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SUSY Dark Matter and Colliders

by

Ben Allanach (DAMTP, Cambridge University) BCA, Lester, hep-ph/0507283; BCA, Belanger, Boudjema, Pukhov, JHEP 0412 (2004) 020, hep-ph/0410091

Talk outline

- SUSY dark matter
- Constraints on SUSY models
- Collider measurements



Electroweak Breaking

Both Higgs get vacuum expectation values:

 $\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \rightarrow \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$ and to get M_W correct, match with $v_{SM} = 246$ GeV: $\underbrace{v_{SM}}_{v_1} \qquad v_2 \qquad \tan \beta = \frac{v_2}{v_1}$

 $\mathcal{L} = h_t \bar{t}_L H_2^0 t_R + h_b \bar{b}_L H_1^0 b_R + h_\tau \bar{\tau}_L H_1^0 \tau_R$ $\Rightarrow \frac{m_t}{\sin \beta} = \frac{h_t v_{SM}}{\sqrt{2}}, \quad \frac{m_{b,\tau}}{\cos \beta} = \frac{h_{b,\tau} v_{SM}}{\sqrt{2}}.$



The Supersymmetric Standard Model

Standard Model particle
quark, spin 1/2
lepton, spin 1/2
higgs, spin 0
gluon, spin 1
Weak bosons, spin 1
graviton, spin 2





The Supersymmetric Standard Model

For every particle present in The Standard Model, we have a heavier supersymmetric copy with the same quantum numbers and couplings to forces but spin differing by $1/2 \bar{h}$.

Standard Model particle	Supersymmetric copy(s)
quark, spin 1/2	2squarks, spin 0
lepton, spin 1/2	2sleptons, spin 0
$2 \times$ higgs, spin 0	higgsinos, spin 1/2
gluon, spin 1	gluinos, spin 1/2
Weak bosons, spin 1	gauginos, spin 1/2
graviton, spin 2	gravitino, spin 3/2



Broken Symmetry

3 components of the Higgs particles are eaten by W^{\pm}, Z^{0} , leaving us with 5 physical states:

 $h^0, H^0(CP+), \qquad A^0(CP-), \qquad H^{\pm}$

SUSY breaking and electroweak breaking imply particles with identical quantum numbers mix:





SUSY Dark Matter

- Galactic rotation curves
- Gravitational lensing effects
- WMAP + large scale structure

Imposing R_P , the neutralino is a good candidate. Must take into account annihilation in the early universe into ordinary matter:





WMAP Results





WMAP Results





SUSY Prediction of Ωh^2

- Assume relic in thermal equilibrium with $n_{eq} \propto (MT)^{3/2} exp(-M/T).$
- Freeze-out with $T_f \sim M_f/25$ once interaction rate < expansion rate (t_{eq} critical)
- We use microMEGAs $: \Omega h^2 \propto 1/<\sigma v >$ to solve coupled Boltzmann equations
- Generate SUSY spectrum with SOFTSUSY linked with SLHA

Belanger *et al*, CPC 149 (2002) 103 BCA, CPC 143 (2002) 305 BCA et al, JHEP0407 (2004) 036



Universality

Reduces number of SUSY breaking parameters from 100 to 3:

- $\tan\beta \equiv v_2/v_1$
- m_0 , the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).

• A_0 , the common trilinear coupling (flavour). **These conditions** should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

 $\propto 1/(16\pi^2) \ln(M_X/M_Z).$



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Also, Higgs potential parameter $sgn(\mu)=\pm 1$.



mSUGRA Regions

After WMAP+LEP2, bulk region diminished. Need specific mechanism to reduce overabundance:

- *τ* coannihilation: small m₀, m_{τ̃1} ≈ m_{χ1}⁰.

 Boltzmann factor exp(-ΔM/T_f) controls ratio of species. *τ*₁χ₁⁰ → τγ, *τ*₁*τ*₁ → τ*τ*.
- Higgs Funnel: $\chi_1^0 \chi_1^0 \to A \to b\bar{b}/\tau\bar{\tau}$ at large $\tan \beta$. Also via h at large m_0 small $M_{1/2}$.
- Focus region: Higgsino LSP at large m_0 : $\chi_1^0 \chi_1^0 \rightarrow WW/ZZ/Zh/t\bar{t}.$
- \tilde{t} coannihilation: high $-A_0, m_{\tilde{t}_1} \approx m_{\chi_1^0}$. $\tilde{t}_1 \chi_1^0 \to gt, \tilde{t}\tilde{t} \to tt$



Constraints on SUSY Models

mSUGRA well-studied in literature: eg Ellis, Olive et al PLB565

(2003) 176; Roszkowski et al JHEP 0108 (2001) 024; Baltz, Gondolo, JHEP 0410 (2004) 052;...







Shortcomings

- Really, would like to combine likelihoods from different measurements
- Typically only 2d scans, but in general we have $\alpha_s(M_Z), m_t, m_b, m_0, M_{1/2}, A_0, \tan \beta$ to vary
- Effective 3d type scan done which parameterises a 2d surface of correct Ωh^2
- Baltz *et al* managed to perform a 4d scan, but lost the likelihood interpretation. They used the impressive *Markov Chain Monte Carlo technique*.



Done in 2d in Ellis *et al*, hep-ph/0310356 Ellis *et al*, hep-ph/0411218



Markov-Chain Monte Carlo

Markov chain consists of list of parameter points x(t)and associated likelihoods $\mathcal{L}^{(t)}$

- Pick a point at random for $x^{(1)}$
- 2. Pick a point around $x^{(t)}$ (say with a Gaussian) width) as the potential new point.
- 3. If $\mathcal{L}^{(t+1)} > \mathcal{L}^{(t)}$, the new point is appended onto the chain. Otherwise, the proposed point is accepted with probability $\mathcal{L}^{(t+1)}/\mathcal{L}^{(t)}$. If not accepted, a copy of $x^{(t)}$ is added on to the chain.

Final density of x points $\propto \mathcal{L}$. Required number of points goes *linearly* with number of dimensions.



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Implementation

Input parameters are: m_0 , A_0 , $M_{1/2}$, $\tan \beta$

- $m_t = 172.7 \pm 2.9 \text{ GeV}$
- $m_b(m_b)^{\overline{MS}} = 4.2 \pm 0.2$ GeV,
- $\alpha_s(M_Z)^{\overline{MS}} = 0.1187 \pm 0.002.$

For the likelihood, we also use

- $\Omega_{DM}h^2 = 0.1125^{+0.0081}_{-0.0091}$
- $\delta(g-2)_{\mu}/2 = (19 \pm 8.4) \times 10^{-10}$
- $BR[b \to s\gamma] = (3.52 \pm 0.42) \times 10^{-5}$





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Convergence

We run 9×1000000 points. By comparing the 9 independent chains with random starting points, we can provide a statistical measure of convergence: an upper bound r on the excepted variance decrease for infinite statistics.







Annihilation Mechanism

Define stau co-annihilation when $m_{\tilde{\tau}}$ is within 10% of $m_{\chi_1^0}$ and Higgs pole when $m_{h,A}$ is within 10% of $2m_{\chi_1^0}$.

Region	likelihood
h^0 pole	$0.02{\pm}0.01$
A^0 pole	$0.41 {\pm} 0.03$
$\tilde{\tau}$ co-an	$0.27{\pm}0.04$
\tilde{t} co-an	$(2.1 \pm 4.8) \times 10^{-4}$

Table 0: Likelihood of being in a certain region of mSUGRA parameter space. Likelihood of chain $\tilde{q}_L \rightarrow \chi_2^0 \rightarrow \tilde{l}_R \rightarrow \chi_1^0$ is $24 \pm 4\%$

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1.5

2

2.5

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0

0

0.5

1

 $m_{A}^{}\left(TeV\right)$

<u>L</u>/L(max)

1

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0







Caveats

- Implicitly assumed that LSP constitutes *all* of dark matter
- Assumed radiation domination in post-inflation era. No clear evidence between freeze-out+BBN that this is the case (t_{eq} changes).
- Examples of non-standard cosmology that would change the prediction:
 - Extra degrees of freedom
 - Low reheating temperature
 - Extra dimensional models
 - Anisotropic cosmologies
 - Non-thermal production of neutralinos (late decays?)

LHC (ATLAS)





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Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.



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Q: Can we measure enough to predict σ ?



Collider SUSY Dark Matter Production

Strong sparticle production and decay to dark matter particles.



Any dark matter candidate that couples to hadrons can Cambridge be produced at the LHC



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LHC vs LC in SUSY Measurement

- LHC (start date 2007) produces strongly interacting particles up to a few TeV. Precision measurements of mass *differences* possible if the decay chains exist: possibly per mille for leptons, several percent for jets.
- ILC has several energy options: 500-1000 GeV, CLIC up to 3 TeV. Linear colliders produce less strong particles but much easier to make precision measurements of masses/couplings.

Q: What energy for LC?Q: What do we get from LHC?

LHC/ILC Working Group Report: hep-ph/0410364



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Coannihilation Slope



$$m_{\tilde{l}_R}^2 \approx m_0^2 + 0.15 M_{1/2}^2, \qquad M_{\chi_1^0} \approx 0.4 M_{1/2}$$

Low enough $M_{1/2} \Rightarrow$ quasi-deegenerate $\tilde{\tau}, M_{\chi}$



Coannihilation Slope



$$m_{\tilde{l}_R}^2 \approx m_0^2 + 0.15 M_{1/2}^2, \qquad M_{\chi_1^0} \approx 0.4 M_{1/2}$$



If we do not assume mSUGRA, we will also have to measure selectron and smuon properties.

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Coannihilation Theory Uncertainties



Expect higher orders to be 100 times smaller than these differences: 3-loop terms could possibly be important!



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Coannihilation Theory Uncertainties



Effect of 2-loop RGE terms suggest a possible effect from 3 loops. Jack and Jones find that it's not significant for the neutralino.



Iterative Procedure

What change in a parameter produces a 10% change in Ωh^2 ?

Take a parameter point with $\omega_{-1} \equiv \Omega h^2$. Change *one* parameter at a time by fraction a_0 . Result is ω_0 , then iterate

$$a_{i+1} = a_i \omega_{-1} \frac{10\%}{w_i - \omega_{-1}}.$$

Small accuracy $a \equiv a_{\infty}$ means the parameter has to be known very accurately in order to predict Ωh^2 to 10%.

For parameters that are zero, we take the absolute value as *a* rather than the fractional value.



Uncertainties

We use two approaches to determine what variation of parameters produce a 10% variation in Ωh^2 :

- **PmSUGRA** variation of weak scale parameters (*not* on mSUGRA trajectory): $m_{\chi_1^0}$, M_A , m_b etc.
- mSUGRA simple variation of mSUGRA parameters and experimental inputs: $m_0, M_{1/2}, \alpha_s(M_Z), m_t$ etc.

mSUGRA theory uncertainties estimated by varying scale at which radiative corrections added to sparticle masses:

$$0.5 < x \equiv \frac{M_{SUSY}}{\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}} < 2, \qquad M_{SUSY} > M_Z$$

mSUGRA Coannihilation Uncertainties



 $a(m_0) \approx a(M_{1/2})$ comes from the sensitivity to $\exp[-(m_{\tilde{\tau}} - M_{\chi_1^0})]$



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mSUGRA Coannihilation Uncertainties



Unknown whether accuracies can be reached - but it looks difficult : $\Delta\Omega h^2 \sim .03$ in diminished bulk region.

Polesello, Tovey, JHEP05 (2004) 071

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PmSUGRA Coannihilation



RHS: Spectrum useful for optimal energy of linear collider. $\tilde{e}_R, \tilde{\mu}_R$ also possible. Cascade $\tilde{q}_L \rightarrow \chi_2^0 \rightarrow \tilde{e}_R \rightarrow \chi_1^0$ available.



PmSUGRA Coannihilation



LHS: Dependences in left-hand plot all come from the effect on LSP mass.

Need to know $M_{\chi_1^0}$ very accurately.



Working group

PmSUGRA Dependencies



LHS: plots of quantities along mSUGRA slope. Below $\Delta M = 1.78$ GeV, no two-body stau decay. LC studies indicate $\Delta M > 5$ GeV is OK.



PmSUGRA Dependencies



 $\tilde{\tau}_1 \chi_1^0 \to \tau \gamma \propto 3 \cos 2\theta_{\tau} + 5$ from coupling of neutralino to $\tilde{\tau}_{L/R}$.





PmSUGRA Dependencies



 $a(M_{\chi})$ found by keeping ΔM constant, δM by just varying stau mass. $m_{\tilde{e}}, m_{\tilde{\mu}}$ needed to about 1.5%





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Slepton Dependence



Accuracy required on $m_{\tilde{l}} - m_{\chi_1^0}$ for WMAP precision. LC studies say this is acheivable, but need more work for $\cos \theta_{\tau}$ (=0.987±0.06 at lower end of slope).



Summary

- Markov chains bring out the multi-dimensionality of the space: is a lot less constrained than in 2d
- Still, current data is constraining
- LHC could produce copious amounts of SUSY dark matter
- Want to measure σ in order to predict Ωh^2 and test cosmological assumptions
- 10% accuracy will require ILC+LHC data
- Can control many uncertainties by measuring additional quantities: Γ_A , $m_{\tilde{\tau}} M_{\chi_1^0}$, ...



- Non mSUGRA case could well be easier.
- Have *not* discussed direct detection yet



Supplementary Material



Likelihood

 $\mathcal{L} \equiv p(d|m)$ is pdf of reproducing data d assuming mSUGRA model m (which depends on parameters).

$$p(m|d) = p(d|m)\frac{p(m)}{p(d)}$$

$$\frac{p(m_1|d)}{p(m_2|d)} = \frac{p(d|m_1)p(m_1)}{p(d|m_2)p(m_2)}$$

Thus, you can interpret the likelihood distribution as relative probabilities if your ratio of priors is 1. Otherwise, convolute it with YOUR priors!

Funnel Slope



$$<\sigma v>^{-1} \sim \frac{4m_{\chi_1^0}\Gamma_A}{g_{m_{\chi_1^0}\tilde{\chi}_1^0A}^2} \left(4\left(\frac{M_A - 2m_{\chi_1^0}}{\Gamma_A}\right)^2 + 1\right).$$



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Funnel Slope



Notice that spectrum is quite *heavy*: need a high energy ILC! Γ_A will be important.



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Funnel Theory Uncertainties



LHS: Γ_A affected by large $m_b^{SM}/(1 + \Delta_{SUSY})$ corrections since $A \to b\bar{b} \propto Ab\bar{b}$ coupling $\propto m_b \tan \beta$, and $\tan \beta = 50$.



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Funnel Theory Uncertainties



RHS: x > 1.5 yielded $M_A^2 < 0$ ie no EWSB. Strong correlation of theory error with its effect on $(M_A - 2M_{\chi^0})/\Gamma_A$ - could measure it!



Funnel Accuracies

LHS: mSUGRA. $a(m_b)$ worrying. $\alpha_s(M_Z)$ dependence comes about through its effect on $m_b(m_b)$. m_0 , $M_{1/2}$ might be feasible at LHC, m_t possible at ILC. tan β looks impossible.





Funnel Accuracies

SM inputs and $\tan \beta$ uncertainties can be controlled by measuring M_A , Γ_A . $A\chi_1^0\chi_1^0$ coupling $\sim 1/\mu$. $\Gamma_A \propto M_A \tan^2 \beta (m_b^2 + m_\tau^2)$ ($\gamma\gamma$ option of LC, $A \rightarrow \mu\mu$ at LHC).





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Funnel Accuracies

Mass of χ_1^0 is important, but not mixing (see $a(\mu)$).







Focus Point Slope



Heavy sfermions and A^0 . $M_1 < \mu < M_2$, ie significant Higgsino component ~ 25%.





 $t\bar{t}$ annihilation predominantly through Z. Coannihilation $\equiv \chi_1^0 \chi_i^0$ or $\chi_1^0 \chi_1^{\pm}$. Several competing channels.





Focus mSUGRA Accuracies



 $\delta m_t = 30$ MeV might be possible at future ILC but $a(m_0) < 0.5\%$ looks completely unfeasible.





Focus PmSUGRA Accuracies



Easier outside of mSUGRA, eg μ no longer sensitive to m_t (\propto coupling to neutral goldstone).





LHC SUSY Measurements



BCA, C Lester, A Parker, B Webber, JHEP 09 (2000) 004



Edge Fitting at S5 and O1



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Edge Positions

endpoint	S5 fit	O1 fit
m_{ll}	109.10 ± 0.13	70.47 ± 0.15
$m_{llq} \ edge$	532.1±3.2	544.1 ± 4.0
lq high	483.5 ± 1.8	$515.8{\pm}7.0$
lq low	321.5 ± 2.3	249.8 ± 1.5
llq thresh	266.0 ± 6.4	182.2 ± 13.5

Best case lepton mass measurements can be as accurate as 1 per mille, but jets are a few percent





Edge to Mass Measurements



Mass differences well constrained, but overall mass scale not so well constrained by LHC



Fitting to SUSY Breaking Model



- Experimenters pick a SUSY breaking point
- They derive observables and errors after detector simulation
- We fit this "data" with our codes

BCA, S Kraml, W Porod, JHEP 0303 (2003) 016

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Other Observables

Often more complicated, eg m_{llq} edge:

 $\max\left[\frac{(m_{\tilde{q}}^2 - m_{\chi_2^0}^2)(m_{\chi_2^0}^2 - m_{\chi_1^0}^2)}{m_{\chi_0^0}^2}, \frac{(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\chi_1^0}^2)}{m_{\tilde{t}}^2}\right]$ $\frac{(m_{\tilde{q}}m_{\tilde{l}} - m_{\chi_2^0}m_{\chi_1^0})(m_{\chi_2^0}^2 - m_{\tilde{l}}^2)}{m_{\chi_2^0}m_{\tilde{l}}}\Big]$ Also m_{la}^{high} , m_{la}^{low} , llq threshold , $M_{T_2}^2(m) =$ $\max[M_{T_2}(m_{\chi_1^0})] = m_{\tilde{l}}$ for dislepton production.

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Statistics Study

- Choose two model-points: S5 $(m_0 = 100, m_{1/2} = 300, A_0 = 300, \tan \beta = 2.1, \mu > 0)$ and O1 $(m_{\tilde{l}} = 177, m_{1/2} = 306, A_{\tilde{q}} = 137, m_{\tilde{q}} = 0, A_{\tilde{l}} = 306, \tan \beta = 10, \mu > 0)$
- Find cuts to measure "signal" endpoints
- Estimate expected accuracy of ATLAS measurement: 100 fb⁻¹
- Perform χ^2 fits of sparticle masses to expected positions of edges expected from an ensemble of experiments
- Interpret results as statistics of measurement on sparticle masses

Cuts Example

We use ATLFAST2.16, HERWIG6.0, ISAWIG and ISAJET7.42. Assume 100 fb⁻¹ of LHC data.

- $|\eta_j| \le 5, p_T^j \ge 15 \text{ GeV}$
- $p_T^e \ge 5, p_T^\mu \ge 6, |\eta_l| \le 2.5$
- *l* isolation: 10 GeV in $\Delta R = 0.2$, $\Delta R(lj) \ge 0.4$. eg for m_{ll} :
 - 2 OSSF leptons, $p_T^{l_1} \ge p_T^{l_2} \ge 10$ GeV.

• $n_{jets} \ge 2$, $p_T^{j_1} \ge p_T^{j_2} \ge 150$ GeV, $p_T > 300$ GeV OSSF-OSDF subtracts well the Standard Model background.



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Uncertainties in Relic Density

Bulk region: $\tilde{B}\tilde{B} \to Z, h \to l\bar{l}$. Coannihilation: $\tilde{\tau}\chi_1^0 \to \tau + X$





Figure 0: Bulk/coannihilation region. Full: SoftSusy, dotted: SPheno.

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Focus Point





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Figure 0: Focus point region. Full: SoftSusy, dotted: SPheno, dashed: SuSpect. Higgsino LSP annihilates into ZZ/WW

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High $\tan\beta$

BCA, Belanger, Boudjema, Pukhov, Porod, hep-ph/0402161. Baer et





Figure 0: High $\tan \beta$ region. Full: SoftSusy, dotted: SPheno, dashed: SuSpect. Get annihilation into A.