when it approaches the microphone (maximum approaching speed),and it abruptly decrease to its minimum value just after the buzzer passed over the microphone (maximum receding speed). When the buzzer is located opposite to the microphone, there is no movement toward or away from it. Therefore, the recorded frequency (f) was the same as that produced by the buzzer at rest (f_0).

4. MEASUREMENTS

For measuring the f_0 of the buzzer we used cobra 3 basic unit [5]. But for measuring f the sound was recorded by mp3 recorder and was then analyzed using the spectrogram software. Time period (T) was measured using a stop watch and analyzed by the receiver available in the lab, but it did not show the kind of graph one must obtain. So we had to again switch to our mp3 recorder setup.

5. OBSERVATION

Theoretically one should obtain the plot of apparent frequency (f) versus time (t) as shown in figure 4, where T is the time period of revolution.

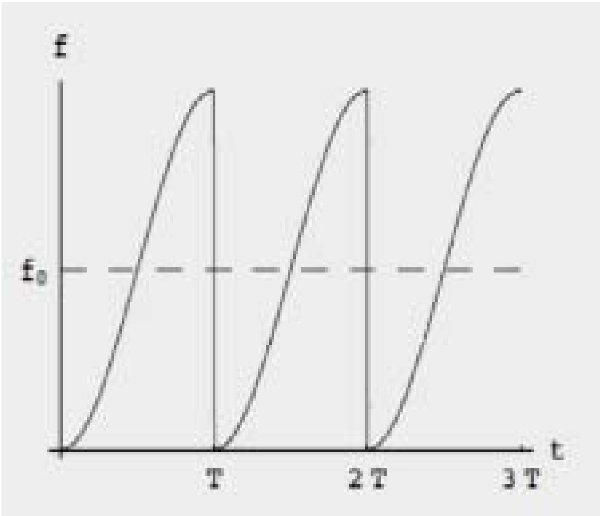

Figure 4. Theoretical plot of apparent frequency(f) versus time (t)

The frequency (f_0) of the buzzer was found out to be 8437 Hz. This was later verified by the spectrogram software. The gradual decrease in the graph is due to switching off of the power supply,see figure 5. The data obtained with the normal frequency receiver available in the lab was not perfect at the beginning when battery was used as a power supply (the frequency counter was set at 0.2 sec). Then the experiment was performed using a dc power supply to the buzzer. The frequency counter was set at 0.1 sec. The graph obtained was somewhat close to the expected result, see figure 6. However, this was discarded as the intensity of the sound produced by the buzzer was not constant.

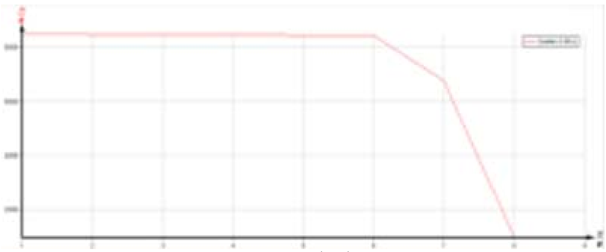

Figure 5. Graph for the frequency at rest (f_0) (Frequency versus number of observations)

So,the average time period is 0.556 sec.

Table 1. Calculating the time period of revolution of the buzzer.

No. of observations	Time taken for 10 revolutions (sec)	Time period (sec)
1	5.57	0.557
2	5.59	0.559
3	5.49	0.549
4	5.56	0.556
5	5.57	0.557
6	5.59	0.559

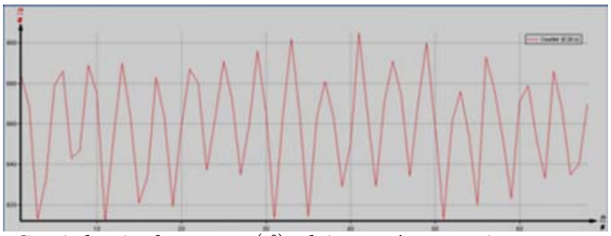

Figure 6. Graph for the frequency (f) of the moving sound source versus number of observations taken from a normal sound receiver

Subsequently, the analyses were done using spectrogram software and the expected results were obtained, see figure 7. From graph the periodic motion was very much clear, and using the theoretical concept one can clearly tell from the graph the position of the buzzer with respect to the observer.

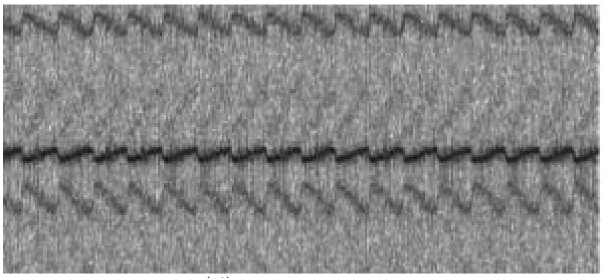

Figure 7. Graph of frequency (f) versus number of observations of the moving source as recorded by the spectrogram

6. RESULTS

Time period of revolution using the spectrogram was found to be 0.564 sec(Table 1), which is quite close compared to the time period measured manually.The periodicity of apparent frequency was (f) clearly seen through the help of the Spectrogram software. The value of f_0 was found to be 8300 Hz. The value of f_{max} and f_{min} (with R= 0.77m) were calculated from the modified Doppler relation and verified from the spectrogram. The apparent frequency f depends upon the velocity of the buzzer and the radius of the circular path.If the radius of the circular path increases then there was a significant change in apparent frequency. In conclusions, it proved to be a good qualitative exercise which helped in extending the well known Doppler formula for the circular motion. The increase in value of R would have led to better results as the frequency change would become much more significant. If one can fix a function generator or any circuit which can be regulated to produce a wide range of frequency then the observations would be more accurate.

In future the project can be extended like changing the radius of motion, effect of medium, oscillatory source etc.

Acknowledgements

We would like to acknowledge Dr. Bipul Pal, Dr. Chiranjib Mitra, Dr. Dhananjay Nandi and Dr. Satyabrata Raj for their continuous help and support.

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P R A Y A S
Students' Journal
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Supercritical Bifurcation: A mechanical model and analogy with phase transition

Rakhee Sharma
Ist Yr. M.Sc., Physics Department, N.I.T. Rourkela, India.

Abstract. We consider a simple mechanical model which exhibits supercritical pitchfork bifurcation. We further discuss symmetry preserving and symmetry broken phases of the model. An analytical time dependent solution is then constructed which interpolates between these two phases. We also discuss the similarities of our model with coninuous phase transition. We argue that an analogue of critical exponents in our model can be constructed via a suitable Landau like expansion of the potential.

Communicated by L. Satpathy

1. INTRODUCTION

Bifurcation theory explains many natural phenomena. The purpose of this project is to analyse a simple mechanical model which exhibits supercritical bifurcation. It also shares some features of symmetry breaking and continuous phase transition.

The phenomena of bifurcation has one common cause: a specific physical parameter crosses a threshold and consequently it forces the system to organize itself to a new state. This specific state differs significantly from the original one. The states of a system generally correspond to the solutions of a nonlinear equation. A state can be observed if it is stable. However, if that state loses its stability when a parameter reaches a critical value, then the state is not observed. The system then generally organizes itself to a new stable state – causing a bifurcation from the original one.

A simple example of bifurcation is known as pitchfork bifurcation. Here the solution of the nonlinear equation bifurcates in pairs and generally the bifurcating state has less symmetry than the original one – often called a symmetry broken state. The simplest of such example is described by the solutions of the following equation:

$$x^3 - \lambda x = 0, \tag{1}$$

where λ is a parameter and x is real. Fo $\lambda \leq 0$, there is only one solution $x = 0$. However, for $\lambda > 0$, two new solutions appear at $x = \pm\sqrt{\lambda}$. It is possible to construct a 'potential' whose extrimization

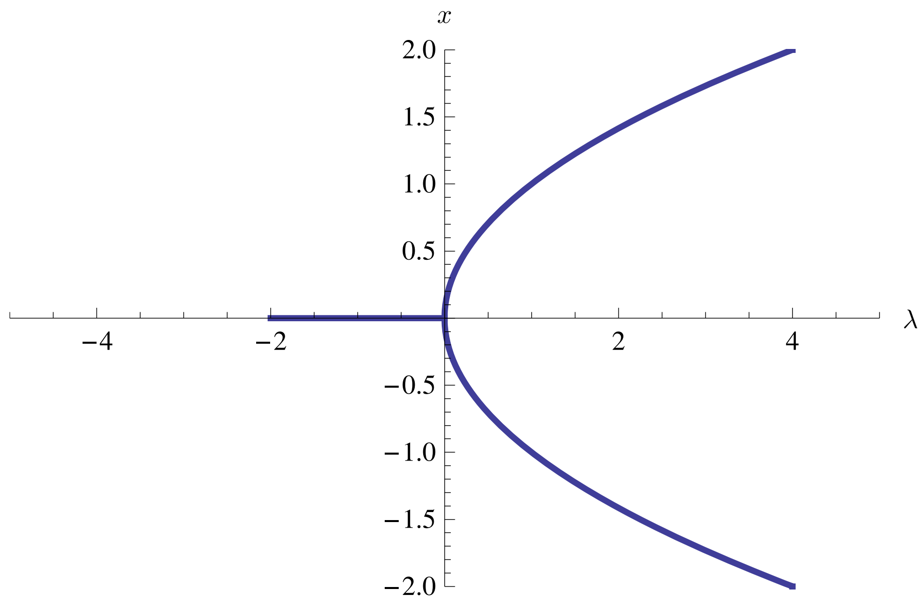


Figure 1. Solutions bifurcate in pairs when λ becomes positive

gives (1)

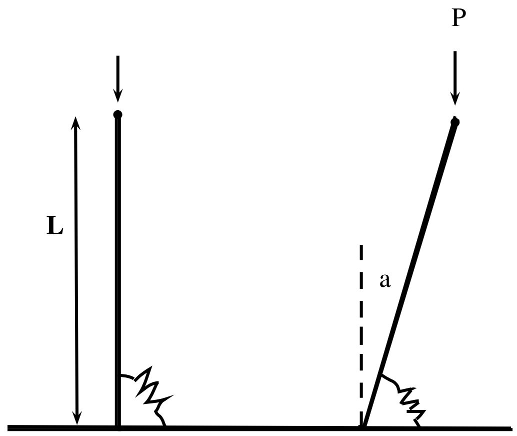
$$V(x,\lambda) = \int (x^3 - \lambda x)dx = \frac{1}{4}(x^4 - 2\lambda x^2). \tag{2}$$

Note that V is symmetric under $x \to -x$. From the structure of the potential, one immediately realises that for $\lambda < 0$, V minimizes at $x = 0$. We call this a symmetric phase. However, this ground state becomes unstable when $\lambda > 0$ and two new minima appear at non-zero values of x symmetrically around $x = 0$ line. As soon as the system reaches one of this states, the $x \to -x$ symmetry gets broken. We call this a symmetry broken phase.

Our purpose is now to introduce a mechanical model which shows such bifurcation. This is what we do in the next section. Subsequently, we study time dependent solutions associated with the crossover from symmetric phase to the symmetry broken phase. We then study as to how our model brings out a simple analogy with second order phase transition. Second order phase transition appears in many thermodynamic systems - ferromagnetic material losing its magnetization with the increase of temperatue is one such example. We end our project with a discussion of our results.

2. THE MODEL

The system is an inverted pendulum where a rigid rod of negligible mass is pivoted about its lower end with a torsion spring [1]. This spring provides the restoring torque proportional to the angular displacement from the equilibrium [1]. See the figure for the details of the system.



A load of weight P is applied vertically on the top of the rod. The equation of motion that this rod of length L has to satisfy can be found from torque ballance equation. This is given by

$$I\frac{d^2a}{dt^2} = -\kappa a + PL\sin a, \tag{3}$$

where I is the moment of inertia of the rod about the axis of rotation and κ is the spring constant. The stable position, the right hand side of the above equation has to be zero. Hence the positions can be found from

$$\kappa a - PL\sin a = 0, \tag{4}$$

or equivalently,

$$\frac{\kappa}{PL}a - \sin a = 0. \tag{5}$$

We define $\frac{\kappa}{PL} = \tilde{\kappa}$. It can be shown that, besides $a = 0$, this equation always has a pair of non-zero solution for a when $\tilde{\kappa} < 1$. However, for $\tilde{\kappa} > 1$, we have $a = 0$ as the only solution. Note that $\tilde{\kappa}$ can be reduced by simply increasing the load P. Let us increase the load P such that it is just above $P_c = \kappa/L$. Now for this, since $\tilde{\kappa}$ is only a little less than one, we expect that non-trivial solutions of a will be close to zero. Hence the term $\sin a$ can be expanded in powers of a. This leads to

$$-(1-\tilde{\kappa})a + \frac{a^3}{6} = 0, \tag{6}$$

where we have neglected the heigher powers of a. This leads to the solution $a_\pm = \pm\sqrt{6(1-\tilde{\kappa})}$ besides the trivial one $a = 0$. We now note the similarity between equation (1) and (6). The role

[1]There are many other mechanical models which show similar bifurcations, see for example [2, 3].

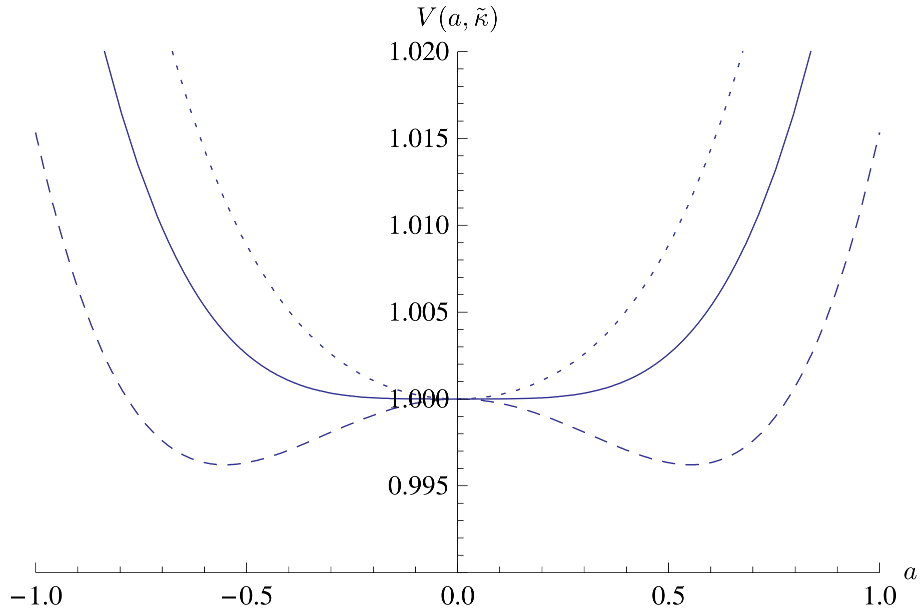


Figure 2. $V(a, \tilde{\kappa})$ for differnt values of $\tilde{\kappa}$. While the solid line is for the critical values of $\tilde{\kappa}$, namely $\tilde{\kappa} = 1$, representing $P = P_c$, the dotted and dashed curves are for $\tilde{\kappa} = 1.05$ and .95 respectively. For $\tilde{\kappa} > 1$, $a = 0$ is the minimum of the potential, This reperesents that the inverted pendulum is sitting staright making an angle zero with the verical axis. $\tilde{\kappa} > 1$, minima are at $a = a_{\pm}$. The pendulum settles at one of these angles, breaking $a \to -a$ symmetry. We refer to this as a symmetry broken phase.

of λ is played by $6(1 - \tilde{\kappa})$ and the role of x is played by a. Consequently, our model is a case of supercritical bifurcation. One can construct a potential whose extremaization gives (5). This is given by

$$V(a, \tilde{\kappa}) = \int da(\tilde{\kappa}a - \sin a) = \frac{\tilde{\kappa}a^2}{2} + \cos a. \quad (7)$$

The plot of the potential for different values of $\tilde{\kappa}$ is shown in figure (2). The stability of the system is decided from the minima of the potential and has been discussed in the caption of figure (2). In the next section we discuss some dynamical issues associated with the model.

3. TIME DEPENDENT INTERPOLATING SOLUTION

As we discussed in the previous section, for $\tilde{\kappa} > 1$ or in other words for $P < P_c$, the stable position is $a = 0$ and for $\tilde{\kappa} < 1$ or for $P > P_c$, $a = 0$ is a maximum and system settles down to either at a_+ or at a_-. The question that we ask in this section is the following: Suppose we suddenly reduce the load from $P > P_c$ to just below P_c, how the system rolls down from the symmetry preserving phase to the symmetry broken one? To analyze this issue in some detail, we go back to equation (5). First we note that this equation can be re-written as

$$\frac{d}{dt}\Big(I(\frac{da}{dt})^2 + \kappa a^2 + \frac{2PL}{I}\text{cos}a\Big) = 0, \tag{8}$$

Note that the experssion inside the big brackets is the total energy of the system. Hence (8) is a statement of energy conservation. A simple integration therfoe gives

$$I(\frac{da}{dt})^2 + \kappa a^2 + \frac{2PL}{I}\text{cos}a = C, \tag{9}$$

where C is a constant. This constants can be fixed by using the boundary condition that when $a = 0$, $\dot{a} = 0$. This leads to

$$C = \frac{2PL}{I}. \tag{10}$$

Substituting this back to (9) and integrating once more, we get

$$\int dt = \pm\sqrt{\frac{I}{PL}}\int \frac{da}{\sqrt{4\text{sin}^2\frac{a}{2} - \tilde{\kappa}a^2}}. \tag{11}$$

It turns out that this integral can only be performed exactly small a. This is only a good approximation when $\tilde{\kappa}$ is close to one, or equivalently P close to P_c. Keeping upto quartic terms in a, we get,

$$\int dt = \pm\sqrt{\frac{I}{PL}}\int \frac{da}{\sqrt{(1-\tilde{\kappa})a^2 - \frac{a^4}{12}}}. \tag{12}$$

This equation can be integrated easily with the result

$$a(\tilde{t}, \tilde{\kappa}) = a_{\pm}e^{\mp\sqrt{1-\tilde{\kappa}}(\tilde{t}-\tilde{t}_0)}. \tag{13}$$

In the above equation, we have defined $\tilde{t} = \sqrt{\frac{PL}{I}}t$, $\tilde{t}_0$ is an integration constant. In writing down the solution, we also used the fact that at $\tilde{t} = \tilde{t}_0$ $a(\tilde{t}) = a_{\pm}$. Note that above solution is real only for $\tilde{\kappa} < 1$. This indicates the fact that rolling down solution does not exist for $\tilde{\kappa} > 1$. We also note that the constant $\tilde{t}_0$ appears due to the time translational invariance of the equation (5). The interpolating solution is plotted in figure (3).

Having analysed the time dependent solution, we now turn our attention to some equilibrium properties of our model. In particular, we would be interested in bringing out some analogies between continuous phase trantion and our model of inverted pendulum.

In the next section, we discuss various static properties of our model and we try argue that our model serves as a crude analogy with second order phase transition.

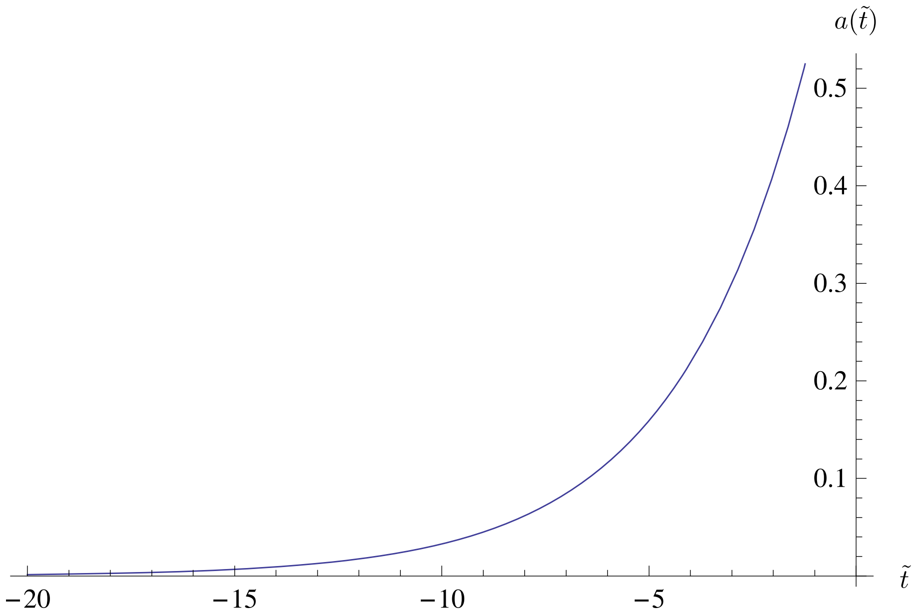


Figure 3. The interpolating solution $a(\tilde{t}, \tilde{\kappa}) = a_+ e^{\sqrt{1-\tilde{\kappa}}(\tilde{t}-\tilde{t}_0)}$ for $\tilde{\kappa} = .9$ and $\tilde{t}_0 = 0$. At very early time the system is at $a = 0$, while at $\tilde{t} = 0$, it reaches a finite value for a.

4. ANALOGY WITH CONTINUOUS PHASE TRANSITION

Let us note that for very small a (that is for P close to P_c), we can expand the potential in (7) as

$$V(a) = \frac{\kappa}{2}a^2 + PL\big(1 - \frac{a^2}{2} + \frac{a^4}{24}\big) + ... \tag{14}$$

or as

$$\tilde{V}(a, \tilde{\kappa}) = \frac{V - PL}{PL} = \frac{a^2}{2}(\tilde{\kappa} - 1) + \frac{a^4}{24} + ... \tag{15}$$

Depending on the value of $\tilde{\kappa}$, it shows three distinct behaviours similar as in figure (2). The occurance of only even powers is a consequence of $a \to -a$ symmetry of the potential. We would like to compare (15) with the expansion of free energy function in terms of order parameter near the vecinity of second order phase transition within the framework of the Landau theory of phase transition [4]. In Landau theory, the phase of a system is characterized by an order parameter. This is a measurable quantity. It is generally zero in the disordered or high temperature phase and acquires a non-zero value in the ordered or the low temperature phase. A common example is the ferromanget. In the absence of any external field, the magnetization of a ferromagnet is zero above a critical temperature T_c. However, for $T < T_c$, its magnetization is non-zero. Therefore, here, magnetization is generally used as the order parameter which distiguishes the high and low temperature phases of a ferromagnet.

When a parameter of the system changes, a system may go through a phase transition. For the case just discussed, this parameter is the temperature. During a phase transition process, order parameter changes either continuously or abruptly. While for a first order transition, the change of the order parameter is disconinuous, for a second order transition, it changes smoothly around T_c. Ferromagnetic transition is an example of a second order phase transition.

To describe a second order phase transition, Landau constructed a free energy function of a system near its critical point. This function, for a ferromagnetic system, has a general form

$$F(M,T) = a_1(T - T_c)M^2 + a_4 M^4 + ..., \tag{16}$$

where, M, the oder parameter, is the avarage magnetization. We note that since $M \to -M$ is expected to be a symmetry of the system, all odd powers of M are absent. Secondly, considering the system close to the critical temperature, we neglect higher powers of M. The equilibrium state of the system is determined by the extimization condition,

$$\frac{\partial F}{\partial M} = 0. \tag{17}$$

Note that the above equation tells us that for $T > T_c$, the only real solution for M is $M = 0$. However, for $T < T_c$ two more real solution develops:

$$M = \pm\sqrt{\frac{a_1(T_c - T)}{2a_4}}, \tag{18}$$

besides $M = 0$. Taking a second derivative on F, one realizes that (18) represents the stable points, while the solution $M = 0$ becomes unstable. From here we see that that the order parameter becomes nonzero and grows at $(T_c - T)^{1/2}$ for temperature below T_c. This leads to a critical exponent β, characterising the phase transition, defined as

$$M \sim (T_c - T)^{\beta}. \tag{19}$$

We get $\beta = 1/2$ for the system in consideration. Similarly, we can define susceptibility as

$$\chi^{-1} = \Big(\frac{\partial^2 F}{\partial M^2}\Big)_{T, M\to 0}. \tag{20}$$

Taking second derivative of (16) with respect to M, keeping T constant and $T > T_c$, we get

$$\chi^{-1} \sim (T - T_c) \tag{21}$$

and for $T < T_c$,

$$\chi^{-1} \sim (T_c - T). \tag{22}$$

These lead to two other critical exponents

$$\gamma = \gamma' = 1. \tag{23}$$

Now we turn our attention to the model we are discussing. We notice the following similarities with ferromagnets.

• F is similar to V in (15).

• Like M in ferromagnet, a behaves as order parameter in our model.

• The load P behaves as temperature which we tune externally.

• The critical temperature T_c is similar to the critical load P_c.

• It can easily be checked that the order parameter in our case behaves near the critical point as

$$a \sim (P - P_c)^{\frac{1}{2}}, \tag{24}$$

giving $\beta = 1/2$.
• Likewise the susceptibility, defined as

$$\chi^{-1} = \Big(\frac{\partial^2 V}{\partial a^2}\Big)_{T, a \to 0}. \tag{25}$$

leads to

$$\chi^{-1} \sim (P - P_c), \quad \text{for} \ \ P > P_c, \tag{26}$$

and

$$\chi^{-1} \sim (P_c - P), \quad \text{for} \ \ P < P_c. \tag{27}$$

Hence, we get

$$\gamma = \gamma' = 1. \tag{28}$$

Though there are quite a few similarities between our model and ferromagnets, there are many differences also. To mention a few, we note that unlike our case, ferromagnetic transition is temperature driven. Morover, M is a local order parameter for ferromagnets. This means that M may change from point to point inside a ferromagnetic material. For our case, a is only a global parameter.

5. CONCLUSION

In this work, we discussed a simple model showing supercritical bifurcation. We constructed a time dependent solution interpolating the symmetric phase and the symmetry broken phase. We also tried to bring out some similarities between our model and ferromagnetic material near its critical temperature using Landau theory.

It would be interesting to set up an experiment to test our results. However, before we do so, the model has to be generalised to take care of other effects. Firstly, we have to do the analysis including the weight of the rod. Secondly, we need to include friction into the system by introducing a term $\beta \dot{a}$ into the equation (5). We hope to look into these areas in the future.

Acknowledgements

I would like to thank Sudipta Mukherji for supervising this work. I would also like to thank Anirban Basu and S.K. Patra for their help during the Summer Student Visiting Programme at Institute of Physics, Bhubaneswar.

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PROBLEMS IN PHYSICS

Readers are invited to submit the solutions of the problems in this section within two months. Correct solutions, along with the names of the senders, will be published in the alternate issues. Solutions should be sent to: H.S. Mani, c/o A.M. Srivastava, Institute of Physics, Bhubaneswar, 751005; e-mail: ajit@iopb.res.in

Communicated by H.S. Mani

Problem set by D.P. Roy

1. Suppose you are standing at the centre of a circular palm groove with uniform density of 4 trees per 100 square meter and the breadth of each tree is half a meter. Find

    i. The radius R of the groove for which your horizon is completely covered by trees,

    ii. The radius $R_{1/2}$of the groove for which half of your horizon is covered by the trees,

    iii. The general formula connecting the radius of the groove R to the fraction of your horizon covered by trees.

    Note: this is the two-dimensional analogue of the famous Olber's paradox, dating back to 1826 - i.e., why is the sky dark at night? the answer came over a hundred years later from the Big Bang cosmology, showing that the visible universe has a finite radius of about 14 billion light years.

Problem set by H.S. Mani

2. Consider a geostationary satellite. What is the maximum lattitue to which signal can reach directly from the satellite? Assume earth surface to be an ellipsoid of revolution with the radius at the equator as 6378Km. and at the pole as 6357 Km.

Solutions to the problems given in Vol.3 No.6

Solutions provided by: H.S. Mani

Problem 1: Consider an infinite parallel plate capacitor made of two plates carrying a surface charge density σ_1 and σ_2 ($\sigma_1 = \sigma_{i1} + \sigma_{o1}$ where σ_{i1}, σ_{o1} are the charges per unit area of the inner and the outer surfaces of the firsst plate with a similar expression $\sigma_2 = \sigma_{i2} + \sigma_{o2}$ for the other plate. Find the charge distribution ($\sigma_{i1}, \sigma_{o1}, \sigma_{i2}, \sigma_{o2}$ on the four surfaces in terms of σ_1 and σ_2. (We are taught the special case when $\sigma_1 + \sigma_2 = 0$, in which case the charge resides only on the inner surface of the parallel capacitor).

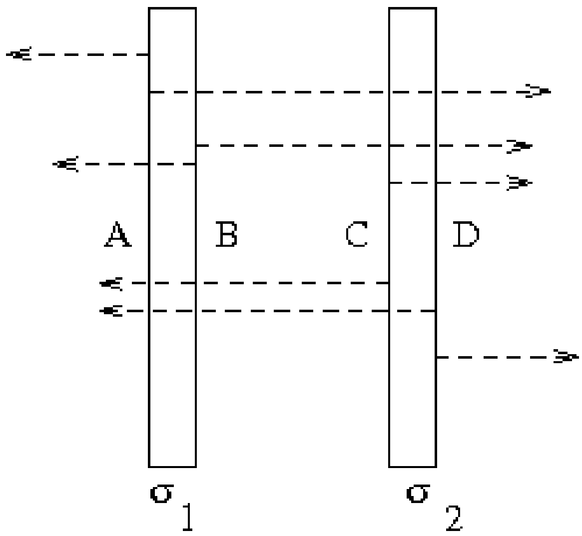

Figure 1.

Solution to Problem 1:
Let A, B, C and D the surface charge densities on the four sides are shown in the figure. (So, $A = \sigma_{o1}, B = \sigma_{i1}, C = \sigma_{i2}$, and $D = \sigma_{o2}$. Then

$$A + B = \sigma_1$$

and

$$C + D = \sigma_2.$$

Each charged surface of charge density σ produces an electric field of magnitude $\sigma/(2\epsilon_0)$ normal to the surface. The direction of the lines of force emenating from each surface is shown in the figure.

The electric field inside each conductor must be zero. Thus we get two equations

$$\frac{(A - B - C - D)}{2\epsilon_0} = 0$$

and

$$\frac{(A + B + C - D)}{2\epsilon_0} = 0$$

Adding the two equations we get

$$A - D = 0$$

This also leads to

$$B + C = 0$$

From the four equations we get

$$A = D = \frac{\sigma_1 + \sigma_2}{2}$$

and

$$B = -C = \frac{\sigma_1 - \sigma_2}{2}$$

Problem 2: Consider two magnetic dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$, the first one pointing along the y-direction (that is $\vec{\mu}_1 = \mu_1\hat{j}$) and kept at the origin $\vec{r}_1 = 0$ and the second pointing along the x-direction (that is $\vec{\mu}_2 = \mu_2\hat{i}$) and is located at $\vec{r}_2 = d\hat{i}$. We know that the torque acting on a magnetic moment $\vec{\mu}$ is given by $\vec{\mu} \times \vec{B}$ where $\vec{B}$ is the extermal magnetic field. Now notice that the magnetic field due to $\vec{\mu}_1$ at $\vec{r}_2$ is along $-\hat{j}$ and hence the torque on $\vec{\mu}_2$is along $-\hat{k}$. The magnetic field due to $\vec{\mu}_2$ at the origin is along $\hat{i}$ and hence the torque on $\vec{\mu}_1$ is also along $-\hat{k}$. This means both the dipoles rotate so that their total angular momentum is along $-\hat{k}$. However initially they are at rest and carry no initial angular momentum. How do you reconcile this with conservation of angular momentum ?

Solution to Problem 2:

For conservation of angular momentum the total torque acting on the system should be zero. The total torque is given by sum of the torque due to the force acting on the magnetic dipoles $(\vec{r} \times \vec{F})$ term and the couple $\vec{\mu} \times \vec{B}$. If the force were central ($\vec{F}$ and $\vec{r}$ are parallel) then this does not contribute. However the force between the dipoles is non central. We will consider the general case of a the dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$ being located at $\vec{r_1}$ and $\vec{r}_2$ respectively. The force $\vec{F}$ on a dipole of moment $\vec{\mu}$ is given by $\vec{F} = (\vec{\mu}.\vec{\nabla}_r)\vec{B}(\vec{r})$. The field at $\vec{r}$ due to $\vec{\mu}_1$ at $\vec{r}_1$ is

$$\vec{B}(\vec{r}) = \frac{\mu_0}{(4\pi)} \left(\frac{3\vec{\mu}_1.(\vec{r} - \vec{r}_1)\,(\vec{r} - \vec{r}_1) - |\vec{r} - \vec{r}_1|^2\,\vec{\mu}_1}{|\vec{r} - \vec{r}_1|^5} \right)$$

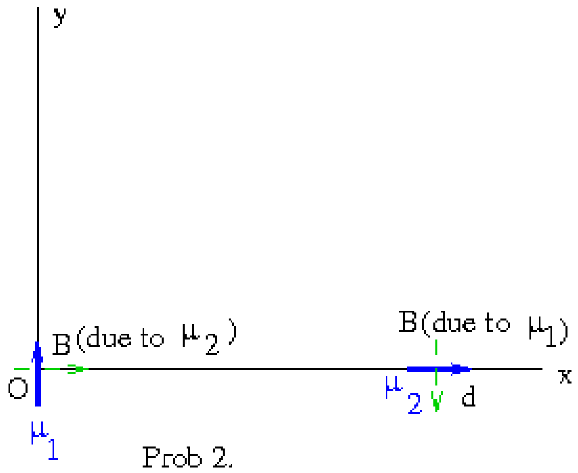

Figure 2.

Thus the force $\vec{F}_{21}$ on the dipole at $\vec{r}_2$ can be obtained by looking at the lth component of the force denoted by F^l_{21}

$$F^l_{21} = \mu_2^m \partial_r^m \, B_{21}(\vec{r})^l|_{\vec{r}=\vec{r}_2}$$

$$= \left(\frac{\mu_0}{4\pi}\right) \mu_2^m \partial^m \left(\frac{3\mu_1^n X^n X^l - X^n X^n \mu_1^l}{R^5}\right)$$

where $\vec{R} = X^n \hat{\epsilon}^n = \vec{r}_2 - \vec{r}_1$.

(here we have used the convention that superscripts l, m, n represent the three compnents of a vector and take the value $1, 2$ and 3). Further we use the convention that repeated indices are summed over $1, 2$ and 3. Also $\vec{R} = X^l \hat{\epsilon}^l, \hat{\epsilon}^l, l = 1, 2, 3$ being the three orthonormal unit vectors. R is the magnitude of $\vec{R}$). On doing the partial differentiation we get

$$F^l_{21} = \frac{\mu_0}{4\pi} \left(\mu_2^m \frac{3\mu_1^n \, X^l \, \delta_{mn} + 3\mu_1^n \, X^n \, \delta_{ml} - 2X^n \, \delta_{mn} \, \mu_1^l}{R^5} \right)$$

$$-5\frac{\mu_0}{4\pi} \left(\mu_2^m \frac{(3\mu_1^n \, X^n \, X^l - X^n \, X^n \, \mu_1^l) X^m}{R^7} \right)$$

This can be simplified to

$$\vec{F}_{21}(\vec{R}) = \frac{\mu_0}{4\pi} \left(\frac{3\vec{\mu}_2.\vec{\mu}_1 R^2 \vec{R} + 3(\vec{\mu}_1.\vec{R}\vec{\mu}_2 + \vec{\mu}_2.\vec{R}\vec{\mu}_1)R^2 - 15\vec{\mu}_1.\vec{R}\vec{\mu}_2.\vec{R}\vec{R}}{R^7} \right)$$

Note that the force $\vec{F}_{21}$ on the dipole 2 due to 1 is opposite in sign due to the force of 2 on 1, given by $\vec{F}_{12}$. This is easily seen by noticing we can get $\vec{F}_{12}$ from $\vec{F}_{21}$ by replacing $\vec{R}$ by $-\vec{R}$ and $\vec{\mu}_1 \leftrightarrow \vec{\mu}_2$. We calculate the torque on 2 about the origin due to the force $\vec{F}_{21}$

$$\vec{r}_2 \times \vec{F}_{21} = \frac{\mu_0}{4\pi} 3 \left(\frac{(\vec{\mu}_1.\vec{R}\vec{r}_2 \times \vec{\mu}_2 + \vec{\mu}_2.\vec{R}\,\vec{r}_2 \times \vec{\mu}_1)}{R^5} \right)$$

$$+\frac{\mu_0}{4\pi} \left(\frac{3\vec{\mu}_1.\vec{\mu}_2 R^2(-\vec{r}_2 \times \vec{r}_1) - 15\vec{\mu}_1.\vec{R}\,\vec{\mu}_2.\vec{R}(-\vec{r}_2 \times \vec{r}_1)}{R^7} \right)$$

The couple due to the $\vec{\mu}_2 \times \vec{B}_{21}(\vec{r}_2)$ term , where $\vec{B}_{21}(\vec{r}_2)$is the magnetic field at $\vec{r}_2$ by the dipole 1 is given by

$$\vec{\mu}_2 \times \vec{B}_{21}(\vec{r}_2) = \frac{\mu_0}{4\pi} \left(\frac{3\vec{\mu}_2 \times \vec{R}\,\vec{\mu}_1.\vec{R} - R^2\,\vec{\mu}_2 \times \vec{\mu}_1}{R^5} \right)$$

The total torque $\vec{T}_{21}$ on dipole 2 is

$$\vec{T}_{21} = \frac{\mu_0}{4\pi} \left(\frac{3\vec{\mu}_1.\vec{R}(-\vec{\mu}_2 \times \vec{r}_1) + \vec{\mu}_2.\vec{R}\,\vec{r}_2 \times \vec{\mu}_1 - R^2\,\vec{\mu}_2 \times \vec{\mu}_1}{R^5} \right)$$

$$+\frac{\mu_0}{4\pi} \left(\frac{3\vec{\mu}_1.\vec{\mu}_2\,R^2(-\vec{r}_2 \times \vec{r}_1) - 15\vec{\mu}_1.\vec{R}\,\vec{\mu}_2.\vec{R}(-\vec{r}_2 \times \vec{r}_1)}{R^7} \right)$$

the torque $\vec{T}_{12}$ on 1 due to 2 can be obtained by the same replacement mentioned earlier $\vec{R}$ by $-\vec{R}$ and $\vec{\mu}_1 \leftrightarrow \vec{\mu}_2$ and $\vec{r}_1 \leftrightarrow \vec{r}_2$.

$$\vec{T}_{12} = -\vec{T}_{21}$$

Thus the total torque obtained from the forces and the couples add up to zero as it should.

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