Fourier Transform of Electric Signal using Kundt's Tube

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Abstract. An experiment to demonstrate the Fourier transform of an electric signal using the Kundt's tube is described. The results of finding the component frequencies and estimating the Fourier coefficients of electric signals composed of two sinusoidal waves is reported. Undergraduate students are expected to better relate to the abstract concept of a Fourier transform with the aid of such mechanical demonstrations.
Keywords: Kundt's Tube, Fourier transform.

1. INTRODUCTION

The mathematical technique of Fourier transform has ubiquitous usage in physics. To demonstrate Fourier transform, there are experiments available which use electronic circuits, computer programs[1],[2] and bass guitar strings[3]. In this paper, we present a *mechanical* means of physically realizing a Fourier transform.

A mechanical demonstration of a Fourier transform can be achieved using a Kundt's tube. Kundt's tube is a simple and an easily available apparatus in undergraduate teaching labs where it is used to measure the speed of sound. In this experiment, we use the resonance property of the Kundt's tube to find the frequencies and amplitudes of component sinusoidal waves which make up an electric signal.

2. THEORY

The representation of a function as a sum of sine and cosine terms is called a Fourier series[4]. That is to say, that the right hand side of

$$f(x) = \frac{a_0}{2} + \sum_{r=0}^{r=\infty} [a_r\ cos(\omega_r x) + b_r\ sin(\omega_r x)] \tag{1}$$

is the Fourier series of the function $f(x)$. a_0, a_r, b_r are called the Fourier coefficients of 'component' frequencies ω_r and can be mathematically calculated using the orthogonality properties of the above trigonometric functions.

Historically, Kundt (1866) had used the concept of stationary waves to measure the speed of sound with better accuracy[5]. Even to this day, we find this tube in undergraduate teaching labs

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to introduce undergraduate students to one of the methods of measurement of speed of sound. The Kundt's tube resonates when sound waves of certain frequencies pass through it. For a tube closed at one end and the source at the other end, the resonating frequencies are,

$$\nu_n = \frac{2n+1}{4}\frac{v_s}{L} \tag{2}$$

where L is length of tube, v_s is speed of sound in the medium encapsulated by tube and $n \geq 0$ is an integer.

3. APPARATUS

The setup consists of the Kundt's tube, a cork which is movable along the length of it and a speaker at other end. Cork dust or thermocol pieces are used as mediums for observing striations in the tube. Open source softwares, *Audacity* and *Praat* are used, for producing input signals and recording the signal from a microphone in the Kundt's tube respectively.

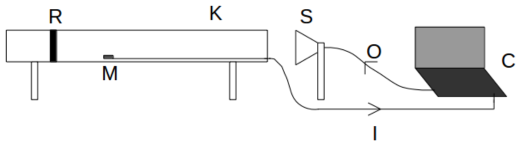

Figure 1. Schematic diagram of setup for the quantitative FT. **Legend -** R: Cork, K: Kundt's tube, M: Microphone, S: Speaker, C: Computer, I: Input from microphone, O: Output to speaker This Kundt's tube is closed only at one end as the air near the speaker-end forms an anti-node forced by the vibrations of speaker's diaphragm.

4. EXPERIMENT

Audacity is used to add two sinusoidal waves to make an electric signal which is then fed to the speaker. This electric signal is treated as a signal known to be composed of two sinusoidal waves but whose frequencies and amplitudes are unknown. The aim of the experiment is a primitive one: Decompose the electric signal to it's component frequencies and amplitudes. The procedure for achieving the former and latter parts of this aim is presented in sections (4.1) and (4.2) respectively.

4.1 *Qualitative Fourier Transform*

The procedure to find the two unknown frequencies of the input signal will be referred to as the qualitative Fourier transform.

Before finding the component frequencies of the electric signal, a length-frequency calibration is necessary. The Kundt's tube is filled with an optimum amount of cork dust and the length of the tube is fixed by the cork. The resonant frequency for this length is estimated by feeding the speaker a range of frequencies using a function generator and judging which frequency sets up the tallest striations in the cork dust. The resonant frequencies for a certain number of lengths are found similarly and plotted (Fig.(2)). Note that for the whole calibration, one should decide upon a single mode n. We chose $n = 1$ and selected frequencies which set up the corresponding pattern in the cork dust.

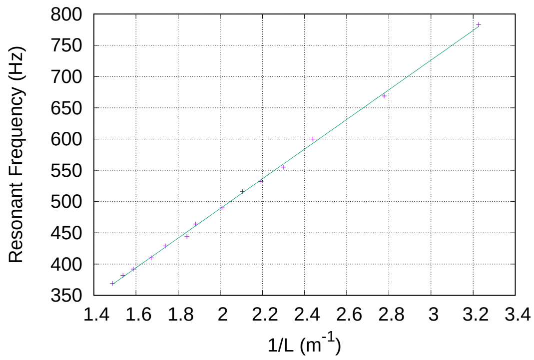

Figure 2. Length-Frequency calibration

Now, the speaker is fed with the unknown signal and the length of the cork is continually changed till the tube resonates at mode $n = 1$. This happens at two different lengths[1] which are noted. The resonant frequencies corresponding to these lengths is found using the calibration plot (2).

This concludes the qualitative Fourier transform as the two unknown frequencies that comprises the signal is found.

In the experiment that we performed, the electric signal fed to the speaker comprised of the frequencies: 358 Hz and 448 Hz and the qualitative FT found resonance at first mode at the lengths of 69 cm and 54 cm corresponding to the frequencies 358 Hz and 448 Hz.

4.2 *Quantitative Fourier Transform*

The procedure to find the amplitudes corresponding to the two unknown frequencies found using the qualitative FT will be referred to as Quantitative Fourier transform.

To quantify the Fourier transform, we need two more calibration curves. The cork is placed at one of the resonant lengths of the unknown signal and the cork-dust which has served its purpose is removed. A microphone is placed at the displacement anti-node. The speaker is fed with the corresponding resonant frequency at different input voltages using *Audacity* and the average sound

[1]This is ensured only if the input signal's frequencies are in the range of frequencies covered in the calibration plot (2).

intensity measured using *Praat* is recorded for each input voltage. An average recorded intensity-input voltage calibration plot is thus obtained.

At this point, when we attempted to verify the shape of the waveform recorded by *Praat*, we found that the waveform was being truncated at the crests and troughs for high input voltages. To remove the associated errors in the average intensity recorded, we scaled down the volume in the computer appropriately.

Once the calibration for a particular resonant length is done, the same is repeated for the other resonant length. The calibration curves obtained are shown in Fig.(3).

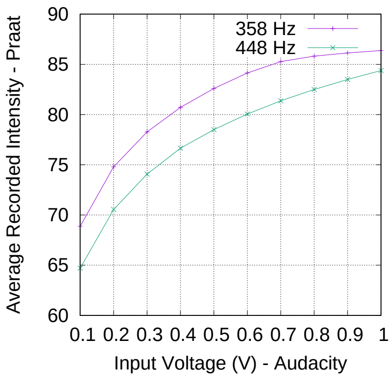

Figure 3. Recorded intensity-input voltagecalibration

To find the amplitudes of the component frequencies of the electric signal, the Kundt's tube is kept at one of the resonant length and the microphone at the displacement anti-node. The unknown signal is fed to the speaker (at the same scaled down computer volume) and the average intensity is recorded. The corresponding input voltage is found using the calibration plots (3). Similar procedure is repeated for the other resonant length. Note that due to the scaling down of computer volume, the amplitudes here correspond to the scaled down electric signal and not the original electric signal which was used in Sec. (4.1).

This concludes the quantitative Fourier transform as the amplitudes of the two component frequencies are estimated.

In the experiment we performed, we fed these two frequencies at different pairs of amplitudes and the quantitative FT's estimated amplitudes are tabulated in Table.

5. DISCUSSION AND CONCLUSIONS

The complete process of finding out the component frequencies of an unknown input signal by scanning the length of Kundt's tube and estimating the amplitudes of the corresponding frequencies using calibration curves can be referred to as taking the Fourier transform of an electric signal using Kundt's tube.

Like Kundt's tube, mechanical systems with discrete resonant frequencies can be considered as candidates for performing a Fourier transform of an electric signal. Once electric signals are transformed to mechanical signals, they can be fed to a resonance capable mechanical system. For example - springs of different resonant frequencies put on a hanger. A vibrator transforms electric signals to mechanical vibrations which are fed to the hanger and spring system. Only those springs whose resonant frequencies match with the frequency of the sinusoidal waves in electric signal will show resonance.

Though unnecessary for the present discussion, it is instructive to look at the Fourier transform that *Audacity* can perform on the sound signal recorded by the microphone when the speaker is fed with a sinusoidal resonant frequency of the tube. Fig.(4) shows such a plot where the speaker was fed with a 367Hz signal which was a resonant frequency for a particular length of the tube.

358 Hz					**448 Hz**			
Sl. No.	Input amp.(V)	Recd. av. intensity	Calib. reading	Error	Input amp.(V)	Recd. av. intensity	Calib. reading	Error
1	0.2	75.08	0.2	0	0.4	77.08	0.42	0.02
2	0.4	80.58	0.39	0.01	0.2	75.53	0.35	0.15
3	0.5	82.49	0.49	0.01	0.5	80.26	0.61	0.11
4	0.5	82.39	0.49	0.01	0.6	81.24	0.69	0.09
5	0.5	82.25	0.48	0.02	0.7	82.16	0.77	0.07
6	0.5	82.09	0.47	0.03	0.8	82.85	0.83	0.03
7	0.6	83.88	0.58	0.02	0.5	80.8	0.65	0.15
8	0.7	85	0.67	0.03	0.5	81.46	0.7	0.2
9	0.8	84.85	0.66	0.14	0.5	82.02	0.75	0.25
10	0.8	82.42	0.49	0.31	0.4	79.24	0.54	0.14
11	0.4	76.89	0.26	0.14	0.4	77.35	0.43	0.03
12	0.4	76.76	0.25	0.15	0.8	81.47	0.71	0.09

Table: The error tabulated is the difference between the expected value of amplitude measurement (Input Amplitude) and the measured amplitude (Calibration Reading). Excluding data points 8, 9 and 10, the error in amplitude measurements is less than 0.15 V.

It can be seen that though the speaker was fed with a single frequency, the constructive interference of certain frequencies contained in the noise are seen as peaks. These are different modes of resonance of the tube at that particular length.

As undergraduate students have better intuition towards sound waves as compared to electronic circuits or computer codes, we expect them to better relate to the meaning of Fourier transforms

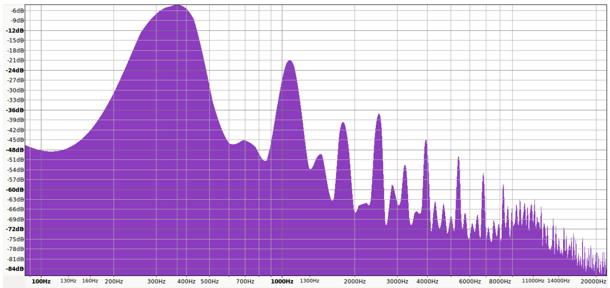

Figure 4. Recorded sound intensity versus frequency for input resonant signal: 367 Hz

with the aid of Kundt's tube.

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