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Editorial

Indian Science and Nobel Prize

The first week of October of each year-when Nobel Prizes are announced by the Swedish Academy-reminds us to introspect as a ritual on the status of Indian Science vis-a-vis the world. A gloom descends on the academic horizon, but vanishes soon without leaving any effect on the future course of Indian science. The question as to why scientists from post- independent India never earn a place on the Nobel list becomes more perplexing with every passing year especially when one considers the growing investment in science and Higher Education by the country. The question often lurks in our mind why India does not qualify even though it could achieve this distinction two decades before its independence through its epoch making discovery of C.V.Raman in experimental Physics with meager facilities. The issue becomes all the more poignant when we recall that pre-independent India produced several luminaries like J.C. Bose, S.N. Bose,M.N. Saha, H.J. Bhabha et al. some of whom the world recognizes as near or equivalent Nobel Laureate. Needless to say that this Nobel glory was not confined to science only but extended to other field as well like literature where Rabindranath Tagore achieved this distinction.

Before 1947 India had about 25 universities devoted mostly to teaching and a couple of research institutes founded and run privately by eminent philanthropic individuals. In the last six decades since independence India has established about 500 universities, a dozen or so IITs and more than a hundred dedicated research institutes, some of which are fully or partly residential with world class infrastructure and facilities. In this scenario, the answers to the above question often paraded on various forums, are more elusive than real. In popular ethos, Nobel Prize has been heralded as the ultimate recognition of the brilliance of a creative mind. The dismal performance of India in this respect may be considered as a pointer to the opposite, inflicting on us self-doubt and loss of self esteem. However the achievement of Indian mind in the last 5000 years points out to the contrary. Rig Veda has been accepted as the oldest book produced by the humanity. India has the rare distinction of establishing the first university in the world at Takshyashila which was flourishing in 500 BC with visiting scholars from other nations. The recent conclusive research by George Ifrah spanning over two decades culminated in the treaties "Universal History of Numbers" translated into 14 languages by scholars from Cambridge and Princeton where he has finally concluded: "While all ancient civilizations struggled for centuries to find a system for writing big numbers India only succeeded in discovering decimal place value system and zero, the very corner stone of human knowledge. Modern science and technology could flourish in the frame work of a number system as revolutionary and efficient as our positional decimal system which originated in India." It has been appropriately termed as "Science of Sciences" by Swami Vivekananda. Such unique contributions, coupled with the fact that four Indians leaving the country after their University education, and while working abroad, could win Nobel Prizes, suggests that the answer to the above question may also partially lie somewhere else in the depth of our consciousness rather than entirely on the external material plane.

Out of many factors contributing to the success of human endeavour, the predominant role of culture is undisputed. Culture is an invisible force, which determines the value system in the society and shapes human thought, empowering it with dynamism and direction. The momentous question is "Does the nation have a scientific culture conducive for achievement of excellence in research?" It has often been alleged, and also normally accepted, that India is not a meritocracy, the primary cause of which is the underlying trait of cronyism and feudalism in national character inherited from our past history. A classic example of the manifestation of this trait is the case of Hargovind Khurana who left India in 1950s and sought his fortune in USA eventually being honoured with Nobel Prize in Biophysics. Needless to say it is a fountain head of many evils polluting the academic and public life as a whole. It may be argued that other countries were also ruled by kings and emperors and had a feudal past like India. However this force has grown feeble and weak and is almost non-existent in most of the European countries who have done away with their monarchy centuries ago; and in USA, the most creative country in the world, it never existed since its inception. A more decisive factor for India has been its long foreign rule. It is probably the only country, which was invaded and ruled by foreigners for about thousand years. It may be recognized that it is easier to fight empire but difficult to fight with the legacy left by it. When a handful of foreigners rule a huge country like India, they have to recruit many natives to the lower level of administration. Those privileged natives serving under the foreign masters eventually are likely to develop the traits of sycophancy and hypocrisy, which practiced for thousand years get ingrained into the character. These traits manifest on the surface as cronyism and feudalism, which may be identified as the invisible evil force plaguing the free play of national sprit.

The second most impeding factor is Indias religious thought. India made pioneering research in various branches of science like astronomy, mathematics, chemistry, metallurgy and medical science etc, in vedic and post-vedic period extending up to seventh century. However with the advent of Sankaracharya and his strong revival of Advaita Vedanta philosophy in eighth century, this momentum got redirected to spirituality by the realization that the ultimate truth does not lie in external nature but in internal spirit whose study would eventually lead to "mokshya". Since then this has been the mainstay and guiding force in national psyche with obvious adverse effect on growth of Indian science.

The third factor is the obsolete Indian Education system based on rote learning and excellence in examination with little stimulation for creativity. This system introduced in mid 1850s designed to produce educated workforce to run colonial rule has almost remained the same defying general law of evolution. The fourth factor is the grinding poverty of India posing a barrier for creating state-of-the art infrastructure and laboratories at competitive pace with the international science. The upsurge of nationalistic spirit during freedom movement commencing from the last decade of nineteenth century, could unleash the national psyche from the strangle-hold of the evil force for a while and could overcome the discomfitures posed by the remaining factors giving rise to the golden era of Indian science and other fields before independence. However soon after independence, this weakened force finding a level playing field has reappeared with renewed strength. Awareness of its existence and conscious effort to eradicate it, and together with the appropriate measures to deal with the other impeding factors may be the national imperative for resurrection of Indian science.

L. Satpathy

TURNING POINTS

Does Space have more than Three Dimensions?

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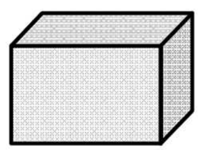
Communicated by: D.P. Roy

Can this be a serious question? Everyone learns quite early in school that space has *three* dimensions, exemplified by the length, breadth and height of a solid object, such as the box in the accompanying illustration. Later, one learns to represent these dimensions by coordinates, or distances from three fixed planes, and then one is able to denote a point in space by a triad of real numbers called coordinates. With a little more mathematical knowledge, we can use any three functions of these coordinates (such as $r = \sqrt{x^2+y^2+z^2}$, $\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$, $\psi = \tan^{-1}\frac{y}{x}$) as coordinates themselves. But the number of coordinates is always taken as three.



Despite this commonsense representation of the space in which we live, from remote antiquity mystic philosophers have speculated on the existence of invisible extra dimensions, where one will be able to find non-mundane entities such as gods, spirits, etc. Scientists generally do not take such ideas seriously – at least from a professional point of view. But nearer home, abstract mathematicians freely use spaces of higher dimension, denoting a point by $(x_1, x_2, \ldots, x_n)$ where n is a (possibly large) integer. Of course, this is believed to be an abstraction not corresponding to a physical reality. In statistical mechanics, physicists borrow the mathematician's concept to talk of a *phase* space, which, for an ideal gas, has the dimension $6N_A$, where $N_A \approx 6.023 \times 10^{23}$ is the Avogadro number. Quantum mechanics is formulated in a space of *infinite* dimensions, though this 'space' is really a system of configurations of the physical system. None of these so-called 'dimensions' are dimensions of *space* – which we shall henceforth call *geometric* dimensions. They just correspond to independent variables describing the system.

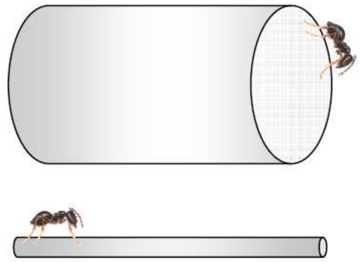
Like so much else in modern physics, the scientific question of higher *geometric* dimensions originated from the transcendent genius of Albert Einstein. In his seminal 1905 paper on the theory of Special Relativity, Einstein showed that, for a moving observer, space and time coordinates transform into one another as

$$\begin{aligned} x \to x' &= \Lambda_{xx}x + \Lambda_{xy}y + \Lambda_{xz}Z + \Lambda_{xt}t \\ y \to y' &= \Lambda_{yx}x + \Lambda_{yy}y + \Lambda_{yz}Z + \Lambda_{yt}t \\ z \to z' &= \Lambda_{zx}x + \Lambda_{zy}y + \Lambda_{zz}Z + \Lambda_{zt}t \\ ct \to ct' &= \Lambda_{tx}x + \Lambda_{ty}y + \Lambda_{tz}Z + \Lambda_{tt}ct \end{aligned}$$

where the coefficients Λ_{xx} etc. are functions only of the velocity v between the two inertial frames, and c, the speed of light in vacuum. Since Special Relativity tells us that all inertial frames are identical from a physical point of view, it is clear that there is no absolute criterion for determining whether the space and time coordinates measured by an observer are "pure" or "mixed" in the above fashion. This can be elegantly expressed as every event taking place in a four-dimensional spacetime continuum with coordinates (x, y, z, ct), a formulation developed by Einstein's old mathematics teacher Hermann Minkowski in 1908. The transformation between moving frames of reference, then, is just like a "rotation" in the four-dimensional spacetime.

Gunnar Nordström

In Minkowski's mathematically pretty spacetime, space and time do get mixed up, but they still retain their separate identities, like partners in an unhappy marriage. This is because, as everyone knows, we cannot go back in time – a principle enshrined in physics as the second law of thermodynamics. Time is not, therefore, a geometric coordinate in the same sense as x, y and z are. This is often expressed by describing spacetime as having 1+3 dimensions, rather than 4 dimensions. However, in 1914, a young Finnish relativist, Gunnar Nordström, showed how it is possible to have extra geometric dimensions, which are genuine extensions of space. It is possible, said Nordström, to have *compact* extra dimensions of very small size – smaller than the smallest object which can be seen by any kind of microscope.



The idea of compactification is as follows. Imagine a flat sheet of paper, and suppose there is an ant crawling along on top of it. The ant is free to move along the length and breadth of the paper and in closed paths if it so desires. If it is an intelligent ant, it will tell us that the space is of two dimensions. Now roll the paper up into a cylinder. The ant can still crawl along the surface in the straight direction, and at the cost of clinging on for dear life, it can crawl right around the circular direction. The latter direction (dimension) is said to be "compact" – this means that the ant can come back to its original position by moving forwards *monotonically* along that direction i.e. without reversing its motion. Of course, the ant will record the existence of two dimensions, even though these two are of somewhat different kinds. However, if we keep rolling up the paper into tighter and tighter cylinders (this is an ideal paper of zero thickness), i.e. reducing the radius of the cylinder (or *radius of compactification*), there will come a time when the ant is no longer able to crawl around the cylinder. It can move only along the straight edge, and it will, therefore, conclude that it is in a space of just one dimension. We say (or rather, the ant says) that the two-dimensional space has become compactified to one dimension.

One can still argue that if we replace the ant by a flea, which is many times smaller, or by a bacterium or a virus, the latter will be able to 'see' the compact dimension, just as the ant was able to in the earlier analogy. However, if we keep shrinking the radius to smaller and smaller sizes – smaller than the smallest probe we can use – then the compact dimension will be invisible for all practical purposes, but can still exist! According to quantum mechanics, the smallest objects are 'seen' when they scatter matter waves of wavelength $\lambda = h/p$, where p is the momentum of the matter particles (typically electrons or protons). For the largest values of p attainable at particle accelerators at present, this wavelength is around $10^{-18}m$, or a nano-nanometre. For radii of compactification below this limit, matter wave diffraction effects will render invisible one, two or any number of compact dimensions which space may have. These are not restricted to be circular, either – any closed (compact) shape will do, so long as we can assign to it a size.

If we cannot 'see' them, of what use are such tiny dimensions? A good deal, it turns out. The point is that *gravity* can see them! This is because Einstein (again!) taught us to regard gravity not as a field encompassing a passive substrate of spacetime, but as the fabric of spacetime itself. Nordström, and his successors Théodore Kaluza (1919) and later Oskar Klein (1926), were able to use this idea with partial success to develop a unified field theory which incorporated both gravitation

Theodore Kaluza

and electromagnetism. The more successful[1] Kaluza-Klein theory required one extra dimension of circular nature – somewhat like the rolled-up side of the cylinder we used above as an illustration. If we consider Einstein's field equations of gravitation (obtainable from his theory of General Relativity) in these *five* dimensions, i.e. 1+3+1 dimensions, where the first 1 is time, the next 3 are the usual non-compact space dimensions and the last 1 is the compact space dimension, then, in the limit when the compact dimension becomes very small, these reduce to (i) the 1+3 dimensional Einstein equations, which describe ordinary gravitation, plus (ii) Maxwell's equations which describe the electromagnetic fields. This is a very beautiful result, often called the "Kaluza-Klein miracle". All that is needed is the existence of a compact dimension, and the assumption that Einstein's postulate of General Relativity holds irrespective of the space dimension. In a letter to Kaluza in April 1919, Einstein wrote "The idea of achieving [a unified field theory] by means of a five-dimensional cylinder world never dawned on me. At first glance I like your idea enormously."

It is hard to believe that a theory as beautiful as the Kaluza-Klein theory can be wrong. But it *is* wrong! The problem arises because of the huge discrepancy in observed strength between gravitation and electromagnetism. The electromagnetic force between two protons is about 10^{38} times stronger than the gravitational force. If we note that 10^{38} is 100 000 000 000 000 000 000 000 000 000 000 000 000 then the point is driven home much more forcefully. In the Kaluza-Klein theory, these strengths, not surprisingly[2], are related by the radius of compactification R_c, and the observed ratio can only be achieved by making R_c mind-bogglingly small – as small as $R_c \sim 10^{-35}m$. Now, if we do make this assumption, it can be shown that all matter waves will have wavelengths of this order, i.e. using de Broglie's relation $\lambda = h/p$ we would predict masses of matter particles to be around $10^{25}eV/c^2$, or about 10 000 000 000 000 000 times the mass of a proton. This so-called *Planck mass*[3] is around tens of micrograms – the weight of a pollen grain or a dust particle. The masses can also be exactly zero. However, the actual masses of protons, electrons,

[1] Nordström's theory used Newtonian gravity and hence was not relativistic.
[2] Since that is the only parameter in the theory.
[3] The Planck mass can be understood in many ways. Perhaps the simplest way is to say that the proton had a mass as great as the Planck mass, then the gravitational force between two protons would equal the electromagnetic force between them.

etc. are neither exactly zero, nor anywhere as large as the Planck mass. It follows, then, that, for all its mathematical elegance, the Kaluza-Klein theory cannot be correct. Barely a month after his earlier letter, a somewhat crestfallen Einstein was writing to Kaluza again: “I respect greatly the beauty and boldness of your idea. But you understand that, in view of the existing factual concerns, I cannot take sides as planned originally. . .”

In the period between 1950 and 1975, a series of discoveries and successful predictions gradually established that the electromagnetic interaction, as well as the two kinds of nuclear forces, can be nicely described by a class of models which go under the name of *gauge theories*. Unlike gravitation, which is intimately connected with the structure of spacetime, these theories have a bunch of fields, defined on a passive spacetime substrate, which mix among themselves in a particular way – the technical name for this is an *internal symmetry*. In order to ensure that the physical world is not changed by this kind of mixing, we require to introduce some extra fields, which can then be shown to act as a cement between the original fields, i.e. give rise to forces, such as electromagnetism and the weak and strong nuclear forces. In fact, for electromagnetism, it can be argued the gauge theory arises quite naturally if we combine the ideas of relativity with the probability interpretation of quantum mechanics. Curiously, Einstein, the pioneer of both relativity and quantum theory, was not willing to accept the probabilistic interpretation of quantum mechanics, and was not, therefore, willing to accept the approach that led to the gauge theory of electromagnetism. He persisted in trying a purely geometric approach, which ultimately failed. This resulted in cutting him off from the mainstream of theoretical physics during the last thirty years of his life. There is a moral in this story: even if you are a genius, you do need to listen to the voices around you.

With the advent of gauge theories, Kaluza-Klein-style unification became obsolete, since the new gauge theories were elegant and worked better in practice – in fact, they work so well that a particular combination of gauge theories goes today by the name of the *Standard Model*. However, the pendulum now swung the other way. It became impossible to unify gravity with gauge theories, so that Einstein’s original dream of having a single unified theory describing all the forces in Nature took a beating. In a kind of desperation, some scientists[4] went a step further and speculated that we must give up the traditional description of matter in terms of elementary particles. Instead, said these theorists, we must imagine the fundamental objects in the Universe to be one-dimensional wriggly little things called *strings*. Different oscillation modes of the strings (like harmonics in a guitar string) would appear as different elementary particles, but underlying the particle description of matter and radiation would be a mass of identical strings. String theories, despite their early promise and obvious attraction, have run into all sorts of technical difficulties over the last thirty years or so. Many of these problems have been solved by invoking more and more esoteric ideas, so that today string theory on its own forms an almost independent branch of physics! It is still a debatable issue whether string theory has really advanced our understanding of the four fundamental interactions. However, the part that interests us here is the fact that it was realized very early that

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[4]Joel Scherk, John Schwartz and Tamiaki Yoneya may be regarded as the pioneers of string theory.

one cannot define a consistent string theory in 1 + 3 dimensions. String theories work either in 1 + 25 dimensions, or in 1 + 9 dimensions. Where are these other dimensions? Obviously, they must be compact and tiny. Thus string theory brought about a revival of the discarded ideas of Nordström, Kaluza and Klein, albeit in a different avatar.

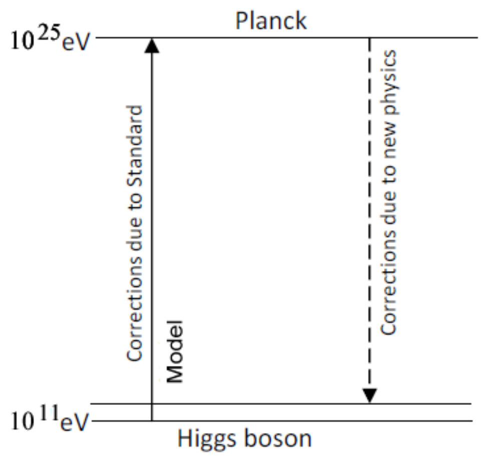
What saves a string theory from the mass problem which killed the Kaluza-Klein theory? This is the fact that string theory claims that all the elementary particles seen so far correspond to the mass*less* Kaluza-Klein modes of a string theory. The fact that the known particles seem to have actually acquired some finite mass is to be attributed to some other source, which would be eventually understood when we understand the dynamics of interacting strings better. In the Nordström and Kaluza-Klein theories, there was no room for any interactions other than gravitation and electromagnetism, so that these models ultimately failed because of their very simplicity. However, though string theory thus sidesteps the mass problem[5], the non-zero masses again lie at the very high Planck mass scale of $10^{25} eV/c^2$, which means that they are unlikely to be ever produced in the laboratory[6]. Until such masses can be produced (i.e. never!), we cannot confirm if string theory is a correct picture of Nature.

Till 1998, none of the speculative ideas of the string theorists were taken with much seriousness by their more hard-boiled colleagues in the particle physics community. In the world of particle physics, where particles streamed round and round in accelerator tubes, collided and annihilated, were created and decayed, leaving telltale tracks in some photographic emulsion or solid state array, everything was still governed purely by the gauge theories developed by the 1970s, or by their successors, which are all grounded solidly in the 1+3 dimensions of Minkowskian spacetime. Unfortunately, despite the well-known successes that have won gauge theorists a clutch of Nobel Prizes, all is not order and understanding in gauge theories. The problem arises because gauge theories have their own kind of mass problem. In a pure gauge theory, such as has been constructed and called the Standard Model, all particles are massless – which, of course, is not the case in reality. To get around this, ingenious minds like Yoichiro Nambu, Peter Higgs, Steve Weinberg and the late Abdus Salam, had introduced *non*-gauge interactions, which go by the technical names of “scalar self-interactions” and “Yukawa interactions”. Moreover, they were forced to include a hitherto undiscovered new particle – the Higgs boson – to mediate this mass generation mechanism. As this article is being written, we are still looking for this Higgs boson. But even assuming it will be found soon, Gerardus tHooft, the Nobel Prize-winning Dutch theorist had pointed out in 1972 that the mass of the Higgs boson is not stable under corrections due to the quantum nature of the theory, and that its only natural value could be – hold your breath – $10^{23} eV/c^2$, a value which we have encountered before as the Planck mass! Such a super-high mass for the Higgs boson would not only drag all the other particle

[5]or sweeps it under the carpet, if you like.

[6]The highest laboratory energy design till now is that of the LHC at CERN, Geneva, which collides protons at the energy of around $10^{12} eV/c^2$. This is still *twelve orders of magnitude* too small than the required to produce massive excitations in a traditional string theory.

masses to the same super-high scale, but also make the quantum mechanical calculations internally inconsistent. This problem goes by the name of the *hierarchy problem*.



The hierarchy problem and its solution

Does this mean that we are back to square one, and were better off with Kaluza-Klein-type theories, which are far simpler and make a smaller number of ad hoc assumptions? Not so, said the hard-headed school of particle physicists. There could be many possibilities. For example, the particles we see now could actually be composites of smaller particles, which would be 'seen' when we go to somewhat higher energies, i.e. long before $10^{23} eV/c^2$. Obviously their masses would be determined by the unknown dynamics which holds these smaller particles together, just as the dynamics of strong interactions determines the masses of the protons and neutrons. This is an attractive idea, but there are technical problems in constructing a realistic model, mainly because the unknown dynamics is, well, unknown. An even more attractive idea is that there is a bunch of hitherto-undiscovered particles which will cancel the intractable quantum corrections to the mass of the Higgs boson. There are two main variants of this idea. In *supersymmetric models*, there are paired bosons and fermions, cancelling each other's contribution to the Higgs boson mass. In *little Higgs models*, there are pairs of bosons (and likewise pairs of fermions) which similarly cancel each other's contribution. Such models are easier to understand, do not interfere with the structure of Minkowskian space-time and have their own connection to string theory – at least supersymmetry does. The only problem is that none of the new particles which induce these convenient cancellations have been found. Searching for these and designing new search strategies at higher energies takes up a great deal of the time and energy of the modern particle physicist.

All this comfortable theorizing received a jolt with the work of Nima Arkani-Hamed, Savas Dimopoulos and Georgi Dvali – collectively referred to nowadays as "ADD" – in 1998. This heroic trio originate from, respectively, Iran, Greece, and Georgia in the erstwhile Soviet Union, all seats of ancient culture, and their collaboration – on the American continent – is a beautiful example of the globalization of science. Their work was based on a simple – but far-reaching – modification of

N. Arkani-Hamed S. Dimopoulos G. Dvali

the original idea of Kaluza and Klein. Recall that the Kaluza-Klein model had been a model which sought to unify gravity with electromagnetism through the agency of an extra dimension. The enormous masses of the Kaluza-Klein particles had actually arisen because electromagnetism is known to be enormously stronger than gravitation. What if the extra force due to the extra dimension is not identified with electromagnetism, but is allowed to be some much, much weaker force? In that case, the masses of the Kaluza-Klein particles could be much, much smaller – as small, in fact, as the observed masses of elementary particles. But, the reader will argue, this would be throwing away the initial motivation of Nordström, Kaluza and Klein, which was to obtain gravitation and electromagnetism from the same theory. Never mind, said ADD[7]. Today we know that electromagnetism comes from a gauge theory, i.e. an internal symmetry of the quantum fields, and we do not need to generate it out of gravitation, i.e. from a spacetime symmetry. Hence, extra dimensions are not needed to understand the Standard Model – after all, we have been doing without them for thirty years!

Once freed from the shackles imposed by the requirement to generate a theory of electromagnetism, how large can the compact dimensions be? For this, we again turn to the experimental tests of the Standard Model, which have been performed to great accuracy at a mass scale of around $10^{11} eV/c^2$, which corresponds to a length scale of around 10^{-18} cm. None of these tests show any evidence whatsoever of extra dimensions. This can be interpreted to mean that either there are no extra dimensions, or, if they exist, they must be compactified to length scales considerably smaller than 10^{-18} m. This is tiny, but already vastly greater than Kaluza's value of 10^{-35} m. However, there is a third alternative, which we owe to the ingenuity of ADD. Suppose we have one or more extra dimensions which are much bigger than 10^{-18} m, but all the particles of the Standard Model (which build up the observable Universe, including our own bodies and instruments) are somehow *confined* within the four canonical dimensions of Minkowski and Einstein? Since all our empirical knowledge comes from these instruments, no ordinary laboratory experiment can show up these extra dimensions. Does this mean that these extra dimensions could be as large as we please, since we do not see them anyway? Not so, said ADD, because gravity can always see the extra dimensions, just as was argued in the case of primitive Kaluza-Klein theory. The limits on the size of any extra

[7]Not pronounced 'add' but as 'ay-dee-dee'.

dimensions should come, in ADD's model of the world, from experiments probing the nature of gravity, or rather the gravitational force.

Are there such experiments, whose results we can borrow? It turns out that such experiments have been done ever since the days of Henry Cavendish in the eighteenth century. For if gravity can propagate in $4+n$ dimensions, and the extra n dimensions are compact with a radius $_c$, then Newton's famous inverse square law of force $V \propto \frac{1}{r}$ would be modified to

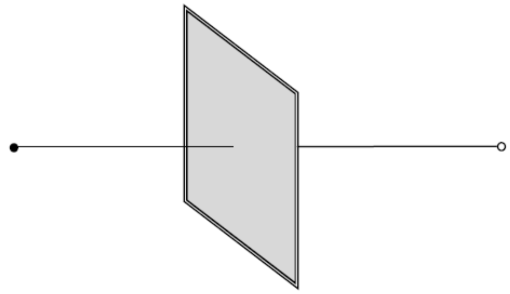
$$V \propto \frac{1}{r}\left(1+\frac{e^{-r/R_c}}{r}\right)$$

i.e. we would have corresponding changes in the gravitational force between two massive objects. These changes clearly become smaller and smaller as $R_c \to 0$. Currently the most accurate measurements of this kind come from the Eöt-Wash experiment at the University of Washington, where a very sensitive torsion balance experiment has been devised by Eric Adelberger and his team of collaborators. Their current results show no sign of any deviation from the exact inverse square law, and enable them to determine that if there are extra spatial dimensions, they will have radii of compactification $R_c < 4.4 \times 10^{-5}$m. This is much, much larger than the figure of 10^{-18} m. The large value is more indicative of the difficulty of gravitational experiments, than of any fundamental principle. The fact remains, however, that there is no obstacle to having extra dimensions as large as 10^{-5} m, so long as the Standard Model particles remain confined to four dimensions.

Eöt-Wash torsion balance

Assuming, then, that we can have an unspecified number of extra compact dimensions as large as 10^{-5}m, how does it matter? How does it affect our four dimensional world, where the Standard Model particles and interactions are confined? Profoundly, as it turns out. The fact is that the gravitational lines of force due to a massive source are now uniformly distributed throughout a space of $3+N$ dimensions, and only a very few of these intersect the wafer-thin sub-space of 3 dimensions, which we call our Universe.

A sketch of the world according to ADD is given above. The horizontal line represents the N extra dimensions, here represented as just one dimension. The full and empty circles at the ends indicate that these two ends are identified, i.e. the extra dimensions are compactified. The thin plane



intersecting this horizontal line orthogonally is the observable three-dimensional Universe. Clearly, its volume forms a very small fraction of the actual volume of space and this is what determines the number of gravitational lines of force intercepted by our Universe. We conclude then, that this small volume is responsible for making the gravitational force extremely weak , i.e. for driving the Planck mass to the extremely high value of $10^{25} eV/c^2$. If we could access the higher dimensions, we would see a much stronger gravitational force, to which corresponds a much smaller Planck mass. In fact, the Planck mass can be shown to reduce drastically in the presence of N large extra dimensions, following the simple formula

$$\tilde{M}_p \sim 10^{\frac{2(25-4N)}{2+N}} \left(\frac{1m}{R_c}\right)^{\frac{N}{2+N}} \mathrm{eV}/c^2$$

If we set $R_c \sim 10^{-5} m$, as is permitted by the Eöt-Wash experiment, then we have

$$\tilde{M}_p \sim 10^{\frac{50-3N}{2+N}} \mathrm{eV}/c^2$$

which is around 10^{16} eV/c^2 for $N = 1$, 10^{11} eV/c^2 for $N = 2$, 10^8 eV/c^2 for $N = 3$ and even smaller for more extra dimensions. Clearly, for $N = 1$, there is still a hierarchy problem, though a less severe one than the original one. For $N = 2$, the Planck scale is now reduced to the precise experimental limit. For $N \geq 3$, this value of the Planck scale is inadmissible, and hence we must have $R_c < 10^{-5} m$. For example, for $N = 6$, having the Planck scale at the experimental limit of $10^{11} eV/c^2$ would require $R_c \sim 10 fm$, i.e. the size of a medium-sized nucleus. The important fact is that by making $R_c \sim 10^{-5} m$, or less, we can reduce the fundamental scale (Planck scale) – at which gravity becomes as strong as the electroweak interaction – to about 10^{11} eV/c^2. This is just beyond the reach of the concluded experiments in particle physics and is about to be tested at the LHC and other machines of comparable energy. Now here is the unique selling point of the ADD model. *Having such a low Planck mass completely solves the hierarchy problem.* Radiative corrections will drive the Higgs boson mass to some fraction of the higher dimensional Plank mass, rather than the four-dimensional Planck mass discovered by Newton. As this higher dimensional Plank mass is not so much higher than the experimentally required value of the Higgs boson mass there are no large cancellations, after all.

We see then, that the new paradigm of ADD is based on the following assumptions:

1. Space has $3 + N$ dimensions, of which the 3 are the usual dimensions of Euclidean geometry and the other N are compact dimensions with a radius of less than $10^{-5}m$ (depending on N);

2. The known particles and forces are confined to a subspace of the 3 usual dimensions, having a thickness not more than $10^{-18}m$ in the new directions;

3. Only gravity can access the entire space, and by doing so its not-so-small strength in three dimensional space becomes very weak;

4. When we go to very small length scales below $10^{-18}m$, the Standard Model of particle physics breaks down, because at this scale its particles begin to access the full space of $3 + N$ dimensions, where strong gravity effects begin to dominate.

Ingenious as they may be, some of the ideas of ADD has been anticipated, in the 1980s, by the Japanese scientist Kei-ichi Akama and by the highly-respected Russian pair of Valéry A. Rubakov and Mikhail E. Shaposhnikov. However, these early precursors had different motivations and had not thought of their models as solutions for the hierarchy problem. The use of extra dimensions to solve the hierarchy problem was one of the two things which enabled the ADD paradigm to take the scientific world by storm. The other was its intimate connection with string theory.

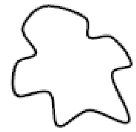
I. Antoniadis

The fact that once we can describe electromagnetism by a gauge theory, we do not need to have very small extra dimensions was known to many workers in the field, but no one really bothered to take it seriously. One researcher who did so was Ignatios Antoniadis, a Greek scientist working in Paris, who like his countryman Dimopoulos, is a living proof that the cradle of Western civilization has not lost her ability to produce first-rate scientific minds. Antoniadis, looking for a possible connection between string theory and experiments done in the laboratory today, was the first person to explore the phenomenological consequences of having large extra dimensions in the context of a string theory. In the early 1990s, he had written a few papers exploring these ideas, some alone and some with collaborators, but none of these had really attracted much attention. Now, after the first ADD paper, he was immediately able to team up with its authors and point out that string theory could readily provide the mechanism by which the Standard Model particles could be confined to a subspace of three dimensions. This arises because of a peculiarly string-theoretic phenomenon called a D-brane, which had been discovered just three years before by the American Joe Polchinski at the University of California at Santa Barbara. The physical idea for this is simple, though the mathematics to describe it is not. Strings, which normally move freely in ten-dimensional space just as atoms and molecules can move freely in three-dimensional space, can conglomerate under their mutual interactions into lower dimensional objects, just as atoms and molecules can clump into sheets and wires. The ends of these congealed string clumps will form a lower dimensional subspace, which we call a D-brane.

This is just like the way in which the ends of a sheaf of wheat stalks, as pictured on the right, form a two-dimensional surface, even though the wheat stalks themselves are like one-dimensional objects which are free to move in three dimensions. Just as the motion of an insect feeding on the cut ends of the stalks would appear as if it were confined to a two-dimensional surface, the behaviour of particles and interactions arising from vibration modes of the strings in the conglomerate would appear to be in the lower dimension. If this dimension happens to be three, then the corresponding D_3-brane could be what we call our Universe. We can now explain why the particles and forces which form the Standard Model of particle physics appear confined to three dimensions – they arise entirely from the vibration modes of open strings which have conglomerated into a D_3-brane. On the other hand, if there is a closed string, like the little loop pictured on the left, then it will be free to move everywhere in the higher dimensional space. The gravitational field has long been known to correspond to vibrational modes of closed strings. Hence we understand why gravity is free to propagate in the higher dimensions.

This combination of a string theoretic mechanism with a neat solution of the hierarchy problem took the scientific world by storm. It related the newest ideas in string theory with the century-old question of why gravity is so much weaker than electromagnetism. Moreover, it indicated, as we shall see, a possibility that the strong gravitational effects lurking just outside the confined of our D_3-brane might actually leak a little into laboratory experiments, leading to small effects which can be verified at current day experimental facilities such as the Large Hadron Collider (LHC) at Geneva. Let us see how this can arise.

Even if we go back to the simple extra dimensional model of Kaluza and Klein, we encounter the



phenomenon of Kaluza-Klein modes. It is not difficult to understand these. According to Einsteins special theory of relativity, the Newtonian relation between energy and momentum, viz.

$$E = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$$

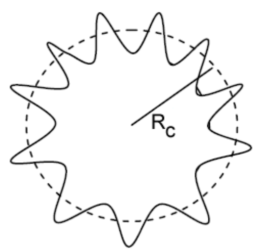
must be replaced by

$$E^2 = p_x^2 + p_y^2 + p_z^2 + m^2$$

in a system of units where the speed of light $c = 1$ (e.g. length is measured in light-seconds). If there is an extra dimension, then this becomes

$$E^2 = p_x^2 + p_y^2 + p_z^2 + p_4^2 + m^2$$

where p_4 is the component of the momentum along the fourth, compact dimension.



Now recall that the wavefunction of a free particle in the extra dimension must describe an integral number of wavelengths around the compact dimension, as shown in the figure on the left. In this case, we can write the circumference of the extra dimension as

$$2\pi R_c = n\lambda$$

i.e. $\lambda = \frac{2\pi R_c}{n}$ where n is an integer. Using the de Broglie relation $\lambda = 2\pi\hbar/p_4$, then, we arrive at

$$p_4 = \frac{n\hbar}{R_c}$$

i.e. the momentum around the compact direction must be discrete, increasing in steps of $\hbar/R_c$. The energy-momentum relation now becomes

$$E^2 = p_x^2 + p_y^2 + p_z^2 + \left(\frac{n\hbar}{R_c}\right)^2 + m^2 = p_x^2 + p_y^2 + p_z^2 + M_n^2$$

which looks like a set of three-dimensional relations with effective (squared) masses

$$M_n^2 = \left(\frac{n\hbar}{R_c}\right)^2 + m^2$$

In most cases of interest, $\hbar/R_c \gg m$, so we can neglect m and write, simply,

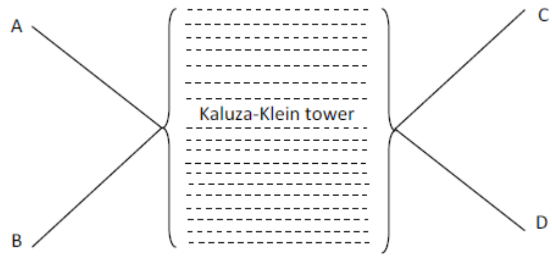
$$M_n = \frac{n\hbar}{R_c} \; .$$

Thus, a single freely-moving particle in three ordinary and one compact dimension, will appear in three dimensions as a whole set of particles, with masses increasing in steps of $\hbar/R_c$. This is often referred to as a Kaluza-Klein *tower of states*, and the individual particles are referred to as Kaluza-Klein modes. The argument is easily extended into N extra dimensions to get

$$M_n \approx \frac{\hbar\sqrt{n_1^2 + n_2^2 + \cdots + n_N^2}}{R_c}$$

How does this matter for the ADD model? Here most of the particles are confined to three dimensions, and they do not have any wavefunction (probability) extending into the fourth (or more) dimensions. However, there is one particle that does go into the extra dimension, and that is the massless graviton – the quantum carrying the gravitational force in the same way as the massless photon carries the electromagnetic force. On the D_3-brane, i.e. in the observable Universe, the graviton will appear, not as a simple massless graviton, but as a whole tower of massive Kaluza-Klein modes of the graviton. If $R_c \sim 10^{-5}$ m, this indicates a mind-boggling 10^{30} modes! The gravitational force between two adjacent particles will then, be not just the force mediated by a single graviton and leading to Newton's law with a strength measured by Newton's constant G_N, but a collective force mediated by literally zillions of Kaluza-Klein modes of the graviton. The net force will be, not the weak Newtonian force predicted between elementary particles, but a much stronger force which may become detectable in scattering experiments performed in the laboratory. A schematic picture of this collective interaction in a two-body scattering process $A + B \to C + D$ is drawn below.

Such collective interactions could, in principle, be expected to lead to observable effects at high energy particle accelerators like the Large Electron Positron (LEP) Collider which ran at CERN, Geneva between 1991 – 2001, at the Tevatron, which is running at Fermilab, USA, since 1994, and at the Large Hadron Collider (LHC) at CERN, which commenced its run last year. Till date, we have not found any evidence whatsoever for gravitational interactions between elementary particles of the kind described above. This tells us that if there are, indeed, extra dimensions as hypothesized by ADD, their size must be small enough to raise the higher-dimensional Planck scale above



$10^{11} eV/c^2$. However, the LHC, currently operating at a collision energy of$7 \times 10^{12} eV/c^2$, could certainly probe the hitherto-inaccessible region and tell us if there are, indeed such large extra dimensions.

What if the LHC does not find any evidence for large extra dimensions, even when it reaches its full energy of $1.4 \times 10^{13} eV/c^2$? This will not invalidate the theory, but merely push the maximum possible size of the extra dimensions to a smaller value. However, it will be a disappointing result, in the sense that the model will then become unverifiable, except perhaps in the realm of ultra high energy cosmic ray studies. Moreover, the ADD construction was discovered, within a year of its proposal, to have a serious flaw, viz. the large size of the extra dimensions is not stable under quantum corrections. In a manner very reminiscent of the way in which the Higgs boson mass is dragged to the Planck scale $10^{25} eV/c^2$ by quantum corrections, the size of the extra dimensions is dragged to $R_c \sim 10^{-35} m$ by analogous effects. This would mean that the Planck scale is $10^{23} eV/c^2$ in the $3 + N$ dimensional space as well as on our D_3-brane, and we would be back to where Kaluza and Klein stood.

Several solutions have been proposed for this problem. One is to invoke supersymmetry to cancel the troublesome quantum corrections, exactly as was done in the case of the Higgs boson. The logic for this is that if the D_3-brane is formed in a string theory, then supersymmetry is a natural ingredient in the theory anyway. On the other hand, if there is supersymmetry, we already have a solution to the hierarchy problem, and then the ADD construction does not serve any useful purpose. This is not to say that there cannot be extra dimensions if there is supersymmetry, but normally science does not assume things unless we need to. A famous principle enunciated by the scholastic philosopher William of Occam (c. 1288 – c. 1348) states: *Entities are not to be multiplied without necessity*, and this is generally known in science as "Occam's Razor". Thus, if we have a supersymmetric solution to the hierarchy problem, the ADD solution would fall foul of Occam's Razor[8]. For this reason, the supersymmetric solution to the problem of stabilizing large extra dimensions has not been very popular, though no one has challenged it as wrong or impossible.

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[8]However, one must not use Occam's Razor blindly. It would be like sitting in Mumbai and arguing that penguins do not exist because they are not needed for the local ecosystem in Mumbai. Nature has surprised us before and will surely surprise us again.

A much more popular alternative to the ADD construction has been a model with two D_3-branes and one extra dimension, proposed by Lisa Randall and Raman Sundrum in 1999. This collaboration, between the all-American Randall and Sundrum, an Australian of Indian origin, is another tribute to the globalization of science, and especially to the US academic system which is a veritable melting pot of nationalities. The Randall-Sundrum (RS) model is a bit too technical to be discussed in an article of this nature, but it succeeds where the ADD model fails, in providing a mechanism to control the quantum corrections to the Higgs boson mass without having recourse to large extra dimensions. However, the ratio between the gravitational force and the electromagnetic force in the RS model is now an extremely sensitive function of the radius of compactification R_c. Small dynamic fluctuations could change this ratio, which is known to be completely stable. Thus, we require a mechanism to keep the size of the extra dimension fixed. There is no such mechanism in the original RS model, but an extension devised by Walter D. Goldberger and Mark B. Wise of Caltech can do the job by introducing an extra scalar field (somewhat like the Higgs boson) which lives in the full five dimensional space of Randall and Sundrum.

Another suggestion which has found favour in the scientific literature is that of a *universal* extra dimension. In this model, there are no D_3-branes. There is just one extra dimension and all the particles and forces of the Standard Model can go into the extra dimension. It differs from the Kaluza-Klein model in that the extra dimension is not a circle, but is like a circle folded about a diameter. In this theory, every particle has Kaluza-Klein modes, and it is predicted that some of these may be discovered at the LHC or other machines, if the radius of compactification R_c is large enough. There are variations to this model, such as a model with *two* universal extra dimensions, but the basic ideas are the same.

To conclude, then, extra dimensions of space have progressed from a metaphysician's dream to an active area of scientific research. Apart from its intrinsic interest, this is a field where various disciplines merge. However, only the future will tell if all this is hard science, or a pretty fiction. At present there is no perfect theory of extra dimensions which explains everything and is completely consistent internally. But this does not mean that we should abandon the search. Saint Augustine, the famous Doctor of the Church, told us long ago that "*A thing is not necessarily false because it is badly expressed, nor true because it is expressed magnificently*". As with all of Western empirical science, the proof of extra dimensions will lie in hard experimental facts acquired in the laboratory. We can only look forward to that exciting era.

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"Absolute" motion of the earth in the universe

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Abstract. In this paper, we determine the velocity of earth with respect to a reference frame in which the distribution of matter in the universe appears isotropic. We use the distribution of distant radio sources to define such a reference frame. In particular we look for departures from isotropy in the angular distribution of radio sources in sky as a result of earth's motion. Our results give a direction of the velocity of earth in agreement with those determined from the Cosmic Microwave Background Radiation (CMBR) measurements by COBE and WMAP satellites.

Communicated by: A.M. Srivastava

1. INTRODUCTION

Our Earth is not at rest. It goes around the sun and the sun along with the earth and the remaining solar system bodies, goes around the centre of our Milky Way. The Milky Way in turn has a motion within the local group of galaxies, which may itself be moving with respect to the Virgo Supercluster and so on. If we add all these velocity vectors and thereby get a resultant vector for the earth's velocity with respect to the largest scale distribution of matter in the universe that may be considered to be fixed in the co-moving co-ordinate of the expanding universe, it may be justifiably called an "absolute" velocity of the earth. Of course it should be clarified that the word absolute here does not imply in any sense the presence of the historical "eather" or some absolute space and time. It is absolute in the sense that there are no further changes in it when we go to still larger scales in the universe. Then we get velocity of earth with respect to a reference frame which is stationary with respect to the average distribution of the matter in the universe and from which, according to the cosmological principle, the universe will appear isotropic without any preferred direction.

The earth's velocity vector in its yearly orbit around the sun is quite accurately known, with the magnitude (~ 30 km/s) and direction at any time well determined. But the same cannot be said of the other velocity vectors. At the same time, while over a year earth's velocity vector around the sun turns by a complete 360° to yield an average value ~ 0, the change in all the other vectors is very minute. For example, in a year the direction of the solar system's velocity in the orbit around the Milky Way changes by only ~ 6 milli-arcsec [1], implying a change of less than half an arcsec over

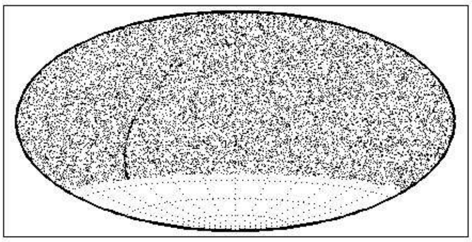

Figure 1. The distribution of strong sources ($S > 300$ mJy) in equatorial co-ordinates

a human life-span (~ 70 years!). Thus it is necessary to find out the sum of the all other vectors alone to know the net velocity vector of the solar system. If need be earth's motion around the sun can always be added to that. In fact the astronomical position calculations routinely take care of the aberration (maximum ~ 20 arcsec) caused by the earth's motion around the sun.

Of course we cannot leave the earth (at least not go very far from it) and make measurements of earth's velocity from some outside points in the universe. All our measurements have to be done confined to the earth from where we may look in different directions in the sky to determine any departures from isotropy. Thus to be able to do this quantitatively, one needs a distribution of some actual quantity which can be measured in various directions of the sky. In the last couple of decades, the Cosmic Microwave Background Radiation (CMBR) has been used as such a quantity and the variation in the temperature distribution of the CMBR has given quite accurate measurements of a dipole anisotropy, supposedly arising from the absolute velocity vector of the earth [2,3].

In this paper we use the angular distribution of distant radio sources in the sky to look for departures from isotropy of the universe. This provides an independent check on the interpretation of CMBR dipole anisotropy being due to earth's motion. Also CMBR provides information about the isotropy of the universe for redshift $z \sim 700$, but the radio source population refers to a much later epoch $z \sim 1 - 2$. Thus it also provides an independent check on the cosmological principle where isotropy of the universe is assumed for all epochs. In past Blake and Wall [4] have done such a study and our attempt though conceptually similar, differs from them in simplicity and directness of the approach.

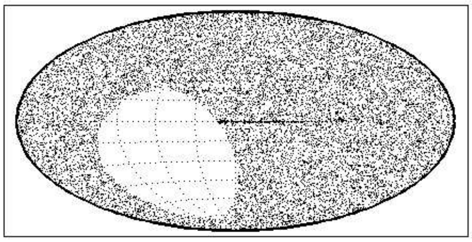

Figure 2. The distribution of strong sources ($S > 300$ mJy) in galactic co-ordinates

2. THE SOURCE CATALOGUE

We have used the NVSS catalogue (NRAO VLA Sky Survey [5]) for our investigations. This survey covers whole sky north of declination $-40°$, a total of 82% of the celestial sphere, at 1.4 GHz. There are about 1.8 million sources in the catalogue with a flux density limit $S > 3$ mJy. We have downloaded the NVSS catalogue files by anonymous FTP from ftp://ftp.aoc.nrao.edu/pub/software/aips/TEXT/STARS. The catalog is available in a compact form, giving for each source right ascension, declination and flux density at 1.4 GHz in a tabular form.

Fig. 1 shows a plot of the relatively strong sources ($S > 300$ mJy) in equatorial co-ordinates. The southern gap is because of the $\delta > -40°$ limit of the survey. The source distribution looks quite uniform accept for a narrow band of enhanced density presumably due to galactic sources. To confirm this, we have plotted in Fig. 2 the source distribution in galactic co-ordinates. The enhanced density is now clearly seen to be lying along the galactic plane.

3. ABERRATION

We assume that to an observer on earth without its motion the sky would have looked isotropic, in particular the radio source distribution would have appeared quite uniform in all directions (ignoring a local enhancement due to galactic sources). The motion of the earth will introduce a dipole anisotropy in this distribution. Due to the aberration of light, the apparent position of a source along angle θ with respect to direction of motion will actually be shifted by $-\beta \sin\theta$, where $\beta = v/c$ is the speed of earth in units of speed of light. Here we have used the non-relativistic formula for aberration as CMBR observations indicate that $\beta << 1$. For $\theta = 90°$ the shift is maximum with a magnitude $\Delta\theta = \beta$. Thus due to the aberration all sources will have a finite angular shift in their position towards the direction of motion of the earth. Now if we divide the sky in two equal hemispheres, one in the forward direction, i.e. centered on the direction of motion of earth, and the second in the backward direction, then due to aberration some sources from the backward hemisphere, lying in a narrow strip of angular width $\Delta\theta = \beta$ at the boundary between the hemispheres, will have shifted to new positions in the forward hemisphere. Thus there will be a larger number of sources N_1 in the forward hemisphere as compared to N_2 in the backward hemisphere. The excess in numbers can be calculated this way. If N_0 is the number density per unit solid angle for the isotropic distribution, then $N_1 = 2\pi\, N_0 + 2\pi\, N_0\, \Delta\theta$ and $N_2 = 2\pi\, N_0 - 2\pi\, N_0\, \Delta\theta$ then the fractional excess in number of sources will be

$$\frac{\Delta N}{N} = \frac{N_1 - N_2}{N_1 + N_2} = \frac{4\pi\, N_0\, \Delta\theta}{4\pi\, N_0} = \Delta\theta = \beta. \tag{1}$$

Thus we see that the fractional excess in number of sources between the two hemispheres could provide a direct measure of the absolute speed of earth. However there are additional complications that need to be considered. The sources in the forward hemisphere will become brighter due to Doppler beaming, while those in the backward hemisphere will become fainter. This will cause a telescope of a given sensitivity limit to detect comparatively a larger number of sources in the forward hemisphere. The integral source counts of extragalactic radio source population show that $N(> S)$, the number density per unit solid angle of sources above a flux density S, is given by a power law $N(> S) \propto S^{-x}$ where index x may depend upon the flux density level. For a Euclidean universe the expected value is $x = 1.5$. From the NVSS data we have determined x to be ~ 1.8 for $S > 1$ Jy and about ~ 1 at weaker levels.

In a non-relativistic case, the frequency ν of photons from a source in direction θ will be shifted by Doppler factor $\delta = 1 + \beta \cos\theta$ and the observed flux density S will be higher than the rest frame value by a factor $\delta^{1+\alpha}$, where α is the spectral index defined by $S \propto \nu^{-\alpha}$. Then as shown in [6], the observed source count due to motion of the earth will show a dipole anisotropy over the sky of magnitude $[2 + x(1 + \alpha)]\beta \cos\theta$. Integrating over the two hemispheres, we get

$$\frac{\Delta N}{N} = \beta \left[1 + \frac{x(1+\alpha)}{2}\right]. \tag{2}$$

Here we see that apart from the term β resulting from aberration as described earlier, there are additional terms arising due to Doppler boosting.

But first we need to find the direction of motion of the earth, otherwise how to know where lies the forward hemisphere and along what great circle to divide the sky in two hemispheres for computing the excess. A hit and trial method could be tried, but that may need too many trials. There is a much neater way of finding the direction of the earth's motion.

We consider all sources to lie on the surface of a sphere of unit radius and let $\mathbf{r}_i$ be the position vector of i^{th} source with respect to the centre of the sphere. An observer stationary at the centre of the sphere will find the position vectors to be randomly distributed in all directions (due to the assumed isotropy of the universe) and therefore should get $\Sigma\mathbf{r}_i = 0$. On the other hand for an observer on moving earth at that location, due to the dipole anisotropy in number density, the sum of all position vectors will give a net vector in the direction of earth's motion, thereby fixing the direction of the dipole.

The NVSS catalogue has a gap of sources below a declination $-40°$. In that case our assumption of $\Sigma \mathbf{r}_i = 0$ for a stationary observer does not hold good. However if we drop all sources from $\delta > 40°$ as well, then there are equal and opposite gaps in source distribution on opposite sides of the celestial sphere and $\Sigma \mathbf{r}_i = 0$ is valid for a stationary observer. Thus we confine ourselves to sources within $\pm 40°$ to determine the direction of motion of the earth. Further we also excluded all sources from our sample which lie in the galactic plane ($|b| < 10°$). This is because the excess of sources in the galactic plane (Fig. 2) is likely to contaminate the determination of the direction of earth's motion. Of course exclusion of such strips, which affect the forward and backward measurements identically, do not affect our results in any systematic manner [6]. We also explored the affect of any excess of radio sources in the super-galactic plane. We found no discernible difference in the determined velocity vector of earth's motion whether we included or excluded sources in the super-galactic plane.

4. RESULTS

Before proceeding with the actual source sample we created an artificial radio sky with about two million sources (similar to the total number of sources in the NVSS catalogue) distributed at random positions in the sky. We took the flux-density values from the actual NVSS sample, but the sky positions were allotted randomly to each source. Then we randomly assigned a velocity vector for earth's motion and superimposed its calculated aberration effects for each source by shifting its position by a small vector $\Delta \mathbf{r}_i = -\beta \sin\theta\, \hat{\mathbf{e}}_\theta$, where θ is the angle of the original source position with respect to the velocity vector assigned to the earth. The resultant artificial sky was then used to calculate the velocity vector of the earth which was compared with the value actually assigned. This not only verified our procedure but also allowed us to make an estimate of errors as a large number of simulations (~ 50) were run starting with different random sky positions and a different velocity vector each time. A realistic estimation of errors was the toughest part of the whole exercise. The simulations also allowed us to verify our assertion that rejection of sources at high declinations ($|\delta| > 40°$) or in galactic plane ($|b| < 10°$) did not have any systematic effects on the direction of the computed velocity vector. However these gaps in the number distribution raised the computed value of $\Delta N/N$ by $\sim 15\%$, resulting in the magnitude of the velocity vector being overestimated by a similar factor.

Our results are presented in Table 1, which is almost self-explanatory. The velocity vector was estimated for samples containing all sources with flux-density levels $> S$, starting from $S = 50$ mJy and going down to $S = 20$ mJy levels. Of course the estimate improves as we go to lower flux-density limits, since the number of sources increases as $N(> S) \propto S^{-x}$. From Table 1 we infer that $x \approx 1$ at these flux-density levels. But we did not go to still lower flux-density levels as we are not sure about the completeness of the NVSS sample at those levels. For calculating β, we took the typical spectral index value of $\alpha = 0.8$. The calculated RA and Dec for the earth velocity vector are

Table 1. Earth's velocity vector determined from samples at various flux-density levels

S (mJy)	N	σ_N ($\sqrt{N}$)	ΔN ($N_1 - N_2$)	$\Delta N/\sigma_N$	$\Delta N/N$ ($\times 10^{-3}$)	RA (°)	Dec (°)	β ($\times 10^{-3}$)
> 50	91597	303	1131	3.7	12.3	171 ± 16	-18 ± 16	5.6 ± 1.5
> 40	115838	340	1218	3.6	10.5	158 ± 14	-19 ± 14	4.8 ± 1.3
> 30	154999	394	1943	4.9	12.5	156 ± 12	-03 ± 12	5.7 ± 1.2
> 25	185477	431	2143	5.0	11.5	158 ± 11	-02 ± 11	5.3 ± 1.1
> 20	229368	479	2836	5.9	12.3	153 ± 10	02 ± 10	5.6 ± 1.0

listed in Table 1 along with the estimated amplitude of the velocity (corrected for the gaps $|\delta| > 40°$, $|b| < 10°$), in units of speed of light. The errors in RA and Dec are estimated from the simulations while that in β are estimated from the expected uncertainty $\sigma_N = \sqrt{N}$ in $\Delta N = N_1 - N_2$, the uncertainty here being that of a binomial distribution, similar to that of the random-walk problem (see, e.g., [7]).

Our estimates of the direction of motion of earth's velocity vector are in quite agreement with those determined from the CMBR (RA= $168°$, Dec= $-7°$, [2,3]), but our estimate of the magnitude of the velocity vector somehow appears much higher than the CMBR value ($\beta = 1.23 \times 10^{-3}$). We are still trying to understand this difference.

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Solar Neutrinos and Neutrino Oscillations*

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Abstract. The following work was carried out in the NIUS 6.2 and 6.3 winter and summer camps. We begin with the neutrino production process in the sun and the solar neutrino anomaly as a motivation for the neutrino oscillation. Assuming non-zero neutrino mass, the formal results of the quantum mechanics of neutrino oscillation in vacuum is stated. Then the effect of the ambient matter on neutrino oscillations is considered. The KamLAND data is then reviewed which pins down the parameters for solar neutrinos. The paper is concluded with the physics of 3-neutrino oscillations. This formalism along with recent data from solar and KamLAND suggests a non zero value of θ_{13} which hints towards a possible discovery of CP violation in the leptonic sector.

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1. NEUTRINOS IN THE STANDARD MODEL

The standard model of particle physics in its simplest form enlists the following properties for neutrinos: i) strict conservation of lepton number, ii) zero mass for neutrinos, and iii) only one helicity state for the neutrinos. Neutrino comes in three flavors, corresponding to the three generations of charged leptons e^-, μ^- and τ^-. These neutrinos namely ν_e, ν_μ and ν_τ are called the flavor or interaction eigenstate. Thus in the standard model we have 3 isospin doublets of left handed leptons

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \tag{1}$$

Neutrino flavor oscillations tend to resolve the long standing solar neutrino anomaly which is explained in the next section. But as we will observe, these flavor oscillations requires that the neutrinos must have a non-zero mass and that they mix. These ideas lie beyond the confines of the standard model of electroweak theory. Therefore the resolution to the solar neutrino problem hints at a new physics beyond the standard model. In the discussion that follows we assume that neutrinos have a tiny but non-zero mass without worrying about its origin.

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*A Review

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2. NEUTRINO PRODUCTION IN THE SUN AND THE SOLAR NEUTRINO ANOMALY

The sun is a main sequence star. It produces an intense flux of electron neutrinos (ν_e). Energy production in the sun takes place through the p-p chain of fusion reactions, which occur at a high temperature of about 1.7×10^7 K inside the sun. Protons fuse to form ^4He nuclei, through various intermediate nuclear reactions producing high energy photons and ν_e whose energy is of the order of MeV. About 99.6% of the total neutrino flux are produced through the pp fusion reaction[1]. The Standard Solar Model(SSM), proposed by J. N. Bahcall in the early sixties, predicts the neutrino fluxes from the various intermediate nuclear reactions[2]. The figure below depicts the neutrino fluxes along with their energy spectra. Most nuclear reactions produces neutrinos with continuous energy spectra but neutrinos that are produced through the *pep* and ^7Be reactions produce neutrino lines, as they correspond to two body final state. The resulting neutrinos in decreasing order of flux are (1) the low energy pp neutrinos, (2) the intermediate energy Be neutrinos and (3) the relatively high energy B neutrinos.

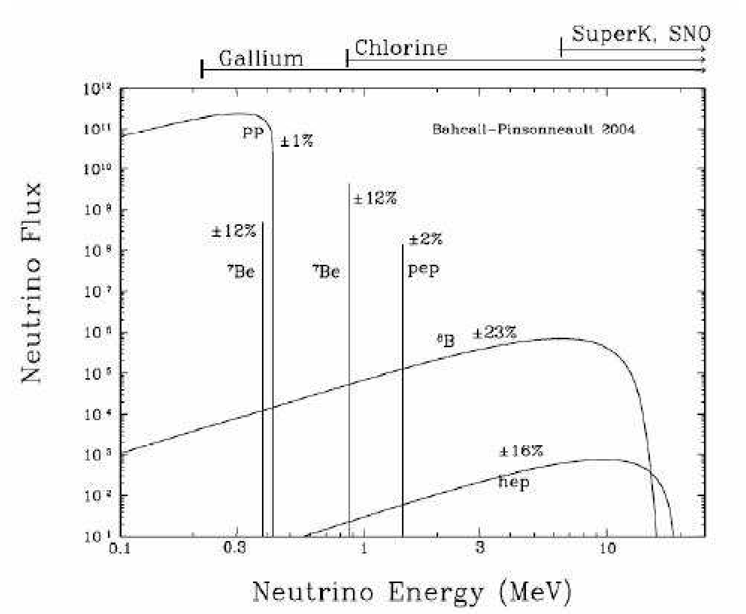

Figure 1. The standard solar model (SSM) prediction for the solar neutrino fluxes shown along with the energy ranges of the solar neutrino experiments.

[1] An overview of the neutrino production process in the sun can be found in Prayas Vol.4, No.1,Jan.-Mar. 2010 *Neutrino Oscillation Phenomenology*

2.1 The Solar Neutrino Problem

Since neutrinos have a very small cross-section, their detection is difficult to achieve. However there are various experimental techniques to detect neutrinos coming from the sun. These are (1) radiochemical detection, (2) water Cerenkov detection, (3) heavy water detection.

Radiochemical Detection(^{37}Cl and ^{71}Ga): In radiochemical method a target material X, on interaction with neutrinos gets converted into a radioactive isotope of another element Y with half-life of several weeks. In a typical run the, the detector is left to absorb neutrinos for a few week. Then the few atoms of the radioactive end product are extracted and counted by using radiochemical techniques. By knowing the production cross-section, we can use the number of produced radioactive nuclei to compute the average neutrino flux. Nuclei that have been used for such experiments are ^{37}Cl and ^{71}Ga. The earliest solar neutrino detection experiment, led by Raymond Davis, used the radiochemical method. The interaction with the solar neutrinos initiates the inverse beta decay charged current reaction:

$$\nu_e +^{37} Cl \rightarrow e^- +^{37} Ar. \tag{2}$$

The Q-value for this reaction is 0.814 MeV[3]. According to the prediction of SSM, the dominant contribution in the chlorine experiment comes from ^8B neutrinos(75%). But as we can see from figure 2 above, the energy of ^7Be neutrinos just scrapes the threshold energy of the experiment. Therefore a further contribution of 15% comes from the ^7Be neutrinos.

^{71}Ga is another element used for solar neutrino detection. In the early 1990s two experiments started producing results using Gallium as an active element in the detector. The reaction is:

$$\nu_e +^{71} Ga \rightarrow e^- +^{71} Ge. \tag{3}$$

The threshold value of this reaction is 0.233 MeV[3].The product ^{37}Ar and ^{71}Ge are radioactive having half lives of 34.8 days and 11.43 days respectively. These radioactive end products are periodically extracted and measured by Geiger-Muller counters, from which the incident neutrino flux is estimated. The value of this observed flux R relative to the SSM prediction gives the ν_e survival probability P_{ee}. The main advantage of the radiochemical method is the low threshold energy of the neutrinos detected. As seen from table 1 below, Gallium detectors have a threshold energy of 0.233 MeV, which enables the detection of the pp neutrinos(55 %). Contribution from other neutrinos are Be(25%) and B(10%). Because of this, the capture rate is quite high in Gallium detectors. Although radiochemical detection method provided the initial data for the solar neutrino fluxes it cannot determine the energy of the neutrino or the direction that it came from. It is also impossible to determine the time when a neutrino was trapped in the detector through these experiments.

Water Cerenkov detection: This technique is used in the Super-Kamiokande(SK). The technique can detect neutrinos with much larger energy. The detector material is water. The neutrino comes and hits the atomic electrons in hydrogen and oxygen atoms. Since the neutrinos have energies in the range of MeVs, the atomic binding energies are negligible ($\sim eV$)and therefore the scattering can be treated as elastic scattering of neutrinos off the free electrons:

$$\nu_e + e^- \rightarrow \nu_e + e^-. \tag{4}$$

The electron recoils with some kinetic energy from the neutrino. If the kinetic energy of the electron is much greater than its mass then it will move at a speed which is larger than the speed of light in water thereby emitting Cerenkov radiation in the process. Detection of this radiation constitutes an indirect detection of the neutrino. This method can be used to detect the direction of the incoming neutrino as the Cerenkov radiation has a strong forward peaked angular distribution. By extrapolating the cone in the backward direction, it can be verified whether the neutrino is coming from the sun. Such detection can be done in 'real-time', i.e., the neutrinos can be detected as soon as they arrive in the detector. This method can detect even ν_μ and ν_τ in a neutral current interaction. However, the efficiency of detection is less as compared to that of ν_e since the cross-sections are different ($\sigma(\nu_{\mu,\tau} + e) \simeq 1/6\sigma(\nu_e + e)$). The interaction rate in terms of survival probability is

$$R = P_{ee} + \frac{\sigma^{NC}}{\sigma^{CC} + \sigma^{NC}}(1 - P_{ee}) \simeq P_{ee} + \frac{1}{6}(1 - P_{ee}). \tag{5}$$

This expression can be inverted to find the corresponding survival probability.

Heavy water detection: The technique is employed at the Sudbury Neutrino Observatory(SNO). The experiment uses 1000 tons of ultra-pure heavy water(D_2O) contained in a spherical acrylic vessel, surrounded by an ultra-pure H_2O shield. Just like the water detectors there are electrons in the atoms whose elastic scattering can be used to detect neutrinos through Cerenkov radiation. However the presence of deuteron opens up more efficient channels of neutrino detection. Deuteron, which is a bound state of a proton and a neutron, has a binding energy of 2.2 MeV, which is in the range of the energy of solar neutrinos. The incoming neutrino can undergo a charged current(CC) reaction with the deuteron as

$$\nu_e + d \rightarrow e^- + p + p, \tag{6}$$

where the information about the neutrino energy and direction can be found by the resulting electron detected via. its Cerenkov radiation. The Q-value for this reaction is -1.4 MeV and the electron energy is strongly correlated with the neutrino energy. Thus the CC reaction provides an accurate measure of the shape of the ^8B solar neutrino spectrum. The contributions from the CC reactions and $\nu_e e$ elastic scattering can be distinguished by using different $\cos\theta_\odot$ distributions where $\theta_\odot$ is the angle of the electron momentum with respect to the direction from the sun to the earth. While $\nu_e e$ have a strong forward peak, the CC events have an approximate angular peak distribution of 1-1/3$\cos\theta_\odot$[3]. A second reaction also takes place in which an incoming neutrino literally breaks up the deuteron into its components. This channel is a neutral current(NC) exchange reaction whose threshold is the binding energy of the deuteron(2.2 MeV)

$$\nu_x + d \rightarrow \nu_x + n + p \tag{7}$$

and is open to all active neutrinos. Detection of the resulting neutron via neutron capture confirms the occurrence of this process. In neutron capture process a photon is emitted. The electron coming from the Compton scattering of this photon is detected through its Cerenkov radiation. Table 1 below summarizes the energy threshold of the above four experiments along with the compositions of the corresponding solar neutrino spectra. It also shows the corresponding survival probability P_{ee} measured by the rates of the charged current reaction relative to the SSM prediction. This

Table 1. The ν_e survival probability P_{ee} measured by the CC event rate R of various solar neutrino experiments relative to the SSM prediction. For SK the P_{ee} obtained after the NC correction is shown in parenthesis.

Experiment	^{71}Ga	^{37}Cl	SK	SNO-I
R	0.55±0.03	0.33±0.03	0.465±0.015 (0.36±0.015)	0.35±0.03
E_{th} (MeV)	0.233	0.814	5	5
Neutrino Composition	*pp* (55%) Be(25%), B(10%)	B(75%) Be(15%)	B(100%)	B(100%)

table shows that the measured solar neutrino flux is less than the SSM prediction. This is the Solar Neutrino Problem. The decrease can be attributed to: i) faulty astrophysics of the sun, or ii) some new physics fundamental to neutrinos. Oscillations among neutrino flavors and solar matter effect on neutrino oscillations are able to explain all the observed solar neutrino interaction rates.

3. NEUTRINO OSCILLATIONS: BASIC RESULTS

Vacuum oscillations

The probability for an electron neutrino to oscillate into other flavour is given by[2]

$$P_{e\mu}(l) = \sin^2(2\theta)\sin^2(1.27\Delta m^2_{\text{sol}} l/E), \tag{8}$$

where θ is the mixing parameter, Δm^2_{sol} is difference between the square of mass eigen values, l is the distance travelled (in meters) and E is the energy (in MeV). Consequently the oscillation wavelength is

$$\lambda = (\pi/1.27)(E/\Delta m^2) \simeq 2.47E/\Delta m^2. \tag{9}$$

Therefore for large mixing angle ($\sin^2 2\theta \sim 1$)the following pattern of neutrino oscillation proba-

Table 2. Conversion probabilities for different values of l.

l	$\ll \lambda$	$\sim \lambda/2$	$\gg \lambda$
$P_{e\mu}$	0	$\sin^2 2\theta \sim 1$	(1/2)$\sin^2 2\theta \sim 1/2$

[2]A detailed walkthrough of the quantum mechanics of two neutrino oscillation can be found in Prayas Vol.4, No.1,Jan.-Mar. 2010 *Neutrino Oscillation Phenomenology*

bility emerges where the factor of 1/2 in the last case comes from averaging over the phase factor.

This formalism alone does not account for the observed rates (Table 1). We see that the survival probability of ν_e is slightly above 1/2 for the low energy solar neutrino but falls to 1/3 at higher energy. To understand its magnitude and energy dependence we have to consider the effect of solar matter on neutrino oscillation.

Matter effect

It was pointed out by Mikheyev, Smirnov and Wolfenstein (MSW) that neutrino oscillation pattern can be significantly affected if the neutrinos travel through a material medium rather than through vacuum. Since normal matter contains electrons and not any muon or tau, any ν_e beam that goes through matter undergoes both charged current and neutral current interaction while ν_μ and ν_τ on the other hand interacts with the electrons only through neutral current interaction. Since the neutral current interaction is common to all neutrino flavors, it has no net effect on neutrino oscillations. On the other hand CC interaction has a profound effect on the neutrino oscillation. Therefore if we include the CC interaction as the potential energy term then the mass eigenvalues become functions of the electron density in the sun:

$$\lambda_{1,2} = \frac{1}{2}[A \mp \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}], \tag{10}$$

where

$$A = 2\sqrt{2} G_F N_e E,$$

and G_F is the Fermi coupling. The electron density at the solar core is $N_0 \sim 6 \times 10^{31} m^{-3}$[2], and it decreases roughly in an exponential manner as we move out of the solar core[3]. The variation of these eigenvalues as functions of the solar electron density is plotted in figure 2. The two eigenvalues however never actually cross. There is minimum gap given by:

$$\Gamma = \Delta m^2 \sin 2\theta \tag{11}$$

This implies that a ν_e produced at the solar core will come out as ν_2 provided the transition probability between the two energy levels remains small. The ν_e produced at the solar core is dominated by the ν_1 component. However at the critical density region, there is a resonant conversion between the two components. After passing through the critical density region, the ν_e continues to follow the upper eigen value curve λ_2 and the neutrino comes out from the sun as ν_2 with

$$P_{ee} = \sin^2 \theta \tag{12}$$

[3]In fact except for the inner 10% of the radius the best fit equation for the solar electron density as a function of the radius R is given by $N_e/N_a = 245 exp(-10.54 R/R_\odot)$[2], where $R_\odot = 6.96 \times 10^8 m$ is the radius of the sun and N_a is the Avogadro's number.

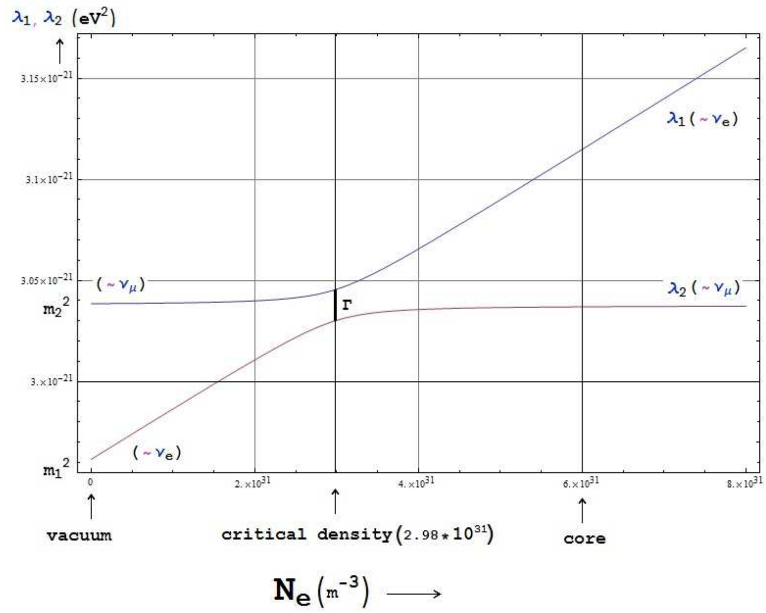

Figure 2. Effective mass(energy) eigenvalues as functions of the solar electron density at small mixing angle.

provided the transition probability between the two levels remains small throughout the propagation. The most important region for this transition is the critical density region, where the gap between the two levels is the smallest. This transition probability is given by the Landau-Zenner formula. It relates the transition rate T to the above gap Γ by the relation

$$T = e^{\frac{-\pi}{2\gamma}} \tag{13}$$

where γ is given by

$$\gamma = \frac{\lambda_c (d\lambda_1/dl)_c}{\Gamma} \propto \frac{\lambda_c (dN_e/dl)_c}{N_e^c} \tag{14}$$

and λ_C represents the oscillation wavelength in matter in the critical density region. If the solar electron density varies so slowly that the resulting variation in the 1^{st} eigenvalue over an oscillation wavelength is small compared to the gap between the two, then $\gamma \ll 1$ and the transition rate is exponentially suppressed. This is called the adiabatic condition. Thus the two conditions for the solar ν_e to emerge as ν_2 together give,

$$\frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2} G_F N_e^0} < E < \frac{\Delta m^2 \sin^2(2\theta)}{2\cos(2\theta)(dN_e/dlN_e)_c}. \tag{15}$$

The figure below shows the triangular region in the $\Delta m^2 - \sin^2 2\theta$ plot satisfying the above condition. The horizontal side of the triangle follows from the first inequality, which gives a practically constant upper limit of Δm^2 in terms of the solar core electron density, since $\cos^2 2\theta \simeq 1$. The second inequality (adiabatic condition) gives a lower limit on $\sin^2 2\theta$, determined by the solar electron density gradient. Moreover since this condition implies a lower limit on the product $\Delta m^2 \sin^2 2\theta$, it corresponds to a diagonal line on the log-log plot. The vertical side of the triangle is simply the physical boundary corresponding to maximal mixing. This is also called the MSW triangle. The

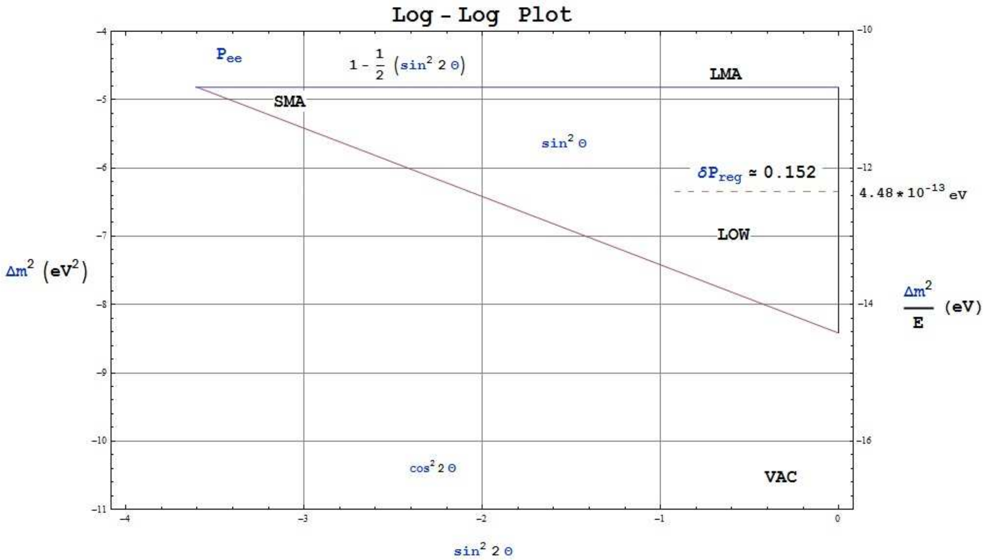

Figure 3. The positions of the MSW triangle, the earth regeneration effect and the vacuum oscillation maximum are shown for E = 1 MeV along with the positions of the SMA, LMA, LOW and VAC solutions.

indicated survival probabilities outside the triangle follows from vacuum oscillation formulae, while that inside corresponds to equation 12. Thus $P_{ee} < 1/2$ inside the MSW triangle and $> 1/2$ outside it, except for the oscillation maximum at the bottom, where the survival probability goes down to $\cos^2 2\theta$. Finally, the earth matter effect gives a small but positive ν_e regeneration probability, which means the sun shines a little brighter at night in the ν_e beam.

3.1 The four solutions: SMA, LMA, LOW and VAC

Figure 3 marks four regions in the mass and mixing parameter space, which can explain the magnitude and energy dependence of the survival probability P_{ee} shown in Table 1. They correspond to the so called Large Mixing Angle (LMA), Small Mixing Angle (SMA), Low Mass (LOW) and Vacuum Oscillation (VAC) solutions. For the LMA and SMA solutions ($\Delta m^2 \sim 10^{-5} eV^2$) the low energy Ga experiment (E $\ll$ 1 MeV) falls above the MSW triangle in Δm^2/E, while the SK and SNO experiments (E $\gg$ 1 MeV) fall inside it. Therefore the solar matter effect can explain the observed decrease of the survival probability with increasing energy. For the LOW solution the low energy

Ga experiment is pushed up to the region indicated by the dashed line, where it gets an additional contribution to the P_{ee} from the earth's regeneration effect. Finally the VAC solution explains the energy dependence of the survival probability via the energy dependence of the oscillation phase in equation 8. Figure 4 shows the predicted survival probabilities for the four solutions as functions of the neutrino energy. The LMA and LOW solutions predict mild and monotonic energy dependence, while the SMA and VAC solutions predict very strong and non-monotonic energy dependence. The

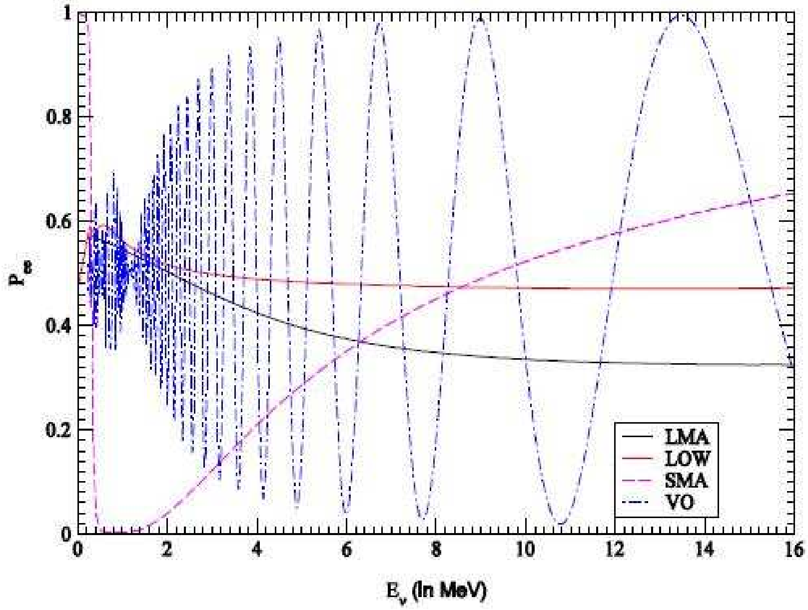

Figure 4. The predicted ν_e survival probabilities for the SMA, LMA, LOW and VAC solutions.

survival rates in Table 1 show a slight preference for a non-monotonic energy dependence, since the intermediate energy Chlorine experiment shows a little lower survival rate than SK. Therefore the SMA and the VAC solutions were the early favorites. However, the situation changed with the measurement of the energy spectrum by SK as shown in the plot below[9]. It shows practically no energy dependence in the 5-15 MeV range in clear disagreement with the SMA and the VAC predictions of figure 4. This was supported by the charged current data from SNO. So the SMA and VAC solutions were ruled out in favor of the LMA and LOW. We also see from Figure 4 that the LOW solution cannot account for the entire drop of the survival probability with energy from 0.55 to 0.35. But we could blame the low survival rate seen by the Cl, SK and SNO CC reactions partly on the large uncertainty in the Boron neutrino flux of the SSM (Fig.1). This changed however with the publication of the neutral current data by SNO. Being flavor independent, the NC reaction is unaffected by neutrino oscillation; and hence it can be used to measure the boron neutrino flux. The measured flux was in agreement with the SSM prediction and significantly more precise than the latter. Using this flux in a global fit to the solar neutrino data essentially ruled out LOW in favor of

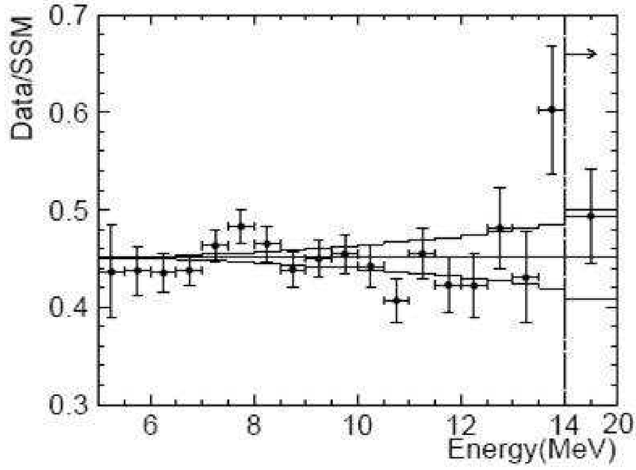

Figure 5. Energy dependence of the SK spectrum

the LMA solution. A further confirmation of the LMA solution came from the reactor anti-neutrino data of the long baseline KamLAND (KL) experiment. It is a kiloton liquid scintillator experiment detecting $\overline{\nu}_e$ from the Japanese nuclear reactor through the charged current interaction

$$\overline{\nu_e} + p \to e^+ + n. \tag{16}$$

It also measures the incident $\overline{\nu_e}$ energy through the visible scintillation energy produced by the positron and its annihilation with the target electron i.e.

$$E_{vis} = E + m_e + m_p - m_n = M - 0.8MeV. \tag{17}$$

The KL data, taken together with the global solar neutrino data gives a precise estimate of both the mass and the mixing angle parameters:

$$\Delta m^2_{sol} = \Delta m^2_{12} = 7.7 \times 10^{-5} eV^2, \quad \sin^2\theta_{\rm sol} = \sin^2\theta_{12} = 0.33 \tag{18}$$

4. THREE-NEUTRNO OSCILLATION

In the quantum mechanics of solar neutrino oscillation, we assume that a ν_e oscillates into a state which is a superposition of ν_μ and ν_τ. However, since, mixing takes place among all the three flavor of neutrinos, we now consider 3-neutrino formalism. The mass eigenstates, ν_i are related to flavor eigenstates, ν_α through a unitary matrix U by the relation,

$$\nu_\alpha = \sum U^*_{\alpha i}\nu_i \tag{19}$$

where U is often denoted as $U_{\rm PMNS}$, after the authors Pontecorvo-Maki-Nakagawa-Sakata. In the general 3-neutrino oscillation case a 3×3 mixing matrix can be parameterized by the three mixing angles θ_a, θ_x, θ_s and three phases δ, ϕ_2, ϕ_3. If neutrinos are Dirac fermions, as assumed by the standard model, one of these phase factors(δ) can be absorbed in the wave function of the neutrino states. However neutrino and anti-neutrino might be two states of the same particle, namely **Majorana particles**. In this case two more phases, which are called Majorana phases ϕ_2 and ϕ_3 are physically observable. These phases are irrelevant for oscillation and matter effects but they become observables in neutrino-less double beta decay. A frequently used parametrization of the U matrix is the following matrix product

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_a & -s_a \\ 0 & s_a & c_a \end{bmatrix} \begin{bmatrix} c_x & 0 & -s_x e^{i\delta} \\ 0 & 1 & 0 \\ s_x e^{-i\delta} & 0 & c_x \end{bmatrix} \begin{bmatrix} c_s & -s_s & 0 \\ s_s & c_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2/2} & 0 \\ 0 & 0 & e^{i(\delta+\phi_3/2)} \end{bmatrix} \tag{20}$$

where c_i denotes $\cos\theta_i$ and s_i denotes $\sin\theta_i$. The angle θ_a, here denoted as θ_{23}, governs the oscillations of atmospheric neutrinos, the angle θ_s (θ_{12}) describes solar neutrino oscillations, and the angle θ_x (θ_{13}) is an unknown angle that is bounded by reactor neutrino experiments at short distances (L $\simeq$ 1 km). Vacuum neutrino oscillations probability are given by[4]

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i U^*_{\alpha i} e^{-\frac{i m_i^2 L}{2E}} U_{\beta i} \right|^2 \tag{21}$$

where the m_i are the neutrino eigenmasses. The oscillation probabilities depend however only on differences of squared neutrino masses. Expanding this expression we get

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2\left(\delta m^2_{ij} \frac{L}{4E}\right) \\ +2 \sum_{i>j} \mathrm{Im}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin\left(\delta m^2_{ij} \frac{L}{2E}\right) \end{aligned}$$

where $\delta m^2_{ij} = m_i^2 - m_j^2$. This is the oscillation probability for *neutrinos*. To obtain the corresponding oscillation probability for *anti-neutrinos* we observe that $\overline{\nu_\alpha} \to \overline{\nu_\beta}$ is the CPT-mirror image of $\nu_\beta \to \nu_\alpha$. Thus if we demand CPT invariance then we have

$$\overline{\nu_\alpha} \to \overline{\nu_\beta} = \nu_\beta \to \nu_\alpha. \tag{22}$$

Now from the above equation for neutrino oscillation probability we see that

$$P(\nu_\beta \to \nu_\alpha; U) \equiv P(\nu_\alpha \to \nu_\beta; U^*). \tag{23}$$

Hence assuming CPT invariance holds,

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}; U) = P(\nu_\alpha \to \nu_\beta; U^*). \tag{24}$$

That is, the probability for oscillation of an anti-neutrino is the same as that for a neutrino, except that the mixing matrix U be replaced by U^*. Therefore the oscillation probability for anti-neutrino becomes

$$P(\overline{\nu_\alpha} \to \overline{\nu_\beta}) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin^2\left(\delta m^2_{ij}\frac{L}{4E}\right)$$
$$-2\sum_{i>j} \mathrm{Im}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin\left(\delta m^2_{ij}\frac{L}{2E}\right)$$

We see that if U is not real, the probabilities for neutrino oscillation $\nu_\alpha \to \nu_\beta$ and for the corresponding anti-neutrino oscillation $\overline{\nu_\alpha} \to \overline{\nu_\beta}$, will in general differ because of the sign difference in the third term of oscillation probability formula. Since $\nu_\alpha \to \nu_\beta$ and $\overline{\nu_\alpha} \to \overline{\nu_\beta}$ are CP-mirror-image processes, this difference will be a violation of CP invariance. Therefore neutrino oscillations provides a definative test of the possible discovery of CP violation in the leptonic sector.
Dnoting the oscillation arguments for the atmospheric and solar phenomena by

$$\Delta_a \equiv \frac{\delta m^2_a L}{4E}, \Delta_s \equiv \frac{\delta m^2_s L}{4E}$$

respectively where $\delta m^2_a = m^2_3 - m^2_1$, $\delta m^2_s = m^2_2 - m^2_1$, the ν_e survival probability for three generation is obtained to be[5]

$$P(\nu_e \to \nu_e) = 1 - sin^2 2\theta_x sin^2\Delta_a - (c^4_x sin^2 2\theta_s + s^2_s sin^2 2\theta_x) sin^2\Delta_s$$
$$+ s^2_s sin^2 2\theta_x(\frac{1}{2} sin 2\Delta_s sin 2\Delta_a + 2 sin^2\Delta_a sin^2\Delta_s)$$

We can see that this formula reduces to Eq. (8) for $\Delta_a = \theta_x = \theta_a = 0$. The measurement of the third unknown leptonic mixing angle is a crucial step toward the possible discovery of CP violation in the leptonic sector. We get the first hints of $\theta_{13} > 0$ from the Solar+KamLAND neutrino data. Oscillation of solar and KamLAND (anti)neutrinos do show a weak dependence on θ_{13}. From the atmospheric neutrino data we know that $\delta m^2_{31} \gg \delta m^2_{21}$, therefore the three-neutrino oscillation survival probability relevant for both solar and KamLAND (anti)neutrinos is approximately given by[6]

$$P^{3g}_{ee} \simeq \cos^4\theta_{13} P^{2g}_{ee} + \sin^4\theta_{13} \tag{25}$$

where P^{2g}_{ee} is the ν_e survival probability in the case of two-neutrino oscillations. P^{2g}_{ee} for high energy ^8B neutrinos is $\sim f_B \sin^2\theta_{12}$ where f_B is the ^8B neutrino flux. For the KamLAND P^{2g}_{ee} is the usual two-generation vacuum oscillation probability $1 - \sin^2(2\theta_{12})\sin^2(\delta m^2_{12}L/4E)$. Thus for solar neutrinos an increase in θ_{13} would imply an increase in θ_{12} whereas for KamLAND an increase in θ_{13} would imply a decrease in θ_{12}. This opposing trend *is* observed in their respective data. The Fig. 6 below shows this trend[7]. This combination of solar data and the long baseline reactor data suggests a weak preference for a non-zero value of $\sin^2\theta_{13}$, which arises as a result of a slight difference between the best-fit values of $\sin^2\theta_{12}$ in the two data sets. The best fit values from the solar neutrino data is[8]

$$\sin^2\theta_{12} = 0.33, \quad f_B = 0.84. \tag{26}$$

whereas the KamLAND data yields

$$\sin^2\theta_{12} = 0.39 \tag{27}$$

As a result of non-zero θ_{13}, θ_{12} of the solar data decreases while that of the KamLAND data

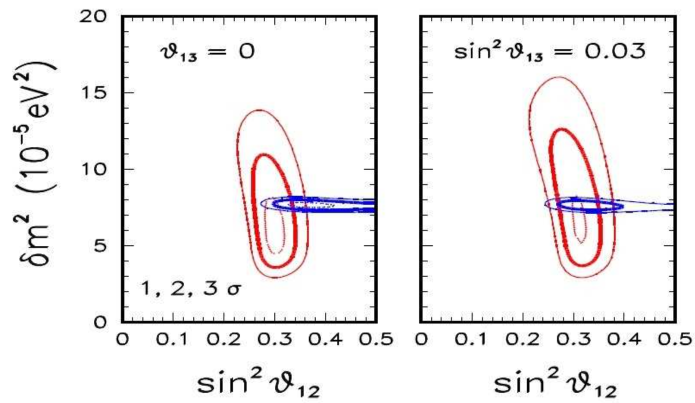

Figure 6. Effect of nonzero θ_{13} on the regions separately allowed by the latest available data(2008) from the solar and KamLAND experiments, at 1, 2 and 3σ levels.

decreases to settle at some intermediate value determined by the magnitude of the third mixing angle. The present standing value of this angle is

$$\sin^2\theta_{13} = 0.021 \pm 0.017 \quad (\text{at } 1\sigma) \tag{28}$$

for the Solar+KamLAND data.

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P R A Y A S
Students' Journal
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PROBLEMS IN PHYSICS

Readers are invited to submit the solutions of the problems in this section within two months. Correct solutions, along with the names of the senders, will be published in the alternate issues. Solutions should be sent to: H.S. Mani, c/o A.M. Srivastava, Institute of Physics, Bhubaneswar, 751005; e-mail: ajit@iopb.res.in

Communicated by H.S. Mani

Problem set by H.S. Mani

1. Consider a hydrogen atom confined inside a thin uncharged conducting shell of radius R. Assuming $R >> a_H$, where a_H is the Bohr radius. The proton (assumed infitely heavy) is at the centre of the shell.

   Find the first nonvanishing correction to

   i. The radius of the hyderogen atom assuming Bohr quantization rule.
   ii. The energy of the ground state.

2. A square cardboard of length L is intially at $x = 0$ has its corners at $(0,0,0),(0,0,L),(0,L,L)$ and $(0,L,0)$ and moves with a velocity $\vec{u} = u\hat{i}$. Rain is coming vertically down at constant velocity $\vec{w} = -w\hat{k}$. If the number of drops per unit volume is N , find the number of drops collected by the cardboard as it travels a distance D.

   Viewing the same from the cardboard's rest frame (assume relativistic velocities), show that you get the same result for the number of drops collected by the card board.

Solutions to the problems given in Vol.4 No.1

Solutions provided by: H.S. Mani

Problem 1: Consider a pencil (length L = .2m, mass 0.05Kg.) standing vertically on its tip, which can be considered as a point.Using the uncertainty principle estimate the time it can stay up without falling. Assume the tip is fixed during the fall.

Solution to Problem 1:
Let the pencil make an angle θ_0 with the vertical and let its initial angular velocity ω_0. Then the center of mass is at

$$\Delta x = \frac{L\theta_0}{2}; \Delta_p = \frac{mL\omega - 0}{2}$$

Unvertainty principle demands

$$\frac{mL^2\theta_0\omega_0}{4} \geq \frac{\hbar}{2}$$

If the pencil makes an angle θ with the vertical the torque about the tip is $mgl\sin(\theta)/2$, g being the acceleration due to gravity and so we have

$$I\frac{d^2\theta}{dt^2} = \frac{mL^2}{3}\frac{d^2\theta}{dt^2} = \frac{mgL\sin(\theta)}{2}$$

Assume the angle is small $\sin(\theta) \approx \theta$ and we get

$$\frac{d^2\theta}{dt^2} = \frac{3g}{2L}\theta$$

Solving the differential equation and using the initial condition to determine the integration constants we have

$$\theta = \frac{\theta_0 + \omega_0\tau}{2}e^{t/\tau} + \frac{\theta_0 - \omega_0\tau}{2}e^{-t/\tau}$$

where $\tau = \sqrt{2L/3g}$

$$\geq \frac{\theta_0 + \frac{2\hbar\tau}{L^2\theta_0 m}}{2}e^{t/\tau} + \frac{\theta_0 - \omega_0\tau}{2}e^{-t/\tau}$$

The minimum of the expression $(\theta_0 + (2\hbar\tau)/(L^2\theta_0 m))$ occurs at $\theta_0 = \sqrt{2\hbar(\tau/L^2m)}$. Thus

$$\theta \geq \sqrt{\frac{2\hbar\tau}{L^2}}e^{t/\tau}$$

Even though we have solved the problem for small $]theta$ we can get the order of magnitude of the time of fall by using it for estimating the time of fall by choosing $\theta = \pi/2$.

$$\frac{\pi}{2} \geq 2\sqrt{2\hbar\tau}L^2 m e^{t/\tau}$$

or

$$t \leq \tau \left(ln(\frac{\pi}{4} + \frac{1}{4} ln \frac{3gm^2l^2}{8\hbar^2} \right)$$

substitution the number the $t \leq\sim 1.7$ sec, which is macroscopic!

Problem 2: Consider a one dimensional motion of a particle along the x-axis under the action of a potential $V(x) = V_0 > 0$ for $x \leq 0$ and $V(x) = 0$ for $x > 0$.If the particle moves to the right from $x < 0$, with an energy $2V_0$, standard quantum mechanics gives for the reflection coefficient at $x = 0$ of the order $0(1)$ (The exact number is $(\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)})^{1/2}$ and the result is independent of the mass of the particle.

Now consider a car travelling with a speed v towards a cliff (height H). From the previous calculation the probability of reflection at $x = 0$ should be $0(1)$ (Assume the kinetic energy of the car to be of the same order as mgH where m is the mass of the car). This is an absurd result. Do a correct modeling for the cliff and obtain a physically reasonable result.

Solution to Problem 2:

The de Broglie wave length of the car $\lambda = \hbar/(mv)$. Since $m \approx$ several hundred kilograms and assuming $H \approx 1m$, the speed of the car is several m/s. Thus $\lambda \approx 10^{-37}m$ and the assumption the potential $V(x)$ drops to zero suggenly at this scale is urealistic. So we model the potential by

$$V(x) = V(0) + xV'(0) + \frac{x^2}{2!}V''(0) + \cdots$$

where a Taylor expansion has been made for the potential about $x = 0$. Further, we assume potential changes continuously at his scale and so $V(0) = V_0$. The wave function $\psi(x)$ for $x \leq 0$ is given by

$$\psi(x) = Ae^{ik'x} + Be^{-ik'x}$$

where $k' = \sqrt{(2m(E - V_0)/\hbar^2}$. The reflextion coefficient is given by $\mid A/B \mid^2$. For $x \geq 0$ we write

$$\psi(x) = e^{ik'x} f(x) = e^{ik'x}(F(0) + xf'(0) + \frac{x^2}{2}f''(0) + \cdots$$

and expand $f(x)$ in a Taylor series. Matching the boundary conditions at $x = 0$, we have

$$A + B = f'(0); ik'(A - B) = ik'f(0) + f'(0)$$

We also hae the Schrödinger's equation in $x \geq 0$

$$\frac{d^2\psi(x)}{dt^2} + [k^2 - U(x)]\psi(x) = 0$$

where $k^2 = 2mE/\hbar^2$ and $U(x) = 2mV(x)/\hbar^2$. Substituting the expansitons for the wave functions and the potentital and equating the coefficients of $x^i, i = 0, 1$ we get

$$f''(0) + 2ikf'(0) = 0; 2ik'f''(0) - U'(0)f(0) = 0$$

These can be used for solving

$$\frac{A}{B} = \frac{U'(0)}{\sqrt{U'(0)^2 + 64k'^4}}$$

Thus the reflection coefficient is

$$R = \mid \frac{A}{B} \mid^2 = \frac{U'(0)^2}{U'(0)^2 + 64k'^4} = \frac{\hbar^4 U'(0)^2}{\hbar^2 U'(0)^2 + 64m^4v^4}$$

Now we make an order of magniture in the MKS system. $\hbar^2 U'(0) = 2mV'(0) \approx 10^3 kg \times (\sim Joules/metre)$ and $mv \sim 10^3 10^2 \sim 10^5$. WE can neglect $\hbar^4 U'(0)^2$ in the denominator compared to m^4v^4 and so we estimate for

$$R \approx \frac{10^{-124}}{10^{22}} \approx 10^{-146}$$

which is an exceedingly small quantity!

Problem 3: A person is dropping stones at a mark on the floor from a height H. Show that the minimum spread of the stones would be

$$\sqrt{\frac{2\hbar}{m}} \left(\frac{2H}{g}\right)^{1/4}$$

where m is the stone's mass and g is the acceleration due to gravity. Calculate the spread for a Cesium atom ($At.wt = 133$) dropping a height of .2m.

Solution to Problem 3:
In classical physics the stone can be made to fall on the mark by dropping it from directly above it. However this can not be done in quantum mechanics as the uncertainity principle demands

$$\Delta(x)\Delta(p) \sim \frac{\hbar}{2}.$$

The time to drop a stone at height H is $T = \sqrt{(2H/g)}$ and so the spread as the stone reaches the floor would be

$$\Delta(x)_{floor} = \Delta(x) + \frac{\Delta(p)}{m}T = \Delta(x) + \frac{\hbar}{2m\Delta(x)}T$$

we can minimise this with respect to $\Delta(x)$ and obtain that the minimum spread occurs when

$$\Delta(x)^2 = \frac{\hbar}{2m}T,$$

which gives the spread as

$$\Delta(x)_{floor} = \sqrt{\frac{2\hbar T}{m}} = \sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}.$$

which is the required answer. For Cs, $m = 133 \times 1.66 \times 10^{-27}$kg and this gives the spread as ~ 25 microns.

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PRAYAS

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