The Jaggery (*Gud*) Mounds of Bijnor

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Abstract. Jaggery (unrefined sugar) is locally made during the sugarcane harvesting season in Bijnor, a major sugarcan growing area in India. As the hot semisolid heaps of jaggery are poured on the workfloor they acquire predictable shapes. We analyse the formation of one such shape employing elementary hydrodynamics. We obtain an interesting relation between the shear stress and the height of the jaggery mound. Using the equation of continuity and plausible assumptions we attempt to explain why the shape of the mound remains invariant. A similar approach can be used to understand a variety of shapes from porous sugar candy to glaciers.

1. INTRODUCTION

Bijnor in western Uttar Pradesh (UP) is arguably the jaggery capital of India. On the bus route from Moradabad to Meerut one can catch sight of huge mounds of jaggery (called *gur* or *gud* in Hindi and *panella* in Cental and South America) during the peak sugarcane harvesting season as one passes by this town. Indeed, the production of jaggery is a cottage industry in almost all areas of the world where sugarcane is grown in abundance.

Sugarcane is crushed and its juice is boiled and evaporated in large shallow pans. Often lime or a chemical is added so that the impurities rise to the top in a frothy mixture and are removed. The semi-solid mixture is yellow to dark brown in colour. It is scooped using small buckets and poured on to a clean floor. The shape acquired by these jaggery mounds are varied but at times one can see the shape as shown in Fig. 1. Two wooden planks or metal sheets are placed at two ends ($y = 0$ and $2L$) and the hot jaggery is poured close to the meridian defined by the y axis at a more or less steady rate. The hot viscous jaggery spreads symmetrically from the centre to the sides ($x = -L/2$ to $x = L/2$). In this article we try to understand the profile of this mound using our knowledge of elementary fluid mechanics.

2. THE PROFILE OF THE MOUND

We can consider the jaggery mound to be an incompressible viscous fluid system. Over short time scales we take the height profile $H(x, y)$ to be fixed and independent of y. In this paper we attempt

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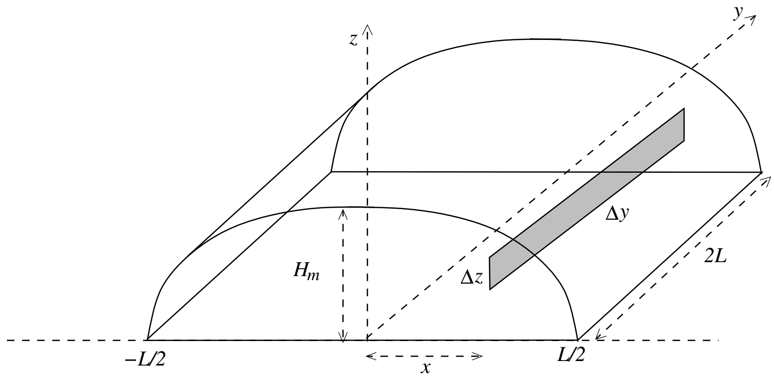

Figure 1. Shape of *gud* mound.

to derive $H(x)$. The maximum height of the *gud* mound is H_m and from the figure it is clear that $H(x = 0) = H_m$. Consider the density (ρ) of the *gur* to be constant. As one may easily verify the pressure inside the *gud* mound is independent of y and at a point (x, z) can be written as [1]

$$\rho g(H(x) - z)$$

where we have neglected the atmospheric pressure.

Consider a vertical slab $\Delta y \Delta z$ (shaded area in Fig. 1) located at x and in equilibrium. Because of the x to $-x$ symmetry (about the yz plane), we will consider only the $x > 0$ side. The horizontal force is in $+x$ direction exerted on the slab due to the pressure exerted by the gur mound can be calculated. A simple integration yields

$$F(x) = \Delta y \int_0^{H(x)} \rho g(H(x) - z)dz = \frac{\rho g H(x)^2}{2} \Delta y \tag{1}$$

The vertical slab $\Delta y \Delta z$ is subjected to two forces, one from the centre side (x) and the other from the peripheral side ($x + \Delta x$). Thus the net horizontal force on the slab is

$$\Delta F = F(x) - F(x + \Delta x) = -\rho g H(x) \frac{dH}{dx} \Delta x \Delta y \tag{2}$$

Now the slab is in equilibrium. We note that the jaggery is in a hot semi-solid viscous state akin to coal tar. Thus there is a shearing stress on the *gud* slab (σ_g) which opposes the net force derived above. This viscous force is $\sigma_g \Delta x \Delta y$ which yields

$$\begin{aligned} \sigma_g &= -\rho g H(x) \frac{dH}{dx} \\ &= -\frac{\rho g}{2} \frac{d}{dx} H(x)^2 \end{aligned} \tag{3}$$

The sign is negative since $H(x)$ is a decreasing function of x. We can solve Eq. (3) with the boundary condition $H(L/2) = 0$ to obtain the dependence of height on coordinate x. Note that you are looking at the $x > 0$ side. For the $x < 0$ side

$$H(x) = \sqrt{\frac{\sigma_g L}{\rho g}\left(1 - \frac{2x}{L}\right)}$$

and the maximum height by inserting $x = 0$ in the above equation. Thus

$$H_m = \sqrt{\frac{\sigma_g L}{\rho g}} \tag{4}$$

The total volume (V) of the (*gur*) mound can be calculated by integrating Eq. (5) and multiplying by the constant factor along the y-axis, namely $2L$. Keeping in mind the symmetry of the mound on either side of the yz plane yields an additional factor of two.

$$\begin{aligned} V &= 4L\int_0^{L/2} H(x)dx \\ &= 4L\sqrt{\frac{\sigma_g L}{\rho g}}\int_0^{L/2}\sqrt{1-\frac{2x}{L}}dx \\ &= \frac{4}{3}L^{5/2}\sqrt{\frac{\sigma_g}{\rho g}} = \frac{4}{3}L^2 H_m \end{aligned} \tag{5}$$

3. DISCUSSION

The Equation (4) for the maximum height could also be derived by dimensional analysis. The shape of viscous fluids is determined by two opposing forces: gravity and the force of viscosity and/or surface tension. In the current case we are in the happy situation that the dimensionless factor is unity, hence the dimensional analysis yields the exact result.

The base area $A \simeq L^2$, so the volume as evidenced in Eq. (5) scales as $V \simeq A^{5/4}$. This scaling relation for the spread of a viscous fluid is perhaps general. For a viscous system with given density and shear stress it is a consequence of the fact that the height decreases parabolically with the spread along the x direction.

A related question is why the shape of the mound remains invariant as the semi-solid jaggery is poured periodically and gently over a span of several hours. This appears to be a case of self-organized criticality (SOC). Additionally the shapes of several viscous substances may also be susceptible to similar analysis. We plan to examine these issues in detail in the future.

The density of the jaggery *gur* found in our house hold is around 1300 kg/m^3 [2]. Taking $L = 4$ m and $H_m = 1$ m, the total mass of a gur mound will be roughly equal to 28 tonnes. This gives $\sigma_g \approx$ 3.25 kPa. We urge the reader to buy jaggery, verify our model and experiment further. Physics is sweet, physics is fun!

4. ACKNOWLEDGMENTS

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