Physics of Extra Dimensions at Colliders

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Quote from Gramsci

Enthusiasm is nothing more than the external adoration of fetishes. The only justifiable enthusiasm is that which accompanies the intelligent will, intelligent activity, the inventive richness of concrete initiatives which change existing reality.

A Dialogue

- String Theory unifies all interactions.

  Does it really provide us a definitive framework to do so?

- Well, it is the only way you can deal with gravity. String Theory is THE theory of quantum gravity.

  Is it the only approach to the quantization of gravity?

- In the heterodox approach to the problem, there is no gravity as we know it. Where is the graviton in that approach?

  Maybe, but where is the graviton in the real world?

- Come on, even a phenomenologist like you should know how important the quantization of gravity is!

  Is it? But we do not have a single compelling observation that tells us that we should worry about it

- But, surely, you have heard of the Bohr-Rosenfeld argument? All classical fields have to be quantised.

  Sure, but the problem is already at the classical level – do we have any evidence for gravitational waves, to begin with?

- Look, why don't we forget all this. I will tell you about realisations of string theory which can have TeV-scale implications.

  Now you are talking! Can we start computing a cross-section for the LHC?

Brief Look at Kaluza-Klein Reduction

- Consider a scalar field in 5 dimensions, $\phi(x, y)$. where y is compactified to a circle of radius R.
- Fourier expansion of the field gives

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \phi_n(x) \exp(iny/R), \tag{1}$$

  where n is an integer and $\phi_n(x)$ are 4-dimensional fields.
- Substituting this in the 5-dimensional Klein-Gordon equation which $\phi(x, y)$ satisfies $\Longrightarrow$ infinite number of equations in 4 dimensions, one for each $\phi_n(x)$ and with a mass $|n|/R$ for every mode n.
- Just as a single 5-dimensional field appears as a tower of states in 4 dimensions, a particle of a given spin in 5 dimensions appears as states of different spins in 4 dimensions.

Strings and Branes

- It has become possible to study string theories at strong coupling as duals of other theories (at weak coupling).
- In the strong coupling limit, other than open and/or closed strings, solitonic modes appear which go under the name of Dp-branes.
- Dp-brane $\Longrightarrow$ a $p+1$ dimensional hyperplane in a higher dimensional spacetime.
- Dynamical rather than rigid, because in the presence of gravity the hyperplane can fluctuate in shape and position as a dynamical object.
- Open strings (to which are associated gauge particles) end on the D-branes and are localised on them
- Closed strings (corresponding to gravitons) are free to propagate in the bulk.
- A new picture of the universe now emerges: the gauge particles of the SM live on a 3-brane, which resides with other branes in a higher dimensional spacetime. While the SM is confined to the brane, only gravity is free to wander over all spacetime including the bulk.

The ADD Model

- Assume that out four dimensional universe is a 3-brane on which the SM is localised and the gravitons are free to move around over the entire space-time.
- If the compactification radius of n extra dimensions is R, then
$$M_{\mathrm{P}}^2 = M_S^{n+2} R^n \ , \tag{2}$$
where M_S is the low-energy effective string scale.
- If R is large then M_S can be chosen to be of the order of a TeV i.e. hierarchically smaller than the Planck scale. For such a value of M_S, $R = 10^{32/n-19}$ m.

n	R (in m)
1	10^{13}
2	10^{-3}
3	4.5×10^{-9}
4	10^{-12}
5	2.5×10^{-13}
6	2.1×10^{-14}

- For $n = 2$ $R \sim 1$ mm, within the possible reach of ongoing gravitation experiments.

The Low-Energy Effective Theory

- Toroidal compactification $\Longrightarrow$ K-K excitations of the graviton corresponding to a tower of spin-2, spin-1 and spin-0 excitations.

- The spin-2 K-K states $\Longrightarrow$ infinite tower of massive gravitons, spin-0 state $\Longrightarrow$ dilaton, spin-1 state $\Longrightarrow$ model-dependent couplings.

- Other light modes $\Longrightarrow$ Y modes which are goldstone bosons associated with the spontaneous breaking of translational invariance in the transverse directions. Effects are subleading.

- Interaction of SM particles with the graviton K-K modes, $G_{\mu\nu}$, is derived from the following Lagrangian:

$$\mathcal{L} = -\frac{1}{\bar{M}_P} G^{(j)}_{\mu\nu} T^{\mu\nu} \ , \tag{3}$$

  where j labels the Kaluza-Klein mode and $\bar{M}_P = M_P/\sqrt{8\pi}$, and $T^{\mu\nu}$ is the energy-momentum tensor.

Graviton Production Processes

- The coupling of the gravitons to the SM particles is suppressed by $1/\bar{M}_P \Longrightarrow$ production of individual graviton states is hopelessly suppressed. But sum over the tower of KK excitations enhances the rate by M_P/M_S.
- Production of gravitons give rise to characteristic missing energy or missing p_T signatures at e^+e^- or hadron colliders resulting in bounds on M_S which are around 500 GeV to 1.2 TeV at LEP2 and around 600 GeV to 750 GeV at Tevatron (for n between 2 and 6).
- Gravitons produced in the core of supernovae in nucleon nucleon bremsstrahlung process can carry away energy from the core. Data on SN1987A from Kamiokande and IMB experiments imply $E \geq 2 \times 10^{53}$ ergs emmitted in the form of a neutrino flux $\Longrightarrow$ bounds on M_S of the order of 50 TeV for n=2 and about 2.5 TeV for n=3.
- Similar graviton production processes studied in the Sun and red giants yield bounds on M_S which are of the order of a TeV for $n = 2$.

Virtual Effects

- The KK excitations will also manifest in collider experiments through their virtual effects.
- The sum over the KK propagators is tricky because it is sensitive to unknown ultraviolet effects and is divergent for $n > 1$.
- Naive approach is to regulate the divergence using a cut-off M_S'. Virtual processes yield bounds on M_S' whose relationship to M_S requires knowledge of the full theory.
- Virtual effects of graviton exchange in the processes listed below have been studied with the bounds on M_S:

  Dilepton production at Tevatron: $M_S =$ 950 GeV to 1100 GeV.

  $t\bar{t}$ production at Tevatron $M_S =$ 650 GeV while at the LHC $M_S =$ 4 TeV.

  Deep-inelastic scattering at HERA: $M_S =$ 550 GeV.

  Dijet production at the Tevatron: $M_S = 1.2$ TeV.

  Pair production of gauge bosons and fermions at LEP2: $M_S =$ 500 GeV – 1 TeV

Minimal Length

- From GR, we know that a particle with momentum p in a volume of space Δx^3 causes a fluctuation of the metric g. Einstein's equations allows us to estimate this perturbation:

$$\frac{\delta g}{\Delta x^2} \sim l_p^2 \frac{p}{\Delta x^3} \tag{4}$$

  leading to a fluctuation of the spacetime to be of $\Delta x' = \delta g \Delta x \sim l_p^2 p$, which becomes important at large momenta.

- To first order, the uncertainty principle is generalised to

$$\Delta x \sim \frac{1}{\Delta p} + l_p^2 p \tag{5}$$

- The motivation for a minimal length is manifold in theories of quantum gravity. One such is string theory where an analysis of string scattering amplitudes suggests the existence of a minimal length. Distances below the string length cannot be probed.

- We work within the context of scenarios where a minimal length l_p exists.

- There are many different ways to realise the minimal length scenario (MLS). Example: modifying dispersion relations. We do not modify dispersion relations but the relationship between p and k.

- Even though k is bounded from above, p can be arbitrarily large. Also to maintain the same dispersion relation the frequency ω is restricted from above while the energy E can go up arbitrarily.

- Standard relations $p = \hbar k$ and $E = \hbar\omega$ have to be modified. Modified relations, known as the Unruh relations:

$$\begin{aligned} l_p k(p) &= \tanh^{1/\gamma}\left[\left(\frac{p}{M_S}\right)^\gamma\right], \\ l_p \omega(E) &= \tanh^{1/\gamma}\left[\left(\frac{E}{M_S}\right)^\gamma\right], \end{aligned} \tag{6}$$

  where γ is a positive constant.

- Interesting union: ADD and MLS. l_p is of the order of TeV^{-1}. Avoiding the UV problem in summing over virtual gravitons.

Transplanckian Effects

- At the LHC, the collision energies are often in the transplanckian domain.

- If the impact parameter in the collision between two partons is smaller than the n-dimensional Schwarzschild radius R, given by
$$R_S = \frac{1}{M_S}\Big[\frac{M_{\rm BH}}{M_s}\Big]^{\frac{1}{n+1}}\Big[\frac{2^n\pi^{\frac{n-3}{2}}\Gamma(\frac{n+3}{2})}{n+2}\Big]^{\frac{1}{n+1}} \qquad (7)$$
then a black-hole maybe formed, if the entropy of the resulting system is sufficiently high.

- The production cross-section, given by the geometrical cross-section πR_S^2, can be huge at the LHC ($\sim$ 400 pb for M_S and $M_{\rm BH}$ of O(1 TeV)).

- The spin of the black hole is important so it may be more appropriate to consider Kerr solutions.

- The decay proceeds mainly through Hawking radiation and primarily into SM particles on the brane.

- Decay proceeds through three phases: a balding phase where the black hole loses hair associated with multipole moments, a spin-down phase where the angular momentum is shed, followed by a longer Schwarzschild phase. One ends up with large multiplicity events with hard jets and leptons.

- Hard processes taking place at short distances are suppressed because of the formation of an event horizon.

The Randall-Sundrum Model

- Out of an attempt to solve the hierarchy problem without introducing a large compactification radius came the model of Randall and Sundrum.

- In this 5-d model, the fifth dimension ϕ of a small radius R_c is compactified on a $\mathbf{S}^1/\mathbf{Z}^2$ orbifold. Two branes are at the orbifold fixed points: a Planck brane at $\phi = 0$ and a TeV brane at $\phi = \pi$.

- The model uses a warped or non-factortisable metric
$$ds^2 = e^{-KR_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + R_c^2 d\phi^2. \tag{8}$$
where K is a mass scale related to the curvature.

- The warp factor acts as a conformal factor for the fields localised on the brane and mass factors get rescaled by this factor.

- So $M_P = 10^{19}$ GeV for the Planck brane at $\phi = 0$ gets rescaled to $M_P \exp(-KR_c\pi)$ for the TeV brane at $\phi = \pi$. Generates a factor of 10^{15} by an exponent of order 30 and solves the hierarchy problem.

- Brane separation is stabilised by a bulk scalar field which generates a stabilising potential. This is the radion which acquires a mass of O(TeV) and coupling 1/TeV.

- The radion couples to massive particles. Very similar to that of the SM Higgs except the vev is of O(TeV). Also the radion has anomalous couplings from the trace anomaly to gluons and photons.

More on Randall-Sundrum

- The zero-mode of the KK tower couples very weakly (suppressed by $\overline{M}_P^{-1}$). The couplings of the massive KK states are enhanced by $e^{\pi K R_c}$ (electroweak strength).

- Continuity requirements of the solutions at the orbifold fixed points yield a simple relation for the masses M_n of the KK excitations in terms of the zeroes, x_n, of the Bessel function of order 1.

$$M_n = x_n K e^{-\pi K R_c} \equiv x_n m_0 \tag{9}$$

  where m_0 sets the scale for the masses of the excitations.

- It is also expedient to define a parameter c_0 as

$$c_0 = \frac{e^{\pi K R_c}}{\overline{M}_P} \frac{m_0}{\sqrt{8\pi}} \tag{10}$$

  The parameters m_0 and c_0 are the free parameters of the model.

Warped Phenomenology

- The distinctive features of the Randall-Sundrum model are as follows: There are no light KK modes. The constraints from real graviton production in current collider experiments do not apply.

- The astrophysical bounds also do not constrain this model because of the inhibition of the graviton production processes.

- In principle, higher energy collider experiments could produce individual graviton resonances and then individual modes can be identified by their decay products.

- Alternatively one probes the RS model by virtual processes. For example, virtual effects in dilepton or diphoton production at Tevatron, fermion pair production at e^+e^- colliders or $t\bar{t}$ production at Tevatron.

- Radion phenomenology: Gluon fusion dominates the production cross-section for the radion. The best hope for the detection of the radion is via the gg initial state at the LHC and to look for decay into WW/ZZ final states.

Other Incarnations of Randall and Sundrum

- The AdS/CFT correspondence tells us that the RS model is dual to a 4-d effective theory incorporating gravity and a strongly coupled sector.
- The dual theory is conformally invariant from the Planck scale down to the TeV scale. The presence of the TeV brane breaks conformal invariance at IR scales.
- The K-K excitations as well as the fields localised on the TeV brane are TeV-scale composites of the strong sector. Since all the SM fields are localised on the TeV brane, the original RS theory is dual to a theory of TeV-scale compositeness of the entire SM.
- Provides insights into model-building $\Longrightarrow$ one can alter the RS model with only the Higgs field localised on the IR brane, so that only the Higgs is a composite.
- Less sensitive to details of UV completion and is a more robust theory when faced with precision tests.
- Distinctive collider signatures result from the enhancement of the Higgs coupling to the gauge boson KK modes $\Longrightarrow$ enhanced scattering of longitudinal W, Z into KK modes.
- In the model, t_R is localised close to the TeV brane and t_R coupling to gluon and right-handed W K-K modes is enhanced. So interesting collider phenomena of gauge boson K-K production from gg-initial states mediated by top loops results.

SUSY Breaking from Extra Dimensions: An Example

- Extra dimensions could provide mechanisms for SUSY breaking and so even very small extra dimensions may have observable consequences if we could measure the sparticle mass spectra that they predict.
- Any attempt to solve the doublet-triplet splitting problem in $SU(5)$ with a global symmetry that forbids the $2-\bar{2}$ mass runs into problems if we have only a finite number of $SU(5)$ multiplets.
- The solution is to bring in an infinite number of fields which have a natural interpretation of K-K towers.
- One such model starts with a M-theory in 11 dimensions and the extra dimensions are compactified on a 7-manifold of $G2$ holonomy.
- With a proper choice of a global symmetry group the $2-\bar{2}$ mass can be forbidden; fermion masses can be generated and proton-decay can be suppressed.
- The spectrum below the GUT scale is anomalous: the Higgs, for example, are in $(\bar{5},1)+(1,5)$. The additional fields $\phi \equiv (5,1)$ and $\tilde{\phi} \equiv (1,\bar{5})$ needed to remove these anomalies have an interpretation as messenger fields of GMSB.
- In the usual GMSB, one would couple the messengers to

a chiral superfield S of the SUSY breaking sector as

$$\int d^2\theta S\phi\tilde{\phi} \tag{11}$$

- In this model, however, it appears as

$$\int d^2\theta(S2\cdot\bar{2}+S'3\cdot\bar{3}) \tag{12}$$

  because $2\cdot\bar{2}$ and $3\cdot\bar{3}$ transform differently under the global symmetry.

- Hence, the mass predictions in this model are not fixed completely as in minimal GMSB. In particular, one can make the slepton light and other sparticles heavy depending on how one tunes these parameters. With R-parity violation, this model predicts interesting $l\gamma\not{E}_T$ signatures which have been studied in the LHC context.

A parting shot

Q: How many string theorists does it take to change a light bulb?

A: $10^{120} + 1$.