

**NEUTRINOS AS A PROBE
OF CP VIOLATION AND LEPTOGENESIS**

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1 – Outline

- **Leptonic Dirac and Majorana CP-violation**: theoretical aspects
- **Determining the leptonic CPV** phases in future experiments:
 - 1) long base-line neutrino oscillation experiments
 - 2) neutrinoless double beta decay experiments
- The see-saw mechanism and **leptogenesis**
- Possible **connection** between Low energy and High energy (leptogenesis) CP-violation
- Conclusions

2 – Dirac and Majorana CPV phases

In the case of 3 neutrino mixing, the unitary lepton mixing matrix can be parametrized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i(\frac{\alpha_{31}}{2} + \delta)} \end{pmatrix}$$

- one universal CPV phase: δ .

It enters both $\Delta L = 0$ and $\Delta L = 2$ processes.

- two Majorana CPV phases α_{21} and α_{31} . They are physical only if neutrinos are Majorana particles.

If CP is conserved we have $\alpha_{21}, \alpha_{31} = 0$ (equal CP-parities) or $\alpha_{21}, \alpha_{31} = \pm\pi$ (opposite CP-parities).

It is possible to define **rephasing invariants** associated with CP-V phases.

- J_{CP} is related to the δ phase.

$$\begin{aligned} J_{CP} &= \text{Im}(U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*) \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \end{aligned}$$

[C. Jarlskog '85; P.I. Krastev and S.T. Petcov '88]

It controls the magnitude of CP and T violating effects in ν -oscillations.

- S_1 and S_2 are linked to the two Majorana CP-violating phases.

$$S_1 = \text{Im}(U_{e1}U_{e3}^*) \quad \text{and} \quad S_2 = \text{Im}(U_{e2}U_{e3}^*)$$

[see e.g., J.F. Nieves and P.B. Pal; J.A. Aguilar-Saavedra and G.C. Branco]

They enter in neutrinoless double beta decay [S.M. Bilenky, S.T. Petcov and S.P.].

3 – Measuring CP-V phases

The δ phase

The Dirac phase δ can be measured in ν -oscillation experiments.

A measure of CP- violating effects is provided by the CP-asymmetry (in vacuum):

$$A_{CP} = \frac{P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})}{P(\nu_l \rightarrow \nu_{l'}) + P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})}$$
$$\propto J_{CP} \propto \sin \delta$$

CP- (T-) violating effects in neutrino oscillations are controlled by $\sin \theta_{13}$.

The CP-asymmetry will be searched for in future **long base-line** experiments, looking for $\nu_\mu \rightarrow \nu_e (\bar{\nu}_\mu \rightarrow \bar{\nu}_e), (\nu_e \rightarrow \nu_{\mu,\tau} (\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}))$.

These oscillations take place in matter (Earth).

The Earth matter (e^- , p and n) is not charge-symmetric \Rightarrow **Matter effects** violate CP ($A \equiv \sqrt{2}G_F\bar{n}_e$).

The probability can be approximated as (for $\Delta m_{12}^2 \sim 0$):

$$P_{e\mu} = \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta_{13}^m L}{2}$$

The mixing angle changes with respect to the vacuum case:

$$\sin 2\theta_m = \frac{(\Delta m^2/2E) \sin 2\theta}{\sqrt{\left(\frac{\Delta m^2}{2E} \sin 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E} \cos 2\theta - A\right)^2}}$$

For $\Delta m^2 > 0$, mixing gets **enhanced** for neutrinos and suppressed for antineutrinos. For $\Delta m^2 < 0$, the opposite happens.

Matter effects imply that

$$P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

If U is complex ($\delta \neq 0, \pi$), we have CP-violation:

$$P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

**It is necessary to disentangle
true CP-V effects due to the δ phase
from the ones induced by matter.**

There are degenerate solutions:

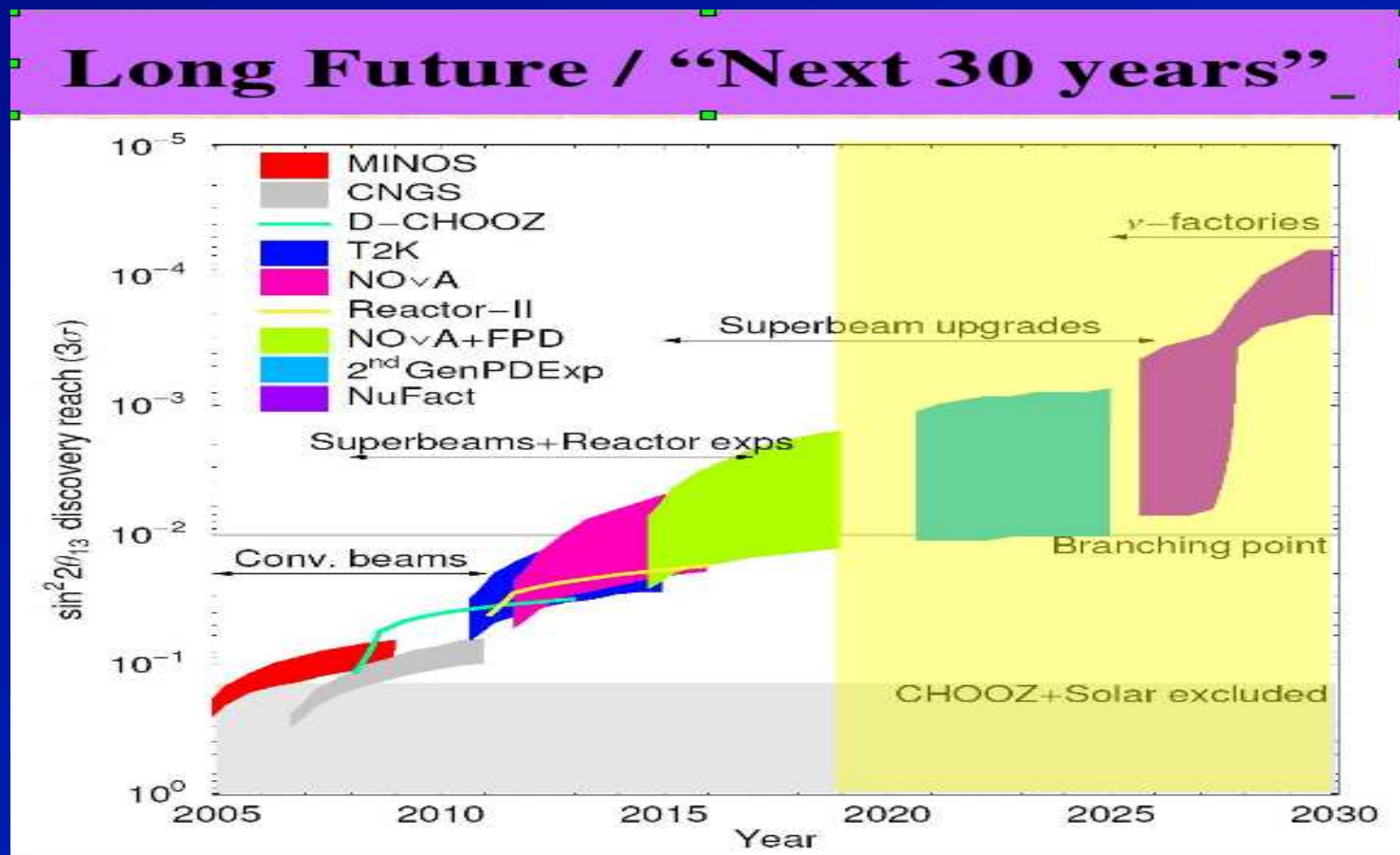
$$\begin{array}{l} \Delta m_{31}^2, \theta_{13}, \delta, \theta_{23} \\ \Delta m_{31}^2, \theta'_{13}, \delta', \theta'_{23} \end{array} \Rightarrow P, \bar{P}$$

Degeneracies worsen the sensitivity of future experiments to θ_{13} , the type of hierarchy and CP-violation. It is crucial to resolve them.

Many approaches have been suggested. For example: combination of different channels, gold ($\nu_{\mu} \rightarrow \nu_e$) + silver channels ($\nu_e \rightarrow \nu_{\tau}$) [Burguet-Castell et al.]; use different energies and/or baselines: NUMI + T2K [Minakata, Nunokawa, Parke; Mena, Parke]; use energy spectrum [Rubbia]; reactors neutrinos + LBL exp. [Minakata, Sugiyama, Yasuda]; atmospheric neutrinos + LBL [Huber, Schwetz, Winter]; use two detectors in the neutrino mode only in NO ν A [Mena, Palomares-Ruiz, SP]; use two detectors in T2K [Hagiwara, Okamura, Senda; Ishitsuka et al.]...

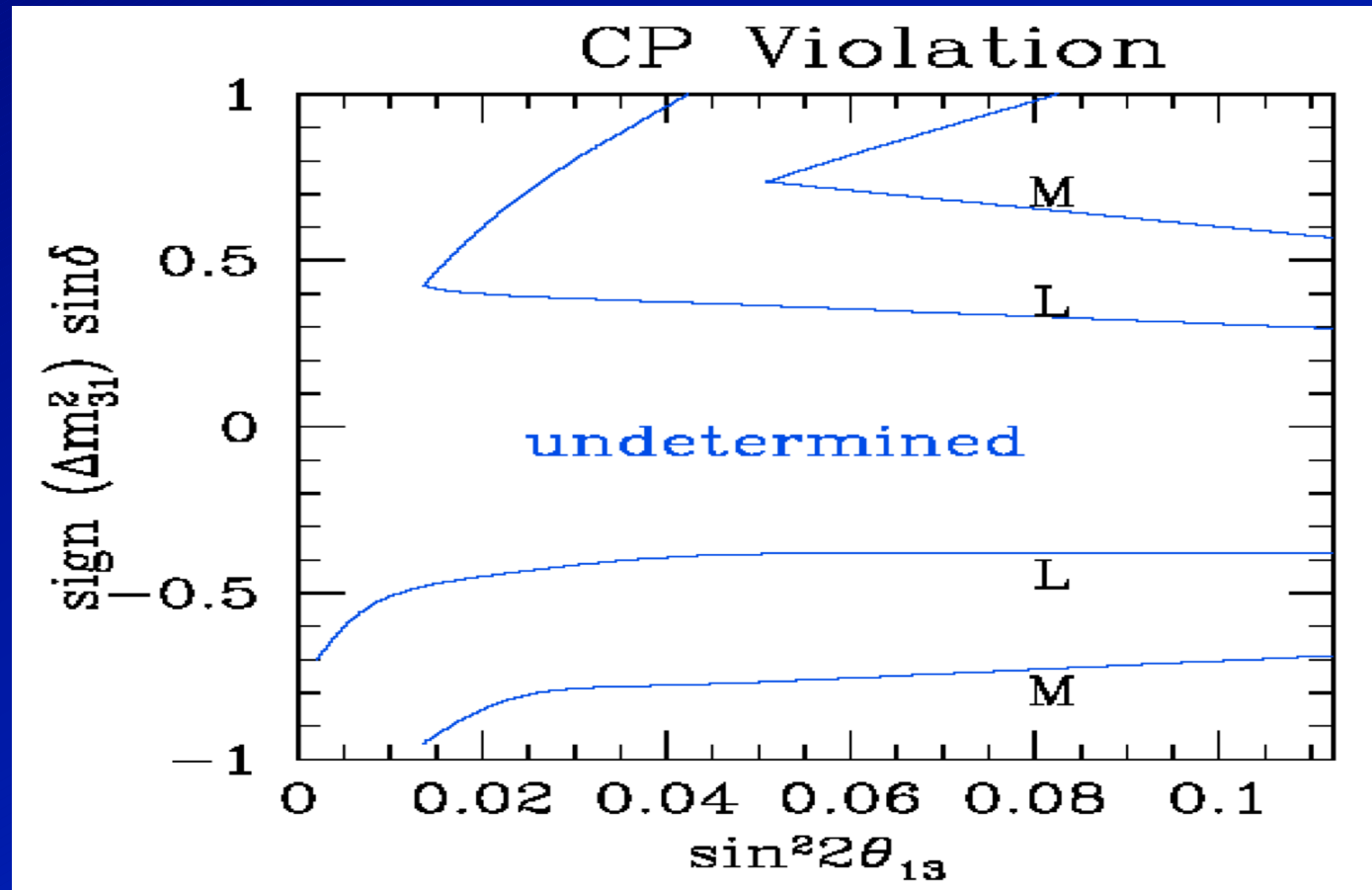
Combining different experiments or choosing specific experimental set-ups, **it may be possible to resolve the degeneracies among different parameters and uniquely determine the existence of CP-V in the lepton sector due to the δ phase.**

1. Superbeams.
2. Neutrino factories.
3. Beta-beams.

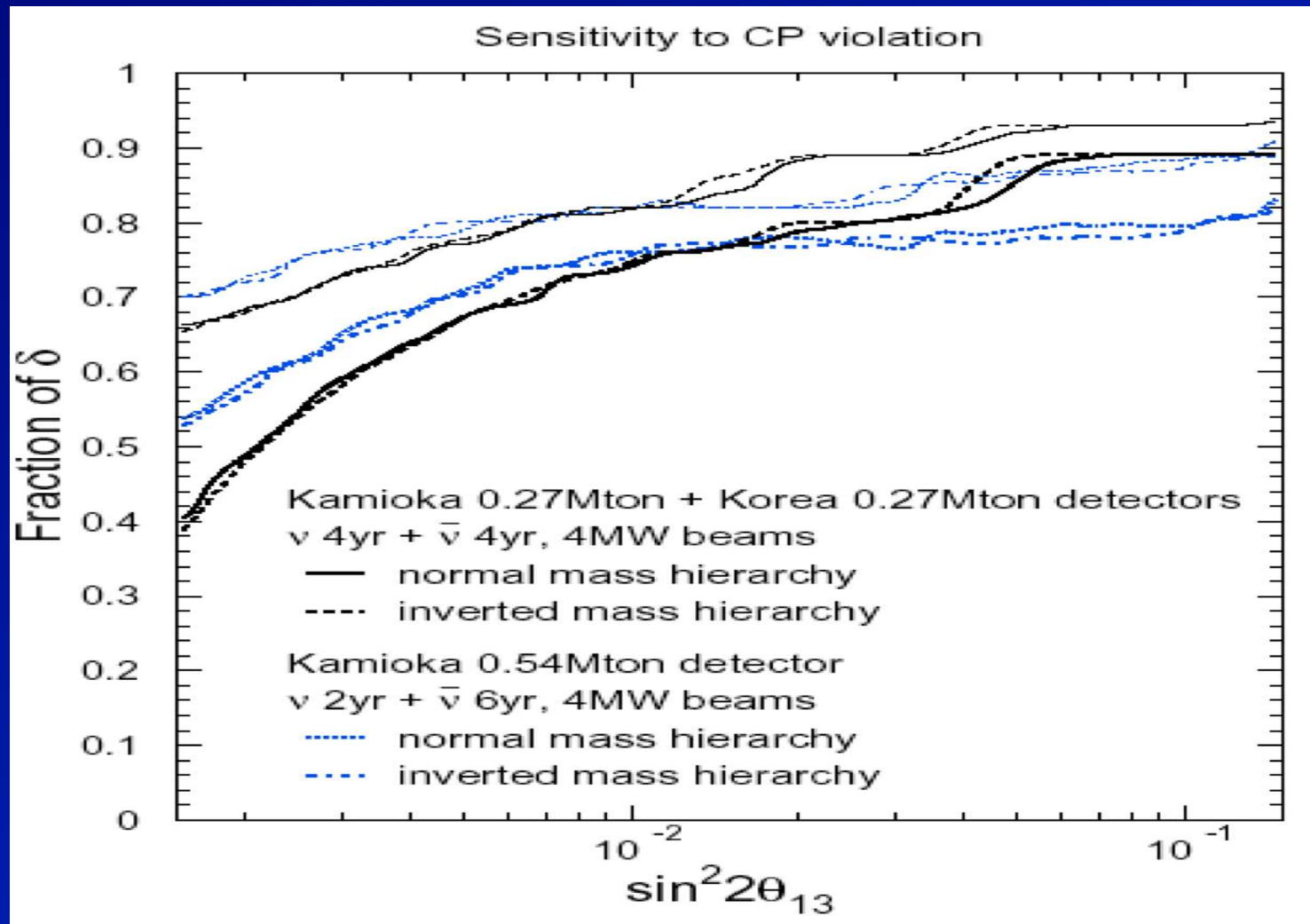


Superbeams

NO ν A: off-axis neutrino oscillation experiment at Fermilab (see B. Choudhary's talk).



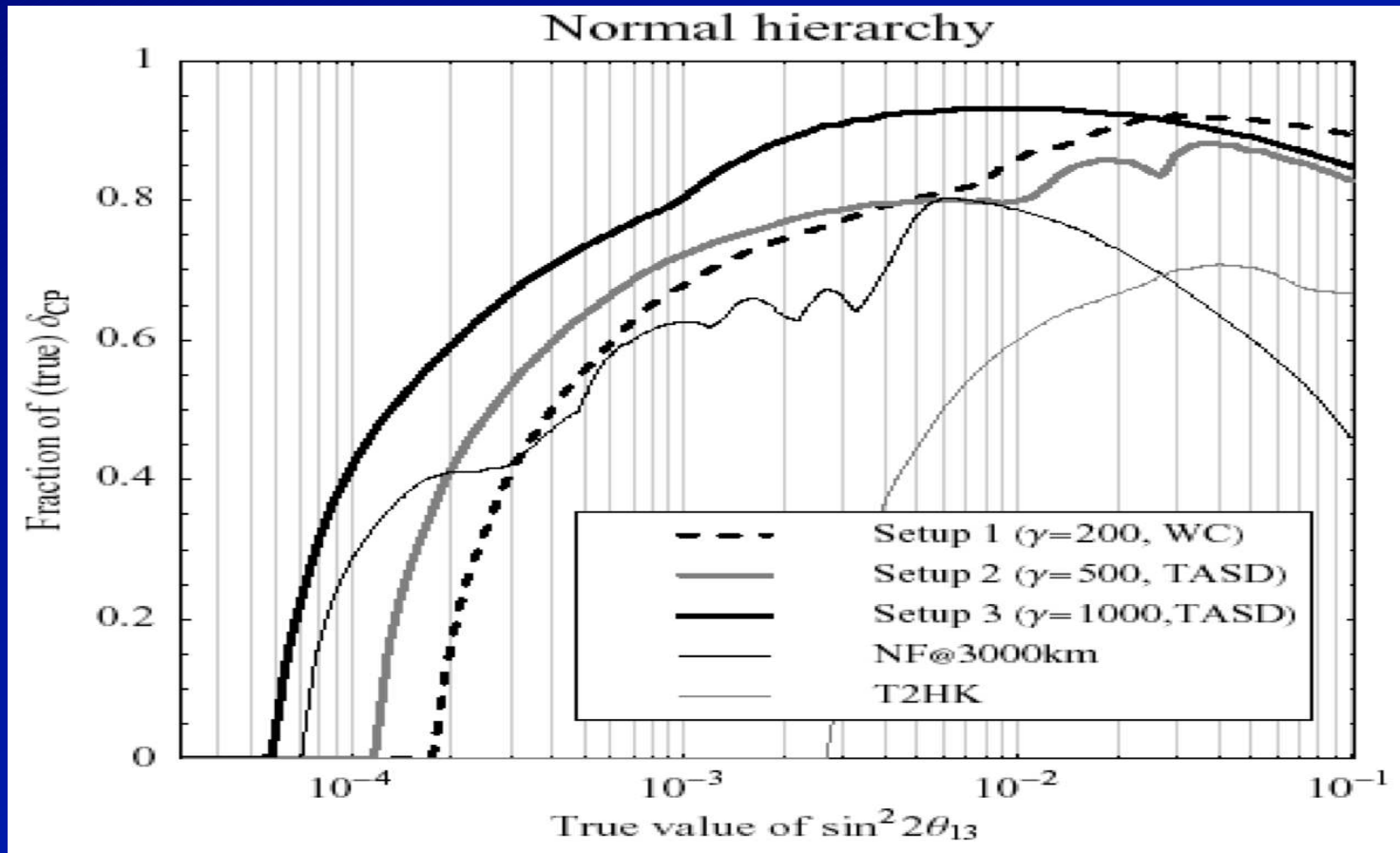
T2K-II: ν_μ off-axis beam in Japan from Tokai to Super-K and Hyper-K (phase II). (see T. Kajita's talk)



[M. Ishitsuka, T. Kajita, H. Minakata and H. Nunokawa]

Neutrino factory and beta-beams

For smaller values of $\sin^2 \theta_{13}$, more sensitive machines are required.



[P. Huber, M. Lindner, M. Rolinec and W. Winter]

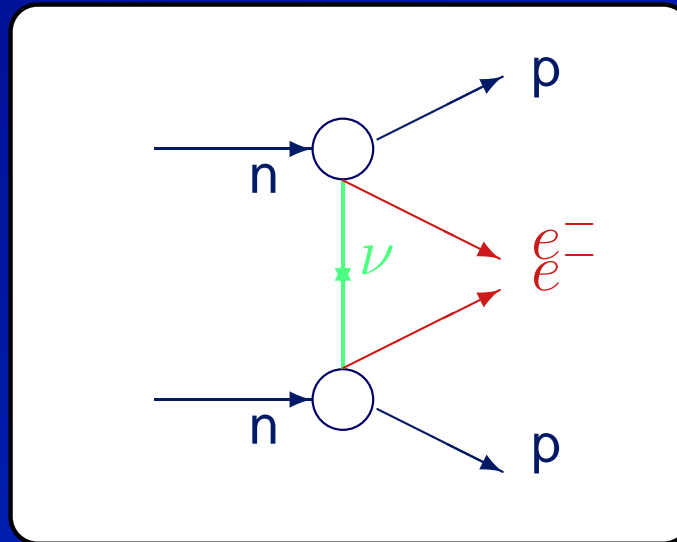
In summary,

- neutrino oscillations experiments are sensitive to δ .
- The search for CP-violation is affected by degeneracies with other unknown parameters which have to be extracted in the analysis of the same data: δ , $\sin^2 \theta_{13}$, $\text{sign}(\Delta m_{13}^2)$, θ_{23} .
- Sensitivity to CP-violation:
 - if $\sin^2 2\theta_{13} > 0.01$ in NO ν A
 - if $\sin^2 2\theta_{13} > 0.003$ in T2K-II
 - if $\sin^2 2\theta_{13} > 10^{-4}$ at a neutrino factory or β -beam.

Majorana phases

Majorana phases can be measured only in processes which violate the lepton number by 2 units ($\Delta L = 2$).

By far, the most sensitive of these processes is **neutrinoless double beta decay**: $(A, Z) \rightarrow (A, Z + 2) + 2e^-$.



$(\beta\beta)_{0\nu}$ -decay has a special role in the study of neutrino properties, as it probes the violation of **global lepton number**, and it might provide information on the **neutrino mass spectrum, absolute neutrino mass scale and CP-V**.

The half-life time, $T_{0\nu}^{1/2}$, of the $(\beta\beta)_{0\nu}$ -decay can be factorized as:

$$\left[T_{0\nu}^{1/2}(0^+ \rightarrow 0^+) \right]^{-1} \propto |M_F - g_A^2 M_{GT}|^2 |\langle m \rangle|^2$$

- M_F, M_{GT} are nuclear matrix elements.
- $|\langle m \rangle|$ is the effective Majorana mass parameter:

$$|\langle m \rangle| \equiv \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}} \right|,$$

For light neutrinos, $|\langle m \rangle|$ contains all the dependence of $T_{0\nu}^{1/2}$ on the neutrino parameters.

U_{ej} are the elements of the lepton mixing matrix U_{PMNS} , m_j the masses of the massive neutrinos ν_j , α_{21} and α_{31} the CP-violating phases.

The present best limit on $|\langle m \rangle|$ reads:

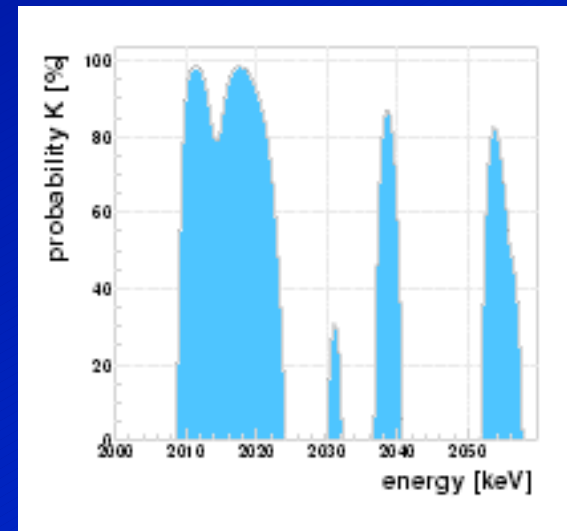
$$|\langle m \rangle| < (350 - 1050) \text{ meV} \quad \text{Heidelberg-Moscow}$$

$$|\langle m \rangle| < (680 - 2800) \text{ meV} \quad \text{NEMO3}$$

$$|\langle m \rangle| < (200 - 1050) \text{ meV} \quad \text{CUORICINO}$$

Recently a claim of $(\beta\beta)_{0\nu}$ decay discovery has been published [Klapdor-Kleingrothaus et al. 2004]. It implies

$$|\langle m \rangle| \simeq 200 - 600 \text{ meV}$$

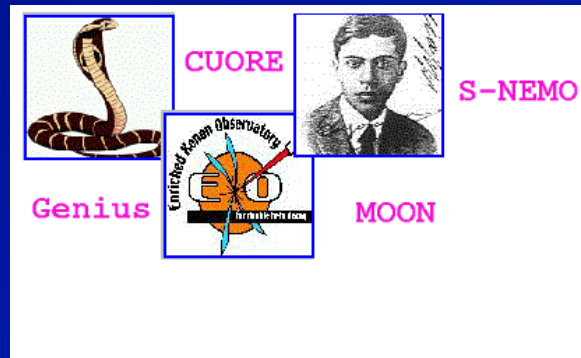


It is still a controversial even if very interesting result.

There are prospects to improve the present limit and test the discovery claim down to $|\langle m \rangle| \sim 200 - 300 \text{ meV}$ in the present experiments



NEMO3 and Cuoricino



and by one order of magnitude,

$$|\langle m \rangle| \sim 10 - 30 \text{ meV} ,$$

in the new generation of experiments which is now under R&D and construction

(CUORE, GENIUS, Majorana, SuperNEMO, EXO, GERDA, COBRA).

Mass quasi degeneracy (QD)

$$m_1 \simeq m_2 \simeq m_3 \equiv m_{\bar{\nu}_e}$$



$$|\langle m \rangle| \simeq m_{\bar{\nu}_e} \left| \left(\cos^2 \theta_{\odot} + \sin^2 \theta_{\odot} e^{i\alpha_{21}} \right) \cos^2 \theta_{13} + \sin^2 \theta_{13} e^{i\alpha_{31}} \right|$$

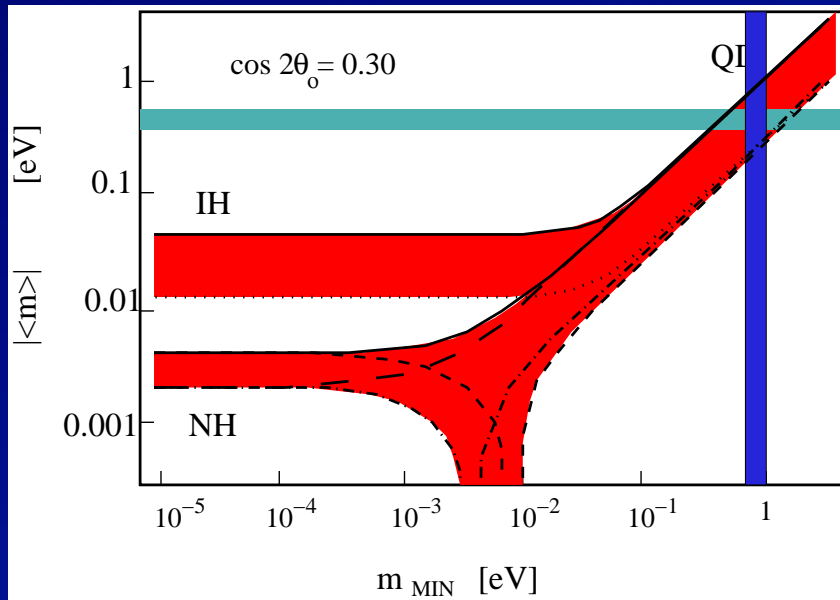
$|\langle m \rangle|$ has both a **lower** and an **upper** bound:

$$0.05 \text{ eV} \leq m_{\bar{\nu}_e} \cos 2\theta_{\odot} |\langle m \rangle| \leq m_{\bar{\nu}_e} < 2.2 \text{ eV}$$

All the allowed range for $|\langle m \rangle|$ is in the range of sensitivity of present and upcoming $(\beta\beta)_{0\nu}$ -decay experiments.

Analogous expressions can be found for the NH ($m_1 \ll m_2 \ll m_3$) and IH ($m_3 \ll m_1 \sim m_2$) mass spectra. The expected values of $|\langle m \rangle|$ are smaller than in the QD case.

3 – Measuring CP-V phases



In principle, a measurement of $|\langle m \rangle|$ combined with a measurement of m_1 (in tritium β -decay exp. and/or cosmology) would allow to establish if

CP is violated

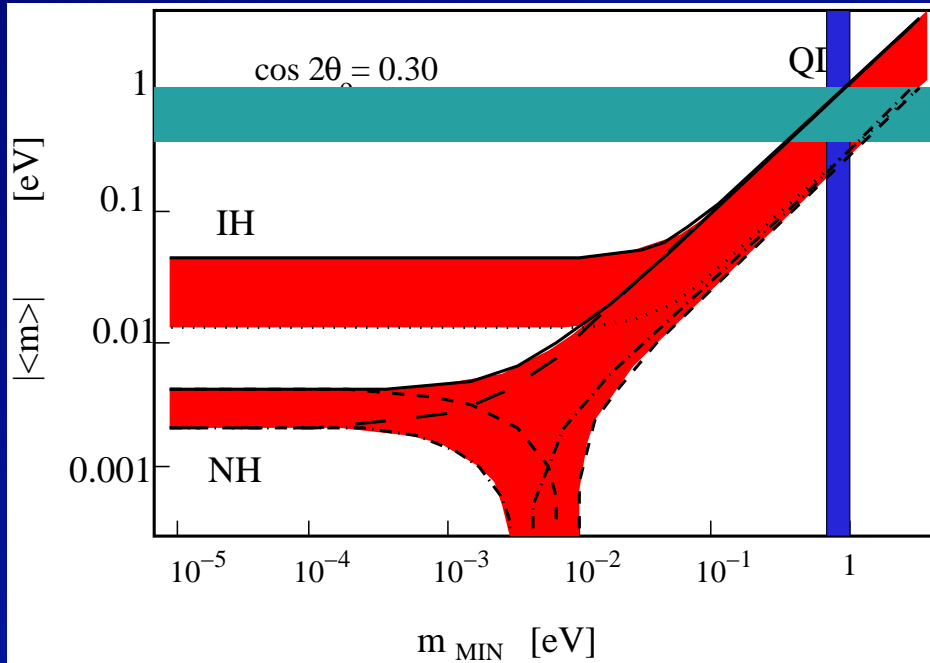
and to constrain the **CPV phases**,

once the ν mass spectrum is known.

For ex., for the QD spectrum, we have:

$$\sin^2 \alpha_{21} / 2 \simeq \left(1 - \frac{|\langle m \rangle|^2}{m_{\bar{\nu}_e}^2} \right) \frac{1}{\sin^2 2\theta_{\odot}}.$$

3 – Measuring CP-V phases

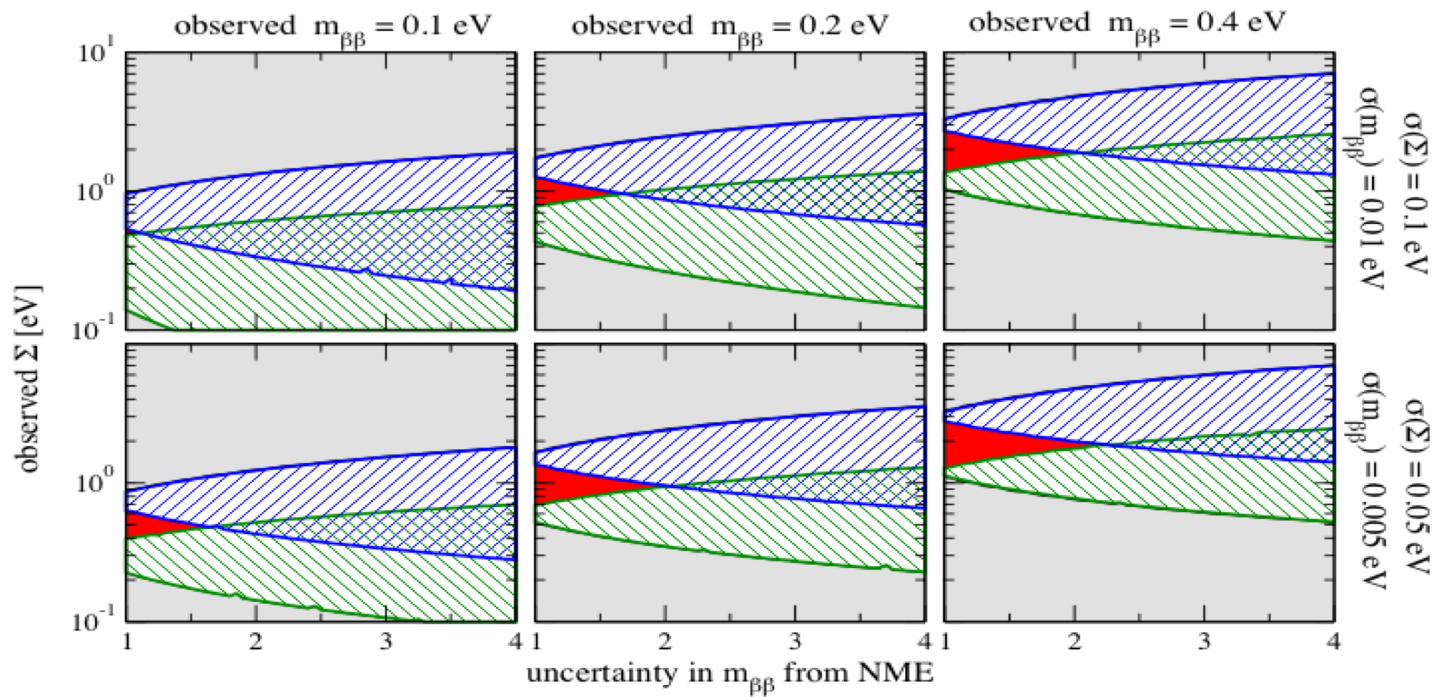



Due to the experimental errors and nuclear matrix elements uncertainties, determining that CP is violated in the lepton sector due to Majorana CPV phases is challenging.


[Barger et al.; S.P., Petcov, Rodejohann; S.P., Petcov, Schwetz]


3 – Measuring CP-V phases


The uncertainty in the nuclear matrix elements plays a crucial role.



 data consistent with $\alpha = \pi$

 data consistent with $\alpha = 0$

 $m_{\beta\beta}$ and Σ inconsistent at 90% CL

 CP violation established at 90% CL

$$\sin^2\theta_{13} = 0 \pm 0.002, \quad \sin^2\theta_{12} = 0.3 \pm 3\%, \quad \Delta m_{21}^2 = 8 \times 10^{-5} \pm 2\%, \quad \Delta m_{31}^2 = 2.2 \times 10^{-3} \pm 3\%$$

[S.P., S. Petcov, T. Schwetz]

In summary,

- $(\beta\beta)_{0\nu}$ -decay experiments are in principle sensitive to one Majorana CP-violating phase.
- Establishing CPV due to Majorana CPV phases is challenging and would require: [S.P., S. Petcov, T. Schwetz]
 - i) small experimental errors on $|\langle m \rangle|$ and neutrino masses;
 - ii) an uncertainty in the NME which amounts to a factor ζ in $|\langle m \rangle|$, $\zeta \ll (\cos 2\theta_{\odot})^{-1}$.

**From probing
leptonic CP-violation at low energy,
which information
can we obtain
about the physics at high energy
and in particular about leptogenesis?**

4 – The see-saw mechanism and Leptogenesis

The see-saw mechanism provides a natural explanation for the smallness of neutrino masses. [Minkovski; Yanagida; Gell-Mann, Ramond, Slansky]

At high energy ($10^9 - 10^{15}$ GeV), **RH neutrinos** are introduced. They are singlets with respect to the gauge group of the SM and possess very heavy Majorana masses:

$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

- **Lepton number is violated.**
- The see-saw mechanism can be embedded in GUT theories (see R. Mohapatra's talk).

At low energy, integrating out the heavy neutrinos, the light neutrino masses are naturally small.

$$\mathcal{L} = (\nu_L^T N^T) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix}$$

$$M_1 \simeq M_R$$

$$m_2 \simeq \frac{m_D^2}{M_R} \sim \frac{1 \text{ GeV}^2}{10^9 \text{ GeV}} \sim 1 \text{ eV}$$

In a 3 neutrino mixing, light masses are given by:

$$m_\nu = U^* d_m U^\dagger \simeq -Y_\nu^T M_R^{-1} Y_\nu v^2$$

- Light neutrinos are predicted to be Majorana particles.

Leptogenesis takes place in the context of see-saw models (see E. Ma's and T. Hambye's talks). As the Universe expands, N 's go out of equilibrium ($T < M / \text{few}$). Their decays produce a lepton asymmetry, which is then converted into a **baryon asymmetry** by sphaleron processes. Leptogenesis can successfully explain the observed baryon asymmetry of the Universe.

[Fukugita, Yanagida; Covi, Roulet, Vissani; Buchmuller, Plumacher]

It requires:

- out of equilibrium;
- L violation;
- C and CP violation.

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Expansion of the Universe

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It requires:

- out of equilibrium;
- L violation; $(\beta\beta)_{0\nu}$ -decay
- C and CP violation.

The baryon asymmetry is given by:

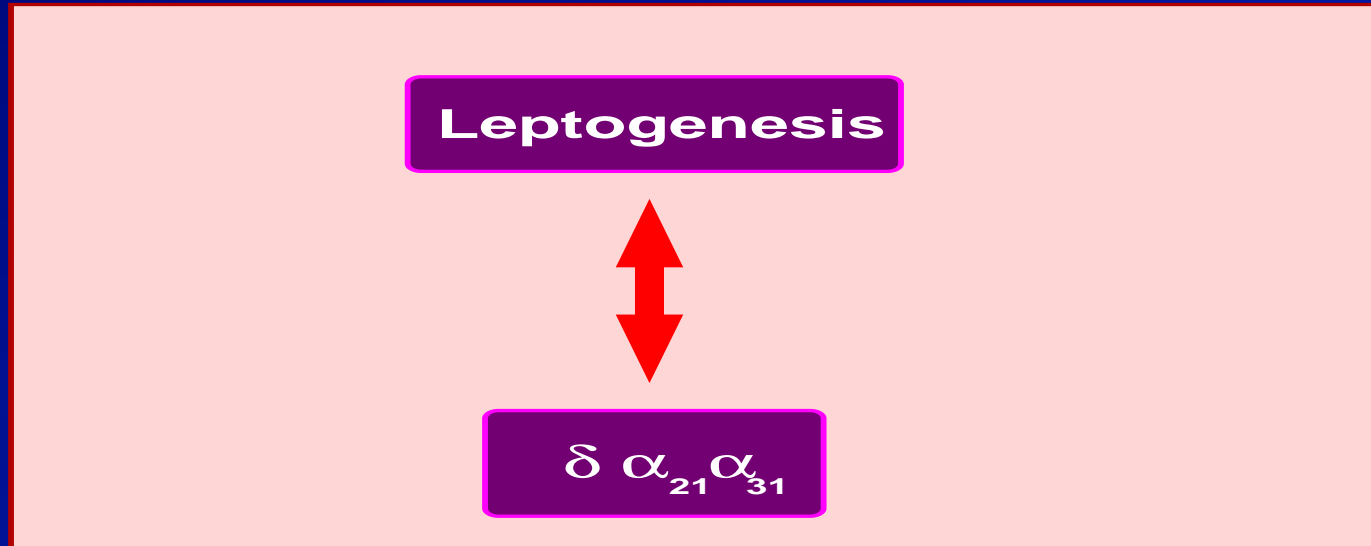
$$\eta_B/s = C\eta_L/s = -10^{-4} \epsilon_1$$

ϵ_1 is the decay asymmetry which depends on the CPV phases in Y_ν :

$$\begin{aligned} \epsilon_1 &\equiv \frac{\Gamma(N \rightarrow lH) - \Gamma(N \rightarrow l^c H^c)}{\Gamma(N \rightarrow lH) + \Gamma(N \rightarrow l^c H^c)} \\ &\propto \sum_j \text{Im}(Y_\nu Y_\nu^\dagger)_{1j}^2 \frac{M_j}{M_1} \end{aligned}$$

5 – Is there a connection between CP-V at low energy and in leptogenesis?

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High energy parameters

Low energy parameters

$$M_R \quad 3 \quad 0$$

$$d_m \quad 3 \quad 0$$

$$Y_\nu \quad 9 \quad 6$$

$$U \quad 3 \quad 3$$

9 parameters are lost, of which 3 phases. In a model-independent way there is **no one-to-one connection** between the low-energy phases and the ones entering leptogenesis. [see, e.g., S.P., MPLA]

In the biunitary parameterization, $Y_\nu = V_R^\dagger(\beta_1, \beta_2, \beta_3)yV_L(\alpha_1, \alpha_2, \alpha_3)$:

$$\begin{aligned} \epsilon_1 &\propto \text{Im}(V_R^\dagger y^2 V_R)_{1j}^2 && \Rightarrow \epsilon_1(\beta_1, \beta_2, \beta_3) \\ m_\nu &= V_L^\dagger y V_R M_R^{-1} V_R^T y V_L^* && \Rightarrow m_\nu(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3) \end{aligned}$$

[Davidson, Ibarra; Ellis et al.; SP, Petcov, Rodejohann]

- ϵ_1 depends only on the mixing in the right-handed sector.

m_ν depends on all the parameters in Y_ν , both the mixing in the left and right-handed sector.

- **Additional information** can be obtained in **LFV charged lepton decays** which depend on V_L (see for example [S. Petcov et al. 2005](#)).

Even if the soft breaking terms are diagonal at the SUSY breaking scale M_X , the radiative corrections due to Y_ν induce off-diagonal soft terms.

$$BR(l_i \rightarrow l_j \gamma) \propto |P_{ij}|^2$$
$$P = Y_\nu^\dagger \log \frac{M_X}{M} Y_\nu \sim V_L^\dagger y^2 V_L.$$



if there is CPV in V_R (leptogenesis), we can expect to have CPV in m_ν .

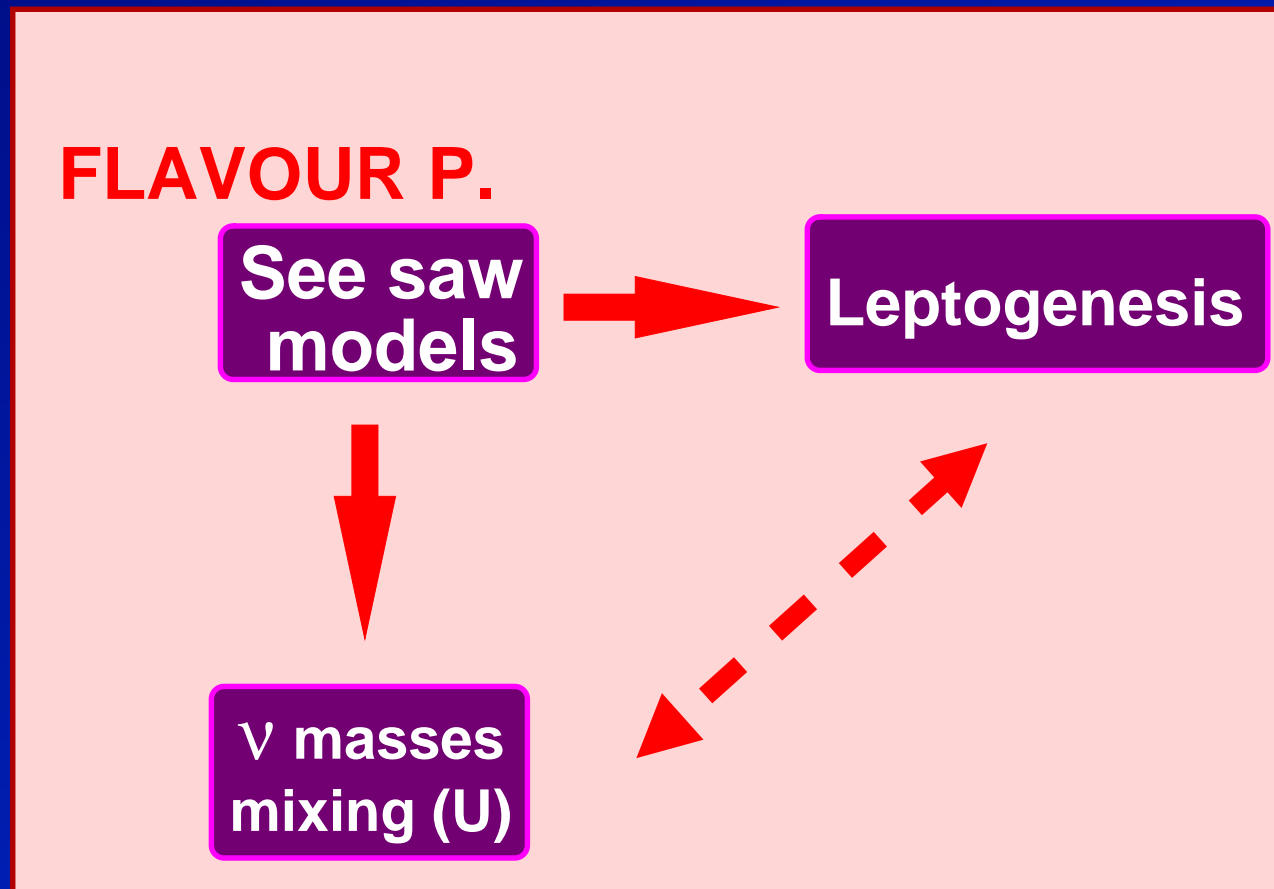
$$\begin{aligned}
 J_{CP} &\propto \text{Im}(h_{12}h_{23}h_{31}) \\
 &\propto \text{Im}\left((V_L^T y V_R^* M_R^{-1} V_R^\dagger y^2 V_R M_R^{-1} V_R^T y V_L)_{12} \right. \\
 &\quad (V_L^T y V_R^* M_R^{-1} V_R^\dagger y^2 V_R M_R^{-1} V_R^T y V_L)_{23} \\
 &\quad \left. (V_L^T y V_R^* M_R^{-1} V_R^\dagger y^2 V_R M_R^{-1} V_R^T y V_L)_{31} \right)
 \end{aligned}$$

[Branco et al.]

Even if $\delta = 0$ at high scale, RGE's effects can generate it at low energy.

In understanding the origin of the flavour structure, the see-saw models have a **reduced number of parameters**. In many cases,

it is possible to link directly Dirac and Majorana phases to leptogenesis.



- Minimal see-saw model:

With only 2 heavy neutrinos, all the high energy parameters (8 + 3) can be reconstructed by using U and P . With additional 2 texture zeros, there is only one phase. It can be shown that:

$$\epsilon_1 \propto \sin 2\delta$$

$$J_{CP} = \text{Im}(Y_{12}Y_{23}Y_{31}) \propto -\sin 2\delta$$

[Frampton, Glashow, Yanagida; Ibarra, Ross; Ibarra]

- Mixing only in the right-handed sector:

$V_L = 1$. In this case the only phases are the ones in V_R .

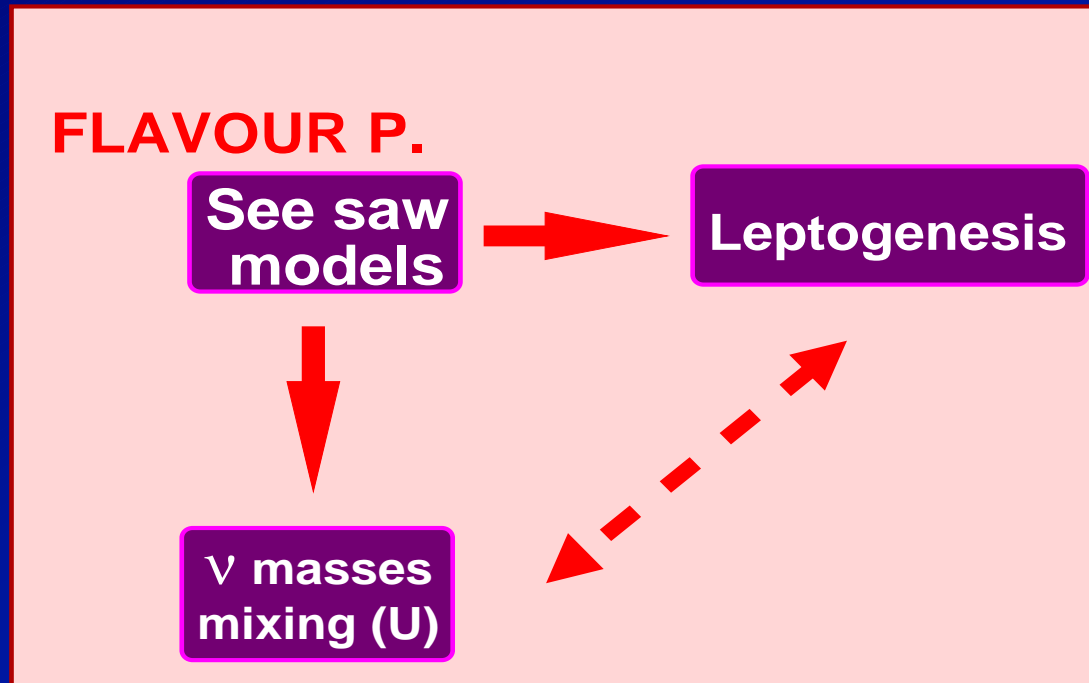
$$\epsilon_1 = \epsilon_1(\beta_1, \beta_2, \beta_3)$$

$$J_{CP} = \text{Im}(Y_{12}Y_{23}Y_{31}) = J_{CP}(\beta_1, \beta_2, \beta_3)$$

$$\alpha_{21,31} = \alpha_{21,31}(\beta_1, \beta_2, \beta_3)$$

[Branco et al.]

6 – Conclusions



The observation of L violation ($(\beta\beta)_{0\nu}$ -decay) and of CPV in the lepton sector (neutrino oscillations and/or $(\beta\beta)_{0\nu}$ -decay) would be a **indication, even if not a proof**, of **leptogenesis** as the explanation for the observed baryon asymmetry of the Universe.

The oscillation probability for $\nu_\mu \rightarrow \nu_e$ is given by:

$$\begin{aligned}
 P(\bar{P}) \simeq & s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{A \mp \Delta_{13}} \right)^2 \sin^2 \frac{(A \mp \Delta_{13})L}{2} \\
 & + \tilde{J} \frac{\Delta_{12}}{A} \frac{\Delta_{13}}{A \mp \Delta_{13}} \sin \frac{AL}{2} \sin \frac{(A \mp \Delta_{13})L}{2} \cos \left(\mp \delta + \frac{\Delta_{13}L}{2} \right) \\
 & + c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2 \frac{AL}{2}
 \end{aligned}$$

We identify 2-, 4- and 8- fold degeneracies [Barger, Marfatia, Whisnant]:

- (θ_{13}, δ) degeneracy [Koike, Ota, Sato; Burguet-Castell et al.] :

$$\delta' = \pi - \delta$$

$$\theta'_{13} = \theta_{13} + \cos \delta \sin 2\theta_{12} \frac{\Delta m_{12}^2 L}{4E} \cot \theta_{23} \cot \frac{\Delta m_{13}^2 L}{4E}$$

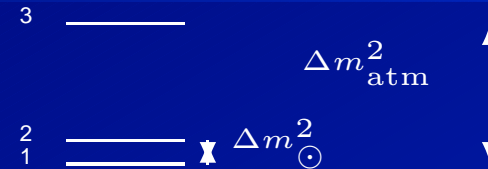
- $(\text{sign}(\Delta m_{13}^2), \delta)$ degeneracy [Minakata, Nunokawa]:

δ'	$\pi - \delta$
$\text{sign}'(\Delta m_{13}^2)$	$-\text{sign}(\Delta m_{13}^2)$

- $\theta_{23}, \pi/2 - \theta_{23}$ degeneracy [Fogli, Lisi].

6 – Conclusions

NH spectrum: $m_1 \ll m_2 \ll m_3$

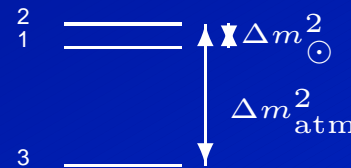


$$|\langle m \rangle| \simeq \left| \sqrt{\Delta m_{\odot}^2} \cos^2 \theta_{13} \sin^2 \theta_{\odot} + \sqrt{\Delta m_{\text{atm}}^2} \sin^2 \theta_{13} e^{i\alpha_{32}} \right|$$

$|\langle m \rangle|$ has both an **upper and lower bound**:

$$\text{few} \times 10^{-4} \text{ eV} \lesssim |\langle m \rangle| \lesssim 8.5 \times 10^{-3} \text{ eV}$$

IH spectrum: $m_3 \ll m_1 \sim m_2$



$$\sqrt{\Delta m_{\text{atm}}^2} \cos 2\theta_{\odot} \leq |\langle m \rangle| \simeq \sqrt{\left(1 - \sin^2(2\theta_{\odot}) \sin^2 \frac{\alpha_{21}}{2}\right) \Delta m_{\text{atm}}^2} \leq \sqrt{\Delta m_{\text{atm}}^2}$$

$|\langle m \rangle|$ has a significant **lower bound**

$$0.01 \text{ eV} \lesssim |\langle m \rangle| \lesssim 0.08 \text{ eV}$$

$|\langle m \rangle|$ is in the range of sensitivity of the upcoming $(\beta\beta)_{0\nu}$ -decay experiments.