Beyond the New Standard Model in Future Neutrino Oscillation Experiments

IX WORKSHOP
ON
HIGH ENERGY PHYSICS PHENOMENOLOGY
03 January - 14 January, 2006

Institute of Physics,
Sachivalaya Marg,
Bhubaneswar-751005,
Orissa, India

Marek Zralek
University of Silesia
Katowice, Poland

Outline

1. INTRODUCTION

Neutrinos are massive particles $\Longrightarrow$ SM must be extended

New Standard Model $\equiv \nu$SM

There is no New Physics (NP) at low energy scale – neutrino masses are related to GUT scale physics

νSM and model of neutrino mass is accompanied by NP at low energy scale (TeV scale).

Such low energy NP can change neutrino oscillation in vacuum and in matter

Many different extensions of SM are considered :

1. Sterile neutrinos,
2. Heavy neutrinos
3. Lepton Number Violation (LNV),

and/or

4. New Vector Current (NVC,)
5. Flavour Changing Neutral Current (FCNC),
6. General Fermi Interaction (GFI→S,P,V,A,T),
7. Mass Varying Neutrinos (MVN),
8. Violation of Lorentz Symmetry (LSV),
9. Principle of General Relativity Violation (PGRV).
10. Large Extra Dimensions (LED),

=> Non Unitary Neutrino Mixing

M.Blennow, T.Ohlsson, W.Winter;
J.W.F.Valle;
S.Bergmann;
A.Friedland, C. Lunardini, C.Pena-Garay;
M.C. Gonzalez, Y. Grossman, A. Gusso, Y. Nir;
P.Huger, J.W.F. Valle;
P.Huber, T.Schwetz, J.W.Valle;
V.Barger, P.Huber, D.Marfatia;
M.Campanelli, A.Romanino;
R.Fardon, A.E.Nelson, N.Weiner,
G.L.Fogli, E.Lisi, A.Mirizzi, D.Montanino;
B.Bekman, J.Holeczek, J.Syska, M.Zralek;
……… and many others.

$\Longrightarrow$ Neutrino interactions in matter are modified,

$\Longrightarrow$ Effective neutrino masses are changed.

- SM works very well $\Longrightarrow$ Existing experimental data gives bound on NP parameters

  $\Downarrow$

  NP effects must be small

- There are also other mechanism which are able to modify (damp) neutrino oscillations:

  1. Neutrino decay
  2. Wave pocket decoherence
  3. Oscillation to strille neutrino
  4. Neutrino absorption
  5. Quantum decoherence

  $\Longrightarrow$ To earch for NP modifications given by damping effects should be known

❑ Experimental data are not precise:

1. experimental oscillation rates,
2. νSM parameters (mixing angles and δm^2's),
3. confusing non-standard neutrino interaction with oscillation.

$\Longrightarrow$ Will it be possible to recognize NP effects?

❑ Oscillation rate $(\Gamma_{P\to D} \equiv \Gamma_\nu)$

In any real experiments, where *relativistic neutrino* oscillate the measure rate $\Gamma_{P\to D}$ is the product of three terms, integrated over neutrino energy (E_ν):

> the flux of produced (P) initial neutrinos $\dfrac{d\Gamma_P}{L^2 dE_\nu}$,

> the probability that, after travelling a distance L, P neutrinos will oscillate into detected (D) neutrinos: $P_{P\to D}(L, E_\nu)$,

and

> the cross section for detection of D neutrinos: $\sigma_D(E_\nu)$

$$\Gamma_\nu = \int dE_\nu \left(\frac{d\Gamma_P}{L^2 dE_\nu}\right) \times \left(P_{P\to D}(L,E_\nu)\right) \times \left(\sigma_D(E_\nu)\right)$$

- We will not consider full oscillation rate, but only $P_{P\to D}(L,E_\nu)$
- We will not concentrate on any specific model,
- Model independent approach
  - neutrino mixing matrix is not unitary – neutrino oscillation in vacuum will be modified, or (and)
  - neutrino interaction in matter is modified by NP,
- We will not discuss any damping effects,
- The impact of NP on the determination of neutrino mixing parameters are not discussed

2. Where should we look for new physics effects?

$$\Gamma_\nu = \int dE_\nu \left(\frac{d\Gamma_P}{L^2 dE_\nu}\right) \times \left(P_{P\to D}(L,E_\nu)\right) \times \left(\sigma_D(E_\nu)\right)$$

- The New Physics (NP) can give corrections to all three factors, but
- The feature of these corrections are quite different

  ---- for production and detection factors the effects are L independent (trivial L^2 factor appears in the production rate),

  ---- the NP in the matter oscillation probability depends on L.
- The oscillation probability depends on tha production $|\nu_P\rangle$ and detection $|\nu_D\rangle$ states which are not necessary flavour states.
- If we assume e.g. that eigenmass (i) neutrino states are produced in the process

$$l + A \to \nu_i + B$$

then the production (and analogously detection) states are defined in the way:

$$\left|\nu_P\right\rangle = \frac{1}{N}\sum_i A(l+A\to\nu_i+B)\left|\nu_i\right\rangle \quad \text{where}$$

$$N = \sqrt{\sum_{i=1}^{3}\left|A(l+A\to\nu_i+B)\right|^2}$$

EXAMPLE

If, for example, the light and heavy neutrinos mix together, and their mixing is described by the $3+n_R$ unitary matrix:

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\\ N_e\\ N_\mu\\ N_\tau\end{pmatrix} = \begin{pmatrix} U_{\alpha i} & V_{\alpha I}\\ V'_{Ai} & U'_{AI}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ N_I\\ N_{II}\\ N_{III}\end{pmatrix}$$

then

$$\left|\nu_{\mathrm{P}}\right\rangle = \frac{1}{\lambda_\alpha}\sum_{\mathrm{i}=1}^{3} \mathrm{U}^*_{\alpha\,\mathrm{i}}\left|\nu_{\mathrm{i}}\right\rangle , \qquad \left|\nu_{\mathrm{P}}\right\rangle \simeq \left|\nu_\alpha\right\rangle$$

with

$$\lambda_\alpha = \sqrt{1-(\mathrm{VV}^+)_{\alpha\alpha}}\ .$$

As elements of matrices V are small it is convenient to parameterize

$$\mathrm{U} = \mathrm{Y}\cdot U$$

where U is the standard 3x3 unitary matrix parameterized in the standard way

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-\mathrm{i}\delta} \\ -s_{12}c_{23}-c_{12}s_{13}s_{23}e^{\mathrm{i}\delta} & c_{12}c_{23}-s_{12}s_{13}s_{23}e^{\mathrm{i}\delta} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}s_{13}c_{23}e^{\mathrm{i}\delta} & -c_{12}s_{23}-s_{12}s_{13}c_{23}e^{\mathrm{i}\delta} & c_{13}c_{23} \end{pmatrix}$$

If we parameterize

$$Y = I - \delta Y,$$

then from unitary condition

$$YY^{+} = I - VV^{+}$$

we get

$$\delta Y = \frac{1}{2}\begin{pmatrix} c_{ee} & 0 & 0 \\ 2c_{\mu e} & c_{\mu\mu} & 0 \\ 2c_{\tau e} & 2c_{\tau\mu} & c_{\tau\tau} \end{pmatrix},$$

where

$$c_{\alpha\beta} = (VV^{+})_{\alpha\beta}$$

From low energy physics bounds on $c_{\alpha\beta}$ exist.

We will work in the flavour SM orthogonal base

$$|\nu_{\bar{\alpha}}\rangle = \sum_{i=1}^{3} U^{*}_{\bar{\alpha} i} |\nu_i\rangle$$

In this base the physical state and effective Hamiltonian are matrices, e.g.

$$\left|\nu_P\right\rangle \rightarrow \begin{pmatrix} \left\langle \nu_{\bar{e}} \middle| \nu_P \right\rangle \\ \left\langle \nu_{\bar{\mu}} \middle| \nu_P \right\rangle \\ \left\langle \nu_{\bar{\tau}} \middle| \nu_P \right\rangle \end{pmatrix}$$

- $P_{P\rightarrow D}(E_\nu, L)$ depends on new physics:
  - In vacuum:

    as $\left|\nu_P\right\rangle$ and $\left|\nu_D\right\rangle$ are NP dependent,
  - in matter:

    --- NP changes coherent neutrino scattering,

    --- effective neutrino masses in matter are changed.
- Effective Hamiltonian in the flavour base:

$$H^{eff} = H^{SM} + H^{NP}$$

where

$$H^{SM} = \frac{1}{2E_\nu}\left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{pmatrix} U^+ + \begin{pmatrix} A_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

and

$$A_e = 2E_\nu \sqrt{2} G_F N_e$$

and

$$H^{NP} = \frac{A_e}{2E_\nu} \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} e^{i\chi_{e\mu}} & \varepsilon_{e\tau} e^{i\chi_{e\tau}} \\ \varepsilon_{e\mu} e^{-i\chi_{e\mu}} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} e^{i\chi_{\mu\tau}} \\ \varepsilon_{e\tau} e^{-i\chi_{e\tau}} & \varepsilon_{\mu\tau} e^{-i\chi_{\mu\tau}} & \varepsilon_{\tau\tau} \end{pmatrix}$$

There are relations between the parameters: $c_{\alpha\beta} \Leftrightarrow \varepsilon_{\alpha\beta}$

To find the transition probability $P_{P\rightarrow D}(E_\nu, L)$, we calculate the transition amplitude $A_{\bar{\alpha} \rightarrow \bar{\beta}}(t)$.

This amplitude satisfy the evolution equation:

$$i\frac{d}{dt}A_{\bar{\alpha}\to\bar{\beta}}(t)=\sum_{\bar{\eta}}H^{eff}_{\bar{\beta}\bar{\eta}}\,A_{\bar{\eta}\to\bar{\alpha}}(t)\,;$$

Having $A_{\bar{\alpha}\to\bar{\beta}}(t)$ the physical amplitudes can be calculated:

$$A_{P\to D}(t)=\langle \nu_D \mid \nu_P(t)\rangle=\sum_{\bar{\beta}}\langle \nu_D \mid \nu_{\bar{\beta}}\rangle\langle \nu_{\bar{\beta}} \mid \nu_P(t)\rangle$$

$$H^{eff}_{\bar{\beta}\bar{\gamma}}(t)\Leftrightarrow\rho(L);$$ Numerically

$$H^{eff}_{\bar{\beta}\bar{\gamma}}(t_0)\Leftrightarrow\rho(L)=\text{const};$$ Analytically

For not very large baseline, ρ=const is good approximation

Methods of approximation

For the Earth: $2\frac{\mathrm{g}}{\mathrm{cm}^3} \le \rho \le 11\frac{\mathrm{g}}{\mathrm{cm}^3}$ and

for neutrino energy in the GeV range two parameters are small:

$$\alpha = \left|\frac{\delta m_{21}^2}{\delta m_{31}^2}\right| \approx 0.03; \qquad \sin^2(2\Theta_{13}) \le 0.05$$

For smaller neutrino energy (MeV) there is also third small parameter:

$$\hat{A}_e = \frac{A_e}{\delta m_{31}^2} \approx 0.001$$

We can resolve the equation of motion perturbatively:

- In the first case if $\hat{A}_e < 1$ or $\hat{A}_e > 1$:

$$\Delta = \delta m^2_{31}$$

$$H^{SM}_{\bar\alpha\bar\beta} = (H_0)_{\bar\alpha\bar\beta} + \alpha\, V_{\bar\alpha\bar\beta} \quad \text{where}$$

$$(H_0)_{\bar\alpha\bar\beta} = \frac{\Delta}{2E}\left(U_{\bar\alpha 3}U^*_{\bar\beta 3} + \hat{A}_e \delta_{\bar\alpha e}\delta_{\bar\beta e}\right), \quad \text{and}$$

$$V_{\bar\alpha\bar\beta} = \frac{\Delta}{2E}\, U_{\bar\alpha 2}U_{\bar\beta 2},$$

- In the second case if $\hat{A}_e << 1$ then

$$H^{SM}_{\bar\alpha\bar\beta} = (H_0)_{\bar\alpha\bar\beta} + V_{\bar\alpha\bar\beta} \quad \text{where now}$$

$$(H_0)_{\bar\alpha\bar\beta} = \frac{\Delta}{2E} U_{\bar\alpha 3}U^*_{\bar\beta 3} \quad \text{and}$$

$$V_{\bar\alpha\bar\beta} = \frac{\Delta}{2E}\left(\hat{A}_e \delta_{\bar\alpha e}\delta_{\bar\beta e} + \alpha\, U_{\bar\alpha 2}U_{\bar\beta 2}\right).$$

The basic Hamiltonian has two degenerate eigenvalues.The perturbative methods for degenerate states must be applied

In the first order of perturbative expansion (in the first case) we have three effective neutrino mass in matter:

$$\lambda_1 = \Delta(\hat{A}_e + \alpha \sin^2\theta_{12} - \frac{\hat{A}_e}{4(1-\hat{A}_e)}\sin^2 2\theta_{13});$$

$$\lambda_2 = \Delta\alpha \cos^2\theta_{13};$$

$$\lambda_3 = \Delta(1 + \frac{\hat{A}_e}{4(1-\hat{A}_e)}\sin^2 2\theta_{13}.$$

and three eigenvectors:

$$|\lambda_1\rangle = \begin{pmatrix} 1 - \frac{\sin^2 2\theta_{13}}{8(\hat{A}_e - 1)^2} \\ \frac{\alpha\cos\theta_{23}\sin 2\theta_{12}}{2\hat{A}_e} - \frac{e^{i\delta}(1+\hat{A}_e(\alpha\sin^2\theta_{12} - 1))\sin 2\theta_{13}\sin\theta_{23}}{2(\hat{A}_e - 1)^2} \\ -\frac{\alpha\sin\theta_{23}\sin 2\theta_{12}}{2\hat{A}_e} - \frac{e^{i\delta}(1+\hat{A}_e(\alpha\sin^2\theta_{12} - 1))\sin 2\theta_{13}\cos\theta_{23}}{2(\hat{A}_e - 1)^2} \end{pmatrix} \quad |\lambda_2\rangle = \begin{pmatrix} \frac{\alpha\sin 2\theta_{12}}{2\hat{A}_e} \\ \cos\theta_{23} + \frac{\alpha(1+\hat{A}_e)e^{i\delta}\sin\theta_{23}\sin 2\theta_{12}\sin 2\theta_{13}}{4\hat{A}_e} \\ -\sin\theta_{23} + \frac{\alpha(1+\hat{A}_e)e^{i\delta}\cos\theta_{23}\sin 2\theta_{12}\sin 2\theta_{13}}{4\hat{A}_e} \end{pmatrix}$$

$$|\lambda_3\rangle = \begin{pmatrix} \frac{(1+\hat{A}_e(\alpha\sin^2\theta_{12} - 1))\sin 2\theta_{13}}{2(\hat{A}_e - 1)^2} \\ \frac{\alpha\hat{A}_e\cos\theta_{23}\sin 2\theta_{12}\sin 2\theta_{13}}{4(1\text{-}\hat{A}_e)} + e^{i\delta}(1 - \frac{\sin^2 2\theta_{13}}{8(\hat{A}_e - 1)^2})\sin\theta_{23} \\ \frac{\alpha\hat{A}_e\sin\theta_{23}\sin 2\theta_{12}\sin 2\theta_{13}}{4(\hat{A}_e - 1)} + e^{i\delta}(1 - \frac{\sin^2 2\theta_{13}}{8(\hat{A}_e - 1)^2})\cos\theta_{23} \end{pmatrix}$$

Using the perturbative methods we can calculate the first order corrections to the eigenvalues and eigenvectors coming from NP Hamiltonian:

$$\lambda_i \to \lambda_i^f = \lambda_i + \langle \lambda_i | H^{NP} | \lambda_i \rangle$$

$$|\lambda_i\rangle \to |\lambda_i^f\rangle = |\lambda_i\rangle + \sum_{k \neq i} \frac{|\lambda_k\rangle\langle\lambda_k | H^{NP} | \lambda_i\rangle}{\lambda_i - \lambda_k}$$

Now the transition amplitudes can be calculated:

$$A_{\bar{\alpha}\to\bar{\beta}}(L=t) = \sum_k \widetilde{W}_{\bar{\beta}k} \widetilde{W}^*_{\bar{\alpha}k} e^{-i\frac{\lambda_k^f}{2E}L}$$

where the $\widetilde{W}$ matrix diagonalizes the full Hamiltonian:

$$\widetilde{W}^+ H^{eff} \widetilde{W} = \mathrm{diag}(\lambda_1^f, \lambda_2^f, \lambda_3^f) \qquad \text{with} \quad \widetilde{W}_{\bar{\beta}i} = \langle \nu_{\bar{\beta}} | \lambda_i^f \rangle$$

If now our production and detection states are constructed as in the example

($|\nu_P\rangle = |\nu_{\bar{\alpha}}\rangle; \quad |\nu_D\rangle = |\nu_{\bar{\beta}}\rangle$) we can write:

$$|\nu_P\rangle = \frac{1}{\lambda_\alpha} \sum_{\bar{\gamma}} Y^*_{\bar{\alpha}\bar{\gamma}} |\nu_{\bar{\gamma}}\rangle; \qquad |\nu_D\rangle = \frac{1}{\lambda_\beta} \sum_{\bar{\delta}} Y^*_{\bar{\beta}\bar{\delta}} |\nu_{\bar{\delta}}\rangle;$$

Then the amplitude $A_{P\to D}(L)$ is:

$$A_{P\to D}(L) = \frac{1}{\lambda_\alpha \lambda_\beta} \sum_k W_{\bar{\beta}k} W^*_{\bar{\alpha}k} e^{-i\frac{\lambda_k^f}{2E}L}$$

where now the matrix:

$$W_{\bar{\beta}k} = (Y\tilde{W})_{\bar{\beta}k}$$

is generally not unitary.

Transition probabilities and results

The transition probability can be calculated:

$$P_{P\to D}(L) \equiv \left|A_{P\to D}(L)\right| = \frac{1}{\lambda_\alpha^2 \lambda_\beta^2}\left(\delta_{\alpha\beta} - 2\,\mathrm{Re}(c_{\alpha\beta}) - \right.$$

$$\left. 4\sum_{i>k} R^{ik}_{\bar{\alpha}\bar{\beta}} \sin^2\Delta_{ik} - 2\sum_{i>k} I^{ik}_{\bar{\alpha}\bar{\beta}} \sin 2\Delta_{ik}\right),$$

where

$$R^{ik}_{\bar{\alpha}\bar{\beta}} = \mathrm{Re}(T^{ik}_{\bar{\alpha}\bar{\beta}}), \quad I^{ik}_{\bar{\alpha}\bar{\beta}} = \mathrm{Im}(T^{ik}_{\bar{\alpha}\bar{\beta}})$$

$$T^{ik}_{\bar{\alpha}\bar{\beta}} = W_{\bar{\alpha}i}\, W_{\bar{\beta}k}\, W^*_{\bar{\alpha}k}\, W^*_{\bar{\beta}i} \;; \qquad \Delta_{ik} = (\lambda^f_i - \lambda^f_k)\frac{L}{4E}.$$

We can decompose the W tensors for three pieces:

$$W = Y\tilde{W} \quad \Rightarrow (I-\delta Y)(W_0+\delta W_0) \Rightarrow \; W_0+\delta W,$$

$$\text{where} \quad \delta W = \delta W_0 - \delta Y\, W_0,$$

$$\text{so} \quad \delta W_0 = \delta W^{\varepsilon};$$

NP interaction in matter

$$-\delta Y\, W_0 = \delta W^{c}.$$

NP change in initial and final neutrino states

As also

$$\Delta_{ik} = \Delta_{ik}^{SM} + \delta\Delta_{ik}^{NP}$$

the full transition probability can be decomposed for two pieces:

$$P_{P\to D}(L) = P_{P\to D}^{SM}(L) + \delta P_{P\to D}^{NP}(L)$$

SM transition probability

Describes corrections coming from NP

The NP part can be once more decomposed for two parts

$$\delta P^{NP}_{P\to D}(L) = \delta P^{c}_{P\to D}(L) + \delta P^{\varepsilon}_{P\to D}(L);$$

Describes NP interaction inside matter. parameterised in the model independent way.

Describes the NP change in the production and detection states. Depends on the model. Survive in vacuum

$$\delta P^{\varepsilon}_{P\to D}(L) = \delta P^{int}_{P\to D}(L) + \delta P^{mass}_{P\to D}(L);$$

Additional NP interaction inside matter

Comes from effective neutrino mass change given by NP

By appropriate decomposition of $P_{P\to D}(L)$ in our example we obtain:

$$\delta P^{c}_{P\to D}(L) = (c_{\alpha\alpha} + c_{\beta\beta})\, P^{SM}_{P\to D}(L) - 2\,\mathrm{Re}(c_{\alpha\beta}) -$$

$$4\sum_{i>k} (\delta R^{c})^{ik}_{\bar\alpha\bar\beta}\ \sin^2\Delta^{SM}_{ik} - 2\sum_{i>k} (\delta I^{c})^{ik}_{\bar\alpha\bar\beta}\ \sin 2\Delta^{SM}_{ik}$$

These tensors are respectively real and imaginary parts of the full tensor $T^{ik}_{\alpha\bar\beta}$ **linear in** $(\delta W^{c})_{\bar\alpha k}$

In the same way the other parts:

$$\delta P^{int}_{P\to D}(L) = -4\sum_{i>k}(\delta R^{\varepsilon})^{ik}_{\bar\alpha\bar\beta}\ \sin^2\Delta^{SM}_{ik} - 2\sum_{i>k}(\delta I^{\varepsilon})^{ik}_{\bar\alpha\bar\beta}\ \sin 2\Delta^{SM}_{ik},$$

Real and imaginary parts of the tensor $T^{ik}_{\bar\alpha\bar\beta}$ linear in $(\delta W^{\varepsilon})_{\bar\alpha k}$

And finally the mass term:

$$\delta P^{mass}_{P\to D}(L) = -\sum_{i>k}\left(4(R^{SM})^{ik}_{\bar\alpha\bar\beta}\ \sin 2\Delta^{SM}_{ik} + 2(I^{SM})^{ik}_{\bar\alpha\bar\beta}\ \cos 2\Delta^{SM}_{ik}\right)\ \delta\Delta^{NP}_{ik}.$$

Now we will concentrate on the model independent “ϵ” part. These corrections can be parameterized by the flavour numbers (e, μ, τ).

The probability can be decomposed for three pieces:

$$P_{\alpha\to\beta} = P^{SM}_{\alpha\to\beta} + \delta P^{int}_{\alpha\to\beta} + \delta P^{mass}_{\alpha\to\beta}$$

$$\mathrm{Pme} = \mathrm{P0me} + \mathrm{dTme} + \mathrm{dSme};$$

$$\mathrm{P0me} = \frac{a^2\, c23^2\, s212^2\, \mathrm{Sin}[A\,\mathrm{Del}]^2}{A^2\, c13^2} - (a\, s212\, s213\, s223\, \mathrm{Cos}[\mathrm{Del}]\, \mathrm{Cos}[\mathrm{delta}]\, \mathrm{Sin}[A\,\mathrm{Del}]\, \mathrm{Sin}[\mathrm{Del} - A\,\mathrm{Del}]) / ((-1+A)\, A\, c13) +$$
$$\frac{s213^2\, s23^2\, \mathrm{Sin}[\mathrm{Del} - A\,\mathrm{Del}]^2}{(-1+A)^2} + (a\, s212\, s213\, s223\, \mathrm{Sin}[\mathrm{Del}]\, \mathrm{Sin}[A\,\mathrm{Del}]\, \mathrm{Sin}[\mathrm{Del} - A\,\mathrm{Del}]\, \mathrm{Sin}[\mathrm{delta}]) / ((-1+A)\, A\, c13);$$

$$\mathrm{dTme} =$$
$$a\left(\frac{2\, c23\, en\, s212\, \mathrm{Cos}[\mathrm{chien}]\, \mathrm{Sin}[A\,\mathrm{Del}]\, (A\, s23^2\, \mathrm{Sin}[(-2+A)\,\mathrm{Del}] + (2\,(-1+A)\, c23^2 + A\, s23^2)\, \mathrm{Sin}[A\,\mathrm{Del}])}{(-1+A)\, A^2} +\right.$$
$$\frac{c23^2\, et\, s212\, s23\, \mathrm{Cos}[\mathrm{chiet}]\, (A\,\mathrm{Cos}[2\,\mathrm{Del}] - A\,\mathrm{Cos}[2\,(-1+A)\,\mathrm{Del}] - 2\,(-2+A)\, \mathrm{Sin}[A\,\mathrm{Del}]^2)}{(-1+A)\, A^2} +$$
$$\frac{c23\, en\, s212\, s23^2\, \mathrm{Sin}[\mathrm{chien}]\, (\mathrm{Sin}[2\,\mathrm{Del}] - \mathrm{Sin}[2\,(1-A)\,\mathrm{Del}] - \mathrm{Sin}[2\,A\,\mathrm{Del}])}{(-1+A)\, A} +$$
$$\left.\frac{c23^2\, et\, s212\, s23\, \mathrm{Sin}[\mathrm{chiet}]\, (\mathrm{Sin}[2\,\mathrm{Del}] - \mathrm{Sin}[2\,(1-A)\,\mathrm{Del}] - \mathrm{Sin}[2\,A\,\mathrm{Del}])}{(-1+A)\, A}\right) +$$
$$s213$$
$$\left(\frac{1}{(-1+A)^2\, A}\right.$$
$$\Big(en\, s23\, \big(-c23^2 + A\, c23^2 + 2\,A\, s23^2 + (-1+A)\, c23^2\, \mathrm{Cos}[2\,\mathrm{Del}] + (c23^2 - A\, c23^2 - 2\,A\, s23^2)\, \mathrm{Cos}[2\,(-1+A)\,\mathrm{Del}] + c23^2\, \mathrm{Cos}[2\,A\,\mathrm{Del}] - A\, c23^2\, \mathrm{Cos}[2\,A\,\mathrm{Del}]\big)$$
$$\mathrm{Cos}[\mathrm{chien} + \mathrm{delta}]\Big) - \frac{c23\, et\, s23^2\, (-1 - A + (-1+A)\, \mathrm{Cos}[2\,\mathrm{Del}] + (1+A)\, \mathrm{Cos}[2\,(-1+A)\,\mathrm{Del}] + \mathrm{Cos}[2\,A\,\mathrm{Del}] - A\,\mathrm{Cos}[2\,A\,\mathrm{Del}])\, \mathrm{Cos}[\mathrm{chiet} + \mathrm{delta}]}{(-1+A)^2\, A} +$$
$$\frac{c23^2\, en\, s23\, (-\mathrm{Sin}[2\,\mathrm{Del}] + \mathrm{Sin}[2\,(1-A)\,\mathrm{Del}] + \mathrm{Sin}[2\,A\,\mathrm{Del}])\, \mathrm{Sin}[\mathrm{chien} + \mathrm{delta}]}{(-1+A)\, A} +$$
$$\left.\frac{c23\, et\, s23^2\, (\mathrm{Sin}[2\,\mathrm{Del}] - \mathrm{Sin}[2\,(1-A)\,\mathrm{Del}] - \mathrm{Sin}[2\,A\,\mathrm{Del}])\, \mathrm{Sin}[\mathrm{chiet} + \mathrm{delta}]}{(-1+A)\, A}\right);$$

$$\mathrm{dSme} = 0;$$

For check we have calculated all probabilities

(α, β) = {(e,e), (e,μ), (e,τ), (μ,e), (μ,μ), (μ,τ), (τ,e), (τ,μ), (τ,τ)}

As NP is parameterized by hermitian Hamiltonian oscillation probabilities are conserved:

$$\sum_{\text{all }\beta} \mathrm{P}_{\alpha\to\beta} = 1, \quad \text{for all} \quad \alpha = \mathrm{e}, \mu, \tau$$

What is more interesting, for each α we have separately:

$$\sum_{\text{all }\beta} \mathrm{P}^{\mathrm{SM}}_{\alpha\to\beta} = 1, \quad \sum_{\text{all }\beta} \delta\mathrm{P}^{\mathrm{int}}_{\alpha\to\beta} = 0, \quad \sum_{\text{all }\beta} \delta\mathrm{P}^{\mathrm{mass}}_{\alpha\to\beta} = 0.$$

The general structure of the SM oscillation probabilities is the next:

$$P^{SM}_{\beta\to\gamma} = \delta_{\beta\gamma} + A^{0}_{\beta\gamma} + \alpha\, A^{\alpha}_{\beta\gamma} + \alpha^2\, A^{\alpha\alpha}_{\beta\gamma} + \sin^2 2\theta_{13}\, A^{s}_{\beta\gamma} + \alpha \sin 2\theta_{13}\, A^{\alpha s}_{\beta\gamma}.$$

The not-suppressed terms, $A^{0}_{\beta\gamma}$ appear in $(\beta\gamma) = \{(\mu\mu), (\mu\tau), (\tau\tau)\}$ channels only, and are the same for all channels:

$$A^{0}_{\beta\gamma} = \pm \sin^2 2\theta_{23}\, \sin^2 \bar{\Delta}; \qquad \text{where} \quad \bar{\Delta} = \delta m^2_{31} \frac{L}{2E}$$

The CP violating phases exist in the appearance channels only (as should be), and are the same for each channel:

$$A^{\alpha s}_{\beta\gamma} = \pm \sin 2\theta_{12}\, \sin 2\theta_{13} \frac{\sin[\bar{\Delta}] \sin[\hat{A}_e \bar{\Delta}] \sin[(1-\hat{A}_e)\bar{\Delta}]}{\hat{A}_e (1-\hat{A}_e)} \sin\delta$$

The NP terms have general structure:

$$\delta P^{\text{int}}_{\beta\to\gamma} = B^{0}_{\beta\to\gamma} + \alpha\, B^{\alpha}_{\beta\to\gamma} + \sin 2\theta_{13} B^{s}_{\beta\to\gamma} + 0(\alpha, \sin 2\theta_{13});$$

$$\delta P^{\text{mass}}_{\beta\to\gamma} = C^{0}_{\beta\to\gamma} + \alpha\, C^{\alpha}_{\beta\to\gamma} + \sin 2\theta_{13} C^{s}_{\beta\to\gamma} + 0(\alpha, \sin 2\theta_{13});$$

Unfortunately, the NP not suppressed terms appear also in $(\beta\gamma) = \{(\mu\mu), (\mu\tau), (\tau\tau)\}$ only and are equal:

$$B^{0}_{\beta\gamma} = \pm \sin 4\theta_{23} \{\sin 2\theta_{23}(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) + 2\cos 2\theta_{23} \varepsilon_{\mu\tau} \cos[\chi_{\mu\tau}]\} \sin^2[\bar{\Delta}];$$

$$C^{0}_{\beta\gamma} = \pm \bar{\Delta} \sin^2 2\theta_{23} \{\cos 2\theta_{23}(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) + 2\sin 2\theta_{23} \varepsilon_{\mu\tau} \cos[\chi_{\mu\tau}]\} \sin[2\bar{\Delta}];$$

The second mass term $C^{0}_{\beta\gamma}$ has different dependence on the baseline L and neutrino energy.

The NP CP violating terms are α or $\sin(2\theta_{13})$ suppressed and appear in the interaction terms, not in the mass terms:

$$\mathrm{B}^{\alpha}_{\beta\to\gamma} = h_0 + h_\mu \varepsilon_{e\mu} \sin[\chi_{e\mu}] + h_\tau \varepsilon_{e\tau} \sin[\chi_{e\tau}];$$

$$\mathrm{B}^{s}_{\beta\to\gamma} = k_0 + k_\mu \varepsilon_{e\mu} \sin[\chi_{e\mu} + \delta] + k_\tau \varepsilon_{e\tau} \sin[\chi_{e\tau} + \delta];$$

The NP CP violating $\chi_{\mu\tau}$ phase, and ε_{ee} term do not appear in the approximation which we consider,

Connections between parameters

Two examples:

- Models with non decoupling heavy neutrinos and left handed current interaction,
- The Left-Right symmetric model.

In the first case:

$$L_{CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{n} \overline{\psi}_\alpha \gamma^\mu (1-\gamma_5)(U_\nu)_{\alpha i} N_i W_\mu^- + h.c.;$$

$$L_{NC} = \frac{e}{4\sin\theta_W \cos\theta_W} \sum_{i,j=1}^{n} \overline{N}_i \gamma^\mu (1-\gamma_5)\Omega_{ij} N_j Z_\mu , \quad \text{where}$$

$$\Omega_{ij} = \sum_{\alpha=e,\mu,\tau} (U_\nu)^*_{\alpha i} (U_\nu)_{\alpha j}$$

In this case the HP Hamiltonian is equal

$$H^{NP} = \frac{1}{2E}(-A_e[\delta Y^{+}E(1) + E(1)\delta Y] + \frac{A_n}{2}[\delta Y^{+} + \delta Y]) \quad \text{where}$$

$$A_n = 2\sqrt{2}\, G_F E\, N_n \quad \text{and} \quad E(1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

Now we can easy find relations between “c” and “ϵ” coefficients:

$$\varepsilon_{ee} = c_{ee}(-1 + \frac{A_n}{2A_e});$$

$$\varepsilon_{\alpha\beta} e^{i\chi_{\alpha\beta}} = \frac{A_n}{2A_e} c_{\alpha\beta} \quad \text{for } \alpha, \beta \neq e$$

In the Left – Right symmetric model

$$\gamma = \frac{M_{Z_1}^2}{M_{Z_2}^2}$$

If neutrinos are Dirac particles

$$H^{NP} = \delta H^{LR}_{Dirac} = \delta H^{HN}\left(\rho \to 1 + \gamma \frac{1 - 2\sin^2\theta_W}{\cos 2\theta_W}\right)$$

If neutrinos are Majorana particles

$$H^{NP} = \delta H^{LR}_{Majorana} = \delta H^{LR}_{Dirac}$$

$$- A_e \beta (\bar{V}^+ E(1) \bar{V}) - A_n \gamma \frac{\cos^4\theta_W}{\cos 2\theta_W}(\delta Y^+ + \delta Y)$$

$$\beta = \frac{M^2_{W_1}}{M^2_{W_2}} \qquad \bar{V} = V' U^+$$

Then connection between the “ϵ” and “c” terms are the next:

For Dirac neutrinos, the same as in the previous example.

For Majorana neutrinos:

$$\varepsilon_{\alpha\beta} = c_{\alpha\beta}\frac{A_n}{A_e}\left(\frac{\rho^{'}}{2} - \gamma\rho^{''}\right) - \beta\left(\overline{V}^{*}_{N_e\alpha}\overline{V}_{N_e\beta}\right);$$

where $\rho^{'} = \rho\left(1+\gamma\frac{1-2\sin^2\theta_W}{\cos 2\theta_W}\right); \ \rho^{''} = \rho\frac{\cos^4\theta_W}{\cos 2\theta_W},$

with $\rho = \frac{M^2_{W_1}}{\cos^2\theta_W M^2_{W_2}}.$

2. Non-unitary neutrino oscillation.

Grossmann;

Gonzalez-Garcia, Grossman, Gusso, Nir;

Czakon, Gluza, Z.M:

Beckman, Gluza, Holeczek, Syska, Z.M;

Bilenky, Giunti.

Three flavour + sterile neutrinos

Matrix U = $(3+n_s) \times (3+n_s)$

$$U_\nu = \begin{pmatrix} U & V \\ V' & U' \end{pmatrix}$$

Matrix U' = $(n_R \ \times n_R)$

Heavy neutrinos

In this lecture we will not consider specific models which can produce non-decoupling for light neutrino.

$$\left|\nu_i\right\rangle \equiv \left|i\right\rangle$$ $$i = 1, 2, 3$$ Eigenmass states in vacuum

$$\left|\nu_\alpha\right\rangle \equiv \left|\alpha\right\rangle$$ $$\alpha = e, \mu, \tau$$ Flavour eigenstates. States for production and detection process

$$\left|\nu_{\bar{i}}\right\rangle \equiv \left|\bar{i}\right\rangle$$ $$\bar{i} = \bar{1}, \bar{2}, \bar{3}$$ Eigenmass states in matter

All bases are orthonormal, so:

$$\left|\bar{i}\right\rangle = \sum_{k=1}^{3} \widetilde{W}_{\bar{i}k}\left|k\right\rangle, \qquad \left|k\right\rangle = \sum_{\bar{i}=\bar{1}}^{\bar{3}} \widetilde{W}^{*}_{\bar{i}k}\left|\bar{i}\right\rangle$$

$$\left|\alpha\right\rangle = \sum_{i=1}^{3} U^{*}_{\alpha i}\left|i\right\rangle, \qquad \left|i\right\rangle = \sum_{\alpha=e,\mu,\tau} U_{\alpha i}\left|\alpha\right\rangle,$$

$$\left|\alpha\right\rangle = \sum_{\bar{i}=\bar{1}}^{\bar{3}} W^{*}_{\alpha\bar{i}}\left|\bar{i}\right\rangle, \qquad \left|\bar{i}\right\rangle = \sum_{\alpha=e,\mu,\tau} W_{\alpha\bar{i}}\left|\alpha\right\rangle$$

$$W\widetilde{W} = U$$

Neutrino states in general case

$$|\nu_i\rangle \equiv |i\rangle \qquad i = 1, 2, 3, \ldots\ldots, n$$

$$|\nu_{\bar{i}}\rangle \equiv |\bar{i}\rangle \qquad \bar{i} = \bar{1}, \bar{2}, \bar{3}, \ldots\ldots, \bar{n}$$

$$|\nu_\alpha\rangle \equiv |\alpha\rangle \qquad \alpha = e, \mu, \tau, ..s, ..N_e, N_\mu, N_\tau.$$

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} (U_\nu^*)_{\alpha i} |\nu_i\rangle = \sum_{i=1}^{3+n_s} \mathrm{U}_{\alpha i}^* |\nu_i\rangle + \sum_{i=3+n_s+1}^{n} \mathrm{V}_{\alpha i}^* |\nu_i\rangle$$

Experimentally produced neutrino flavour states:

$$\left|\tilde{\nu}_\alpha\right\rangle = \frac{1}{\lambda_\alpha}\sum_{i=1}^{3+n_s} U^*_{\alpha i}\left|\nu_i\right\rangle \equiv \sum_{i=1}^{3+n_s} \tilde{U}^*_{\alpha i}\left|\nu_i\right\rangle$$

$$\alpha = e, \mu, \tau$$

$$\tilde{U}_{\alpha i} = \lambda_\alpha^{-1} U_{\alpha i}$$

This mixing matrix is not unitary

$$\tilde{U}\tilde{U}^{+} \neq I \qquad \tilde{U}^{+}\tilde{U} \neq I$$

$$\left\langle\tilde{\nu}_\alpha\middle|\tilde{\nu}_\beta\right\rangle \neq 1 \ \text{ for } \ \alpha \neq \beta; \qquad \left\langle\tilde{\nu}_\alpha\middle|\tilde{\nu}_\alpha\right\rangle = 1$$

As before we parameterize;

$$\lambda_\alpha = \sqrt{1-(VV^+)_{\alpha\alpha}}$$

and

$$\mathrm{U} = \mathrm{Y}\cdot U \qquad \tilde{\mathrm{U}} = \Lambda \mathrm{Y} U; \quad \Lambda = \mathrm{diag}(1/\lambda_e, 1/\lambda_\mu, 1/\lambda_\tau)$$

where U is the standard 3x3 unitary matrix parameterized in the way:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}.$$

And once more

$$Y = I - \delta Y,$$

then from unitary condition of the full mixing matrix

$$YY^{+} = I - VV^{+}$$

we can parameterize

$$\delta Y = \frac{1}{2}\begin{pmatrix} c_{ee} & 0 & 0 \\ 2c_{\mu e} & c_{\mu\mu} & 0 \\ 2c_{\tau e} & 2c_{\tau\mu} & c_{\tau\tau} \end{pmatrix}$$

where

$$c_{\alpha\beta} = (VV^{+})_{\alpha\beta}$$

From charge lepton proceses there are bounds on the c parameters

$$(\Gamma_Z)_{\text{invisible}} \rightarrow N_\nu = \frac{\Gamma_{\text{invisible}}}{\Gamma_{\nu\bar{\nu}}} = 2.9840 \pm 0.0082$$

$$\mu \to e\,\nu_e\,\nu_\mu \qquad \tau \to \mu\,\nu_\mu\,\nu_\tau \qquad \mu \to e\,\gamma$$

$$\pi \to e\nu_e, \quad \pi \to \mu\nu_\mu,$$

$$\mu \to e\,\nu_e\,\nu_\mu, \qquad \tau \to \mu\,\nu_\mu\,\nu_\tau,$$

K_{l3} decay, β decay,

- Langacker, London;
- Nardi, Roulet, Tommasini;
- Bornaboim, Bernabeu, Jorlskog, Tommasini;
- Bergman, Kagan;
- Ilana, Rieman;

$$c_{ee}^2 \le 0.0054; \quad c_{\mu\mu}^2 \le 0.0094; \quad c_{\tau\tau}^2 \le 0.016;$$

$$|c_{e\mu}|^2 \le 0.0001; \quad |c_{\mu\tau}|^2 \le 0.01.$$

Neutrino Propagation in Matter

Based on: B.Bekman, et. al.,

Phys. Rev.,D66(2002)093004.

Everything is much more convenient to consider in the neutrino mass basis

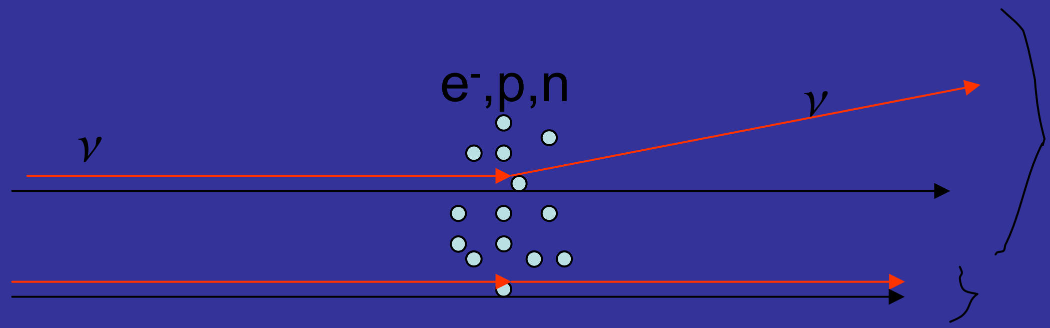
Let us assume that our neutrino interactionin has general form:

$n = 3 + n_s + n_R$

$$L_{CC} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{n} \overline{\psi}_\alpha \gamma^\mu (1-\gamma_5)(U_\nu)_{\alpha i} N_i W_\mu^- + h.c.;$$

$$L_{NC} = \frac{e}{4\sin\theta_W\cos\theta_W} \sum_{i,j=1}^{n} \overline{N}_i \gamma^\mu (1-\gamma_5)\Omega_{ij} N_j Z_\mu , \quad \text{where}$$

$$\Omega_{ij} = \sum_{\alpha=e,\mu,\tau} (U_\nu)^*_{\alpha i} (U_\nu)_{\alpha j}$$

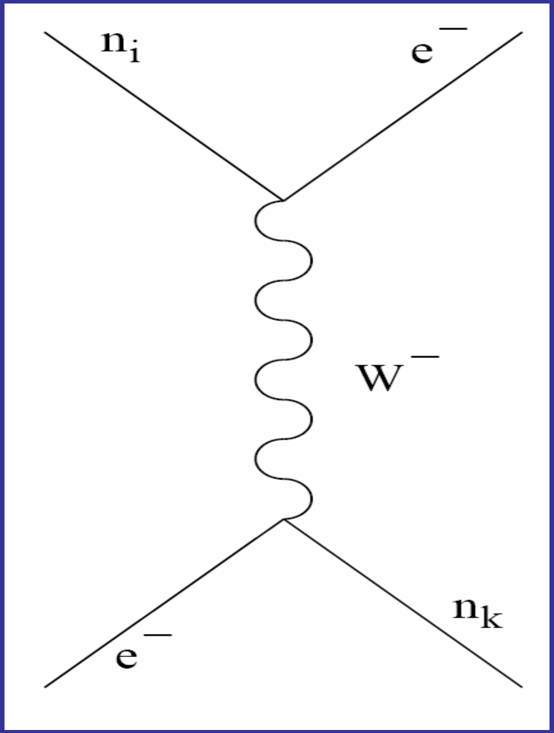


No interference

Interference

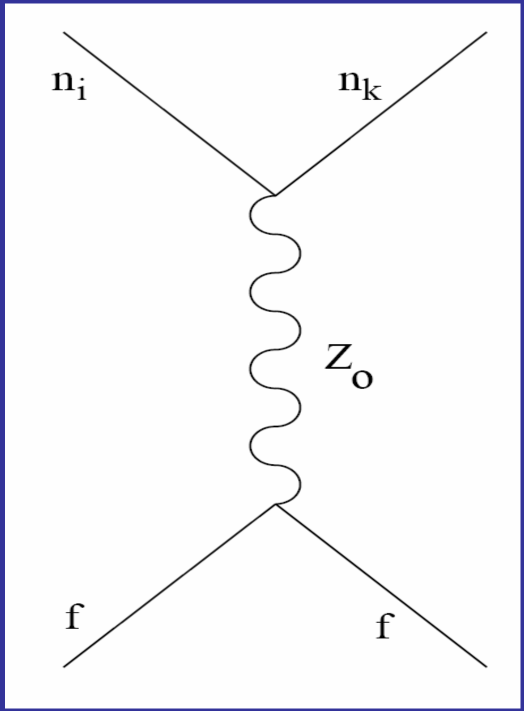
Neutrino interaction are described by the L_{CC} and L_{NC} Lagrangians

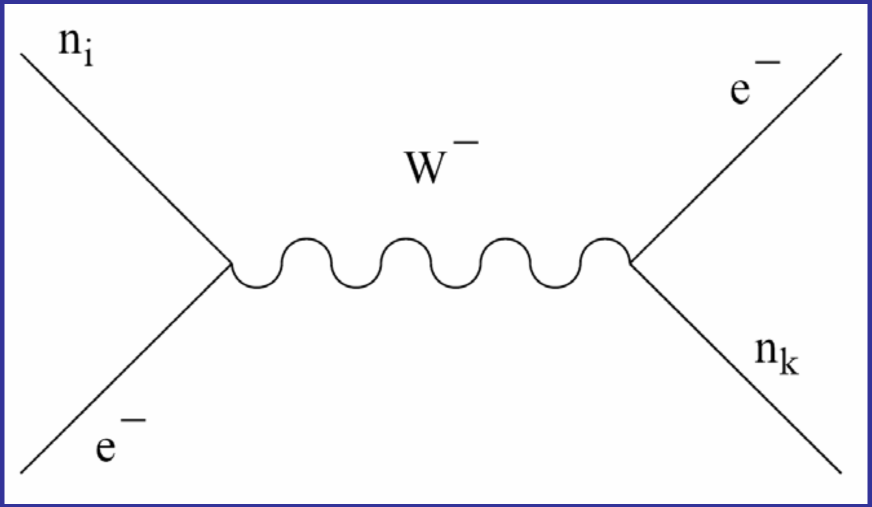
COHERENT NEUTRINO SCATTERING IS DESCRIBED BY THREE TYPES OF FEYNMAN DIADRAMS



All three diagrams contribute to neutrino - electron scattering:

$$n_i + e^- \rightarrow n_k + e^-$$





Only this diagram contributes to neutrino- nucleon scattering:

$$n_i + f \rightarrow n_k + f; \qquad f = \ p, n$$

Bergmann, Grossman, Nardi

For $q^2 << M_W^2, M_Z^2$, the effective interaction of light neutrinos with background particles can be written in the form:

$$H_{int}^{f}(x) = \frac{G_F}{\sqrt{2}} \sum_{i,k=1}^{3} \sum_{a=V,A} [\overline{n}_i \Gamma_a n_k][\overline{\psi}_f \Gamma^a (g_{fa}^{ki} + \overline{g}_{fa}^{ki} \gamma_5) \psi_f]$$

where

$$g_{eV}^{ki} = -\bar{g}_{eA}^{ki} = \mathcal{U}_{ek}^{*} \mathcal{U}_{ei} + \rho \Omega_{ki} \left(-\frac{1}{2} + 2 \sin^2 \Theta_W \right)$$

$$\bar{g}_{eV}^{ki} = -g_{eA}^{ki} = -\mathcal{U}_{ek}^{*} \mathcal{U}_{ei} + \frac{1}{2} \rho \Omega_{ki},$$

$$g_{fV}^{ki} = -\bar{g}_{fA}^{ki} = \rho \Omega_{ki} \left(T_{3f} - 2 Q_f \sin^2 \Theta_W \right),$$

$$\bar{g}_{fV}^{ki} = -g_{eA}^{ki} = -\rho \Omega_{ki} T_{3f},$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \Theta_W}, T_{3p} = -T_{3n} = 1/2, Q_p = 1, Q_n = 0$$

Amplitudes for forward scattered fermions f (their momenta and spins are untouched) is defined as:

$$\left(M_a^f\right)_{ki} = \langle f, \vec{p}, \vec{\lambda} | \bar{\Psi}_f \Gamma_a \left(g_{fa}^{ki} + \bar{g}_{fa}^{ki} \gamma_5\right) \Psi_f | f, \vec{p}, \vec{\lambda} \rangle$$

Let $\rho_f(\vec{\mathrm{p}}, \vec{\lambda})$ describes a distributions of momentum (p) and spins (λ) of the background particles f, normalized in such a way that N_f, defined as

$$N_f \equiv \sum_{\vec{\lambda}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\vec{p}, \vec{\lambda})$$

is the number of fermions f in a unit volume (V=1).

Let us define the average amplitude for fermions (f) forward scattering:

$$\left(V_a^f\right)_{ki} = \frac{G_F}{\sqrt{2}} \sum_{\vec{\lambda}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\vec{p}, \vec{\lambda}) \left(M_a^f\right)_{ki}$$

And the effective sum over all fermions:

$$V_{ki} = \sum_f V_{ki}^f = \sum_f \sum_a \Gamma^a \left(V_a^f\right)_{ki}$$

Then the global effect of matter – light neutrino interaction can be described by the Hamiltonian

$$H_{int}(x) = \sum_{i,k=1}^{3} \overline{n}_k V_{ki} n_i$$

The amplitudes M^f can be calculated ($u^+u = 1$):

$$\left(M_V^f\right)_{ki}^{\mu} = -\left(M_A^f\right)_{ki}^{\mu} = g_{fV}^{ki}\left(\frac{p^\mu}{E_f}\right) + m_f \bar{g}_{fV}^{ki}\left(\frac{S_f^\mu}{E_f}\right)$$

where E_f, m_f and $S_f^\mu = \frac{1}{m_f}\left(\vec{p}\vec{\lambda}, \vec{\lambda} m_f + \frac{\vec{p}(\vec{p}\vec{\lambda})}{m_f+E_f}\right)$ are energy, mass and spin four – vector of the f fermion.

So finally, for the effective potential V we obtain:

$$V^{f}_{ki} = \left(A^{\mu}_{f}\right)_{ki} \gamma_{\mu} P_{L}$$

where

$$\left(A^{\mu}_{f}\right)_{ki} = \sqrt{2} G_{F} N_{f} \left[g^{ki}_{fV} \langle \frac{p^{\mu}_{f}}{E_{f}} \rangle + m_{f} \bar{g}^{ki}_{fV} \langle \frac{S^{\mu}_{f}}{E_{f}} \rangle \right]$$

and the symbol $\langle z \rangle$ is defined as:

$$< z >= \frac{1}{N_{f}} \sum_{\vec{\lambda}} \int \frac{d^{3}p}{(2\pi)^{3}} \rho_{f}(\vec{p}, \vec{\lambda}) z(\vec{p}, \vec{\lambda})$$

For the (k,i) matrix of the element of the Hamiltonian (in the eigenmass base) we obtain:

$$H_{ki}^{int} = < \nu_k | \int_{V=1} d^3x \; H_{int}(x) \; | \nu_i > = \begin{cases} A_{ki}^{\mu} \bar{u}_k \gamma_\mu P_L u_i & \text{for Dirac neutrinos,} \\ -(A_{ki}^{\mu})^* \bar{u}_k \gamma_\mu P_R u_i & \text{for Dirac antineutrinos,} \\ A_{ki}^{\mu} \bar{u}_k \gamma_\mu P_L u_i - (A_{ki}^{\mu})^* \bar{u}_k \gamma_\mu P_R u_i & \text{for Majorana neutrinos,} \end{cases}$$

$$A_{ki}^{\mu} = \sum_f \left(A_f^{\mu} \right)_{ki}$$

For relativistic neutrinos:

$$\bar{u}_k \gamma_\mu P_L u_{i|\lambda=-1} = \bar{u}_k \gamma_\mu P_R u_{i|\lambda=+1} = \left(1, -\frac{\vec{k}}{|\vec{k}|}\right)$$

$$\bar{u}_k \gamma_\mu P_L u_{i|\lambda=+1} = \bar{u}_k \gamma_\mu P_R u_{i|\lambda=-1} = \left(0, \vec{0}\right)$$

We obtain the final, general Hamiltonian for neutrino interaction with background fermions:

$$H_{ki}^{int} = \begin{cases} A_{ki}^0 - \frac{\vec{k}}{|\vec{k}|} \vec{A}_{ki} & \text{for Dirac and Majorana neutrinos with } \lambda = -1, \\ -(A_{ki}^0)^* + \frac{\vec{k}}{|\vec{k}|} (\vec{A}_{ki})^* & \text{for Dirac antineutrinos and Majorana neutrinos with } \lambda = +1 \end{cases}$$

We see that our Hamiltonian has the properties:

$$H^{int}_{particle} = -[H^{int}_{antiparticle}]^*$$

Valid for V-A neutrino interaction

So the most general Hamiltonian for light relativistic neutrinos which interact in the V-A way, propagating in any background medium has the form:

$$H^{int}_{ki} = A^0_{ki} - \frac{\vec{k}}{|\vec{k}|}\vec{A}_{ki}$$

$$A^\mu_{ki} = \sqrt{2}G_F \sum_{f=e,p,n} N_f \left(g^{ki}_{fV} \left\langle \frac{p^\mu_f}{E_f} \right\rangle + m_f \bar{g}^{ki}_{fV} \left\langle \frac{S^\mu_f}{E_f} \right\rangle \right)$$

NORMAL MATTER

unpolarized: $\langle \vec{\lambda}_f \rangle = 0$

isotropic: $\langle \vec{p}_f \rangle = 0$

electrically neutral: $N_e = N_p \neq N_n$

We arrive to the simple Hamiltonian which has $(3 + n_s) \times (3 + n_s)$ dimension

$$H^{int}_{ki} = \sqrt{2} G_F [N_e U^*_{ek} U_{ei} - \frac{1}{2} \rho N_n \Omega_{ki}],$$

Which in the standard case ($n_s = n_R = 0$) reproduce (in the flavour base) the well known Hamiltonian (α, β = e, μ, τ):

$$H^{int}_{\alpha\beta} = \sqrt{2} G_F N_e \delta_{e\beta} \delta_{e\beta} \Rightarrow \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

If a single sterile neutrino is present (n_s=1, α, β = e, μ, τ, s)):

$$H_{\alpha\beta} = \left(U \frac{\Delta m^2}{2E} U^+ \right)_{\alpha\beta} + \sqrt{2} G_F [N_e \delta_{\alpha e} \delta_{\beta e} + \frac{1}{2} N_n \delta_{\alpha s} \delta_{\beta s}];$$

WE WILL CONSIDER CASES WHEN AT LEAST ONE NON-DECOUPLING HEAVY NEUTRINO EXIST;

$n_R \geq 1$

The energy and momentum conservation do not allow heavy neutrinos to be produced or detected

$$\begin{matrix} & 3+n_s & n_R \end{matrix}$$

$$H^{int} \Rightarrow \begin{pmatrix} H^{int} & 0 \\ 0 & 0 \end{pmatrix}$$

If no sterile neutrinos are present ($n_s=0$) but $n_R = 1,2$ ..

$$H_{\alpha\beta} = \left(U \frac{\Delta m^2}{2E} U^+ \right)_{\alpha\beta} + \sqrt{2} G_F [N_e \{\delta_{\alpha e}\delta_{\beta e} - c_{\alpha\beta}(\delta_{\alpha e} + \delta_{\beta e}) + c_{e\alpha} c_{e\beta}\}$$

$$+ \frac{1}{2} N_n \{2c_{\alpha\beta} - \sum_{\gamma=e,\mu,\tau} c_{\alpha\gamma} c_{\gamma\beta}\}];$$

If a single sterile neutrino ($n_s = 1$) and some heavy neutrinos exist ($n = 4 + n_R$)

$$H_{\alpha\beta} = \left(U \frac{\Delta m^2}{2E} U^+ \right)_{\alpha\beta} + \sqrt{2} G_F [N_e \{\delta_{\alpha e}\delta_{\beta e} - c_{\alpha\beta}(\delta_{\alpha e} + \delta_{\beta e}) + c_{e\alpha} c_{e\beta}\}$$

$$+ \frac{1}{2} N_n \{\delta_{\alpha s}\delta_{\beta s} + c_{\alpha\beta}(2 - \delta_{\alpha s} - \delta_{\beta s}) - \sum_{\gamma=e,\mu,\tau} c_{\alpha\gamma} c_{\gamma\beta}\}];$$

And generally in the full flavour base:

$$H_{\alpha\beta} \Rightarrow \begin{pmatrix} UH^{eff}U^{+} & UH^{eff}V'^{+} \\ V'H^{eff}U^{+} & V'H^{eff}V'^{+} \end{pmatrix}_{\alpha\beta}$$

Even we consider the problem of light neutrino propagation only, in the flavour basis we have to deal with ($3 + n_s + n_R$) dimensional matrices

No heavy mass eigenstates can experimentally be produced → the properly normalized light neutrino states produced in real experiment:

$$\left|\tilde{\nu}_\alpha\right\rangle = \frac{1}{\lambda_\alpha}\sum_{i=1}^{3+n_s} U^{*}_{\alpha i}\left|\nu_i\right\rangle \equiv \sum_{i=1}^{3+n_s} \tilde{U}^{*}_{\alpha i}\left|\nu_i\right\rangle$$

Usual notation of flavour neutrino loose its meaning. Neutrino created with electron can produce muon and tau.

Propagation equation in the experimentally accessible states:

$$i\frac{d}{dt}\langle\tilde{\nu}_\alpha|\psi(t)\rangle = \sum_\beta \tilde{H}_{\alpha\beta}\langle\tilde{\nu}_\beta|\psi(t)\rangle;$$

where the Hamiltonian is not hermitian:

$$\tilde{H} = \tilde{U}H\tilde{U}^{-1} = \frac{1}{2E}\tilde{U}\begin{pmatrix} 0 & 0 & 0 & 0 & .. \\ 0 & \delta m^2_{21} & 0 & 0 & .. \\ 0 & 0 & \delta m^2_3 & 0 & .. \\ 0 & 0 & 0 & \delta m^2_{41} & .. \\ .. & .. & .. & .. & .. \end{pmatrix}\tilde{U}^{-1} + \sqrt{2}G_F\tilde{U}\tilde{U}^{+}\lambda^2\begin{pmatrix} (N_e - \frac{N_n}{2}) & 0 & 0 & 0 & .. \\ 0 & -\frac{N_n}{2} & & 0 & .. \\ 0 & 0 & -\frac{N_n}{2} & 0 & .. \\ 0 & 0 & 0 & 0 & .. \\ .. & .. & .. & .. & .. \end{pmatrix};$$

If we assume that the initial neutrino states is:

$$|\psi(0)\rangle = |\tilde{\nu}_\alpha(0)\rangle$$

The initial condition:

$$\langle\tilde{\nu}_\beta|\tilde{\nu}_\alpha(0)\rangle = (\lambda_\alpha\lambda_\beta)^{-1}(\delta_{\alpha\beta} - c_{\alpha\beta});$$

In the eigenmass basis the propagation equation:

$$i\frac{d}{dt}\langle \nu_k | \tilde{\nu}_\alpha(t)\rangle = \sum_{i=1}^{3+n_s} H_{ki}\langle \nu_i | \tilde{\nu}_\alpha(t)\rangle;$$

is simple , Hamiltonian is hermitian:

$$H = \frac{1}{2E}\begin{pmatrix} 0 & 0 & 0 & 0 & .. \\ 0 & \delta m_{21}^2 & 0 & 0 & .. \\ 0 & 0 & \delta m_3^2 & 0 & .. \\ 0 & 0 & 0 & \delta m_{41}^2 & .. \\ .. & .. & .. & .. & .. \end{pmatrix} + \sqrt{2}G_F U^+ \begin{pmatrix} (N_e - \frac{N_n}{2}) & 0 & 0 & 0 & .. \\ 0 & -\frac{N_n}{2} & & 0 & .. \\ 0 & 0 & -\frac{N_n}{2} & 0 & .. \\ 0 & 0 & 0 & 0 & .. \\ .. & .. & .. & .. & .. \end{pmatrix} . U$$

The initial condition:

$$\langle \nu_k | \tilde{\nu}_\alpha(0)\rangle = \lambda_\alpha^{-1} U_{\alpha k}^*$$

Nonorthogonality of neutrino states has some impact on theoretically calculated amplitudes:

$$A(\nu_\alpha X \to l_\beta^- Y) = \lambda_\alpha^{-1} \sum_{i=1}^{3+n_s} U_{\alpha i}^* A(\nu_i X \to l_\beta^- Y) \cong \lambda_\alpha^{-1} \sum_{i=1}^{3+n_s} U_{\alpha i}^* U_{\beta i} A(\nu_{m=0} X \to l_\beta^- Y)$$

and cross sections:

$$\sigma(\nu_\alpha X \to l_\beta^- Y) = \lambda_\alpha^{-2} \left| \delta_{\alpha\beta} - c_{\alpha\beta} \right|^2 \sigma^{SM}(\nu_\beta X \to l_\beta^- Y)$$

Oscillation of light Neutrino

Oscillation of 3 light (no sterile) with at least one heavy will be considered:

For one heavy neutrino:

$$V = \begin{pmatrix} \varepsilon_e \\ e^{-i\chi_\mu} \varepsilon_\mu \\ e^{-i\chi_\tau} \varepsilon_\tau \end{pmatrix};$$

$$V' = \left(V'_{R1}, \quad V'_{R2}, \quad V'_{R3} \right)$$

$$V'_{Ri} = -U_{ei}\varepsilon_e - U_{\mu i} e^{i\chi_\mu} \varepsilon_\mu - U_{\tau i} e^{i\chi_\tau} \varepsilon_\tau$$

Then full neutrino interaction Hamiltonian in the eigenmass basis:

$$H^{int}_{ki} = \frac{1}{2E}(m_i^2 \delta_{ik} + \sqrt{2} G_F E [N_e U^*_{ek} U_{ei} - \frac{1}{2} N_n \Omega_{ki}]),$$

With the Ω matrix:

$$\Omega_{ik} = \delta_{ik} - V'^*_{Rk} V'_{Ri}, \quad i,k = 1, 2, 3.$$

For neutrino propagation in a uniform density medium equation of motion can be solved analytically.

Effective Hamiltonian can be diagonalized:

Effective neutrino mass in matter

$$H^{eff} = \frac{1}{2E}\tilde{W}^{+}\mathrm{diag}(\tilde{m}_i^2)\tilde{W};$$

and the propagation equation takes the form:

$$i\frac{d}{dt}\Psi^{\alpha}(t) = \frac{1}{2E}\tilde{W}^{+}\mathrm{diag}(\tilde{m}_i^2)\tilde{W}\Psi^{\alpha}(t);$$

where:

$$\Psi^{\alpha}(t) = \begin{pmatrix} \langle \nu_1 | \tilde{\nu}_{\alpha}(t) \rangle \\ \langle \nu_2 | \tilde{\nu}_{\alpha}(t) \rangle \\ \langle \nu_3 | \tilde{\nu}_{\alpha}(t) \rangle \end{pmatrix}$$

The equation of motion together with the initial condition give:

$$\Psi_k^{\alpha}(t) = \sum_i (\widetilde{W}^+)_{ki}\, e^{-i\frac{\widetilde{m}_i^2}{2E}} (\widetilde{W}\widetilde{U}^+)_{i\alpha}$$

and the amplitude for the $\nu_\alpha \longrightarrow \nu_\beta$ oscillation after travelling a distance L can be calculated:

$$A_{\alpha\rightarrow\beta}(L) = \left\langle \widetilde{\nu}_\beta \middle| \widetilde{\nu}_\alpha (L=t) \right\rangle = \sum_{i=1}^{3} \overline{W}_{\beta i}\, \overline{W}_{\alpha i}^{*}\, e^{-i\frac{\widetilde{m}_i^2}{2E}L}$$

Where the new tensors are defined by:

$$\overline{W}_{\alpha i} = (\widetilde{U}\widetilde{W}^+)_{\alpha i} = \lambda_\alpha^{-1} \sum_k U_{\alpha k} \widetilde{W}_{ik}^{*} \equiv \lambda_\alpha^{-1} W_{\alpha i}$$

Now we can calculate the transition probability:

$$P_{\alpha\to\beta}(L) = \left|A_{\alpha\to\beta}(L)\right|^2;$$

And we have:

$$P_{\alpha\to\beta}(L) = \frac{1}{\lambda_\alpha^2\lambda_\beta^2}\{(\delta_{\alpha\beta} - |c_{\alpha\beta}|)^2 - 4\sum_{i>k} R_{\alpha\beta}^{ik}\sin^2\Delta_{ik} + 8 I_{\alpha\beta}^{12}\sin\Delta_{21}\sin\Delta_{31}\sin\Delta_{32} + \\ + 2[A_{\alpha\beta}^{(1)}\sin 2\Delta_{31} + A_{\alpha\beta}^{(2)}\sin 2\Delta_{32}]\}$$

Where:

$$R_{\alpha\beta}^{ik} = \mathrm{Re}[W_{\alpha i} W_{\beta k} W_{\alpha k}^* W_{\beta i}^*]$$

$$I_{\alpha\beta}^{ik} = \mathrm{Im}[W_{\alpha i} W_{\beta k} W_{\alpha k}^* W_{\beta i}^*]$$

$$A_{\alpha\beta}^{(i)}(c_{\alpha\beta}) = I_{\alpha\beta}^{ik} = \mathrm{Im}[W_{\alpha i}^* W_{\beta k} c_{\alpha\beta}^*]$$

$$\Delta_{ik} = 1.267\frac{(\tilde{m}_i^2 - \tilde{m}_k^2)[eV^2]L[km]}{E[GeV]}$$

The transition probability for Dirac antineutrino or Majorana neutrino with λ=+1 can be obtained after the replacement:

$$P_{\bar{\alpha}\rightarrow\bar{\beta}}(L) = P_{\alpha\rightarrow\beta}(L; U \rightarrow U^{*}, G_F \rightarrow -G_F)$$

We can find now the CP and T asymmetries:

$$\Delta P^{CP}_{\alpha\rightarrow\beta} \equiv P_{\alpha\rightarrow\beta} - P_{\bar{\alpha}\rightarrow\bar{\beta}} =$$

$$-2\frac{1}{\lambda_\alpha^2\lambda_\beta^2}\{2\sum_{i>k}[R^{ik}_{\alpha\beta}(G_F)\sin^2\Delta_{ik}(G_F) - R^{ik}_{\alpha\beta}(-G_F)\sin^2\Delta_{ik}(-G_F)]$$

$$-4[I^{12}_{\alpha\beta}(G_F)\sin\Delta_{21}(G_F)\sin\Delta_{31}(G_F)\sin\Delta_{32}(G_F) +$$

$$I^{12}_{\alpha\beta}(-G_F)\sin\Delta_{21}(-G_F)\sin\Delta_{31}(-G_F)\sin\Delta_{32}(-G_F)] -$$

$$[A^{(1)}_{\alpha\beta}(G_F)\sin 2\Delta_{31}(G_F) + A^{(2)}_{\alpha\beta}(G_F)\sin 2\Delta_{32}(G_F) +$$

$$A^{(1)}_{\alpha\beta}(-G_F)\sin 2\Delta_{31}(-G_F) + A^{(2)}_{\alpha\beta}(-G_F)\sin 2\Delta_{32}(-G_F)]\}$$

$$\Delta P^{T}_{\alpha\to\beta} \equiv P_{\alpha\to\beta} - P_{\beta\to\alpha} = 4\frac{1}{\lambda_\alpha^2\lambda_\beta^2}[\,4I^{12}_{\alpha\beta}\sin\Delta_{21}\sin\Delta_{31}\sin\Delta_{32} + A^{(1)}_{\alpha\beta}\sin2\Delta_{31} + A^{(2)}_{\alpha\beta}\sin2\Delta_{32}\,]$$

Nonunitarity of mixing matrix produce two types of effects:

- R and I tensors depend on ϵ and χ parameters ,
- Additional new terms proportional to A appear.

Seen in numerical analysis

More spectacular, survive in vacuum

Non - Unitary Effects in Neutrino Oscillation

In vacuum $G_F \rightarrow 0$

$$\Delta P^{CP}_{\alpha\rightarrow\beta}(vacuum) = \Delta P^{T}_{\alpha\rightarrow\beta}(vacuum)$$

For $\alpha = \beta$ $$A^{(i)}_{\alpha\alpha} = 0$$

$$\Delta P^{T}_{\alpha\rightarrow\alpha}(\text{matter}) = \Delta P^{T}_{\alpha\rightarrow\alpha}(\text{vacuum}) = \Delta P^{CP}_{\alpha\rightarrow\alpha}(\text{vacuum}) = 0$$

Normal medium is matter (not antimatter)

$$\Delta P^{CP}_{\alpha\rightarrow\alpha}(matter) \neq 0$$

and heavy neutrinos make this effect stronger:

$$\Delta_{LBL} \approx \Delta_{31} \cong \Delta_{32} \cong 0(1), \qquad \Delta_{21} \cong 0$$, and

for unitary oscillation in vacuum, the CP and T asymmetries for LBL disappear. Contrary, in the non-unitary oscillation:

$$\Delta P^{T}_{\alpha\to\beta}(vacuum) = \Delta P^{CP}_{\alpha\to\beta}(vacuum) \neq 0$$

and

$$\Delta P^{T}_{\alpha\to\beta}(\mathrm{vacuum}) = 4\frac{1}{\lambda_\alpha^2\lambda_\beta^2}\left(A^{(1)}_{\alpha\beta} + A^{(2)}_{\alpha\beta}\right)\sin 2\Delta_{LBL} = - 4\frac{1}{\lambda_\alpha^2\lambda_\beta^2}\mathrm{Im}(W^*_{\alpha 3}W_{\beta 3}c^*_{\alpha\beta})\sin 2\Delta_{LBL}$$

CP and T asymmetries appear even for two flavour neutrino transitions.

For unitary 3 flavour transitions, moduli of all Jarlskog invariants are equal and all T and CP asymmetries in vacuum are equal, eg.

$$\Delta P^{T}_{e\to\mu} = \Delta P^{T}_{\mu\to\tau} = \Delta P^{T}_{\tau\to e}$$

In unitary case, if any elements of the mixing matrix is small (vanish) then above asymmetries are also small (vanish), eg.

$$\Delta P^{T}_{\alpha\to\beta} = \Delta P^{CP}_{\alpha\to\beta} \approx \sin^{2}(2\theta_{13})$$

In the non-unitary case the Jarlskog invariants are not equal, and:

$$I^{ik}_{e\mu} = I^{ik}_{\mu\tau} + \mathrm{Im}[W_{\mu i} W^{*}_{\mu k} (\tilde{W} V'^{T} V'^{*} \tilde{W}^{T})_{ik}] =$$
$$I^{ik}_{\tau e} - \mathrm{Im}[W_{ei} W^{*}_{ek} (\tilde{W} V'^{T} V'^{*} \tilde{W}^{T})_{ik}]$$

And

$$I^{12}_{\alpha\beta} = I^{23}_{\alpha\beta} - \mathrm{Im}[W^{*}_{\alpha 2} W_{\beta 2} c^{*}_{\alpha\beta}] = - I^{13}_{\alpha\beta} + \mathrm{Im}[W^{*}_{\alpha 1} W_{\beta 1} c^{*}_{\alpha\beta}]$$

The above relations and terms proportional to A tensors imply:

$$\Delta P^{T}_{e \to \mu} \neq \Delta P^{T}_{\mu \to \tau} \neq \Delta P^{T}_{\tau \to e}$$

We have calculated the differences ΔP and asymmetries defined by:

$$A^{CP}_{\mu\to\tau} = \frac{\Delta P^{CP}_{\mu\to\tau}}{P_{\mu\to\tau} + P_{\bar{\mu}\to\bar{\tau}}} \qquad A^{T}_{\mu\to\tau} = \frac{\Delta P^{T}_{\mu\to\tau}}{P_{\mu\to\tau} + P_{\tau\to\mu}}$$

P
A
R
A
M
E
T
E
R
S

- Standard, best fit values for mass square differences and mixing angles (we do not consider error for them),
- L= 295 km and L = 732 km,
- Energy E = 0.1 – 30 GeV,
- madium density ρ = const = 2.6 g/cm^3,
- Y_e= 0.494,
- The ϵ parameters:

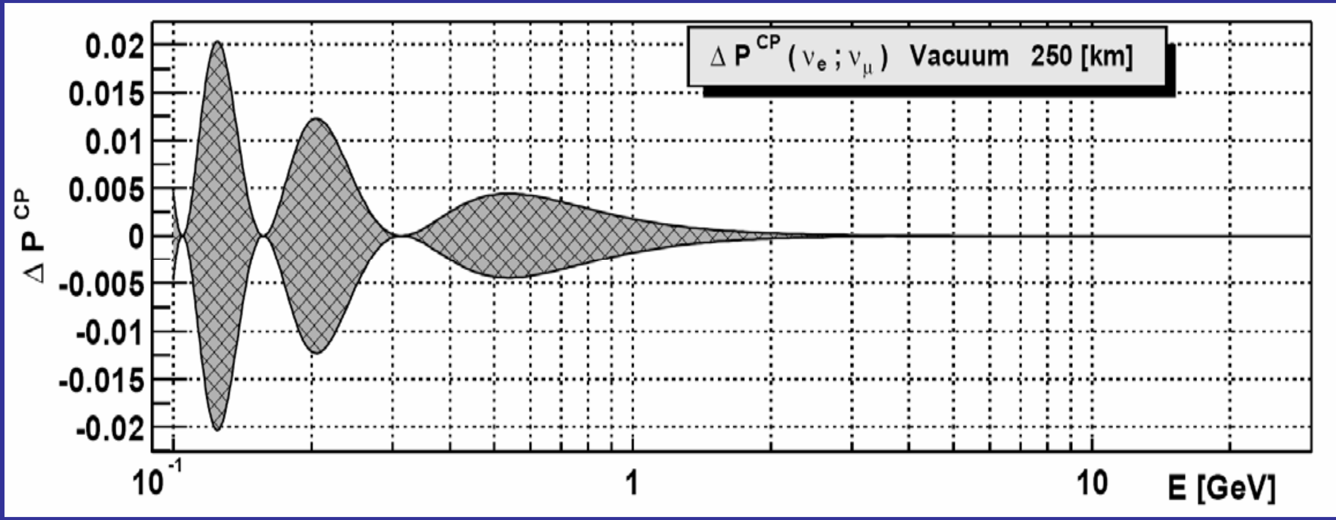
A) ϵ_e ~0.001, $\epsilon_\mu = \epsilon_\tau$ ~ 0.1;

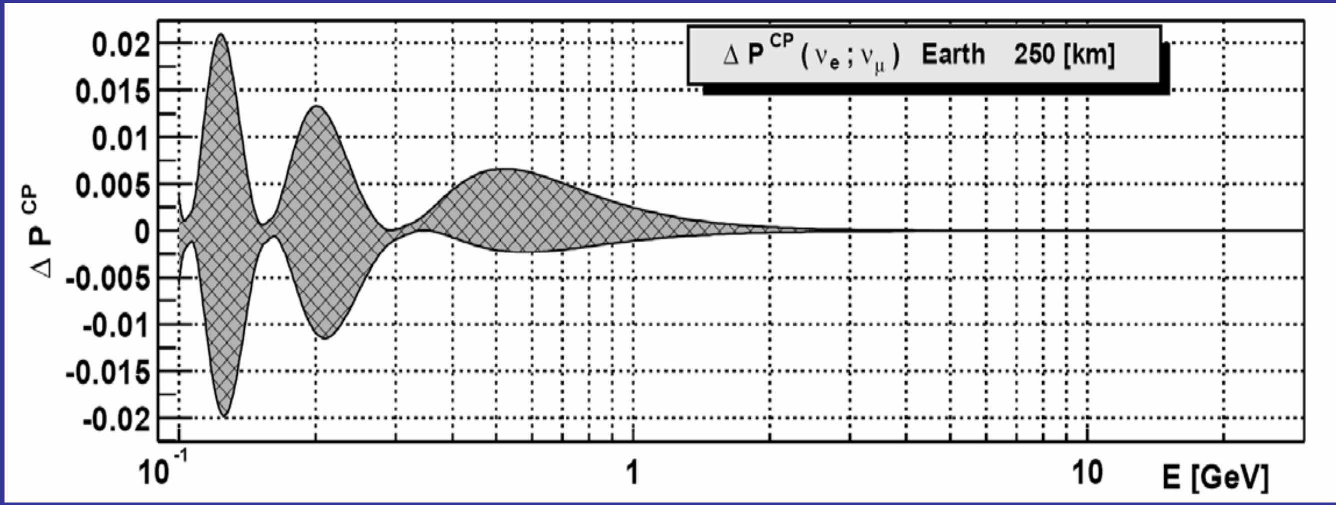
B) $\epsilon_e = \epsilon_\mu$ ~ 0.01, ϵ_τ ~ 0.1

$$A_e[eV^2] = 2\sqrt{2}G_F N_e E = 7.63x10^{-5}[\frac{\rho}{g/cm^3}][\frac{Y_e}{0.5}][\frac{E}{GeV}]$$

$$A_n[eV^2] = 2\sqrt{2}G_F N_n E = 7.63x10^{-5}[\frac{\rho}{g/cm^3}][1-Y_e][\frac{E}{GeV}]$$

Marek Zralek
Univ. of Silesia





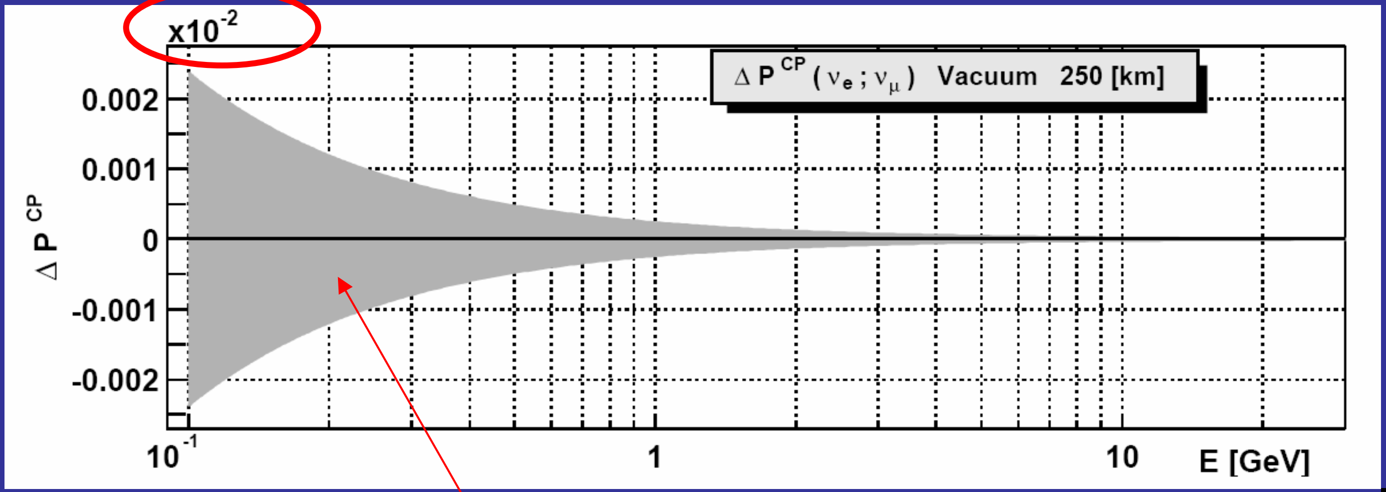
Hatched region

$0 < \delta_{CP} < 2\pi$

Shaded region

$0 < \chi_i < 2\pi$

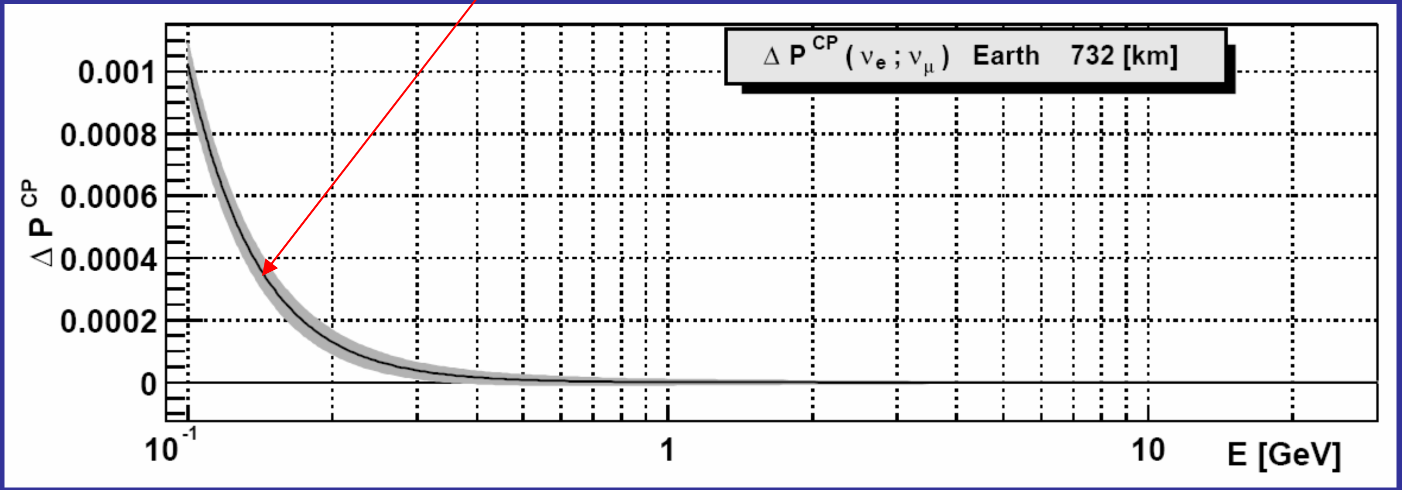
$\tan^2(2\theta_{13}) = 0.005$



$\Theta_{13} = 0$

Shaded region - effect of nonunitarity

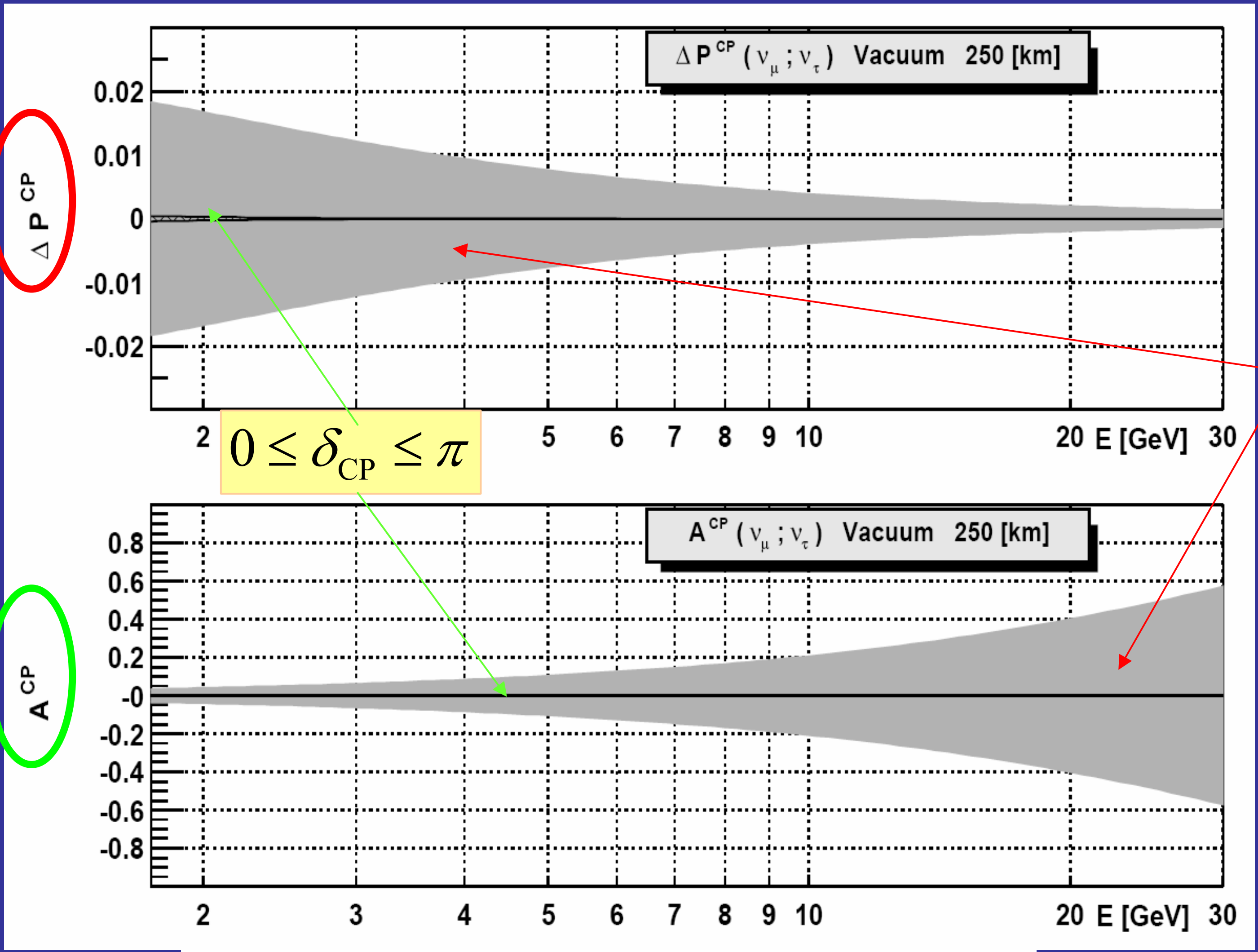
Shaded region:
$0 < \chi_i < 2\pi$



Earth effect: ($\Delta P \neq 0$) even for unitary neutrino oscillation

A) ϵ_e ~0.001,

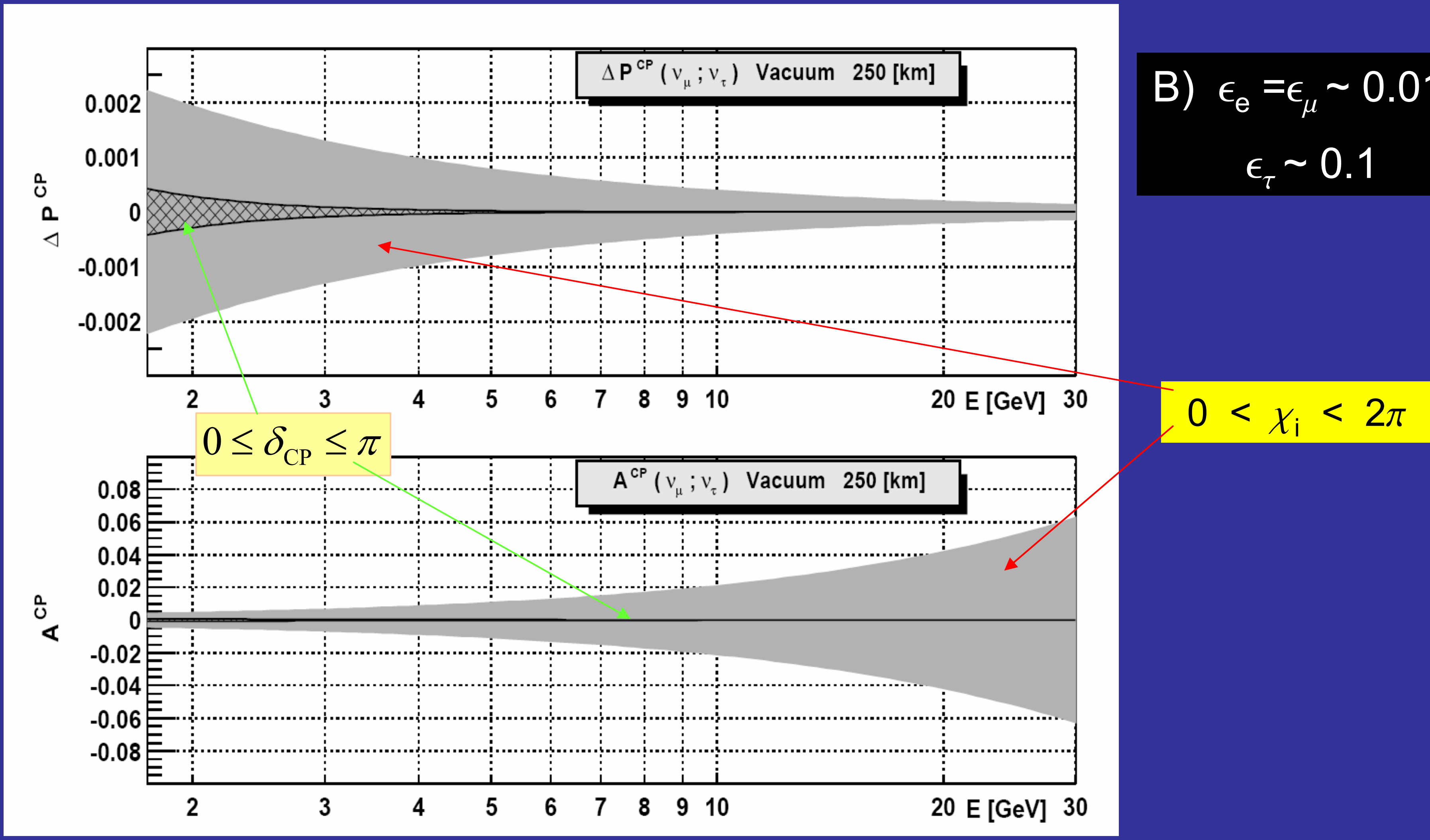
$\epsilon_\mu = \epsilon_\tau$ ~ 0.1;



$0 \le \delta_{CP} \le \pi$

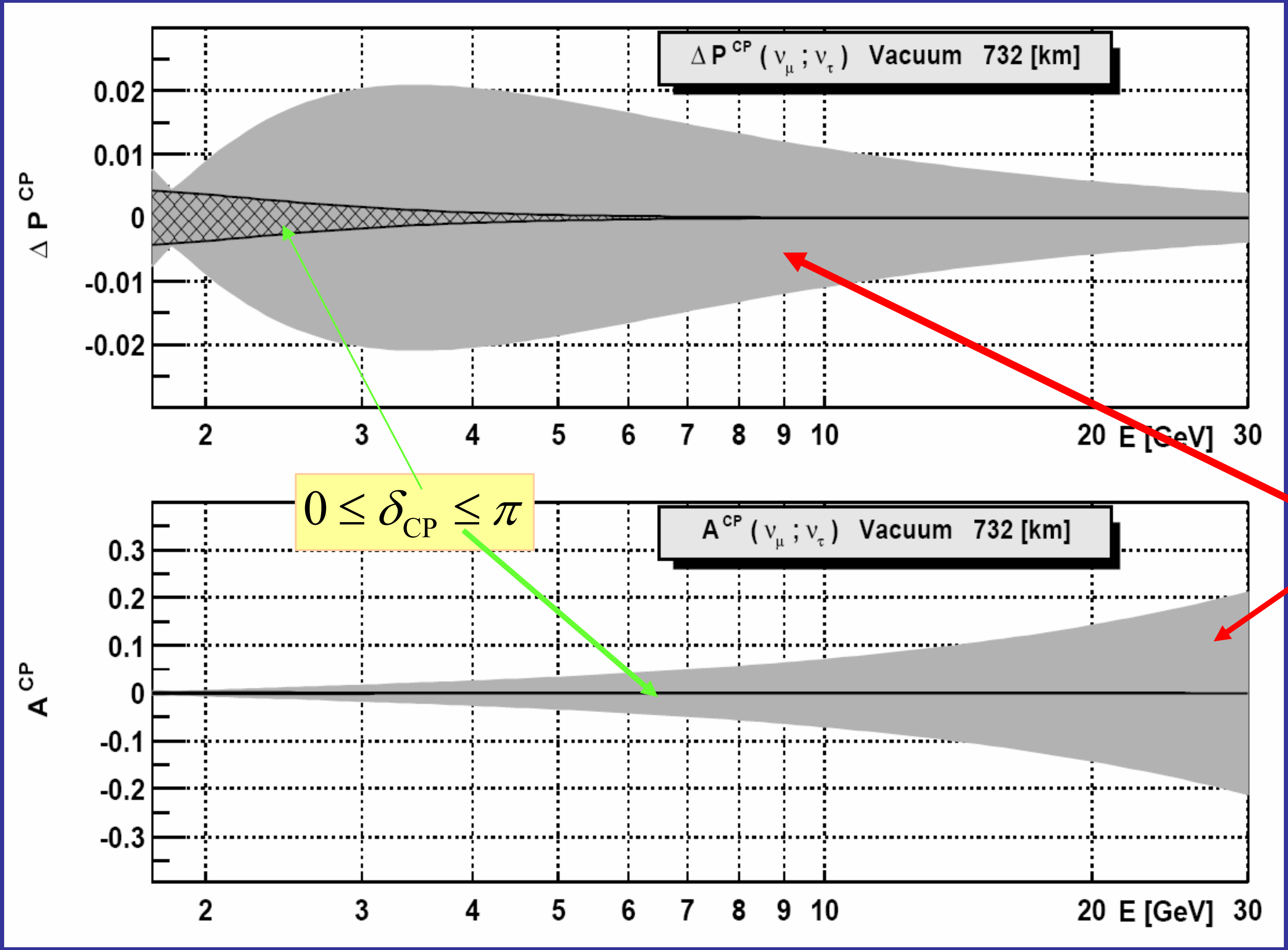
$0 < \chi_i < 2\pi$

$\mu \rightarrow \tau$ transition in vacuum for 250 km



B) $\epsilon_e = \epsilon_\mu \sim 0.01$, $\epsilon_\tau \sim 0.1$

$\mu \rightarrow \tau$ transition in vacuum for 250 km

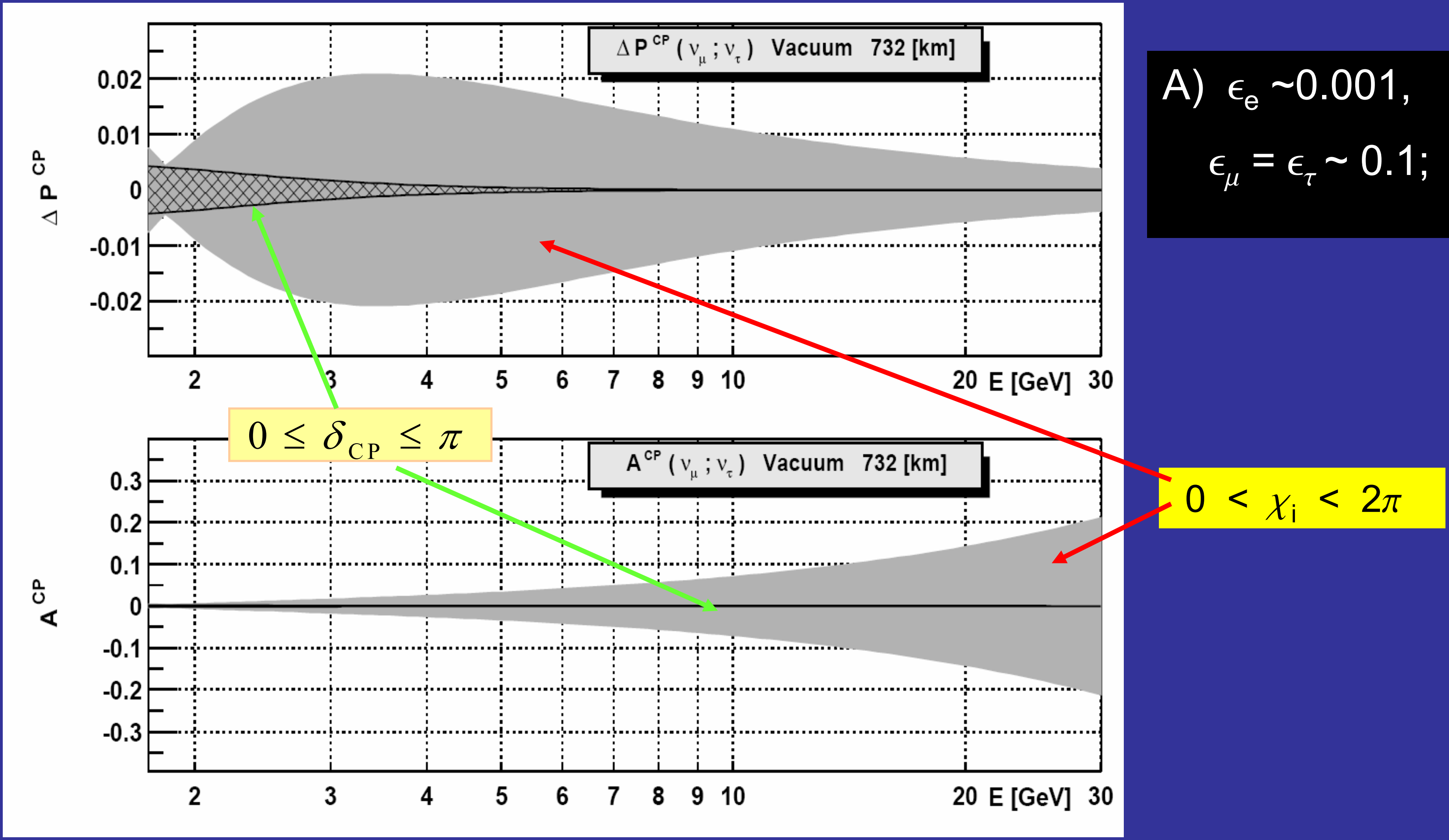


A) $\epsilon_e \sim 0.001$,

$\epsilon_\mu = \epsilon_\tau \sim 0.1$;

$0 < \chi_i < 2\pi$

$\mu \longrightarrow \tau$ transition in vacuum for 732 km



A) $\epsilon_e \sim 0.001$,

$\epsilon_\mu = \epsilon_\tau \sim 0.1$;

$\mu \longrightarrow \tau$ transition in vacuum for 732 km

Resonance phenomena,

For unitary neutrino oscillations, the effective mixing angle sometimes has resonance behaviour:

$$\sin 2\theta_{\text{eff}} = \frac{\sin 2\theta_{12}}{\sqrt{\left(\frac{2\sqrt{2}G_F|\vec{k}|N_2}{\Delta m^2_{21}} - \cos 2\theta_{12}\right)^2 + \sin^2 2\theta_{12}}}$$

And we see, that for:

$$\frac{2\sqrt{2}G_F|\vec{k}|N_e}{\Delta m^2_{21}} \to \cos 2\theta_{12};$$

Θ_{12} mixing angle bacomes maximal:

$$\sin 2\theta_{\text{eff}} \to 1$$

We have checked the resonance behaviour in the case of two non-unitary mixing neutrinos

F. del Aguila & M.Z.
Acta Phys. Pol. B33(2002)2585

The effective mixing angle in this case is given by:

$$\sin 2\theta_{\text{eff}} = c_{13}^2 \frac{A}{\sqrt{(B - \cos 2\theta_{12})^2 + A^2}}$$

with

$$A^2 = \left[\sin 2\theta_{12} + \frac{\sqrt{2} G_F |\vec{k}| N_n}{\Delta m_{21}^2} s_{13} \sin 2\theta_{23} \cos\delta\right]^2 + \left(\frac{\sqrt{2} G_F |\vec{k}| N_n}{\Delta m_{21}^2}\right)^2 s_{13}^2 \sin^2 2\theta_{23} \sin^2\delta \ ,$$

$$B = \frac{\sqrt{2} G_F |\vec{k}| (2N_e - N_n)}{\Delta m_{21}^2} c_{13}^2 + \frac{\sqrt{2} G_F |\vec{k}| N_n}{\Delta m_{21}^2} (c_{23}^2 - s_{13}^2 s_{23}^2)$$

There are two kinds of new effects

The resonance has a new position:

$$\frac{2\sqrt{2}G_F\left|\vec{k}\right|}{\Delta m_{21}^2}\left(1-\frac{\varepsilon^2}{2}\right) \rightarrow \cos 2\theta_{12}$$

The effective mixing is not maximal:

$$\sin 2\theta_{eff} \quad \rightarrow \quad c_{13}^2 = 1 - \varepsilon^2 \neq 1$$

4. Effects of Right – Handed Currents.

Let us assume that the effective Hamiltonian which descsibes the coherent neutrino – background matter particle scattering has the following form

$$\mathcal{H}^{int}(x) = \sum_a \sum_{j,i} \Big((z_a)_{ij} \; (\bar{\nu}_i \Gamma^a \nu_j) + (z_a^*)_{ij} \; (\bar{\nu}_j \bar{\Gamma}^a \nu_i) \Big) \;,$$

$$\Gamma^a = I,\; \gamma_5,\; \gamma^\mu,\; \gamma^\mu \gamma_5 \text{ or } \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

Neutrino transition n → m matrix element:

$$\mathcal{M}_{n\to m}(x) = \langle \nu_m | \mathcal{H}^{int} | \nu_n \rangle = \sum_a \sum_{j,i} < 0 | A_m ((z_a)_{ij} \, \bar{\nu}_i \Gamma^a \nu_j + (z_a^*)_{ij} \, \bar{\nu}_j \bar{\Gamma}^a \nu_i) A_n^\dagger | 0 > ,$$

$$\mathcal{M}_{n\to m} = \begin{cases} \mathcal{M}^D_{n\to m} = \sum_a \{(z_a)_{mn}\ \bar{u}_m \Gamma^a u_n + (z_a^*)_{nm}\ \bar{u}_m \overline{\Gamma}^a u_n\} & \text{for Dirac neutrinos,} \\ \mathcal{M}^D_{\bar{n}\to\bar{m}} = -\sum_a \{(z_a^*)_{mn}\ \bar{v}_n \bar{\Gamma}^a v_m + (z_a)_{nm}\ \bar{v}_m \Gamma^a v_n\} & \text{for Dirac antineutrinos,} \\ \mathcal{M}^M_{m\to n} = \quad \mathcal{M}^D_{n\to m} + \mathcal{M}^D_{\bar{n}\to\bar{m}} & \text{for Majorana neutrinos.} \end{cases}$$

Or more explicitely

$$\mathcal{M}^D_{n\to m} = Z_S\, \bar{u}_m u_n + Z_P\, \bar{u}_m \gamma_5 u_n + Z_{V\mu}\, \bar{u}_m \gamma^\mu u_n + Z_{A\mu}\, \bar{u}_m \gamma^\mu \gamma_5 u_n + Z_{T\mu\nu}\ \bar{u}_m \sigma^{\mu\nu} u_n$$

$$\mathcal{M}^D_{\bar{n}\to\bar{m}} = Z^*_S\, \bar{u}_m u_n - Z^*_P\, \bar{u}_m \gamma_5 u_n - Z^*_{V\mu}\, \bar{u}_m \gamma^\mu u_n + Z^*_{A\mu}\, \bar{u}_m \gamma^\mu \gamma_5 u_n - Z^*_{T\mu\nu}\ \bar{u}_m \sigma^{\mu\nu} u_n$$

$$\begin{aligned} \mathcal{M}^M_{n\to m} \quad &= \quad (2\, ReZ_S)\, \bar{u}_m u_n + (2\, i\, ImZ_P)\, \bar{u}_m \gamma_5 u_n + \\ &+ \quad (2\, i\, ImZ_{V\mu})\, \bar{u}_m \gamma^\mu u_n + (2\, ReZ_{A\mu})\, \bar{u}_m \gamma^\mu \gamma_5 u_n + (2\, i\, ImZ_{T\mu\nu})\ \bar{u}_m \sigma^{\mu\nu} u_n \end{aligned}$$

($Z = z + z^+$ for hermitian: S,V,A,T) ($Z = z - z^+$ for antyhermitian: P)

For relativistic neutrinos

$$u_\lambda(k_\nu) = \begin{pmatrix} \sqrt{\frac{E_\nu + \lambda|\vec{k}_\nu|}{2E_\nu}} \\ \sqrt{\frac{E_\nu - \lambda|\vec{k}_\nu|}{2E_\nu}} \end{pmatrix} \chi_\lambda$$

$$\vec{p}_n \approx \vec{p}_m =: \vec{k}_\nu \text{ and } p_n^0 \approx p_m^0 =: E_\nu$$

$$\bar{u}_m u_n \approx 0 \,,$$

$$\bar{u}_m \gamma^5 u_n \approx 0 \,,$$

$$\bar{u}_m \gamma_\mu u_n \approx \kappa_\lambda^\mu \,,$$

$$\bar{u}_m \gamma^\mu \gamma_5 u_n \approx \lambda \, \kappa_\lambda^\mu \,,$$

$$\bar{u}_m \sigma^{\mu\nu} u_n \approx 0 \,,$$

$$\kappa_\lambda^\mu = (1, \lambda \frac{\vec{k}_\nu}{|\vec{k}_\nu|}) \quad \text{with} \quad \lambda = -1 \quad \text{or} \quad +1$$

$$\mathcal{M}^D_{n\to m}(\lambda) \quad = \quad (Z_{V\mu} + \lambda Z_{A\mu})_{mn} \;\; \kappa_\lambda^\mu$$

$$\mathcal{M}^D_{\bar{n}\to \bar{m}}(\lambda) \quad = \quad (-Z^*_{V\mu} + \lambda Z^*_{A\mu})_{mn} \;\; \kappa_\lambda^\mu$$

$$\mathcal{M}^M_{n\to m}(\lambda) \quad = \quad 2\,(\lambda Re Z_{A\mu} + i\, Im Z_{V\mu})_{mn} \;\; \kappa_\lambda^\mu$$

For V-A

$$(Z_{V\mu}) = -(Z_{A\mu})$$

$$\mathcal{M}^{D}_{n\to m}(\lambda=-1) = \mathcal{M}^{M}_{n\to m}(\lambda=-1) = 2(Z_{V\mu})_{mn}\,\kappa^{\mu}_{-}$$

$$\mathcal{M}^{D}_{\bar{n}\to\bar{m}}(\lambda=+1) = \mathcal{M}^{M}_{n\to m}(\lambda=+1) = -2(Z^{*}_{Z\mu})_{mn}\,\kappa^{\mu}_{+}$$

If both V – A and V+A exist

$$Z_{V\mu} \neq -Z_{A\mu}$$

$$\mathcal{M}^{D}_{j\to k}(\lambda=-1) - \mathcal{M}^{M}_{n\to m}(\lambda=-1) = \left(Z^{*}_{V\mu} + Z^{*}_{A\mu}\right)_{mn}\,\kappa^{\mu}_{-}$$

$$\mathcal{M}^{D}_{\bar{j}\to\bar{k}}(\lambda=+1) - \mathcal{M}^{M}_{n\to m}(\lambda=+1) \;=\; -\left(Z_{V\mu} + Z_{A\mu}\right)_{mn}\,\kappa^{\mu}_{+}$$

Let us assume the general interaction Lagrangian in the form:

$$\mathcal{L}_C = \frac{e}{2\sqrt{2}\sin\theta_W}\sum_i \bar{\nu}_i\gamma^\mu\left[\varepsilon_L^C(1-\gamma_5)+\varepsilon_R^C(1+\gamma_5)\right] U_{\alpha i}^*\, e_\alpha\, W_\mu^+ + h.c.$$

$$\begin{aligned}\mathcal{L}_N &= \frac{e}{4\sin\theta_W\cos\theta_W}\{\sum_{i,j}\bar{\nu}_i\gamma^\mu\left[\varepsilon_L^{N\nu}(1-\gamma_5)\delta_{ij}+\varepsilon_R^{N\nu}(1+\gamma_5)\Omega_{ij}\right]\nu_j + \\ &+ \sum_\alpha \bar{f}_\alpha\gamma^\mu\left[\varepsilon_L^{Nf}(1-\gamma_5)+\varepsilon_R^{Nf}(1+\gamma_5)\right] f_\alpha\}Z_\mu\ ,\end{aligned}$$

SM is reconstructed for: $\varepsilon_{\mathrm{L}} = 1,\quad \varepsilon_{\mathrm{R}} = 0,\quad U_{\alpha i} = \delta_{\alpha i}$

So we assume: $\varepsilon_L \approx 1 \text{ and } \varepsilon_R \ll 1$

$$\varepsilon_L^{Nf} = 2T_{3f} - q_f sin^2\Theta_W + \delta_L^f \qquad \varepsilon_R^{Nf} = -q_f sin^2\Theta_W + \delta_R^f$$

We can calculate effective Hamiltonian which describe neutrino propagation in matter (uncharged, unpolarized, constant density) :

$$\mathsf{H}^{eff}_{ij} = (k + \frac{m_i^2}{2k})\ \delta_{ij} + \mathsf{H}_{ij}$$

$$\mathsf{H}^{D}_{\alpha\beta}(\lambda = -1) = \sqrt{2} G_F N_e\ \delta_{e\alpha}\ \delta_{e\beta}$$

$$\mathsf{H}^{M}_{\alpha\beta}(\lambda = -1) = \sqrt{2} G_F \{N_e\ \delta_{e\alpha}\ \delta_{e\beta} - N_e[|\varepsilon^{Ce}_R|^2 \delta_{e\alpha}\ \delta_{e\beta} + \frac{1}{2}(\varepsilon^{N\nu}_R)^*(\delta^{e*}_L + \delta^{e*}_R + 2\delta^{u*}_L + 2\delta^{u*}_R + \delta^{d*}_L + \delta^{d*}_R)\Omega^{R*}_{\alpha\beta}] - N_n \frac{1}{2}(\varepsilon^{N\nu}_R)^*(-1 + 2\delta^{d*}_L + 2\delta^{d*}_R + \delta^{u*}_L + \delta^{u*}_R)\Omega^{R*}_{\alpha\beta}\}\ ,$$

$$\Omega^{R}_{\alpha\beta} = \sum_{i,j} U_{\alpha i}\ \Omega^{R}_{ij}\ U^*_{\beta j}$$

$$\mathsf{H}^{M}_{\alpha\beta}(\lambda = -1) = \sqrt{2} G_F N_e \{\ \delta_{e\alpha}\ \delta_{e\beta} + \frac{N_n}{N_e}\frac{1}{2}(\varepsilon^{N\nu}_R)^*\Omega^{R*}_{\alpha\beta}\}$$

$$\Omega^R = \begin{pmatrix} 1 & \eta & \eta^2 \\ \eta & 1-\chi & \eta \\ \eta^2 & \eta & 1-\omega \end{pmatrix}$$

$$\Delta P = P^D - P^M \qquad \eta \to 0 \;\Rightarrow\; \Delta P \to 0$$

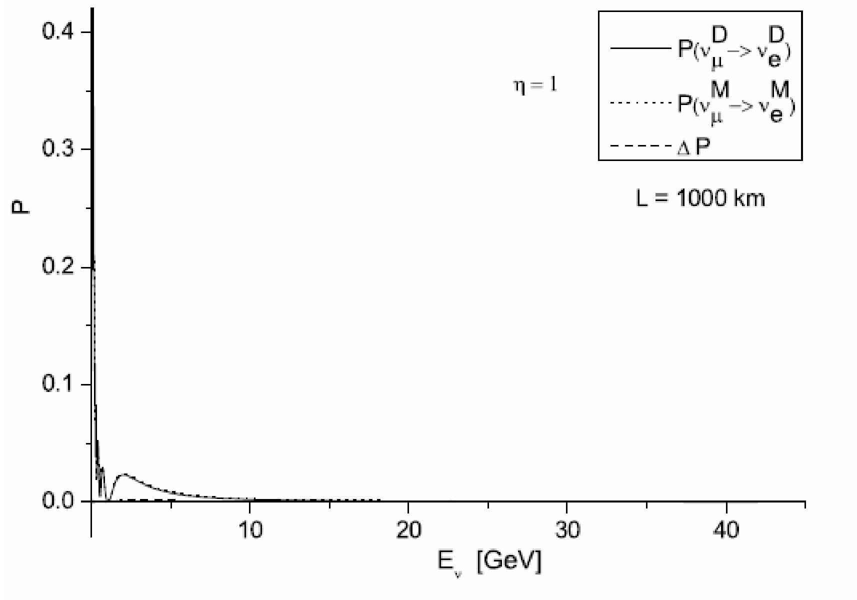
In the example which we consider:

Present experimental values for θ_{ij} and δm^2_{ij} and their errors;

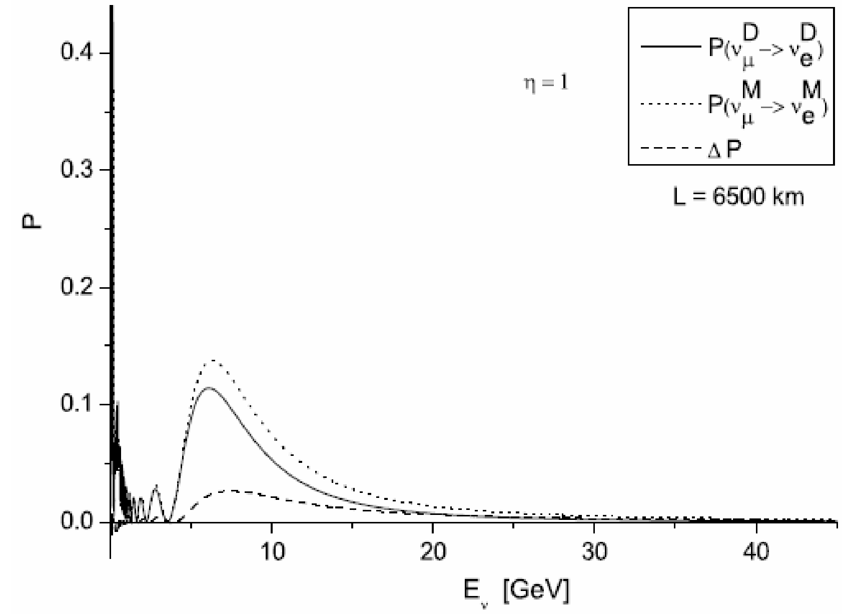
Future experimental errors ---1%

$$\varepsilon_R^{N\nu} \approx 10^{-2}, \quad \eta \le 1, \quad 0 \le \chi, \omega \le 1$$

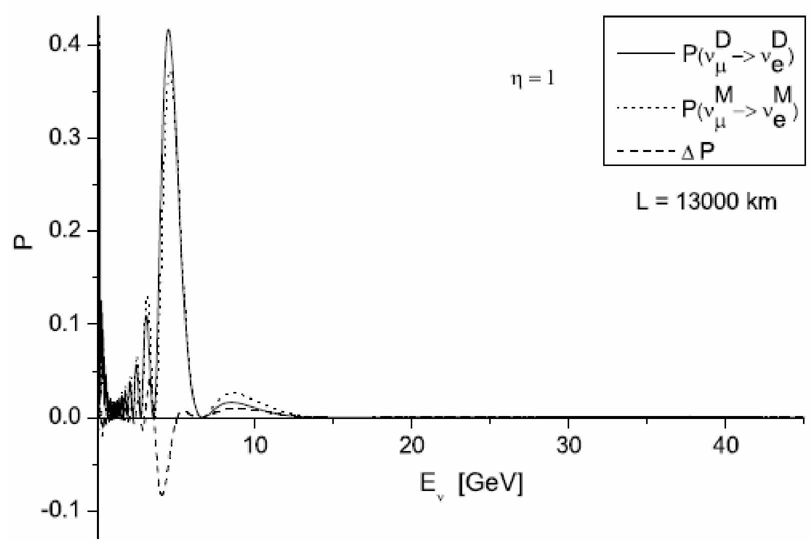
$$\rho[g/cm^3] = \langle \rho \rangle = \left\langle \begin{array}{l} 3 \text{ for } L = 1000 \text{ km} \\ 5 \text{ for } L = 6500 \text{ km} \\ 7 \text{ for } L = 13000 \text{ km} \end{array} \right.$$



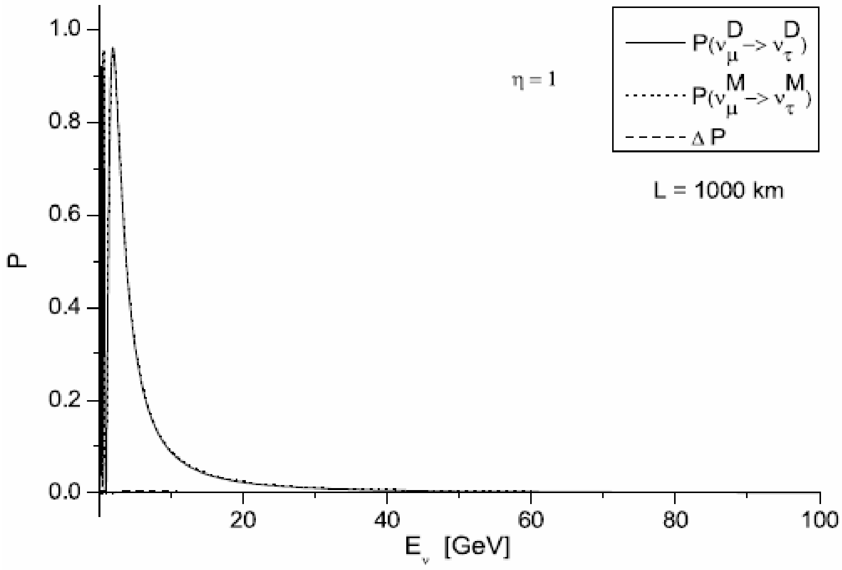
$P(\nu_\mu^D -> \nu_e^D)$
$P(\nu_\mu^M -> \nu_e^M)$
ΔP
$\eta = 1$
L = 1000 km
P
E_ν [GeV]



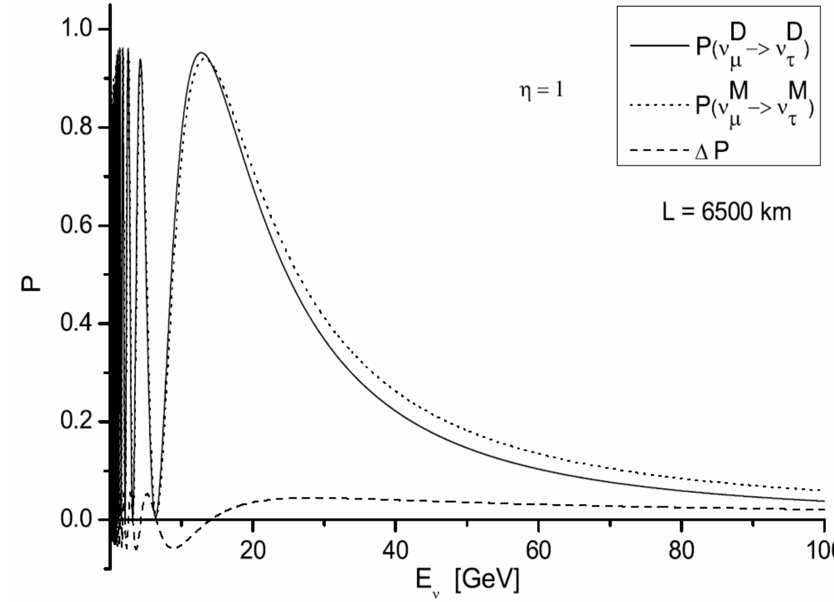
$P(\nu_\mu^D -> \nu_e^D)$
$P(\nu_\mu^M -> \nu_e^M)$
ΔP
$\eta = 1$
L = 6500 km
P
E_ν [GeV]



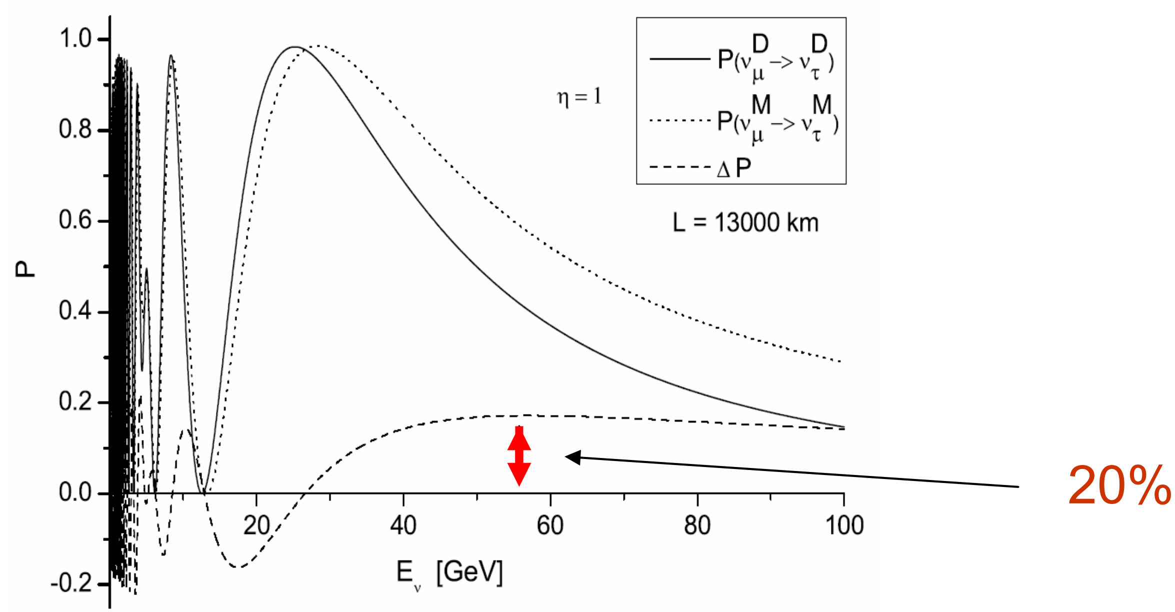
$P(\nu_\mu^D -> \nu_e^D)$
$P(\nu_\mu^M -> \nu_e^M)$
ΔP
$\eta = 1$
L = 13000 km
P
E_ν [GeV]



1.0
0.8
0.6
0.4
0.2
0.0
P
20
40
60
80
100
E ν [GeV]
η = 1
P(ν μ D -> ν τ D)
P(ν μ M -> ν τ M)
Δ P
L = 1000 km



1.0
0.8
0.6
0.4
0.2
0.0
P
20
40
60
80
100
E ν [GeV]
η = 1
P(ν μ D -> ν τ D)
P(ν μ M -> ν τ M)
Δ P
L = 6500 km



1.0
0.8
0.6
0.4
0.2
0.0
-0.2
P
20
40
60
80
100
E ν [GeV]
η = 1
P(ν μ D -> ν τ D)
P(ν μ M -> ν τ M)
Δ P
L = 13000 km
20%

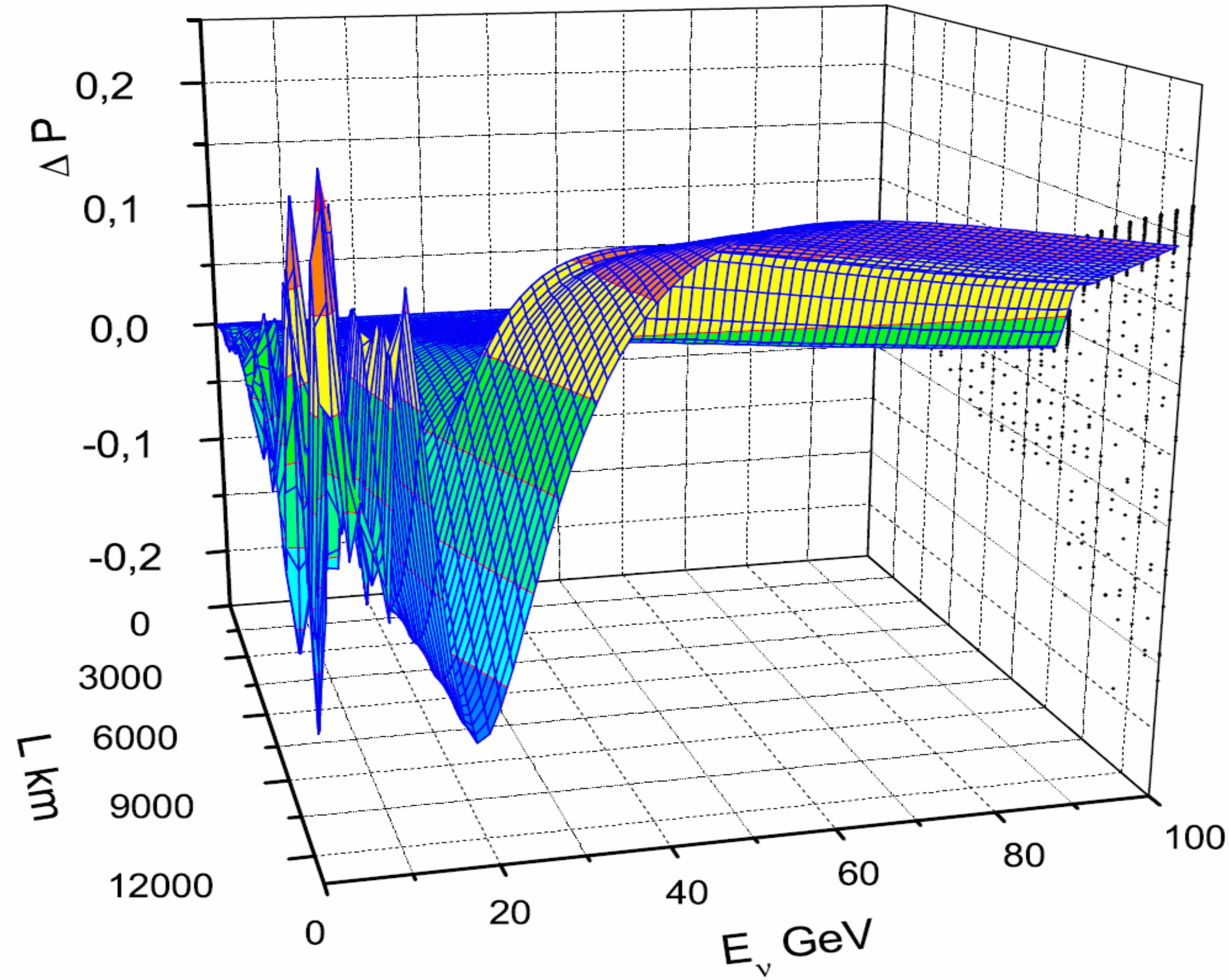


Figure 3: *The transition probabilities difference* ΔP *as the function of the neutrino energy* E_ν *in GeV and* L *in km, for the process* $\nu_\mu \to \nu_\tau$, $\eta = 1$. *The calculations were performed for the matter densities equal to 3, 4 and 7* $[g/cm^3]$ *for the travelling distances* L *between* $0 - 4500$, $5000 - 9000$ *and* $9500 - 13000$ *km, respectively. The projections on the* L-ΔP *plane were marked by the set of the black points.*

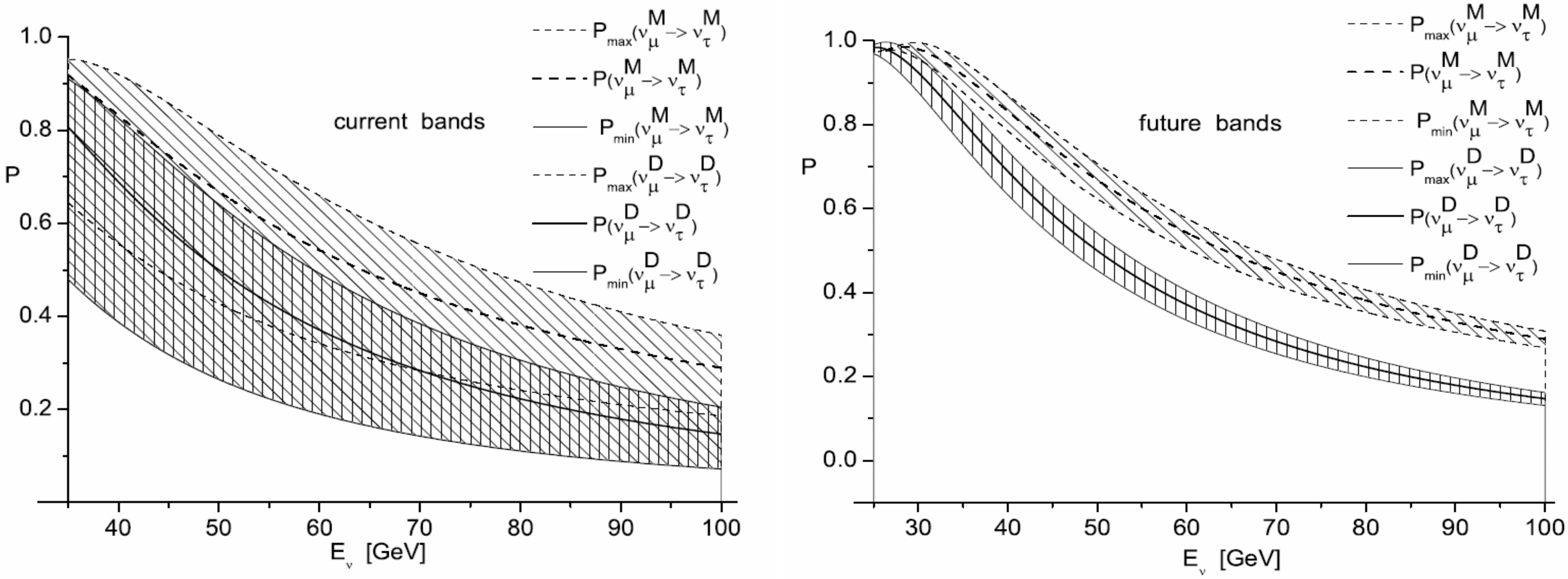


Figure 4: **Panel**: *(a) Upper and lower bands for the transition probabilities for the Dirac and the Majorana neutrino cases for current 95% confidence intervals of the oscillation parameters [6] as the function of the neutrino energy E_ν in GeV for $L = 13000$ km for the process $\nu_\mu \to \nu_\tau$, $\eta = 1$. (b) Upper and lower bands for the transition probabilities for the Dirac and the Majorana neutrino cases for future 95% confidence intervals of the oscillation parameters as the function of the neutrino energy E_ν in GeV for $L = 13000$ km for the process $\nu_\mu \to \nu_\tau$, $\eta = 1$;*

5. Summary

1. In all channels without electron neutrino the New Physics effects are not suppressed by powers of α and $\sin 2\theta_{13}$,

2. NP CP violation effects are suppressed by powers of α and $\sin 2\theta_{13}$,

3. In the case of non-unitary mixing, CP is violated even with no mixing between first and third families ($\theta_{13} \longrightarrow 0$),

4. Other effect, like the sum of probabilities not adding to 1 or modified resonance effects will be difficult to discriminate,

5. The NP additional V+A neutrino interaction potentially gives chance to distinguish between Dirac and Majorana neutrinos