Decaying Dark Energy and Emergence of FRW Universe

Athira Sasidharan[1*] and Titus K Mathew[2]**
[1]M.Sc Student 2011-13 Batch, Department of Physics, Cochin University of Science and Technology, Kochi-22, India.
[2]Department of Physics, Cochin University of Science and Technology, Kochi-22, India.

Abstract. We consider a universe started in a de Sitter phase, with time varying holographic dark energy equivalent to a time varying cosmological term. The time varying dark energy and the created matter are consistent with the Einstein's equation. The general conservation law for the decaying dark energy and the created matter is stated. By assuming that the created matter is in relativistic form, we have analyzed the possibility of evolving the universe from de Sitter phase to Friedman-Robertson-Walker(FRW) universe.
Keywords: Dark energy, Friedmann Universe, Cosmology.

1. INTRODUCTION

Current astrophysical data shows that the present universe is accelerating in expansion[1]. This indicates that the present universe is dominated by some kind of very smooth form of energy with negative pressure and is called, dark energy, which accounts for about 73% of the total energy density of the present universe. Various models have been proposed to explain this phenomenon, for example there are models based on the dynamics of scalar or multi-scalar filed, called quintessence models [2]. Another dark energy candidate is the cosmological constant, which was initially introduced by Einstein. In the cosmological constant model, the dark energy density, ρ_Λ, remains constant throughout the entire history of the universe, while the matter density decreases during the expansion. The equation of the state for cosmological constant as dark energy is $w = p/\rho_\Lambda = -1$. While in Phantom models [7], it is possible to have an equation of state with $w < -1$.

An alternative approach to dark energy is from holographic principle. According to the principle of holography the number of degrees of freedom in a bounded system should be finite and has relations with area of its boundary. By applying the principle to cosmology, one can obtain the upper bound of the entropy contained in the universe. For a system with size L and UV cut-off Λ, without decaying into a black hole, it is required that the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3\rho_\Lambda \leq LM_P^2$. The largest L allowed is the one saturating this inequality, thus

$$\rho_\Lambda = 3c^2M_P^2L^{-2} \tag{1}$$

** tituskmathew@gmail.com, titus@cusat.ac.in

where c is numerical constant having value close to one, we will take it as one in our analysis. and M_P is the reduced Planck Mass $M_P^{-2} = 8\pi G$. When we take the whole universe into account,the vacuum energy related to this holographic principle can be viewed as dark energy. L can be taken as the large scale of the universe, for example Hubble horizon, future event horizon or particle horizon which were discussed by many [5–9].

In this paper we assume a decaying cosmological term. We also assume that the universe is started in de-Sitter phase. While in the de-Sitter phase the universe is completely dominated with the cosmological term. As the universe expands the dynamical cosmological term decaying in to matter and the universe will subsequently enter the Friedman phase. As it expands further, the universe enter a matter dominated phase with decelerated expansion. In section two we have shown that the decaying cosmological dark energy and created matter are consistent with the Einstein's equation. In section 3, we have obtained the Friedmann equations for the decaying dark energy, and analyzed the possibility of the evolution of the universe in to the Friedmann phase. We have also obtained the time evolution of the decaying dark energy and its equation of state. In section 4, we presented a comprehensive discussion of our analysis.

2. DYNAMICAL DARK ENERGY AND HORIZON

In the presence of cosmological constant the Einstein's field equation is

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu}_{total} \tag{2}$$

where $G^{\mu\nu}$ is the Einstein tensor, $R^{\mu\nu}$ is Ricci tensor, R is the Ricci scalar (except in this equation, we will refer R as the scale factor of the expanding universe)and $T^{\mu\nu}_{total}$ is the total energy momentum tensor comprising matter and cosmological term, and is

$$T^{\mu\nu}_{total} = T^{\mu\nu} + \rho_\Lambda g^{\mu\nu} \tag{3}$$

in which $T^{\mu\nu}$ is the energy momentum tensor due to matter in perfect fluid form and ρ_Λ is the density due to cosmological term, given as,

$$\rho_\Lambda = \frac{c^4\Lambda}{8\pi G} \tag{4}$$

with Λ as the so called "cosmological constant".

Einstein's equation satisfies the covariant conservation condition,

$$\nabla_\mu G^{\mu\nu} = 0 \tag{5}$$

In the conventional case this implies that, $\nabla_\mu T^{\mu\nu} = 0$. As such this condition doesn't give any time conserved charge. If the matter is being created from an independent source, say form the cosmological term, the conservation law will then take the general form

$$\nabla_\mu \left(T^{\mu\nu} + \rho_\Lambda g^{\mu\nu}\right) = 0 \tag{6}$$

This conservation law implies that the energy and momentum of matter alone is not conserved, but energy and momentum of matter and cosmological term or dark energy are together be conserved. This general conservation law allows the exchange of energy and momentum between matter and dark energy. It is acting as a controlling condition for this exchange. The existing theories predicts a very large value for the cosmological term [3, 4] in the early stage of the universe, but the present observations points towards a very low value for the cosmological term for the late universe. In this light it is inevitable to consider that, there must be a transference of energy form the dark energy or cosmological term sector to the matter sector.

Let us assume that, the term Λ correspondingly ρ_Λ is a function of time, since a space dependent Λ will lead to an anisotropic universe. The covariant conservation law will then give the equations,

$$\nabla_\mu T^{\mu i} = 0 \tag{7}$$

and

$$\nabla_\mu T^{\mu 0} = -\frac{c^3}{8\pi G}\frac{d\Lambda}{dt} \tag{8}$$

where $i = 1, 2, 3$ for the spatial part and $i = 0$ for the time part.

In reference [10] authors have considered the energy transference between decaying cosmological term and matter. It is important to realize that the covariant conservation law given above is drastically different from that appearing in some quintessence model [11–13], where energy-momentum tensor of the scalar field that replaces the cosmological term is itself covalently conserved, but no matter creation. In the present paper we have considered that the cosmological term decaying into matter and is consistent with the Friedmann model of the universe.

The energy density ρ_Λ corresponds to the time varying cosmological term is taken as the holographic dark energy as defined in equation (1). A simple holographic dark energy model by taking $L = H^{-1}$, where H is the Hubble's constant was considered by Hsu et al [5] and they have shown that the Friedmann model with $\rho_\Lambda = 3c^2 M_p^2 H^2$ makes the dark energy behave like ordinary matter rather than a negative pressure fluid, and prohibits accelerating expansion of the universe. We adopt an equation for holographic dark energy energy, where the future event horizon (R_h) is used instead of the Hubble horizon as the IR cut-off L, which was shown to lead a accelerating universe by Li [14]. Thus the time varying cosmological energy density is

$$\rho_\Lambda = 3c^2 M_p^2 R_h^{-2} \tag{9}$$

where c is a constant with value $O(1)$ and the event horizon $R_h(t)$, a function of cosmological time, is given by

$$R_h(t) = R(t)\int_t^\infty \frac{dR(t^{'})}{H(t^{'})R(t^{'})^2} \tag{10}$$

where $R(t)$ is the expansion factor and $H(t)$ is the Hubble constant.

3. COSMIC EVOLUTION OF DARK ENERGY AND FRIEDMANN UNIVERSE

Let us consider an empty universe in de Sitter phase, with very large [15] decaying cosmological term, corresponds to dark energy density as given by equation (9). If the matter from the decaying dark energy, are themselves distributing in a homogeneous and isotropic manner, then the geometry of the universe can be taken to be of Friedmann-Robertson-Walker form,

$$ds^2 = c^2 dt^2 - R^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right] \tag{11}$$

where k is the curvature parameter R is the scale factor of expansion, t is the cosmological time and (r, θ, ϕ) are the co-moving coordinates. By taking the energy momentum tensor of matter as

$$T^\mu_\nu = (\rho + p) u^\mu u_\nu - p \delta^\mu_\nu \tag{12}$$

where ρ is the energy density of the created component due to the decay of cosmological term and p is it's pressure. Under these conditions, the covariant conservation law (8) leads to (here we consider only one component of matter)

$$\frac{d\rho_m}{dt} + 3H \left(\rho_m + p_m \right) = -\frac{d\rho_\Lambda}{dt} \tag{13}$$

where $H = \frac{dR}{dt}/R$ is the Hubble parameter, ρ_m is the density of the created matter and p_m is its pressure. This equation obtained form the general conservation law is found to be followed from the combinations the standard Friedmann equations,

$$\left(\frac{dR}{dt} \right)^2 = \frac{8\pi G}{3c^2} \left(\rho_m + \rho_\Lambda \right) R^2 - kR^2 \tag{14}$$

and

$$\frac{d^2 R}{dt^2} = \frac{8\pi G}{3c^2} \left(\rho_\Lambda - \frac{1}{2} \left(\rho_m + 3p_m \right) \right) R \tag{15}$$

provided the cosmological term ρ_Λ is time dependent. If one assumes ordinary pressureless matter as

$$\rho_m = \rho_{m0} R^{-3} \tag{16}$$

where ρ_{m0} is the present density of matter. Then equation (13) will lead to the result that, the cosmological term will be independent of time. On the other hand this shows that the time dependent cosmological term does not decay in to pressureless matter.

Let us assume that the cosmological term can possibly decay into some form of matter with equation of state $p_m = \omega_m \rho_m$, where the parameter ω_m is assumed to be in the range $0 \leq \omega_m \leq 1$, the exact value depend on the particular matter component which is being created. In this paper we are considering only one component of matter. The covariant conservation law (8) can now be written for the possibility of cosmological term decaying into matter as

$$\frac{d\rho_m}{dt} + 3H\left(1+\omega_m\right)\rho_m = -\frac{d\rho_\Lambda}{dt} \tag{17}$$

Since density behavior of ordinary pressureless matter does not work for a varying cosmological dark energy, we will assume the form for ρ_m which is slightly different from its canonical form, as [16, 17]

$$\rho_m = \rho_{m0} R^{-3+\delta} \tag{18}$$

where ρ_{m0} is the present value of ρ_m and δ is a parameter which is effectively depends upon the state of the universe. From equation (18) and (9), equation (17) become,

$$\rho_\Lambda \left(\frac{HR_h - 1}{HR_h}\right) = \left(\frac{3\omega_m + \delta}{2}\right)\rho_m \tag{19}$$

where we have assumed a vanishing integration constant and also with the condition that limit t→ ∞, $R(t) \to \infty$ and equation for time rate of horizon can be cast into the differential form

$$\frac{dR_h}{dt} = HR_h - 1. \tag{20}$$

Equation (19) suggest that, depending on the parameters δ and ω_m the energy densities ρ_m and ρ_Λ may eventually be of the same order, as suggested by the present observations [18]. This relation also gives the relation between dark energy and matter. When $\delta = 3$, the equation implies a constant matter density which corresponds to an equilibrium between matter creation and expansion of the universe.

In general event horizon is not exist in Friedmann universe. But in de Sitter universe there exists a event horizon, which satisfies the relation $R_h \sim H^{-1}$. Consequently for de Sitter universe, $HR_h \sim 1$ which implies that in de Sitter phase the energy density is almost completely dominated by the cosmological term or dark energy. For Friedmann universe we will hence take HR_h as a very large value. In general we will take the value of HR_h is equal to one or large.

3.1 *Friedmann Universe*

Friedmann universe is a homogeneous and isotropic universe, satisfying the conditions (14) and (15). With the relation between decaying cosmological term and created matter (19), the second Firedmann equation becomes,

$$\frac{d^2R}{dt^2} = \frac{\left(\left(1+3\omega_m\right)\beta^2\right.}{2}\left[\frac{3\omega_m+\delta}{1+3\omega_m}\left(\frac{HR_h}{HR_h-1}\right) - 1\right]R^{\delta-2} \tag{21}$$

where $\beta^2 = 8\pi G\rho_{mo}/3c^2$. For de Sitter phase the acceleration is very large, for which $HR_h \sim 1$. As it enters the Friedmann phase by the decay of cosmological term or the dark energy, the acceleration can be negative or positive, which depends on the value of the term in the parenthesis of the right hand side of the above equation. The condition for acceleration is,

$$\frac{3\omega_m + \delta}{3\omega_m + 1} > 1 - \frac{1}{HR_h} \tag{22}$$

The factor $1 - \frac{1}{HR_h}$ is in the range $0 \leq 1 - \frac{1}{HR_h} \leq 1$. The extreme limits are corresponds to de Sitter phase and matter dominated universes respectively. For the transition period from de Sitter phase to Friedmann phase, HR_h is near to one, and assuming that the created matter have the equation state $\omega_m = \frac{1}{3}$, where the created matter is in relativistic form then

$$\delta > 2\alpha - 1 \tag{23}$$

where $\alpha = 1 - \frac{1}{HR_h}$ having value less than one. This implies that during the period of decay of the cosmological dark energy term the density of created matter is diluted slowly as the universe expand, and this decrease is faster than the decreasing of the density of non-relativistic matter in the Friedmann universe. This shows that even in the Friedmann universe it is possible to have an initial accelerating phase, where the cosmological dark energy is start its decay into matter and is still dominating over matter. As universe proceeds, the created matter will subsequently dominate and hence the universe will come to a matter dominated phase, at which the universe is expanding with deceleration.

For decelerating expansion, where matter is dominating over the cosmological term, the condition in the limit where HR_h is very large is,

$$\delta < 1 \tag{24}$$

This condition is true irrespective of whether the created matter is relativistic or non-relativistic. However as the universe enter the decelerating phase, the matter will become non-relativistic, satisfying the extreme condition that $\delta \rightarrow 0$ so that $\rho_m \sim R^{-3}$

3.2 *Flat Universe*

For flat universe, where the curvature parameter $k = 0$, the Friedmann equation (14) become

$$\left(\frac{dR}{dt}\right)^2 = \beta^2 \left(\frac{3\omega_m + \delta}{2} \frac{HR_h}{HR_h - 1} + 1\right) R^{\delta - 1} \tag{25}$$

On integration and avoiding the integration constant, the solution is,

$$R \sim t^{\frac{2}{3-\delta}} \tag{26}$$

By considering the relation for matter creating out of decaying dark energy, i.e. $\rho_m = \rho_{m0} R^{-3+\delta}$, Then the relation between dark energy density and matter evolves as

$$\rho_m \sim \rho_\Lambda \sim t^{-2} \tag{27}$$

This is the time dependence of ρ_Λ for any value of ω_m and δ. This time dependence shows that ρ_Λ diverge at the initial time, which implies the existence of initial singularity.

3.3 *An equation of state for the decaying dark energy*

An equation of state for the time decaying cosmological term can be written as [19]

$$\omega_\Lambda^{eff} = -1 - \frac{1}{3}\left(\frac{dln\rho_\Lambda}{d\ln R}\right) \tag{28}$$

With the equation for the relation between decaying dark energy and creating matter [19], the equation of state become

$$\omega_\Lambda^{eff} = -\frac{\delta}{3} - \frac{1}{3\left(HR_h - 1\right)^2} \tag{29}$$

This shows that for large values of HR_h the equation of sate become $\omega_\Lambda = -\frac{\delta}{3}$. From the above analysis, it is seen that for an accelerating universe ω_Λ is less than $-\frac{1}{3}$, but for a decelerating Friedmann universe it is greater than $-\frac{1}{3}$, which is similar to the latest analysis by many.

4. DISCUSSION

In the presence of a time varying cosmological term, assumed to be of holographic dark energy form, it is possible that the universe may begin with the de Sitter phase, exhibiting horizon and where the energy density is completely dominated by the dark energy. If the horizon R_h is assumed to be equal to the plank length at very early stage, then Λ, the cosmological constant would have a value of the order

$$\Lambda \sim 10^{66}\, cm^{-2} \tag{30}$$

the corresponding dark energy density would be $\rho_\Lambda \sim 10^{112}\, ergcm^{-3}$. This enormous dark energy decay into matter as the universe evolves to the Friedmann phase, and the dark energy reached the present value

$$\rho_\Lambda^0 = 10^{-8}\, erg\, cm^{-3} \tag{31}$$

The evolution of the de Sitter phase in to the Friedmann universe is in such a way that the total energy density comprising the dark energy, created matter and the gravitational field together be conserved. In this paper we have considered the decay of the dark energy into matter. During the initial phase of decay, the universe might be in the accelerating phase, where the parameter δ characterizing the equation $\rho_m \sim R^{-3+\delta}$, is greater than one. This implies that the dilution of the density of the created matter is slower compared to the non-relativistic matter. As the universe proceeds with expansion, the matter created will come to dominate, and the universe eventually go over to matter dominated phase, with the condition $\delta < 1$. In the extreme limit this condition may go the limit $\delta \to 0$, which emphasizes that, the created matter will eventually become non-relativistic, with behavior, $\rho_m \sim R^{-3}$. In section 3.3, the equation of the state of the decaying holographic dark energy is given and it shows that, in the early phase of universe too, $\omega_\Lambda < -\frac{1}{3}$ as like the equation of state in late accelerating universe [20]. To explain why ρ_Λ and ρ_m are of the same order today, it is essential to have a specific time evolution for dark energy. We argued that a dynamical dark energy, endowed with an appropriate time evolution can contain the possibility of the development of a Friedmann universe from a de Sitter universe. As the de Sitter phase evolves to the Friedmann universe, the value of the dark energy is decreased gradually to a low value, which eventually leads to matter dominating phase with decelerating expansion. But on the other hand, recent observations indicating that the present universe is in a accelerating expansion phase. This fact indicates the possibility that at present the dark energy is increasing at the expense of decaying matter. This time increasing dark energy, in other words, implies that the universe may evolves in to state where the whole energy density would dominate completely by the dark energy. The ultimate clarity regarding these, of course be given by the proper analysis on the quantum effects, which is still an open question.

References

[1] Perlmutter, S. et al.: Measurements of Omega and Lambda form 42 High-redshift supernovae, *Astrophys. J.* **517** 565 (1999)
Knop, R. et al.: New Constraints on Ω_M Ω_Λ, and ω from an independent Set of Eleven Hogh-Redshift Supernovae observed with *Astrophys. J.* **598** 102 (2003) [SPIRES]
Riess, A. G. et al.: Type Ia Supernovae Discoveries at $Z > 1$ From the Hubble Space telescope Evidence of Past Deceleration and Constraints on Dark Energy Evolution *Astrophys. J.* **607** 665 (2004) [SPIRES]
Jassal, H., Bagla, J., and Padmanabhan, T. : Observational Constraints on Low Redshift Evolution of Dark Energy: How consistent are Diffrent Observations *Phys. Rev.* **D 72** 103503 (2005) [SPIRES]

[2] Ratra, B., Peebles, P. J. E. : Cosmological Consequences of a Rolling Homogeneous Scalar field *Phys. Rev.* **D 37** 3406 (1998) [SPIRES]
Zlatev, I., Wang, L., Steinhardt, P. J. : Quintessence, Cosmic Coincidence and Cosmological Constant *Phys. Rev. Lett.* **82** 896 (1999) [SPIRES]
Steinhardt, P. J., Wang, L., Zlatev, I. : Cosmological Tracking Solutions 1999 *Phys. Rev.* **D 59** 123504 (1999) [SPIRES]

[3] Weinberg, S. : Rev. Mod. Phys. The Cosmological Constant Problem, **61** 1 (1989)

[4] Linde,A. : Particle Physics and Inflationar Cosmology, Harwood, New York, 1990.

[5] Hsu,S.D.H. : Entropy bounds and dark energy, *Phys. Lett.* **B 594** 13 (2004) [hep-th/0403052] [SPIRES].

[6] Li, M. : A model of holographic dark energy, *Phys. Lett.* **B 603** 1 (2004) [hep-th/0403127] [SPIRES];

[7] . Huang, Q.-G., Li,M. : Anthropic principle favors the holographic dark energy, *JCAP* **03** 001 (2005)[hep-th/0410095] [SPIRES];
Huang, Q. -G., Li, M : The holographic dark energy in a non-at universe, *JCAP* **08** 013 (2004) [astro-ph/0404229] [SPIRES];
Chen,B., Li, M., Wang, Y. : Ination with Holographic Dark Energy, *Nucl. Phys.* **B 774** 256 (2007) [astro-ph/0611623] [SPIRES];
Zhang,J.-f., Zhang, X., Liu, H.-y.: Holographic dark energy in a cyclic universe, *Eur. Phys. J.* **C 52** 693 (2007) [arXiv:0708.3121] [SPIRES]

Nojiri, S., Odintsov,S.D.: Unifying phantom ination with late-time acceleration: Scalar phantom-non-phantom transition model and generalized holographic dark energy, *Gen. Rel. Grav.* **38** 1285 (2006) [hep-th/0506212] [SPIRES].

[8] Horava, P., Minic, D. : Probable values of the cosmological constant in a holographic theory, *Phys. Rev. Lett.* **85** 1610 (2000) [hep-th/0001145] [SPIRES].

[9] Horvat,R. : Holography and variable cosmological constant, *Phys. Rev.* **D 70** 087301 (2004) [astro-ph/0404204] [SPIRES].

[10] Aldrovandi,R., Beltran Almeida,J.P., Pereira,J.G. : Time-Varying Cosmological Term: Emergence and Fate of a FRW Universe, *Grav. Cosmol.* **11** 277 (2005) , arXiV:gr-qc/0312017v3 12 Apr 2005.

[11] Ratra,B., and Peebles,P.J.E. : Cosmological Consequences of a Rolling Homogeneous Scalar field *Phys. Rev.* **D 37** 3406 (1998) [SPIRES]

[12] Friman,J., Hill,C.T., Watkins,R. : Late Time Cosmological Phase Transitions,1. Particle Physics Models and Cosmic Evolution *Phys. Rev* **D 46** 1226 (1992)

[13] Sahni, V., Starobisky, A.A. : The Case for a Positive Cosmological Lambda-term *Int. J. Mod. Phys.* **D9** 373 (2000) .

[14] Huang, Q.-G., Li, M. : Holographic Dark Energy in Non-Flat Universe *J. Cosmol. Astropart. Phys*. JCAP **0408** 013 (2004) [SPIRES]

[15] Mashhoon, B., Wesson,P.S. : Gauge Dependent Cosmological Constant *Class. Quant. Grav.* **21**, 3611 (2004)

[16] B.Guberina, R.Horvat and Hrvoje Nikolic, Non-Structured Holographic Dark Energy, *JCAP* **0701** (2007) 012

[17] B.Guberina, R.Horvat and Hrvoje Nikolic, Dynamical Dark Energy with a constant vacuum energy density, *Phys. Lett*.**B 636**: (2006) 80-85

[18] S M Carrol, why is the universe Accelerating, in - Measuring and modelling the universe. ed. by W.L.Freeman, Cambridge University Press, Cambridge, 2003 [astro-ph/0310342]

[19] C J Fang, Holographic Cosmological Constant and Dark Energy, *Phys. Lett.* **B663** (2008) 367.

[20] E J Copeland, M Sami, S Tsujikawa, Dynamics of Dark Energy, *Int. J. Mod. Phys.* **D15** (2006) 1753

[21] S.Weinberg, Gravitation and Cosmology, Wiley, New York, 1972