What have we learnt about inflation from WMAP?

Courtesey: NASA/WMAP Science Team

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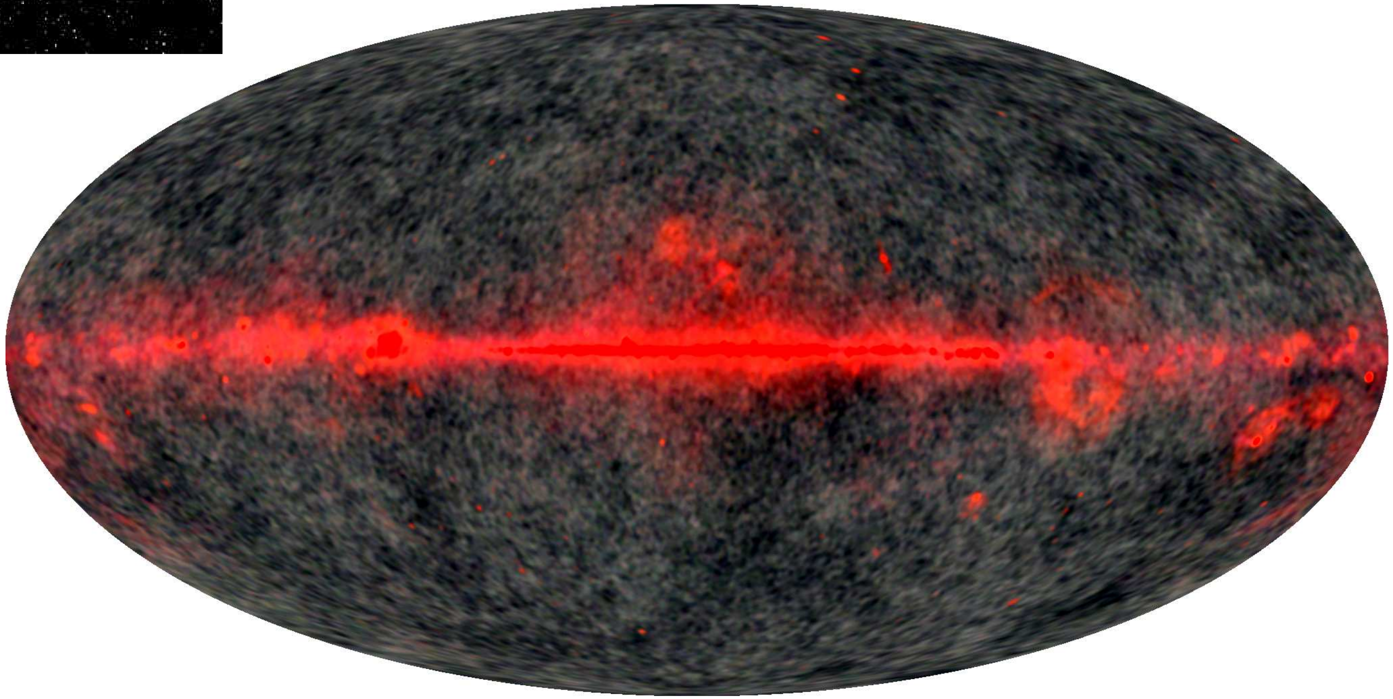
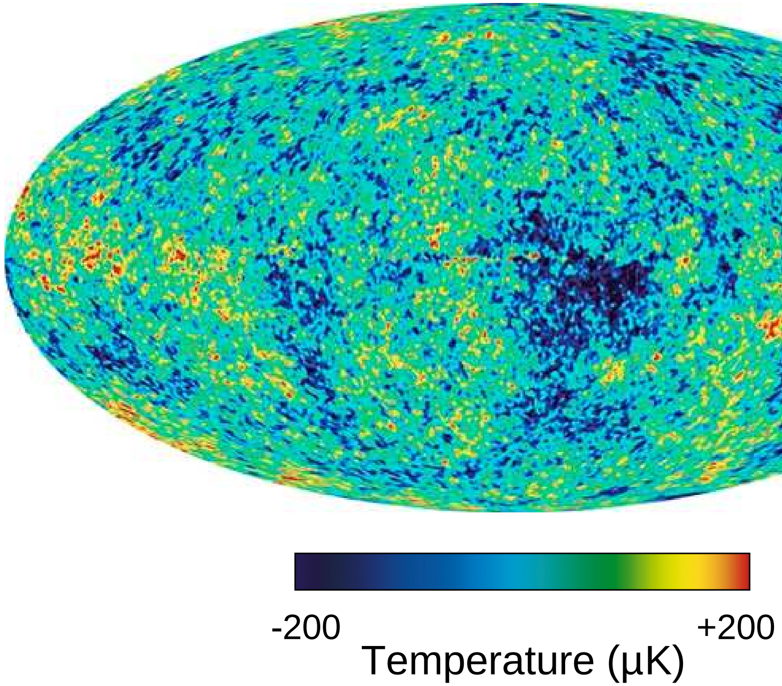


WHEPP 9, Bhubaneshwar, 9 January 2006

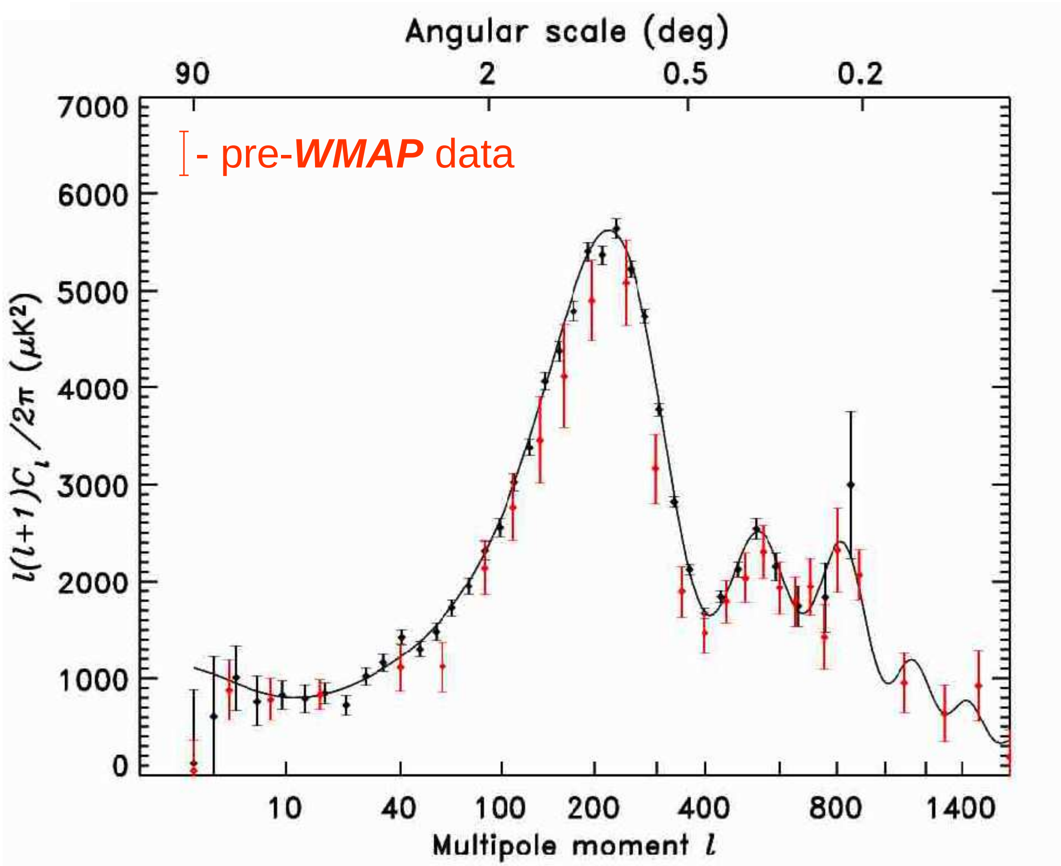
Wilkinson Microwave Anisotropy Probe – 1st yr data (2003)



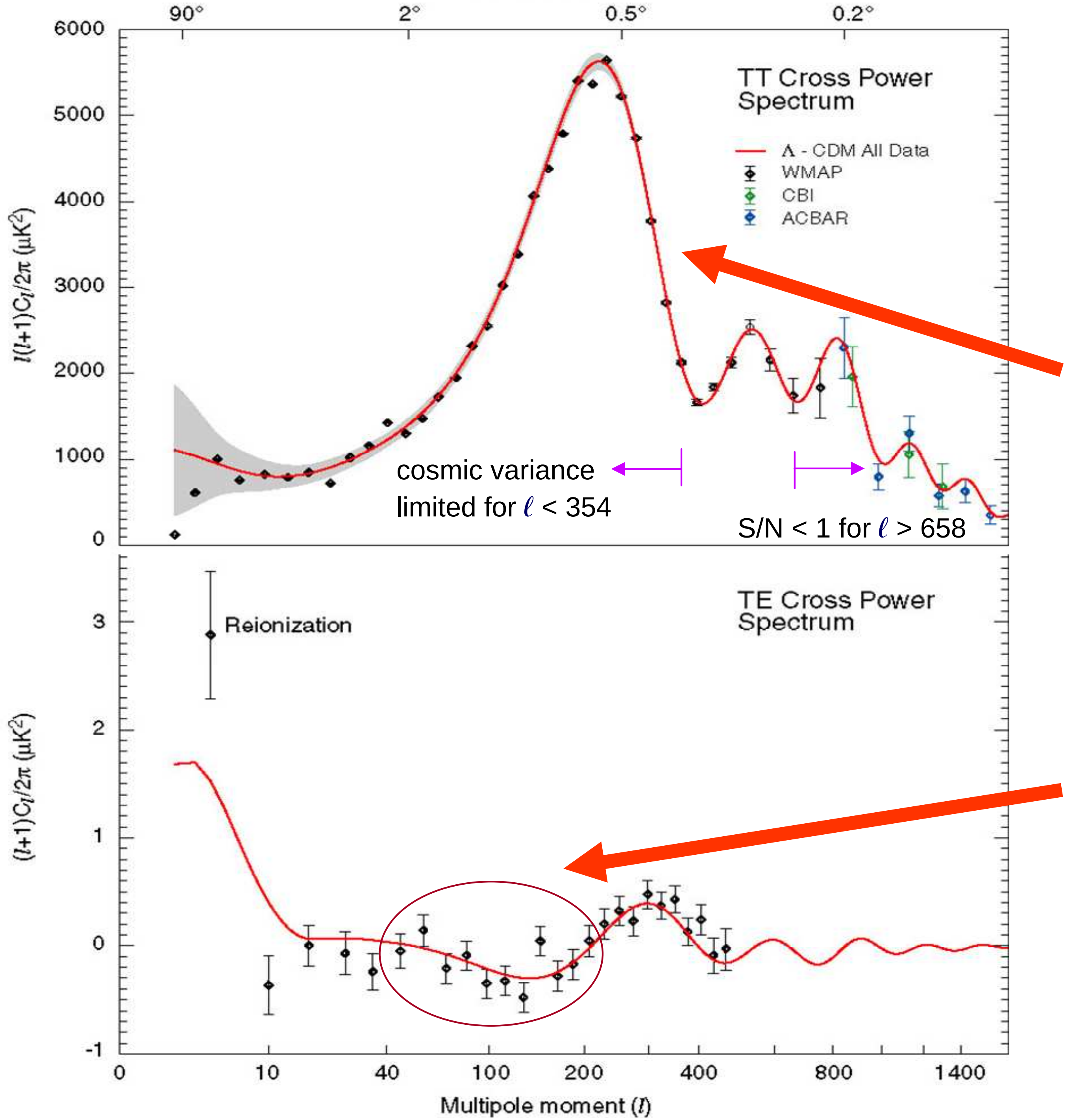
Decomposition in spherical harmonics

(C_l's mildly correlated since only ~85% of sky is mapped)

Coherent oscillations in photon-baryon plasma, excited by primordial density perturbations on super-horizon scales



WMAP does provide strong support for inflation



The characteristic features of scalar density perturbations generated during a (quasi-) de Sitter phase of expansion:

(a) ***Coherence*** of the Fourier modes → clean ‘acoustic peak’ structure on angular scales ($< 1^0$) which were sub-Hubble radius at last scattering ($z \sim 10^3$)

(b) Dipole out-of-phase with the monopole → ***negative cross-correlation*** between temperature and (electric) polarization on (*super*-Hubble radius) scales $\sim 1\text{-}5^0$

So the observed CMB anisotropy is ***consistent*** with having been generated by adiabatic, scale-invariant primordial density perturbations

But no ***tensor*** perturbations have yet been detected (through the expected B-mode polarization on large angular scales) … sets limit: $r \equiv T/S < 0.9$

$\Rightarrow$ Bound on inflationary energy scale: $V^{1/4} < 3 \times 10^{16}$ GeV

… thus no specific clue to the physics driving inflation

(GUT-scale? Hidden-sector scale? Electroweak scale?)

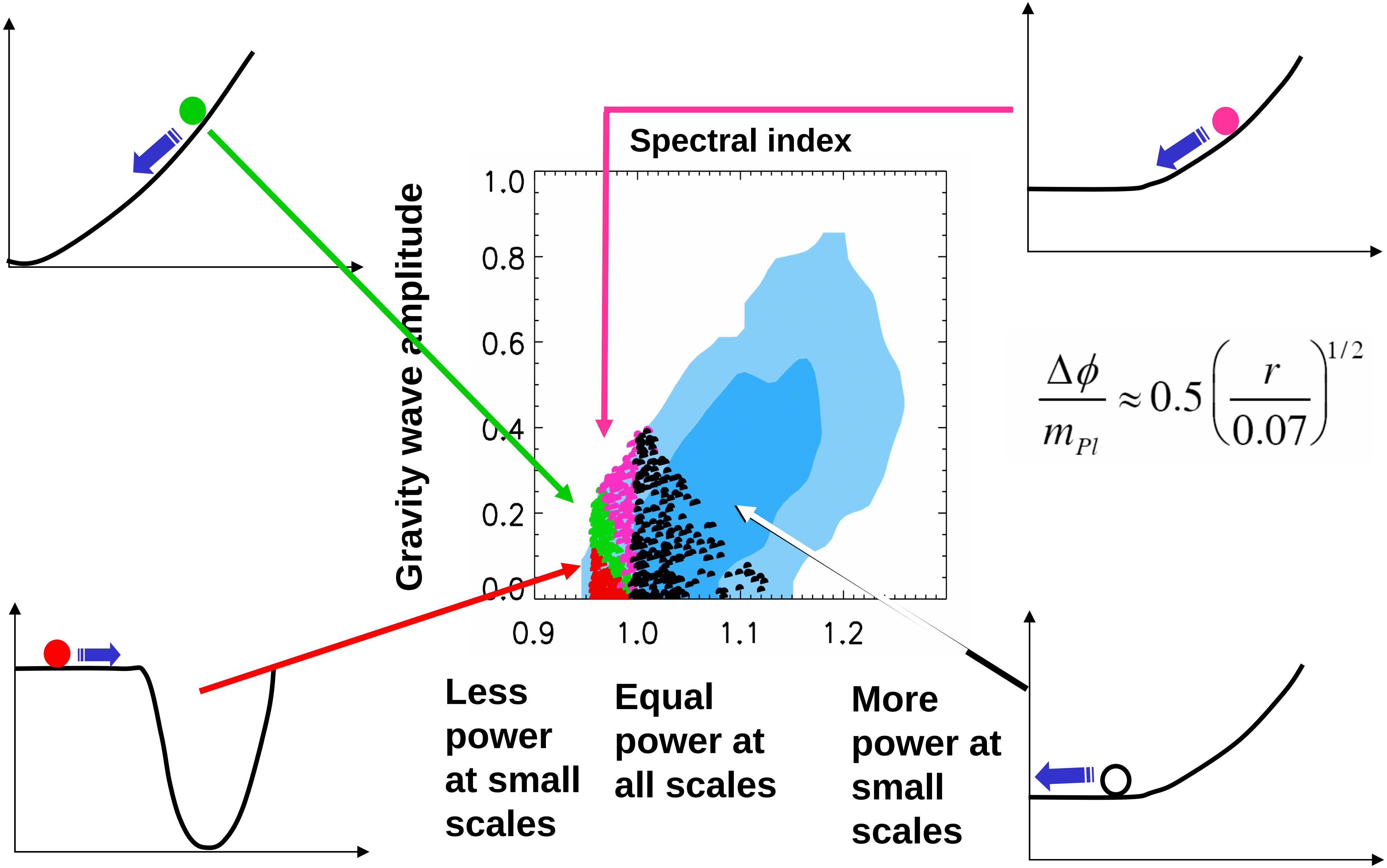
Can at best attempt to rule out 'toy models' (e.g. $V = \lambda\phi^4$) where inflation occurs at $\phi > M_P$ hence a large gravitational wave signal is predicted …

but such models have no ***physical*** basis in any case … require $\lambda \sim 10^{-12}$ in order to generate the correct magnitude ~10^{-5} of density perturbations!

Is there any signature in the data of the physics responsible for inflation?

… can discuss this only in the context of an effective field theory i.e. with $\phi << M_P$.

Constraining (single-field) inflationary models with *WMAP*



Less power at small scales

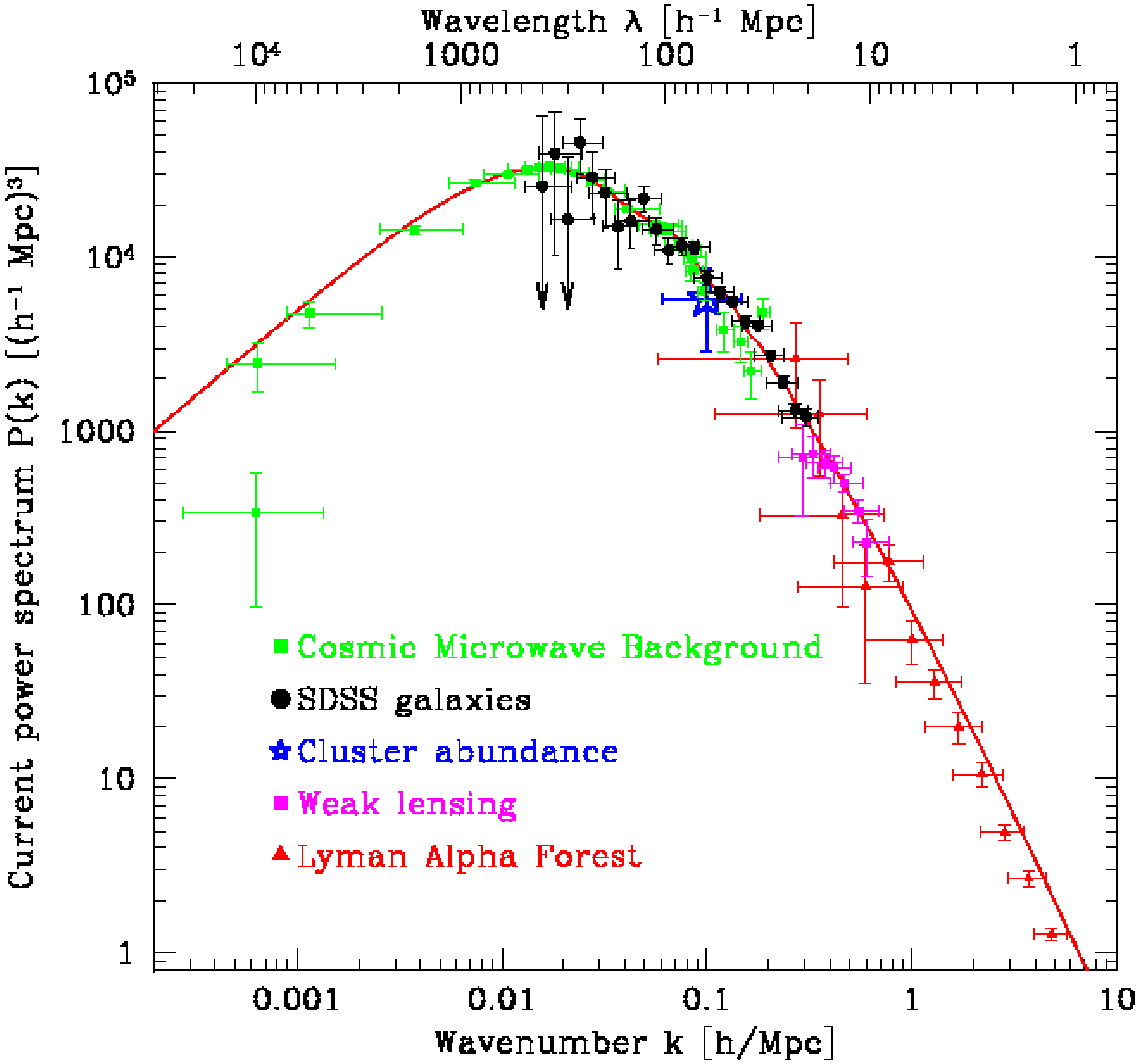
Equal power at all scales

More power at small scales

$$\frac{\Delta\phi}{m_{Pl}} \approx 0.5\left(\frac{r}{0.07}\right)^{1/2}$$

Observations of large-scale structure are *consistent* with the ΛCDM model if the primordial fluctuations are ***adiabatic*** and ***scale-invariant***

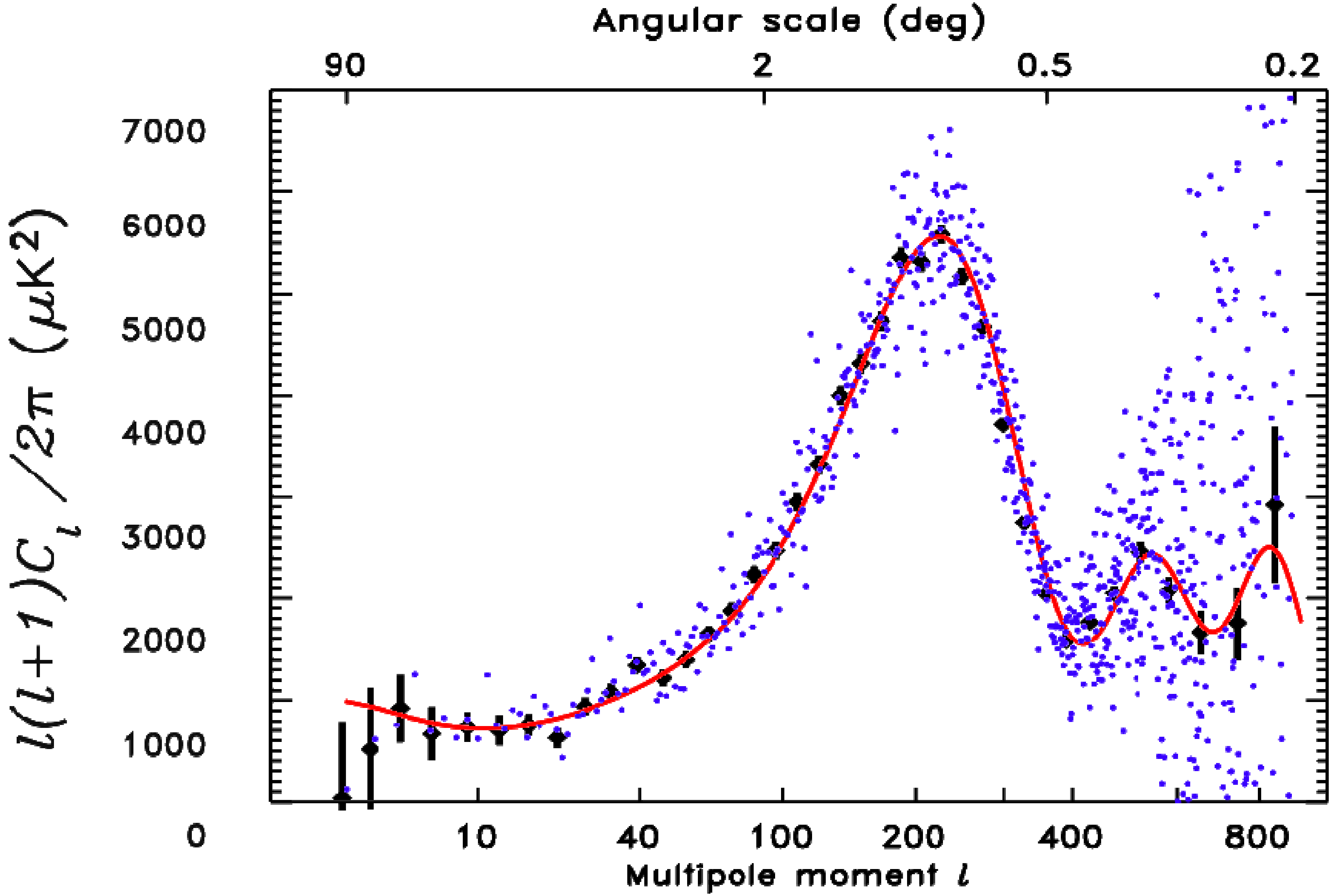
(as is apparently “expected in the simplest models of inflation”)



Tegmark. (2004)

However on closer examination, the ‘concordance model’ fit to WMAP is ***not*** so good

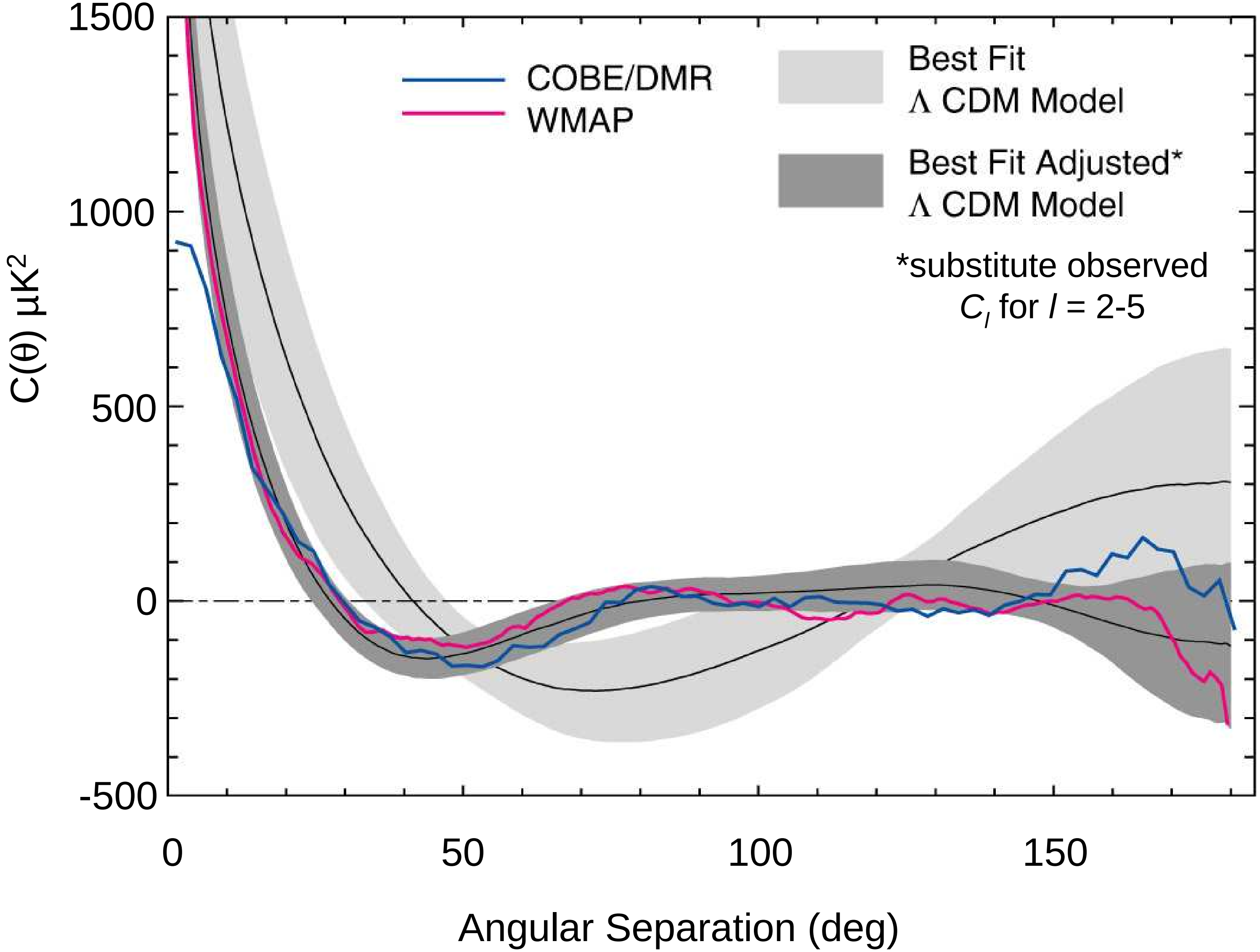
Best-fit: $\Omega_m h^2 = 0.14 \pm 0.02$, $\Omega_B h^2 = 0.024 \pm 0.001$, $h = 0.72 \pm 0.05$, $n = 0.99 \pm 0.04$



But the χ^2/dof = 973/893 ⇒ probability of only 3% that this model is correct!

The lack of power on large angular scales has received most attention

… although this is claimed to be ***not*** too unlikely after taking cosmic variance and uncertainties in foreground subtraction into account → chance probability of $O(1\%)$?



$$C(\theta) = \langle T(n_i) T(n_j) \rangle$$

$$n_i \cdot n_j = \cos\theta$$

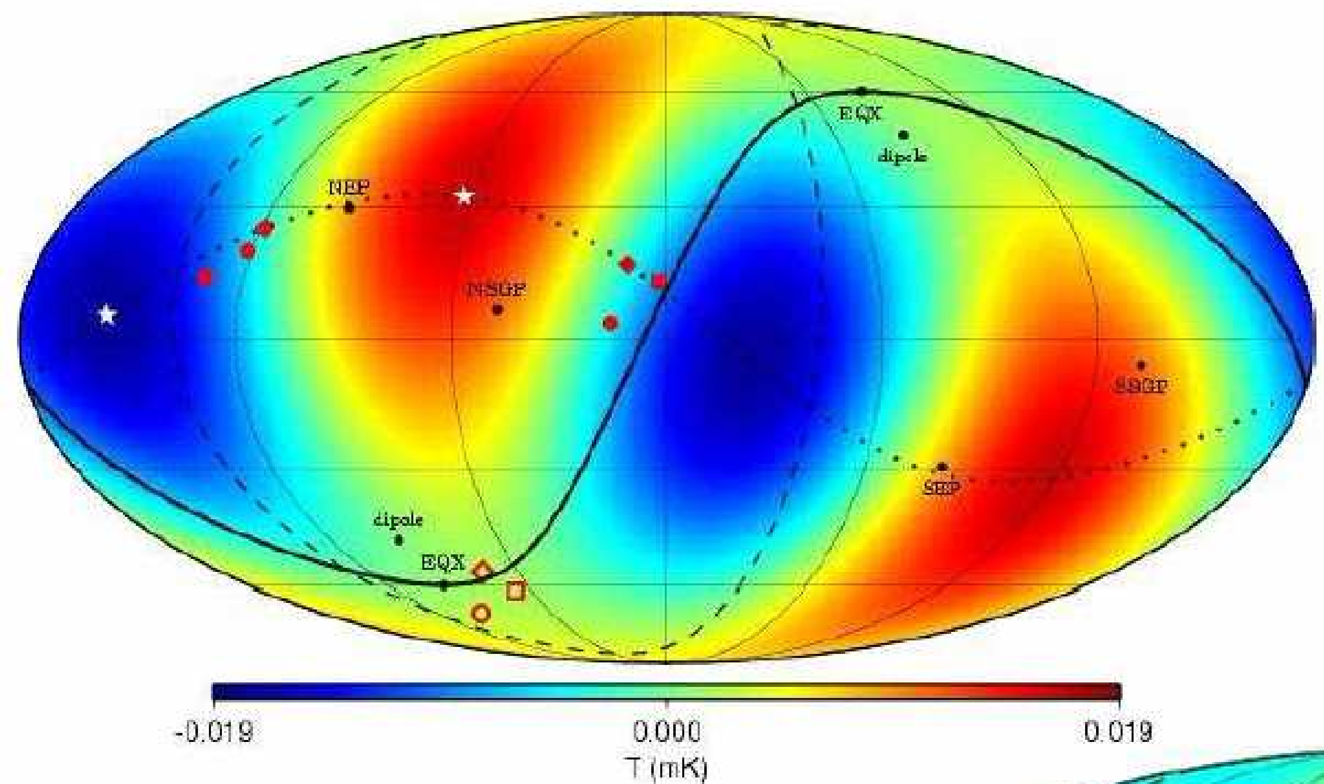
$$S = \int_{60^\circ}^{180^\circ} C(\theta)^2 d\theta$$

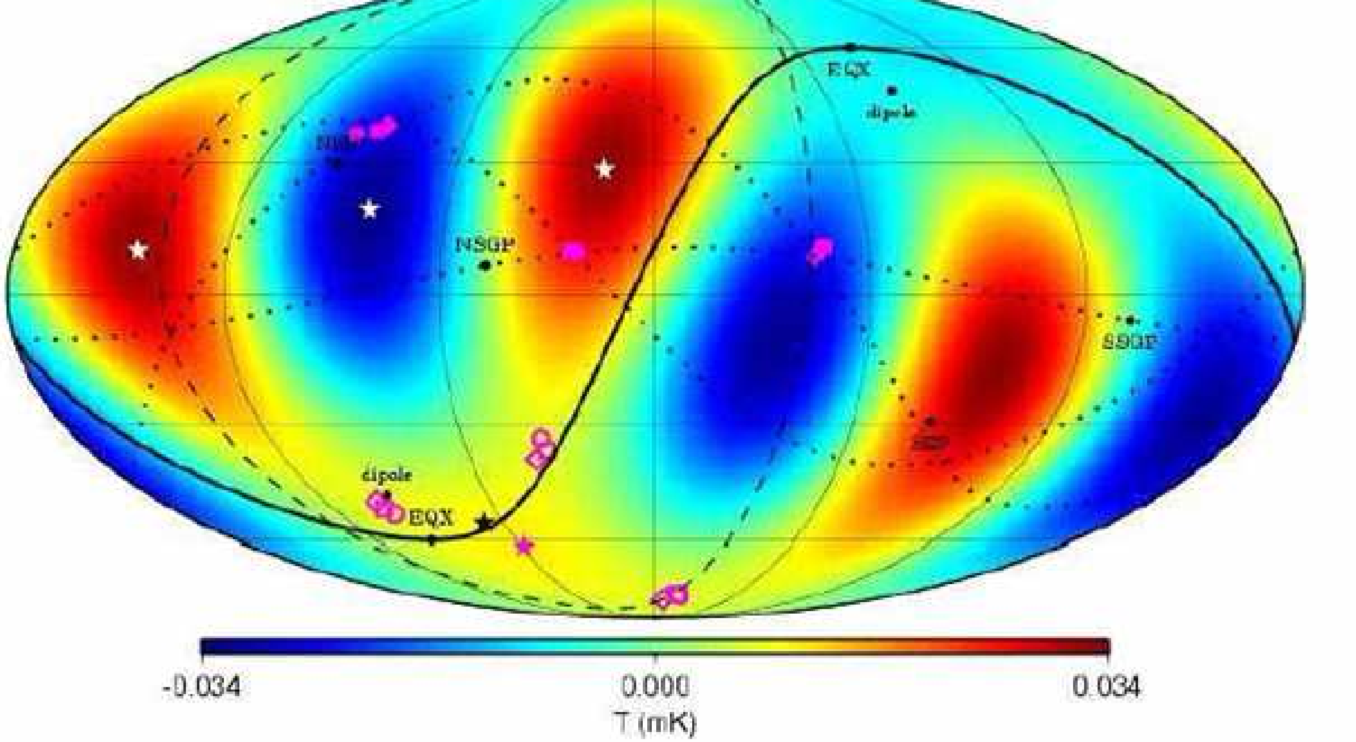
A posteriori likelihood of observed S is only 0.15 - 0.3 %

However there are unexpected alignments of low multipoles (with ecliptic plane), a cold spot, differences between North and South ecliptic hemispheres … is the low-ℓ CMB of local origin?

Alignment of the quadrupole and the octupole … along the ecliptic

Copi, Schwarz & Starkman (2004)





ANOMALIES SUMMARY (Kate Land)

★ Axis of Evil ~(260,60)

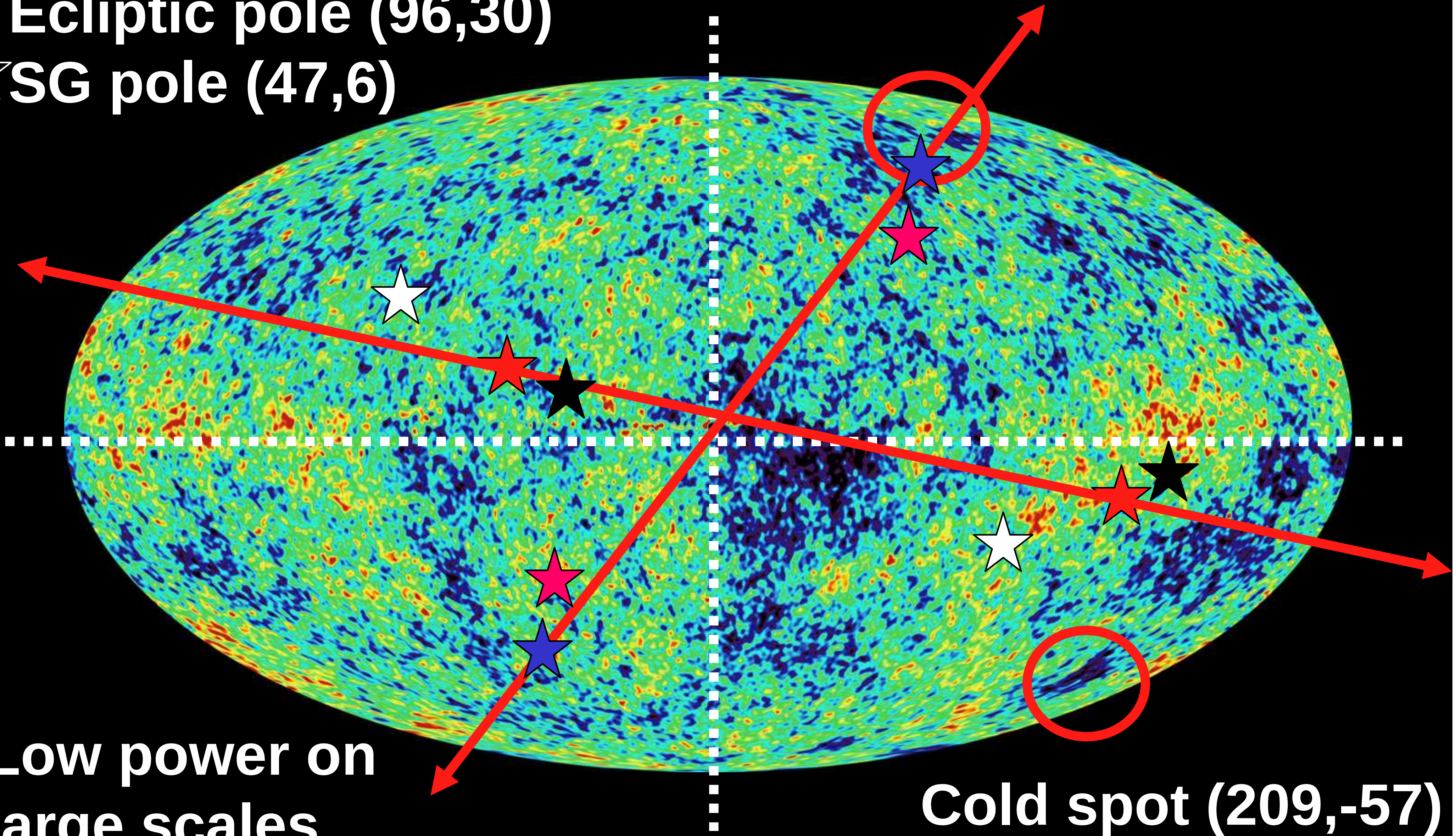
★ Dipole (264,48)

Virgo ~(260,70)

★ Max asym axis (57,10)

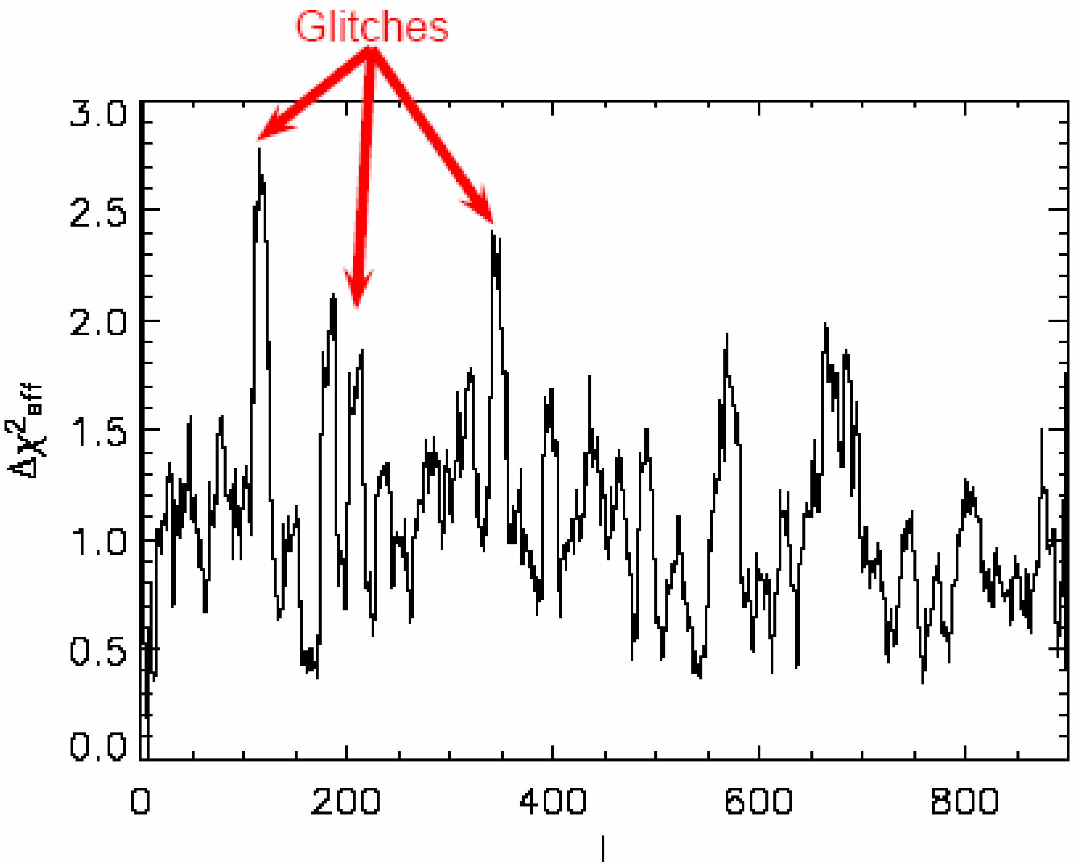
★ Ecliptic pole (96,30)

☆ SG pole (47,6)



Low power on large scales

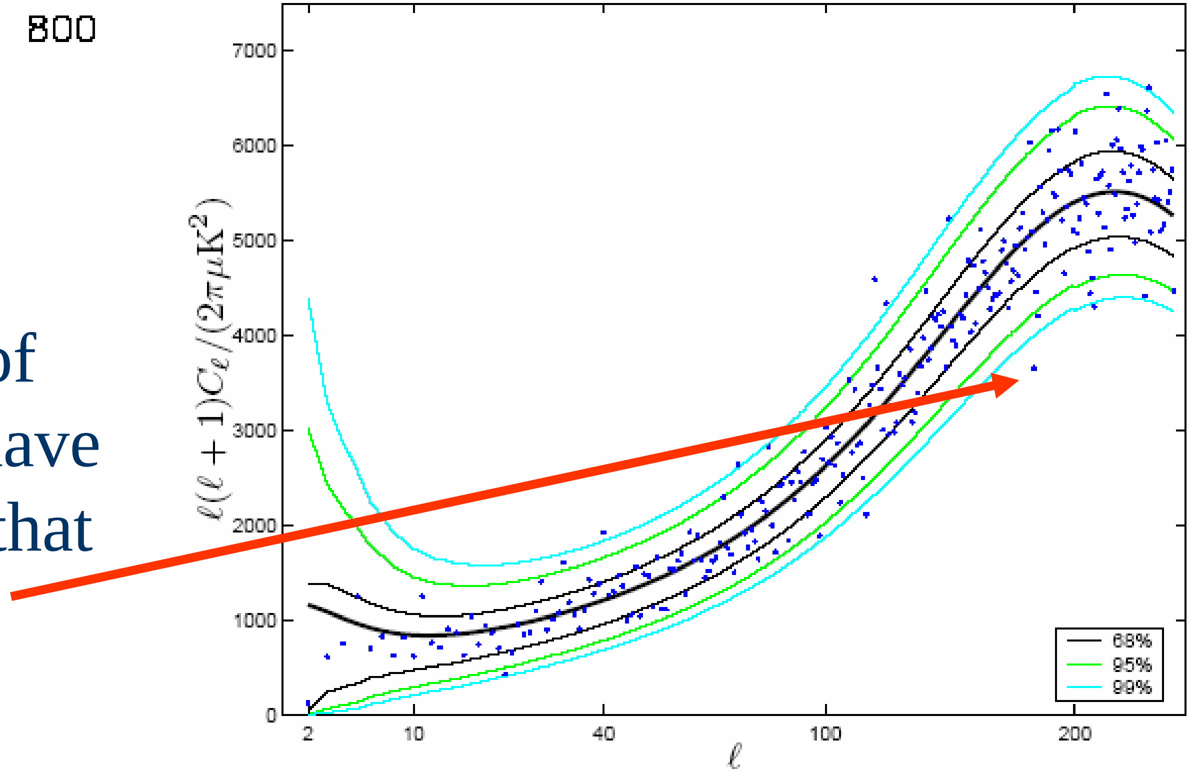
Cold spot (209,-57)



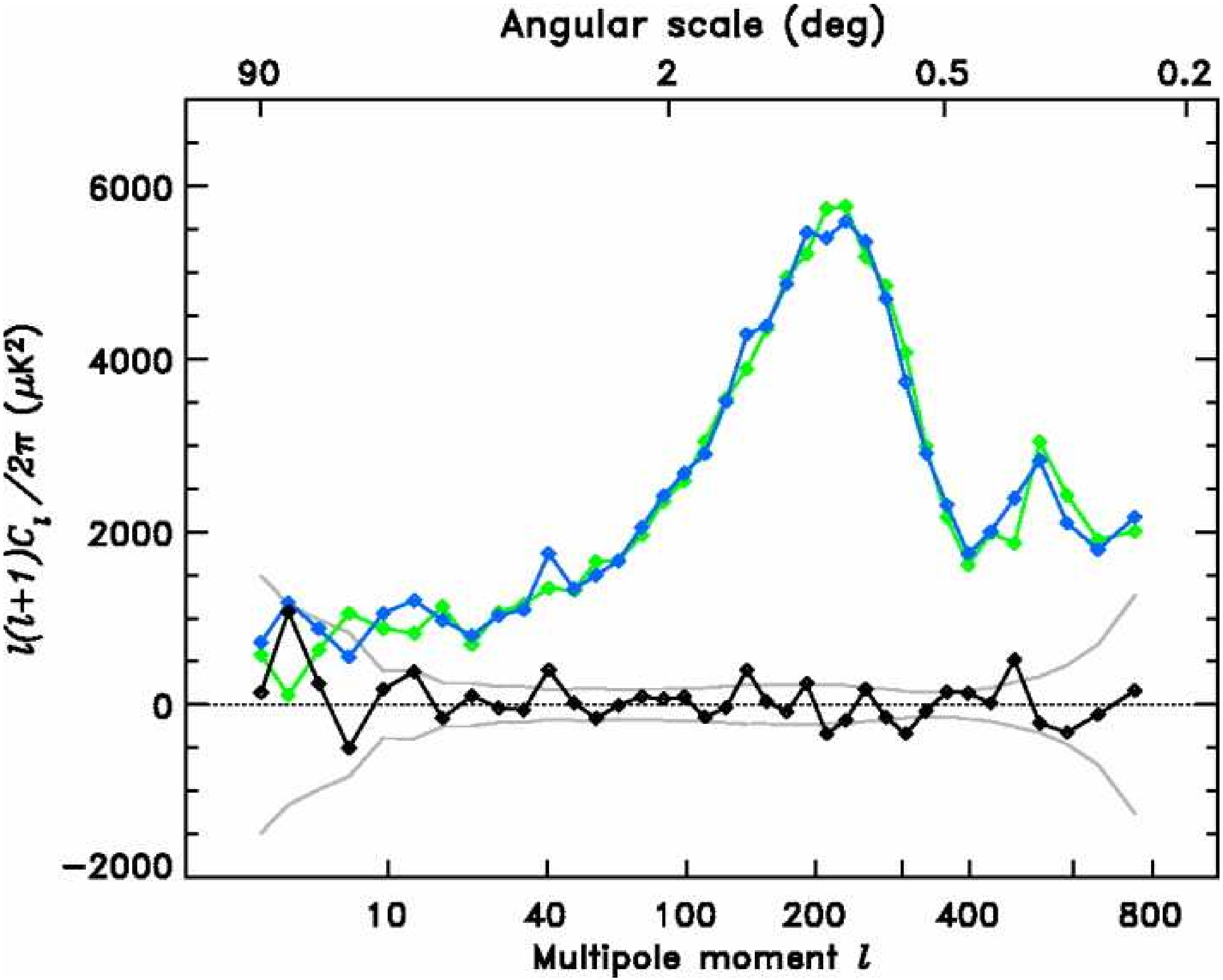
But the excess χ^2 comes mostly from the ‘glitches’ (>3σ outliers in TT spectrum)

hard to tell *by eye* from binned C_ℓ’s since neighbouring data points are ***correlated*** …

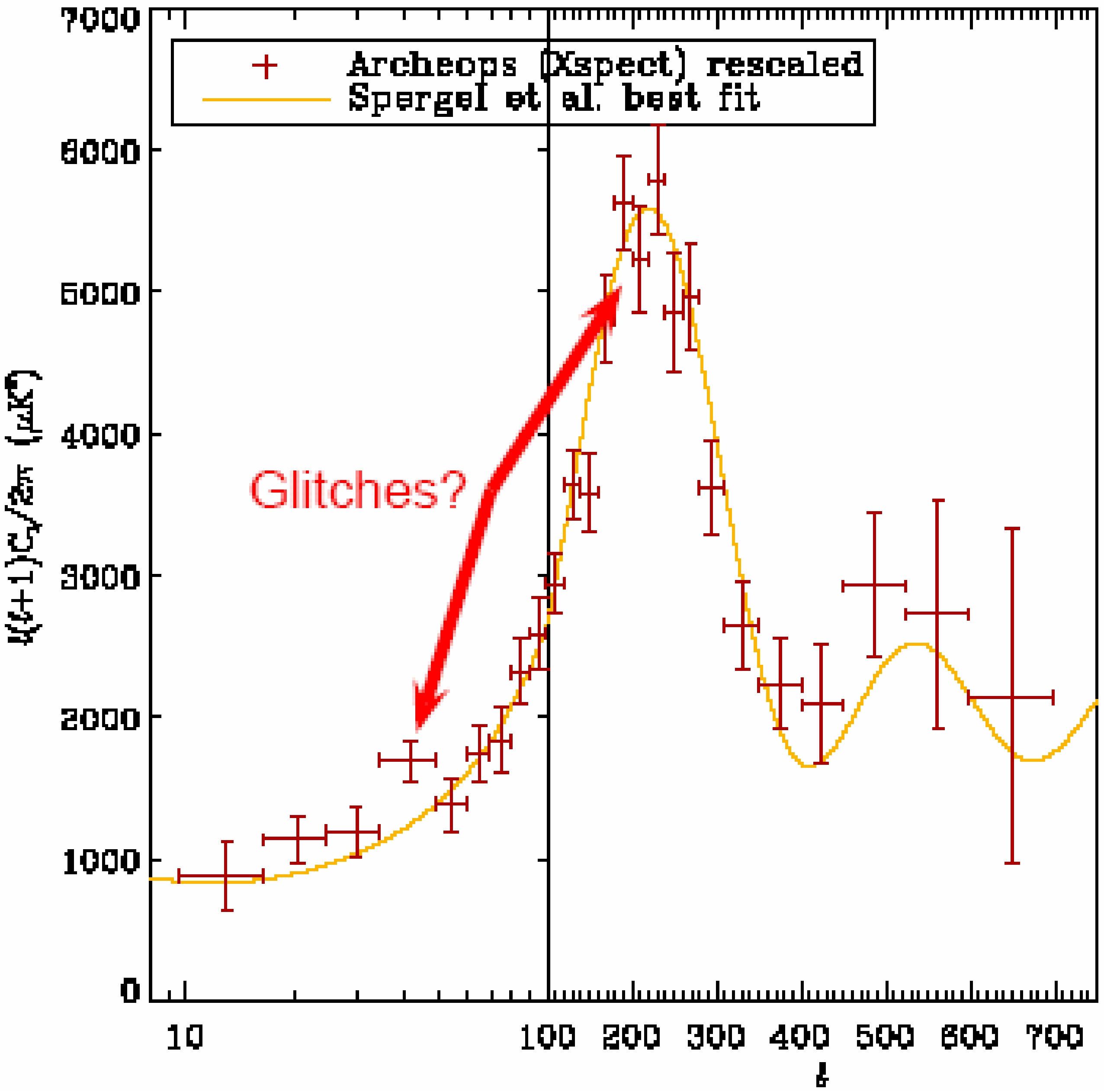
For example only 3 out of 16000 simulations would have a lower value of C_{181} than that observed (Lewis 2004)



Some of the ‘glitches’ are admittedly seen only in the angular power spectrum towards the ecliptic poles (blue), not in the ecliptic plane (green) so may not be of cosmic origin

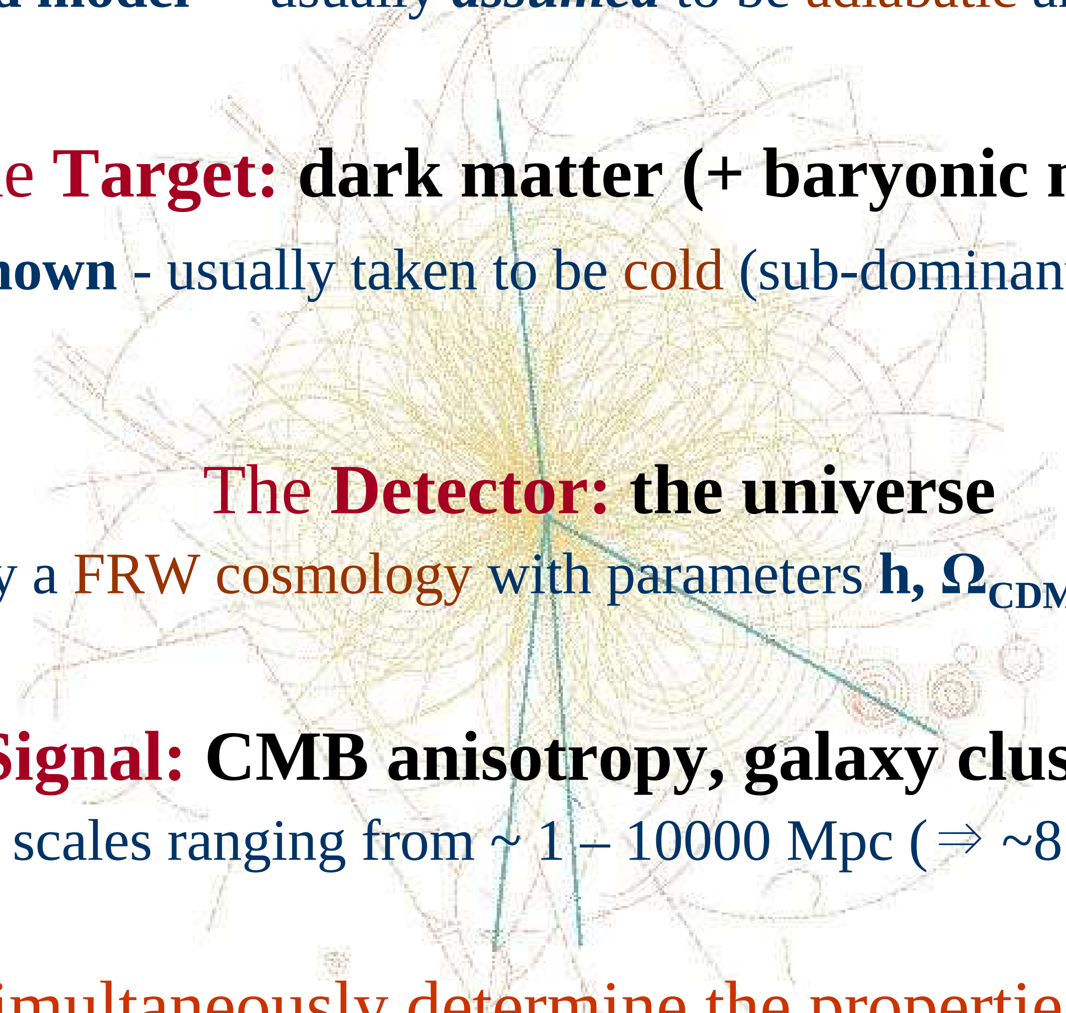


Similar ‘glitches’ have also been seen by ***Archeops*** at ℓ ~ 40, 220 (although not as significant)



suggests that the primordial density perturbation has ‘features’ – can we ***prove*** this?

The formation of large-scale structure is akin to a scattering experiment



The **Beam:** **inflationary density perturbations**

No 'standard model' – usually ***assumed*** to be adiabatic and ~scale-invariant

The **Target:** **dark matter (+ baryonic matter)**

Identity unknown - usually taken to be cold (sub-dominant 'hot' component?)

The **Detector:** **the universe**

modelled by a FRW cosmology with parameters **h, Ω_{CDM} , Ω_B , Ω_Λ , Ω_K ...**

The **Signal:** **CMB anisotropy, galaxy clustering …**

measured over scales ranging from ~ 1 – 10000 Mpc (⇒ ~8 e-folds of inflation)

We cannot simultaneously determine the properties of ***both*** the **beam** and the **target** with unknown **detector**… hence must adopt suitable 'priors' (e.g. on **h, Ω_{CDM}** , …) in order to break parameter **degeneracies**

Astronomers have traditionally ***assumed*** a Harrison-Zeldovich spectrum:

$$P(k) \propto k^n, \;\; n = 1$$

But models of inflation generally predict departures from scale-invariance

… even in **single-field slow-roll models**: $n = 1 + 2V''/V - 3\,(V'/V)^2$

Since the potential $V(\phi)$ steepens towards the end of inflation, there will be a ***scale-dependent spectral tilt*** on cosmologically observable scales:

e.g. in simple F-term supergravity model: $V(\phi) \simeq V_o - \alpha\phi^2 + \ldots \;\Rightarrow\; n \simeq 1 - 2/N_* \sim 0.96$

where $N_* \approx 50 + \ln\,(k^{-1}/3000h^{-1}\ \mathrm{Mpc})$ is the #-e-folds from the ***end*** of inflation

In **hybrid models**, inflation is ended by the 'waterfall' field, ***not*** by steepening of $V(\phi)$, so spectrum can be quite close to scale-invariant …

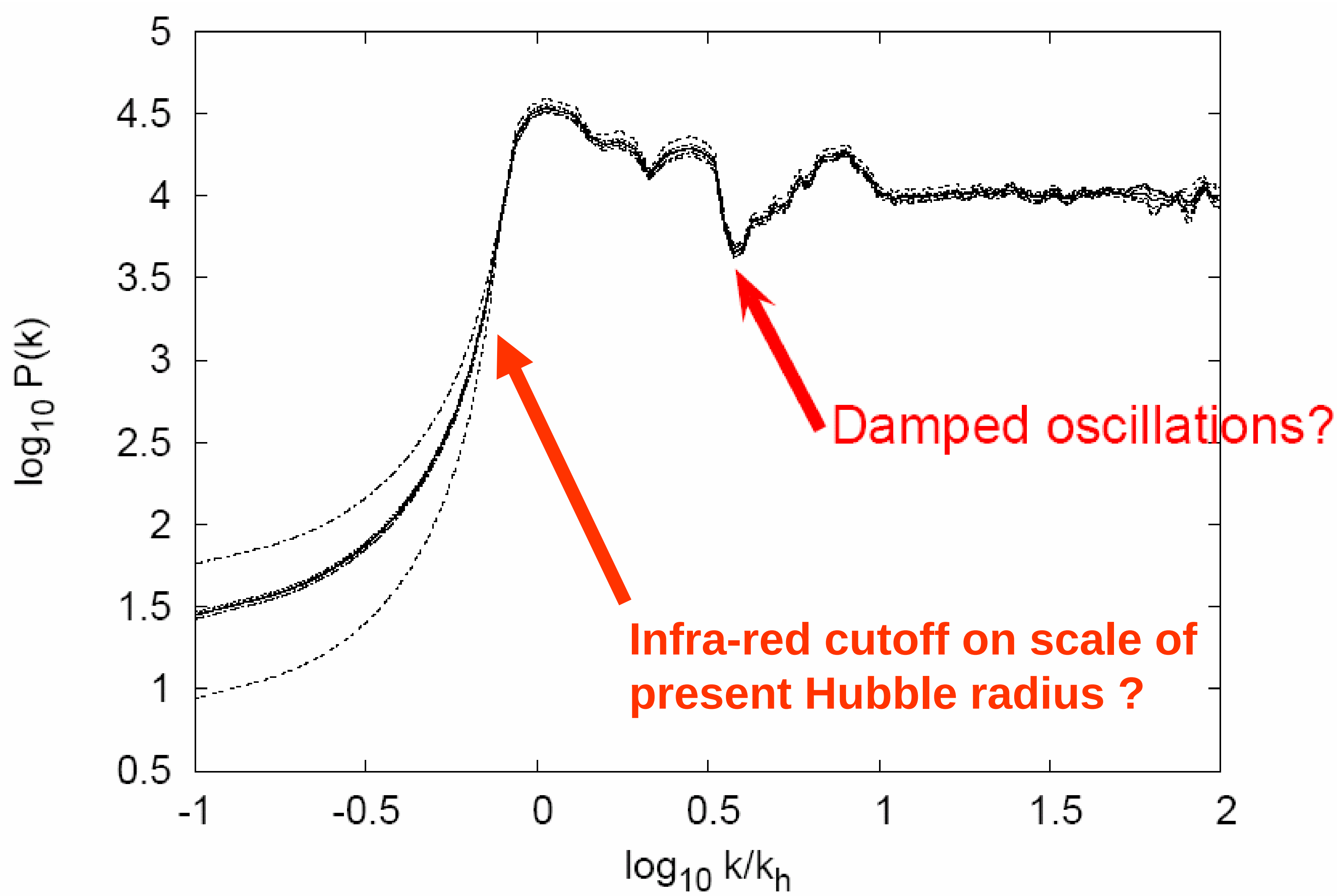
However, in general there are ***many*** other fields present, whose dynamics may ***interrupt*** the inflaton's slow-roll (rather than terminate it altogether)

→ can generate features in the spectrum ('steps', 'oscillations', 'bumps' …)

Many people have attempted to reconstruct the primordial spectrum (***assuming*** ΛCDM)

Bridle, Lewis, Weller & Efstathiou 2003; Cline, Crotty & Lesgourgues 2003, Mukherjee & Wang 2003; Hannestad 2004; Kogo, Sasaki & Yokoyama 2004; Tocchini-Valentini, Douspis & Silk 2004, …

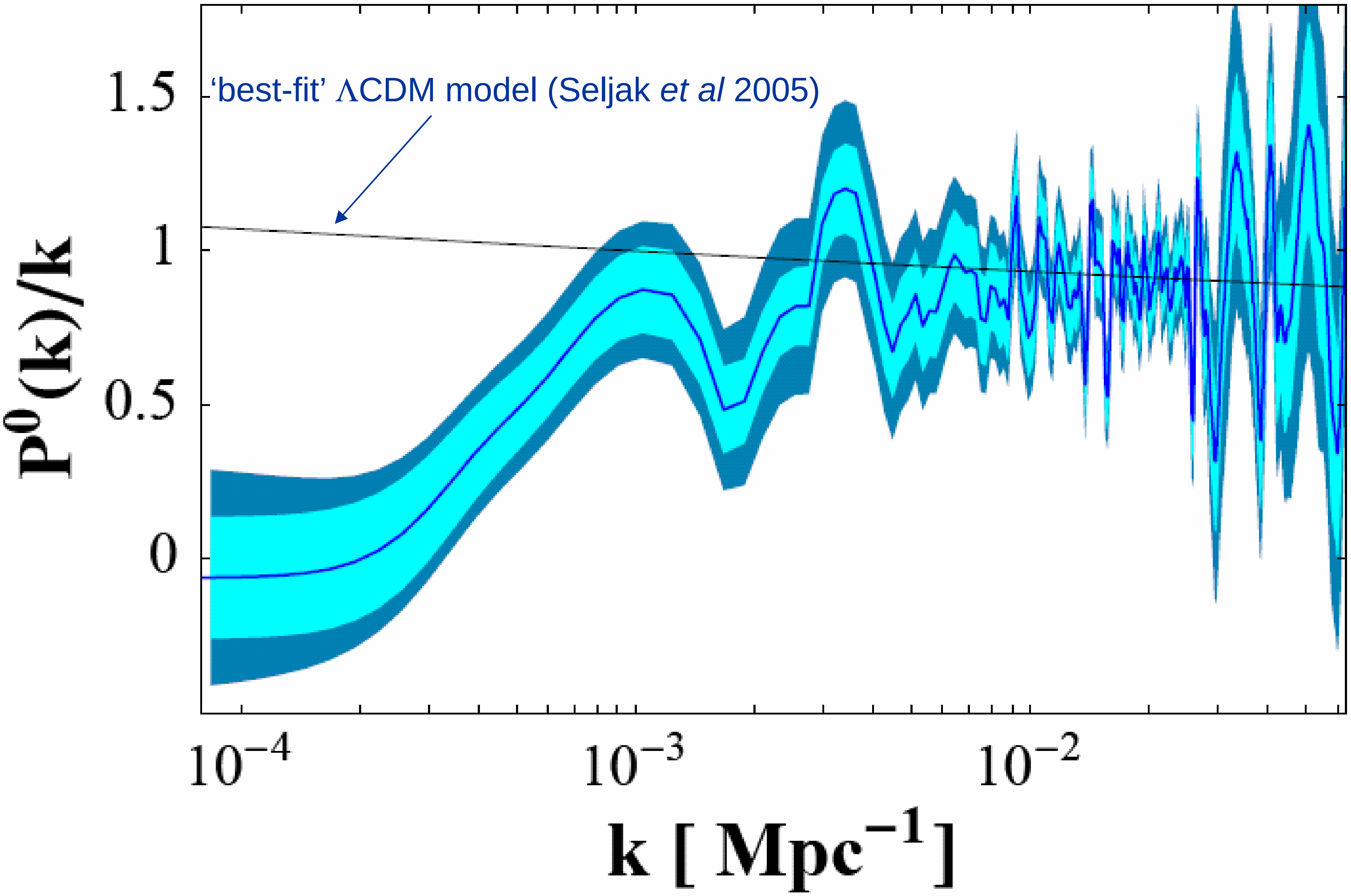
Shafieloo & Souradeep (2004) used Richardson-Lucy inversion on WMAP data to get:



Damped oscillations?

Infra-red cutoff on scale of present Hubble radius ?

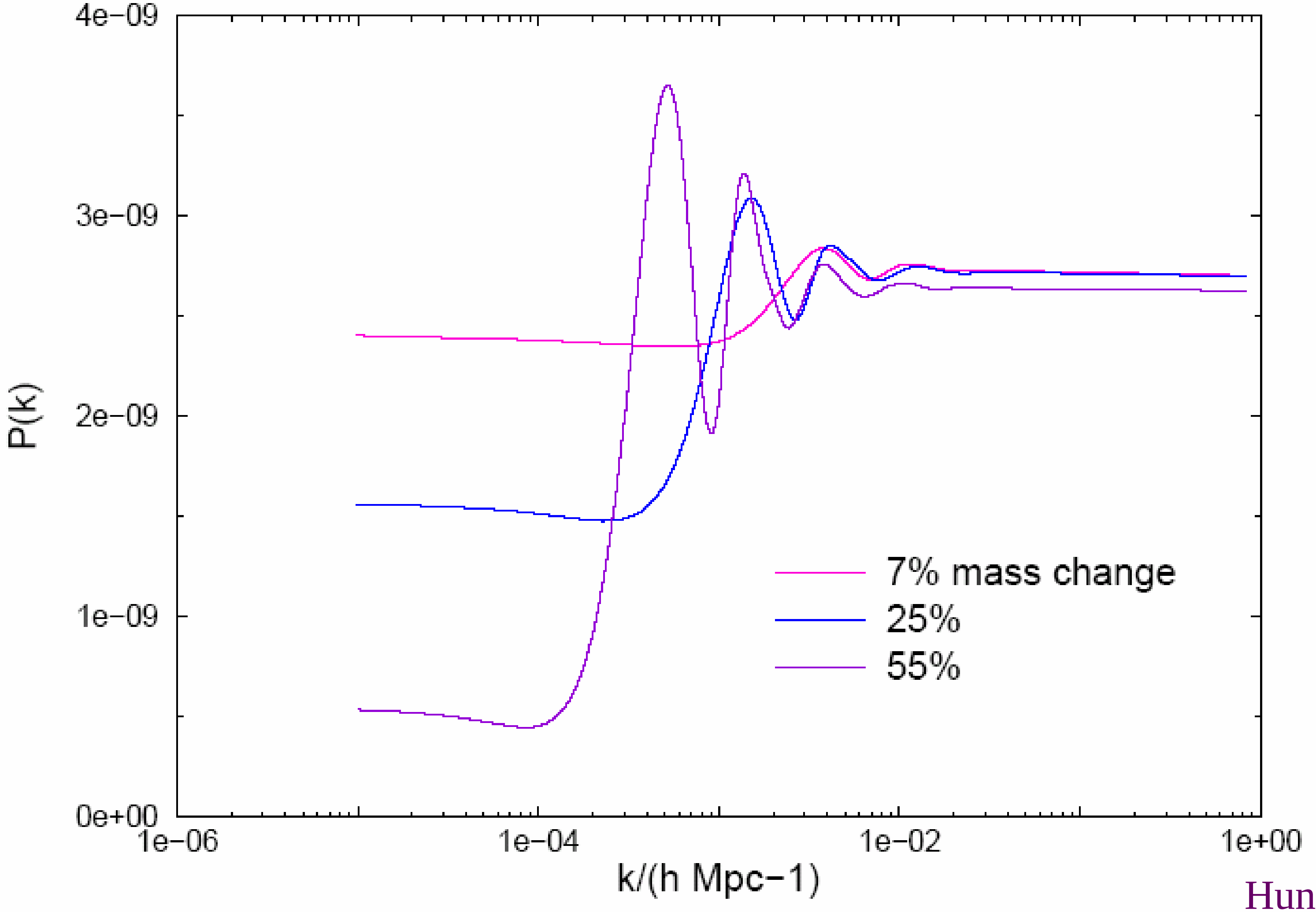
Similar results have been obtained by other (non-parametric) reconstruction methods:



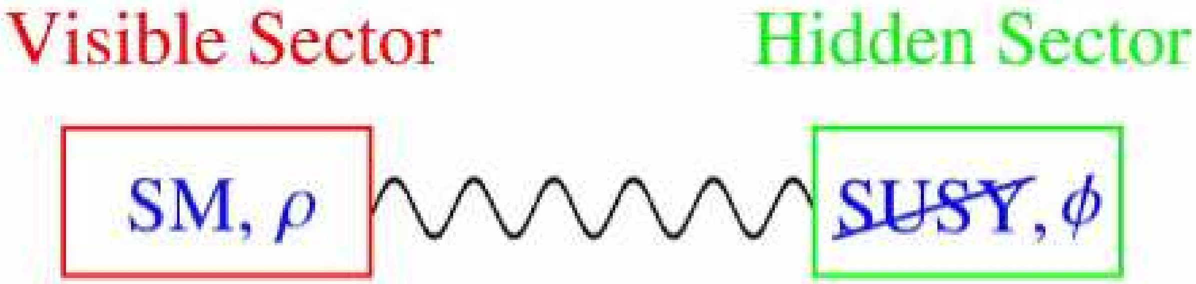
Tochhini-Valentini, Hoffman & Silk (2005)

Such spectra arise ***naturally*** if the inflaton mass changes suddenly, e.g. due to its coupling (through gravity) to a field which undergoes a fast symmetry-breaking phase transition in the rapidly cooling universe (Adams, Ross & Sarkar 1997)

This must happen as cosmologically interesting scales (today) 'exit the horizon' ... not unlikely if (last phase of) inflation did not last much longer than ~50 e-folds



Hunt & Sarkar (2005)


Visible Sector

Hidden Sector

SM, ρ

~~SUSY~~, ϕ

Visible sector could be important during inflation if gauge symmetry breaking occurs.

SUSY theories contain flat directions in field space where the potential vanishes in unbroken SUSY limit.

Flat directions lifted by

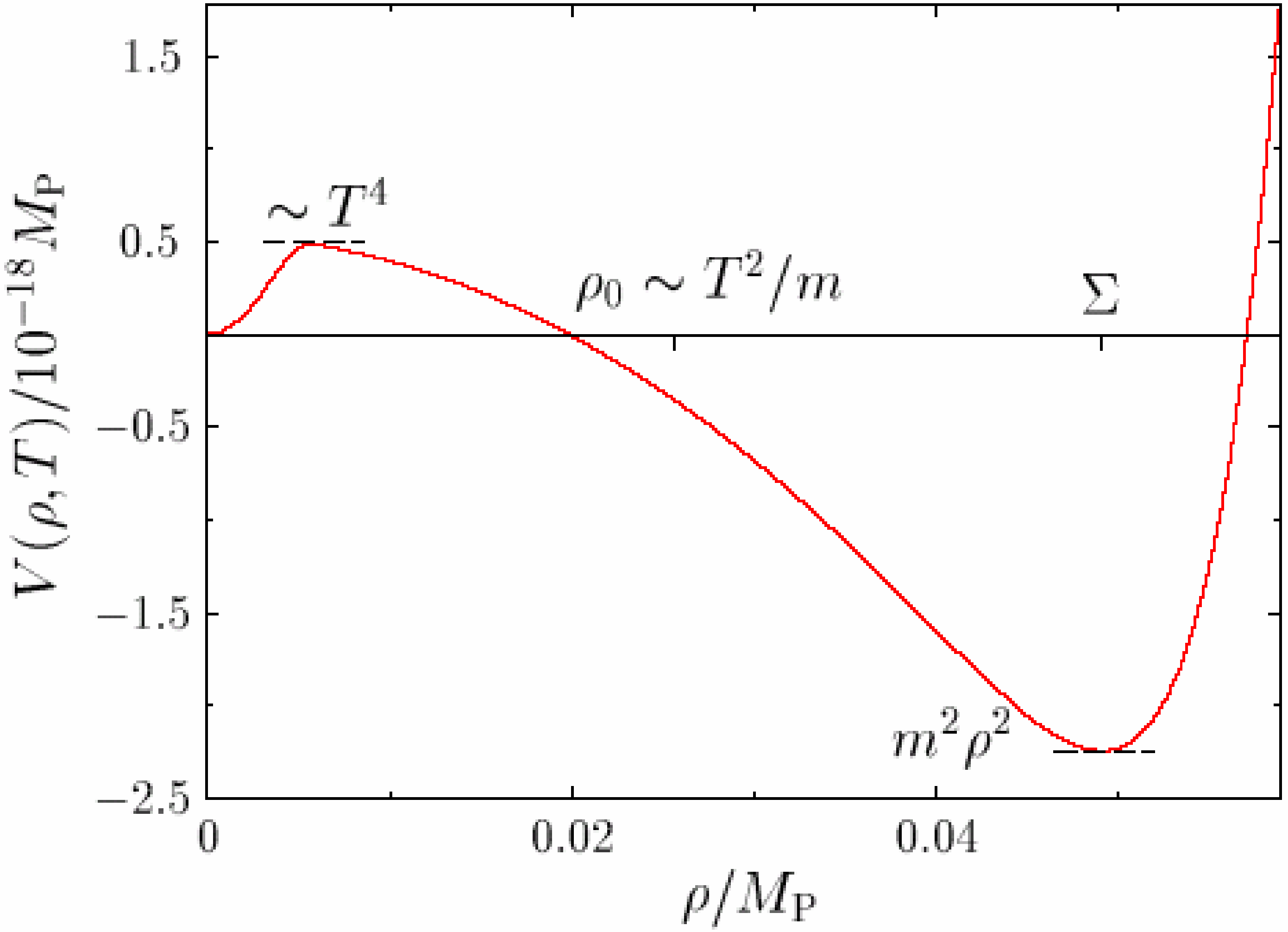
- ~~SUSY~~.
- Higher dimensional operators $\rho^n/M_{\mathrm{P}}^{n-4}$.

If m^2 is negative, ρ is stabilised at $\Sigma \sim \mathcal{O}\left(M_{\mathrm{P}}^2\left|m^2\right|\right)^{1/(n-4)}$, by $\rho^n/M_{\mathrm{P}}^{n-4}$ terms

Assume that in the era preceding observable inflation, all fields (with gauge and/or Yukawa couplings) are in thermal equilibrium

Including the one-loop finite temperature correction

$$V(\rho,T) \simeq \begin{cases} C_1 T^2 \rho^2, & \text{for} \quad \rho \ll T \\ -m^2\rho^2 + \frac{1}{90}\pi^2 N_{\mathrm{h}}(T) T^4 + \frac{\gamma\rho^n}{M_{\mathrm{P}}^{n-4}}, & \text{for} \quad T \ll \rho < \Sigma \end{cases}$$

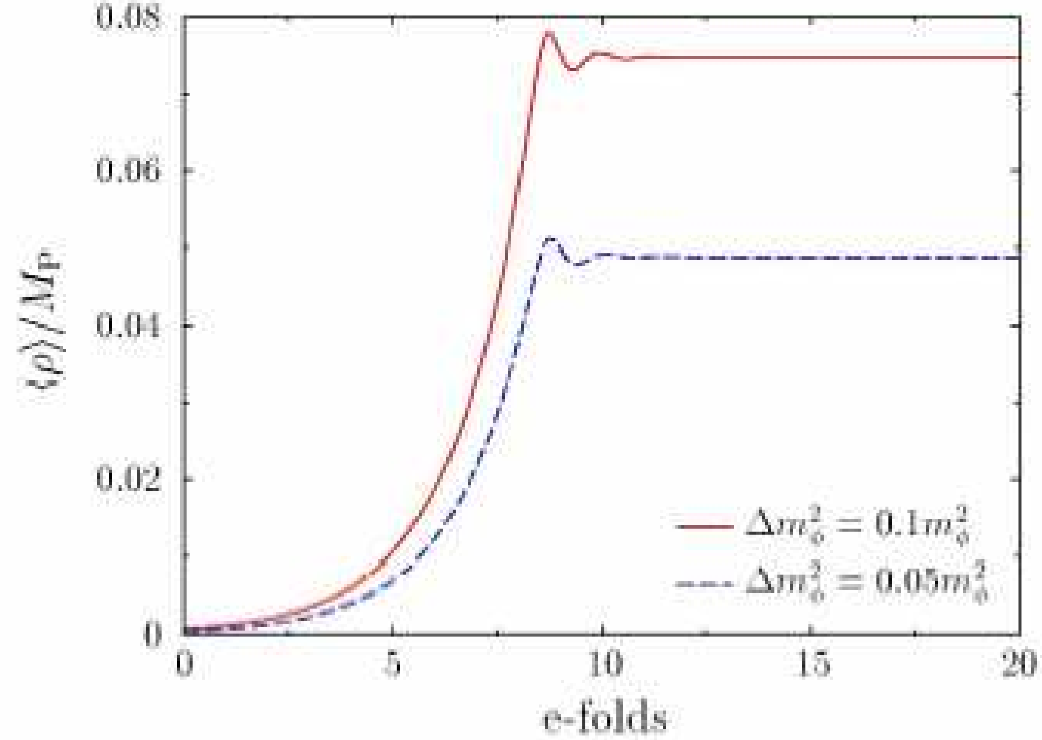


Tunneling rate through thermal barrier between $\rho = 0$ and $\rho \sim T^2/m$ is negligible $\Rightarrow$ $\rho = 0$ until $T \sim m$ when barrier disappears.

ρ evolves to the global minimum at Σ as

$$\ddot{\rho} + 3H\dot{\rho} = -\frac{dV}{d\rho}.$$

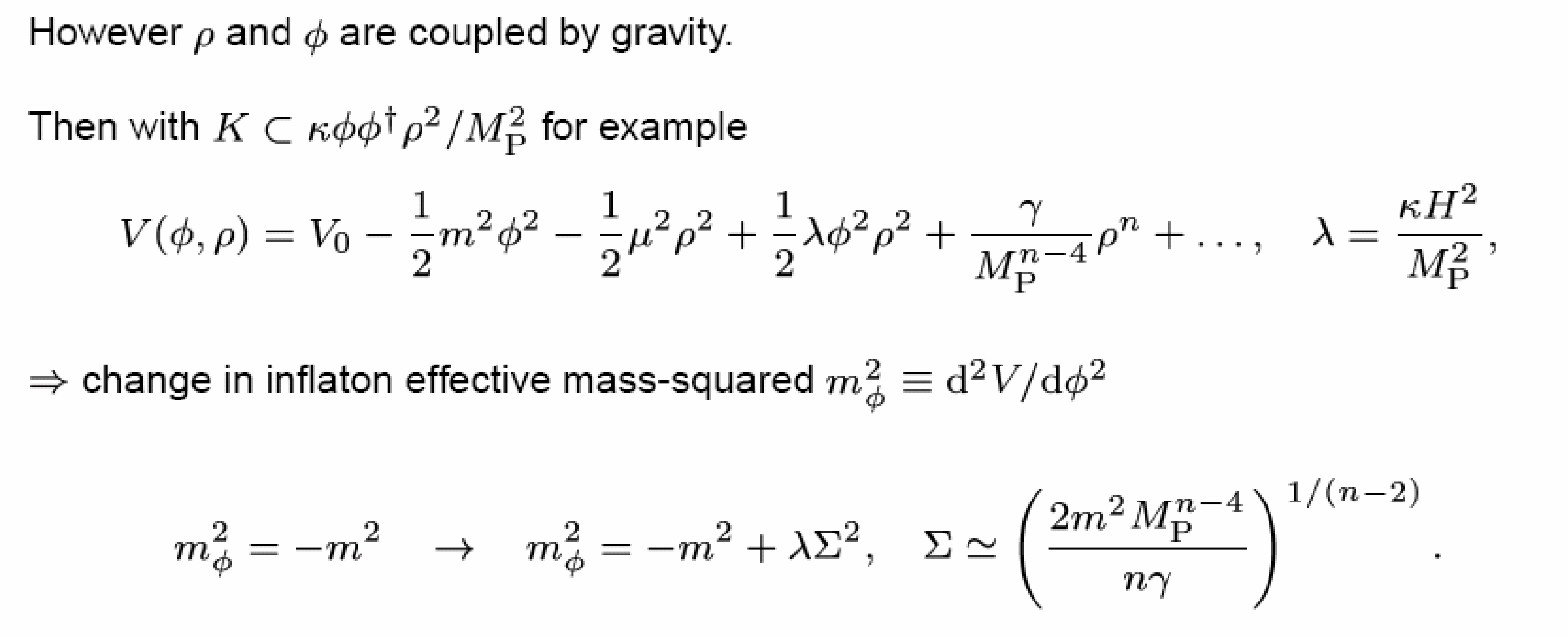
$$\Rightarrow \rho \quad \simeq \quad \begin{cases} \rho_0 \exp\left[\frac{3Ht}{2}\left(\sqrt{1+\frac{8m^2}{9H^2}}\right)-1\right], & \langle\rho\rangle \ll \Sigma \\ \Sigma + K_1 \exp\left(-\frac{3Ht}{2}\right)\sin\left[\frac{3Ht}{2}\sqrt{(n-2)\frac{8m^2}{9H^2}-1}+K_2\right], & \langle\rho\rangle \sim \Sigma \end{cases}$$



After the phase transition slow-roll inflation continues but at a reduced scale

$$V(\phi) \quad \rightarrow \quad \left[1-(\Sigma/M_{\rm P})^2\right] V(\phi)$$

For $\Sigma \ll M_{\rm P}$ the change is negligible and so H can be taken to be sensibly constant



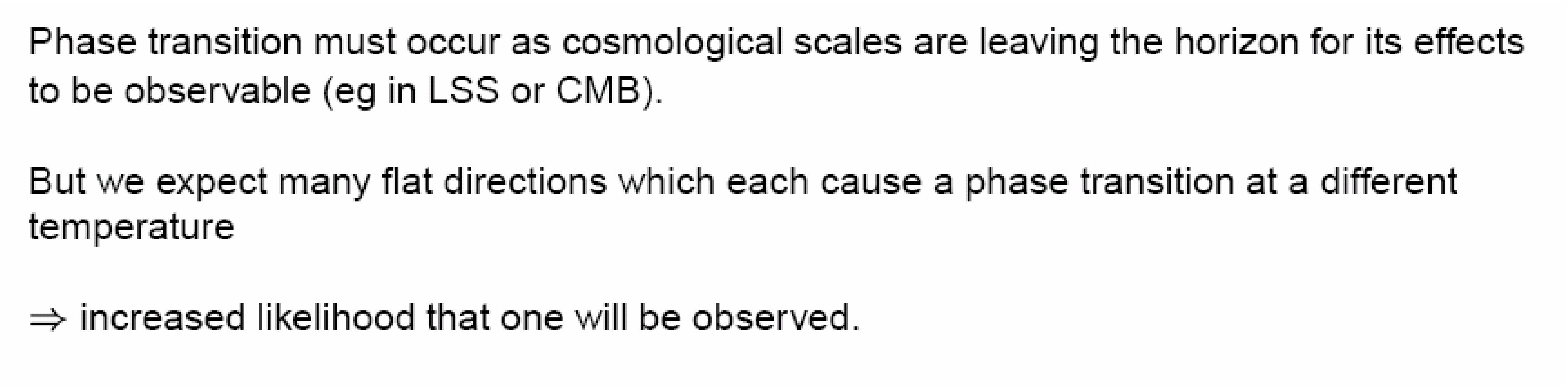
However ρ and ϕ are coupled by gravity.

Then with $K \subset \kappa\phi\phi^\dagger\rho^2/M_{\rm P}^2$ for example

$$V(\phi,\rho) = V_0 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\mu^2\rho^2 + \frac{1}{2}\lambda\phi^2\rho^2 + \frac{\gamma}{M_{\rm P}^{n-4}}\rho^n + \ldots, \qquad \lambda = \frac{\kappa H^2}{M_{\rm P}^2},$$

$\Rightarrow$ change in inflaton effective mass-squared $m_\phi^2 \equiv {\rm d}^2V/{\rm d}\phi^2$

$$m_\phi^2 = -m^2 \quad \rightarrow \quad m_\phi^2 = -m^2 + \lambda\Sigma^2, \quad \Sigma \simeq \left(\frac{2m^2M_{\rm P}^{n-4}}{n\gamma}\right)^{1/(n-2)} .$$



Phase transition must occur as cosmological scales are leaving the horizon for its effects to be observable (eg in LSS or CMB).

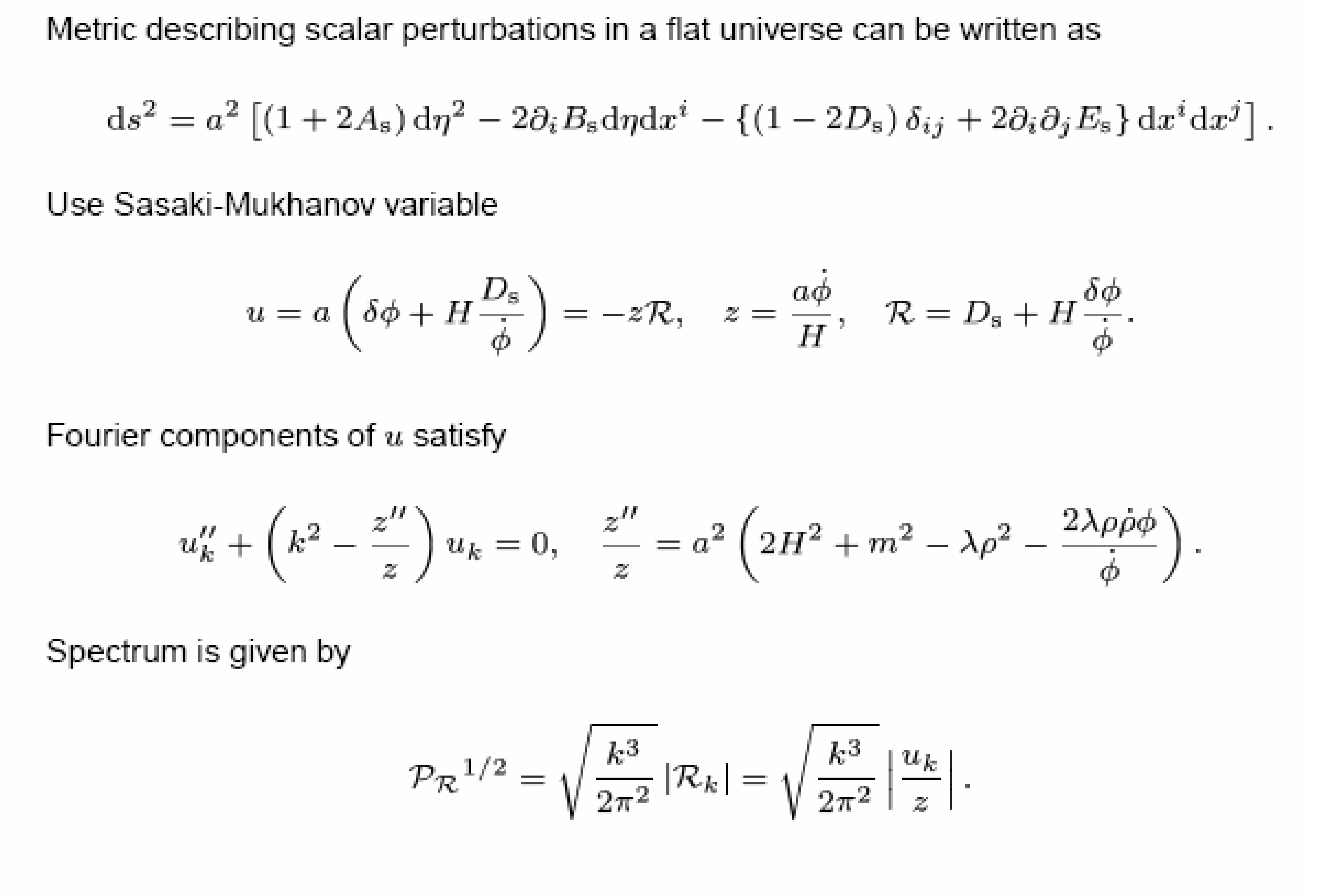
But we expect many flat directions which each cause a phase transition at a different temperature

$\Rightarrow$ increased likelihood that one will be observed.

This assumes that the **initial conditions are thermal** (so ρ is at the origin) and (this *last* phase of) **inflation lasts just ~50 e-folds** so as to create our present Hubble volume

Seems fine-tuned but the ***data does indicate an IR cut-off*** at the present Hubble radius

The Spectrum



Metric describing scalar perturbations in a flat universe can be written as

$$\mathrm{d}s^2 = a^2\left[(1+2A_\mathrm{s})\,\mathrm{d}\eta^2 - 2\partial_i B_\mathrm{s}\mathrm{d}\eta\mathrm{d}x^i - \left\{(1-2D_\mathrm{s})\,\delta_{ij} + 2\partial_i\partial_j E_\mathrm{s}\right\}\mathrm{d}x^i\mathrm{d}x^j\right].$$

Use Sasaki-Mukhanov variable

$$u = a\left(\delta\phi + H\frac{D_\mathrm{s}}{\dot{\phi}}\right) = -z\mathcal{R}, \quad z = \frac{a\dot{\phi}}{H}, \quad \mathcal{R} = D_\mathrm{s} + H\frac{\delta\phi}{\dot{\phi}}.$$

Fourier components of u satisfy

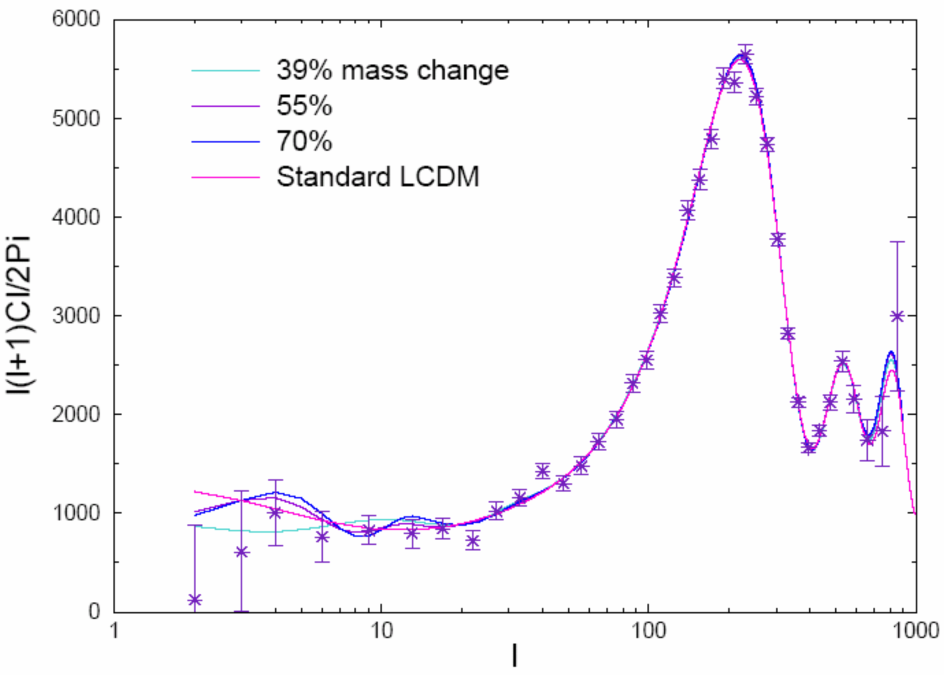
$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0, \quad \frac{z''}{z} = a^2\left(2H^2 + m^2 - \lambda\rho^2 - \frac{2\lambda\rho\dot{\rho}\phi}{\dot{\phi}}\right).$$

Spectrum is given by

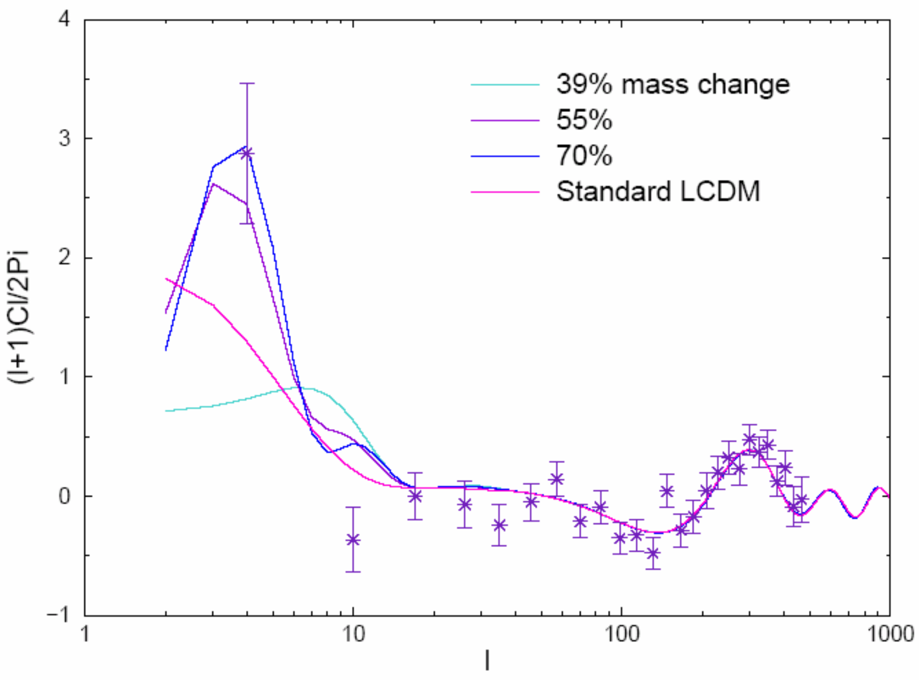
$${\mathcal{P}_\mathcal{R}}^{1/2} = \sqrt{\frac{k^3}{2\pi^2}}\,|\mathcal{R}_k| = \sqrt{\frac{k^3}{2\pi^2}}\left|\frac{u_k}{z}\right|.$$

Use WKB method (Martin & Schwarz 2003) to obtain $\boldsymbol{P_R}$ when slow-roll is violated …

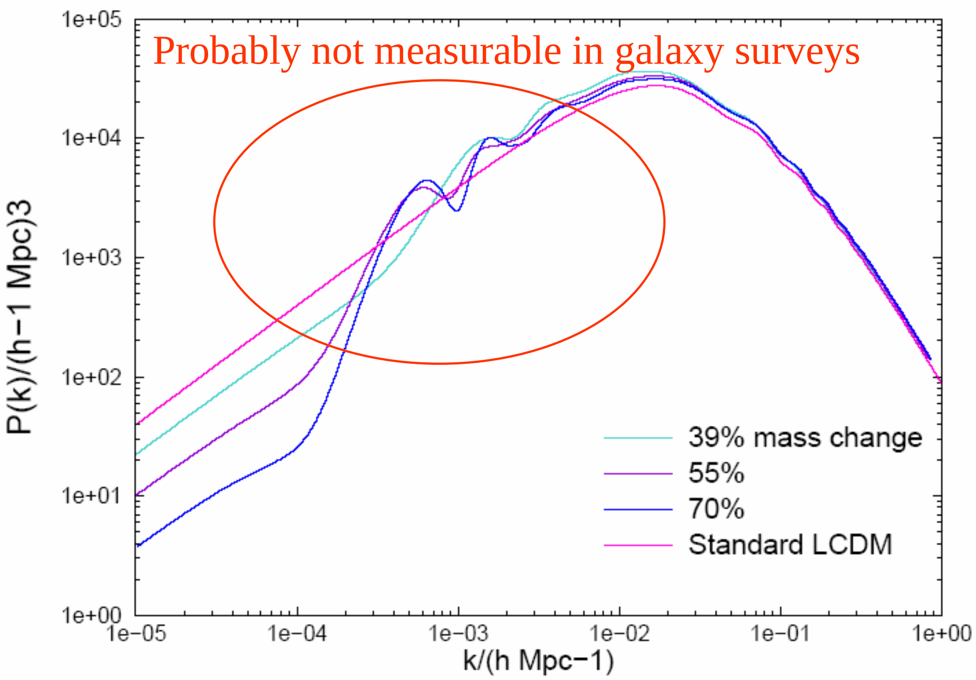
TT spectra of best fit models.



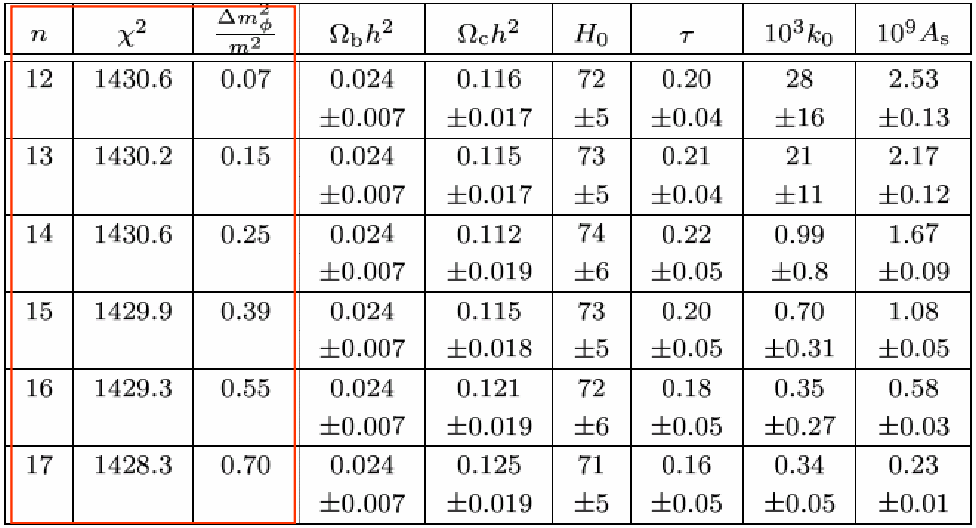
TE spectra of best fit models.



Matter power spectra of best fit models.


Probably not measurable in galaxy surveys

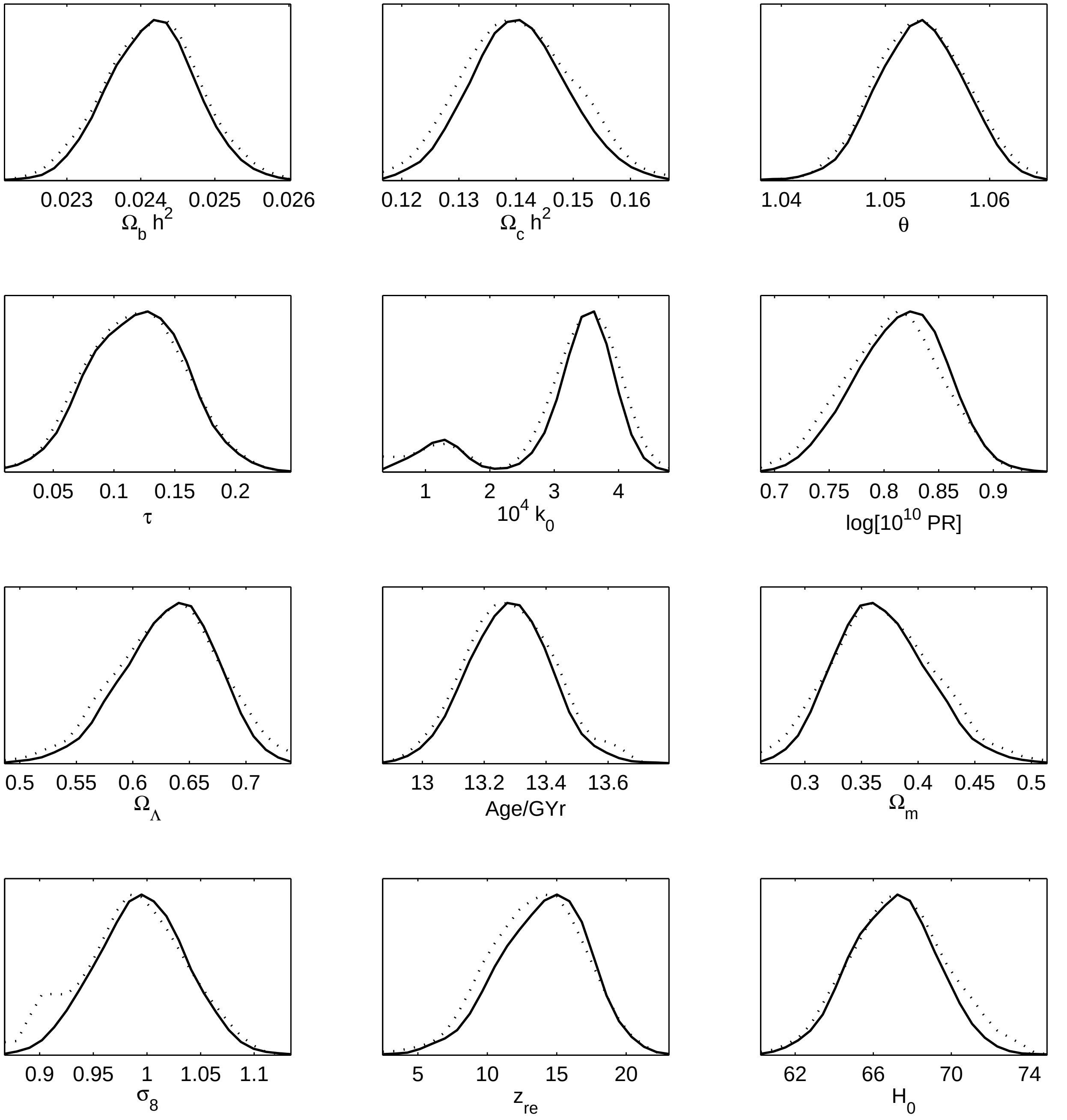
Fits all acceptable … cosmological parameters change little but now have probe of new physics!



n	χ^2	$\frac{\Delta m_\phi^2}{m^2}$	$\Omega_b h^2$	$\Omega_c h^2$	H_0	τ	$10^3 k_0$	$10^9 A_s$
12	1430.6	0.07	0.024 ±0.007	0.116 ±0.017	72 ±5	0.20 ±0.04	28 ±16	2.53 ±0.13
13	1430.2	0.15	0.024 ±0.007	0.115 ±0.017	73 ±5	0.21 ±0.04	21 ±11	2.17 ±0.12
14	1430.6	0.25	0.024 ±0.007	0.112 ±0.019	74 ±6	0.22 ±0.05	0.99 ±0.8	1.67 ±0.09
15	1429.9	0.39	0.024 ±0.007	0.115 ±0.018	73 ±5	0.20 ±0.05	0.70 ±0.31	1.08 ±0.05
16	1429.3	0.55	0.024 ±0.007	0.121 ±0.019	72 ±6	0.18 ±0.05	0.35 ±0.27	0.58 ±0.03
17	1428.3	0.70	0.024 ±0.007	0.125 ±0.019	71 ±5	0.16 ±0.05	0.34 ±0.05	0.23 ±0.01

Hunt, Morgan & Sarkar (2005)

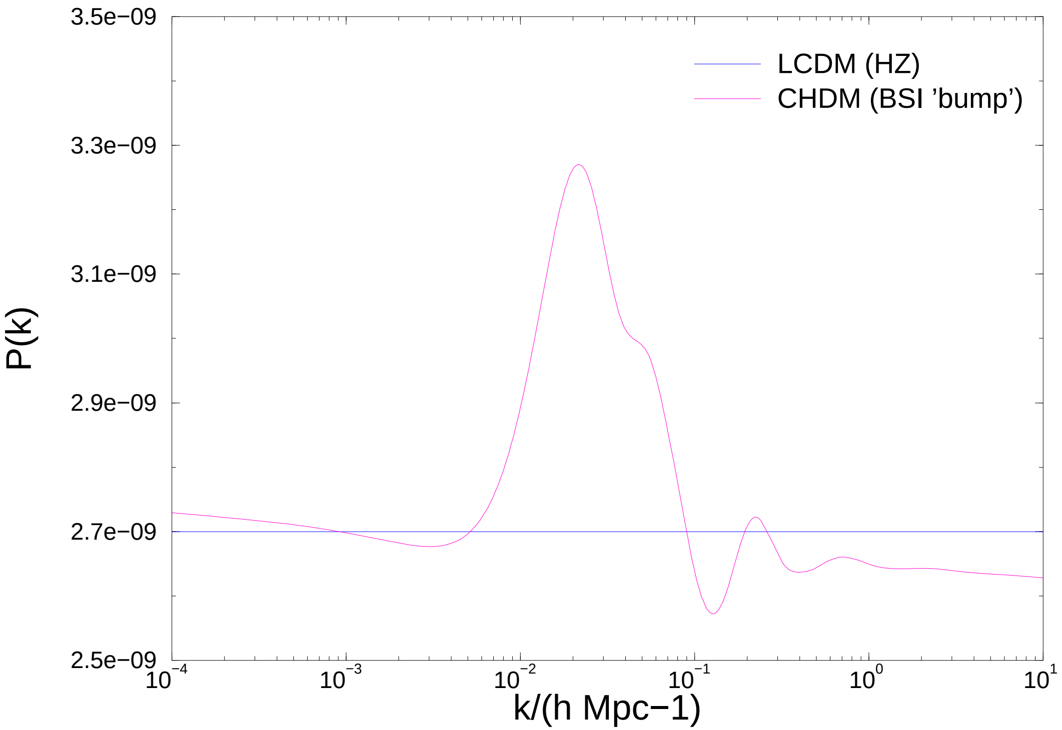
MCMC likelihood distributions for ΛCDM (BSI ‘step’)

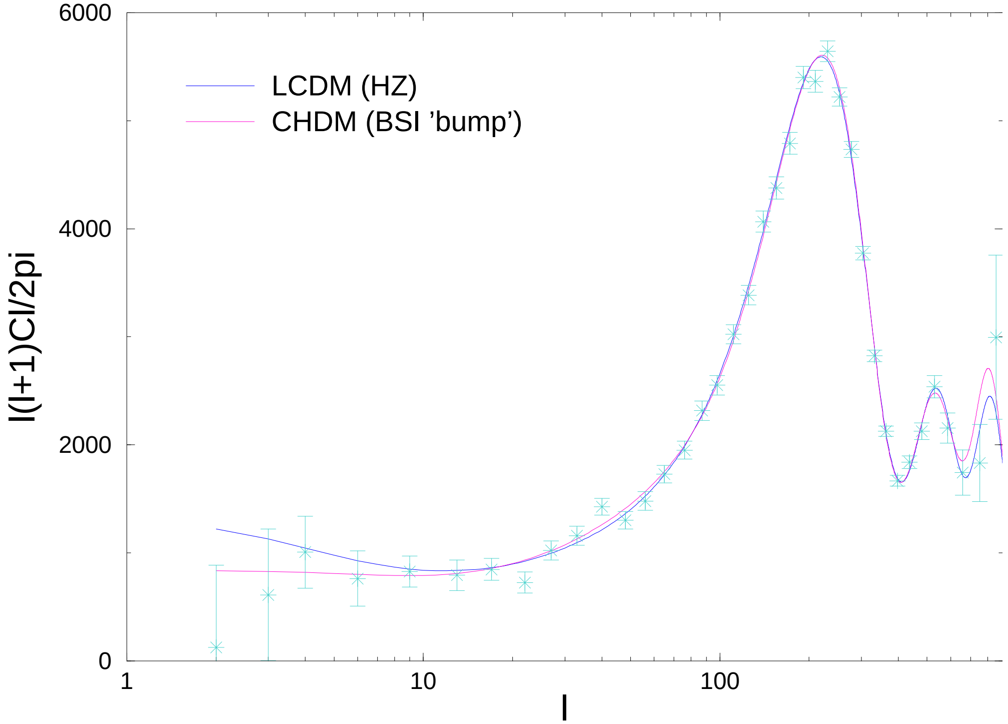


… similar to standard *Λ*CDM

Hunt, Morgan and Sarkar (2005)

But if there are *many* flat direction fields, then two phase transitions may occur in quick succession, creating a ‘bump’ in the primordial spectrum on



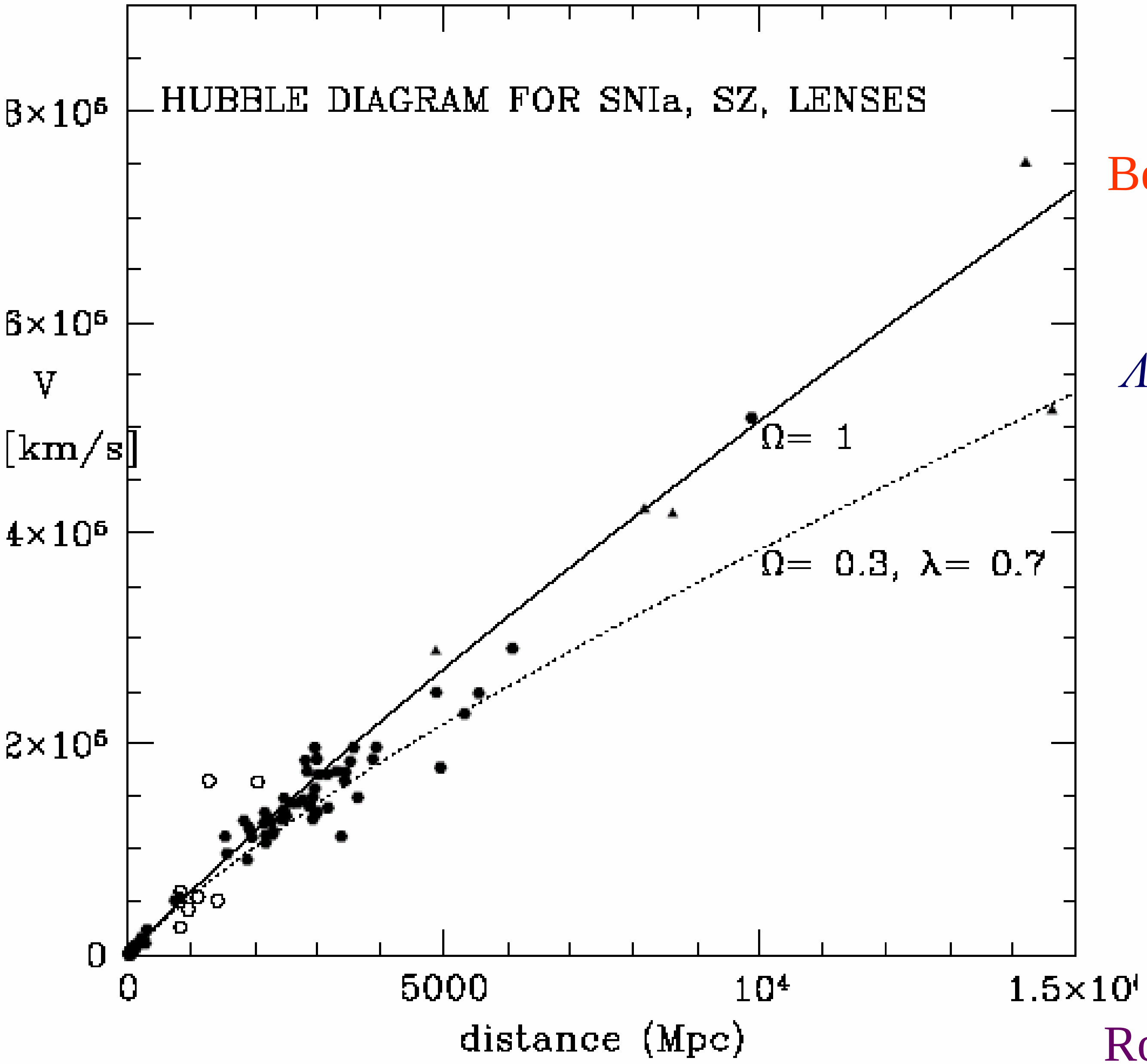


Such a spectrum fits the *WAMP* data with ***no need for dark energy*** ($\Omega_m = 1$) … requires only that $h \approx 0.5$

h ~ 0.5 is *inconsistent* with Hubble Key Project value (*h* = 0.72 ± 0.08)

but is in fact ***indicated*** by direct (and deeper) determinations:

SZ effect in clusters (*h* = 0.54 ± 0.03), gravitational lens time delays (*h* = 0.48 ± 0.03)



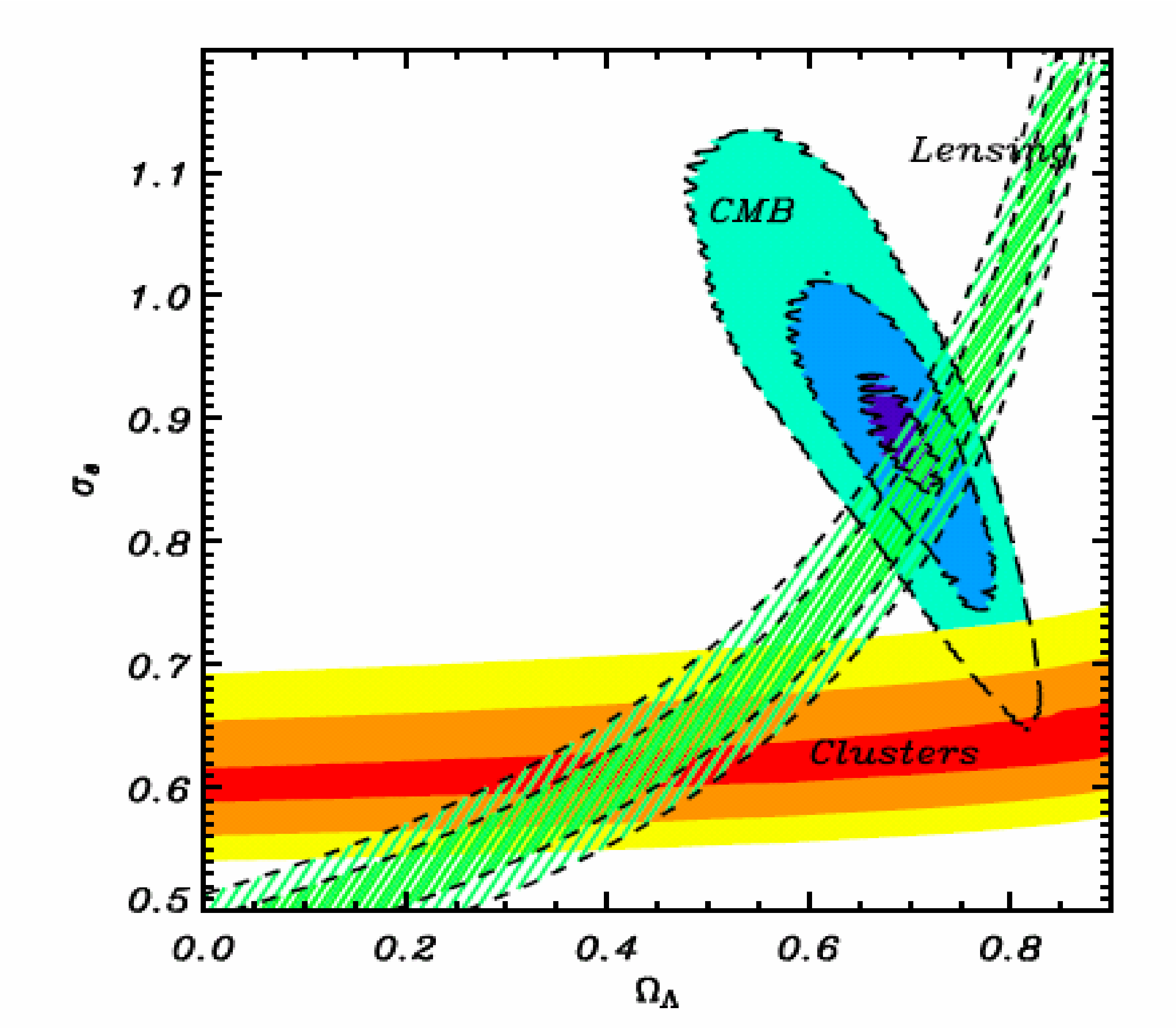
Best fit E-deS model
(*h* ~ 0.6)

*Λ*CDM model
(*h* ~ 0.7)

Rowan-Robinson (2002)

Discrepancy between amplitude of (dark) matter fluctuations deduced from CMB data and from galaxy clusters suggests there may be a small component of hot dark matter

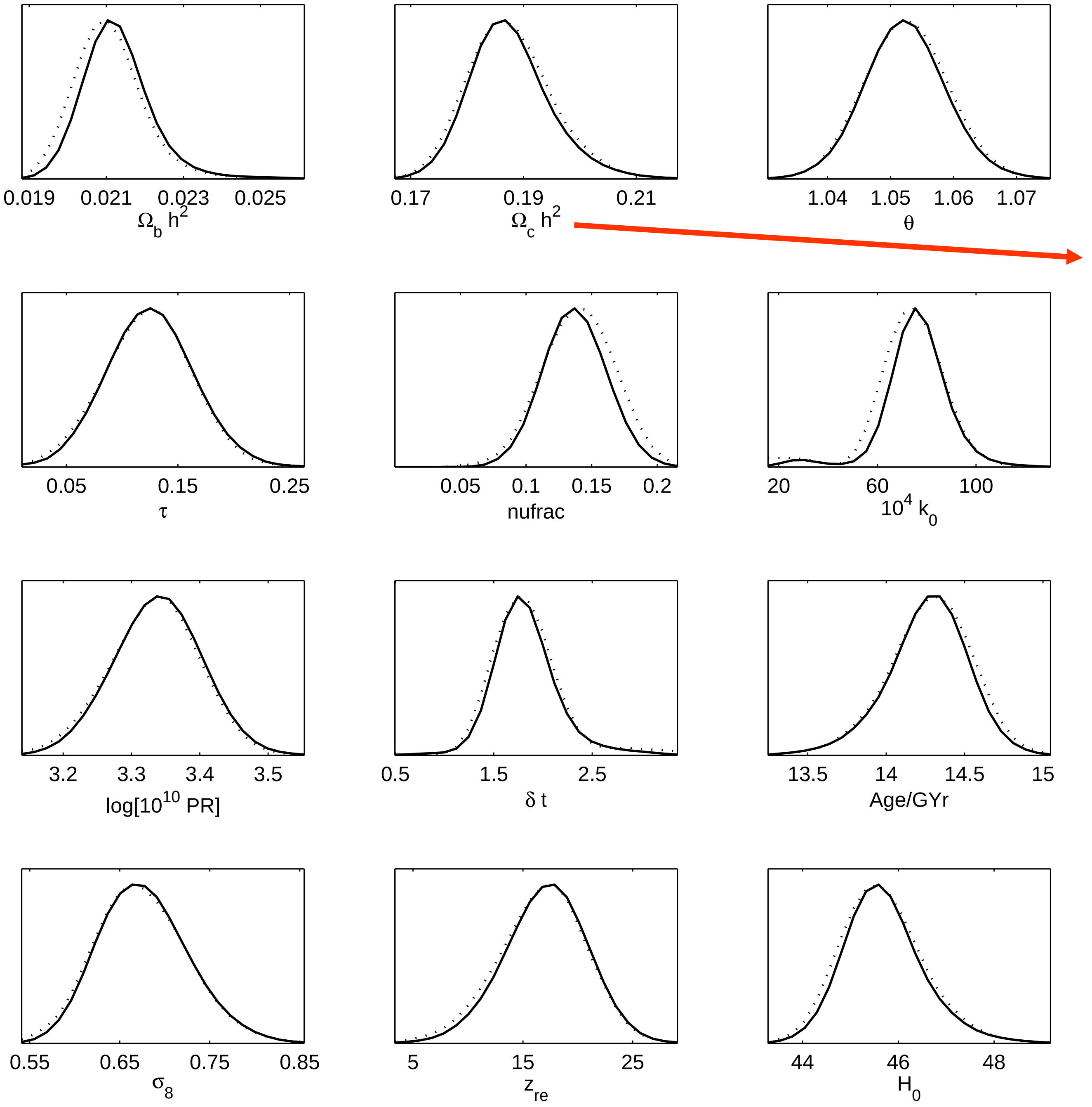
(Blanchard & Douspis 2005)



Including 3 neutrinos of mass ~1 eV (⇒ $\Omega_\nu \sim 0.14$) allows good match to large-scale structure

Fits give $\Omega_B h^2 \sim 0.021$ ⟶ BBN √ ⇒ baryon fraction in clusters predicted to be ~12% √

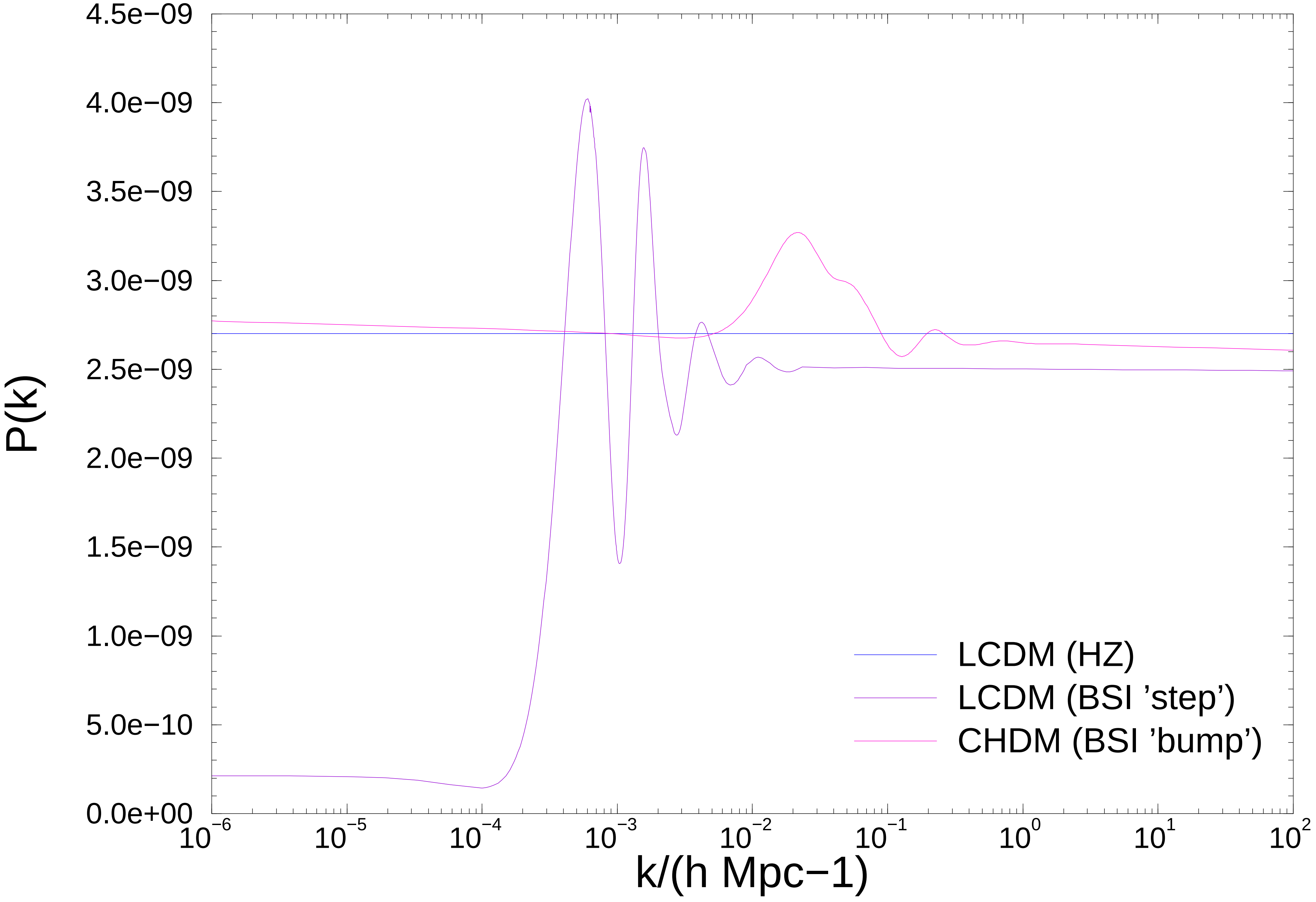
MCMC likelihood distributions for CHDM (BSI ‘bump’)



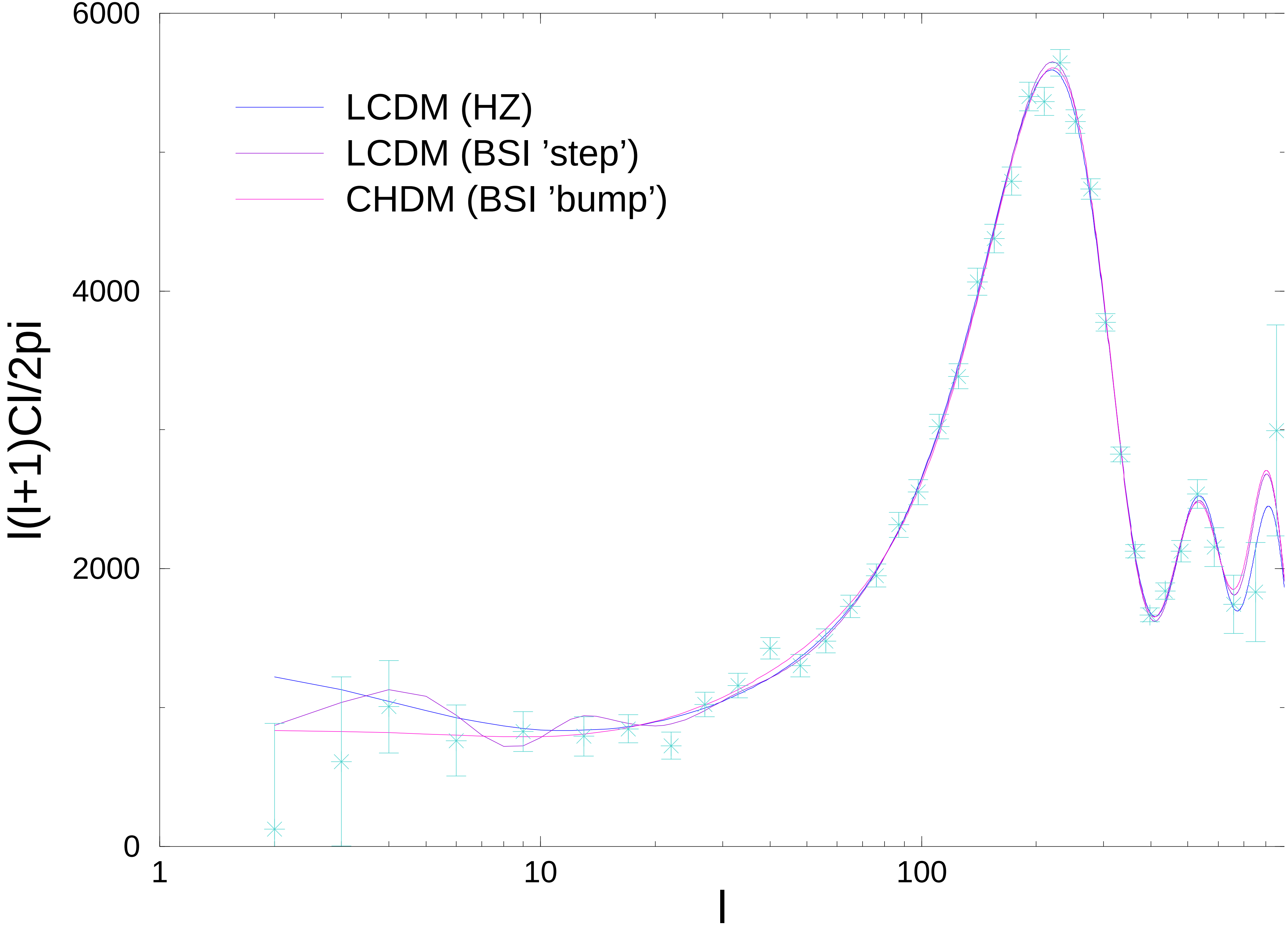
Note this is ~50% higher than the ‘WMAP value’ used for dark matter studies

Hunt, Morgan and Sarkar (2005)

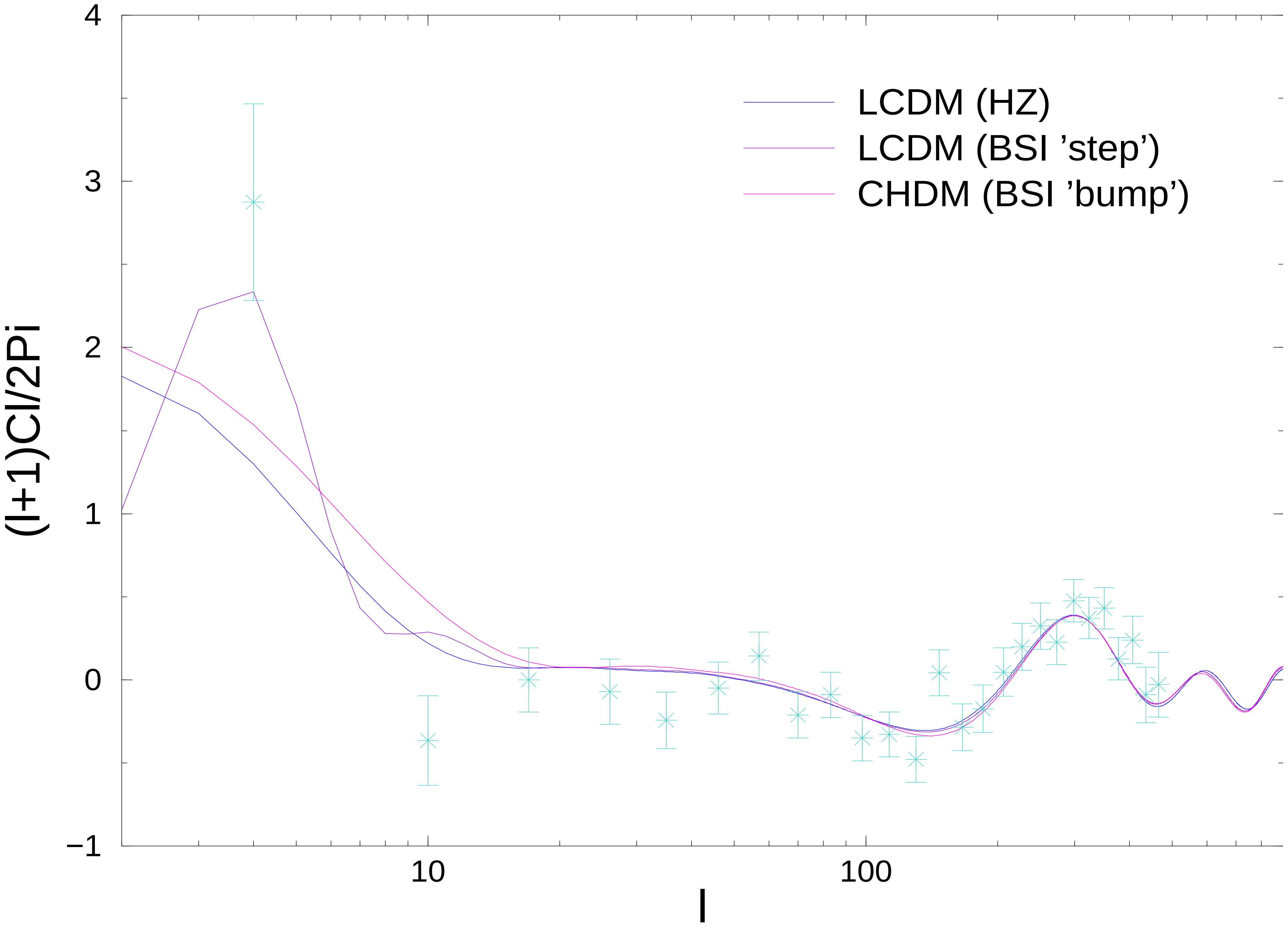
Some possibilities for the primordial spectrum



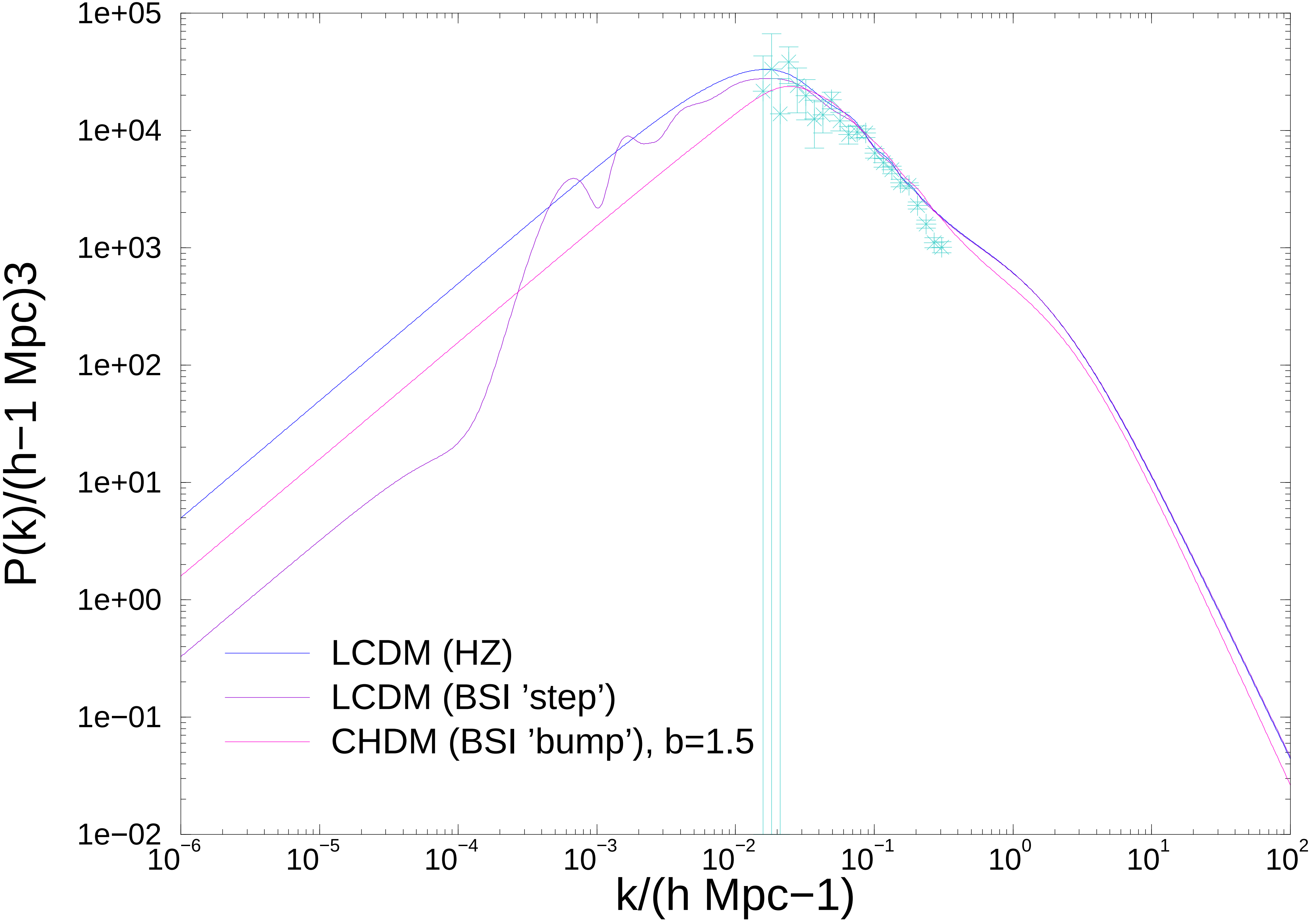
Fits to the *WMAP* TT power spectrum



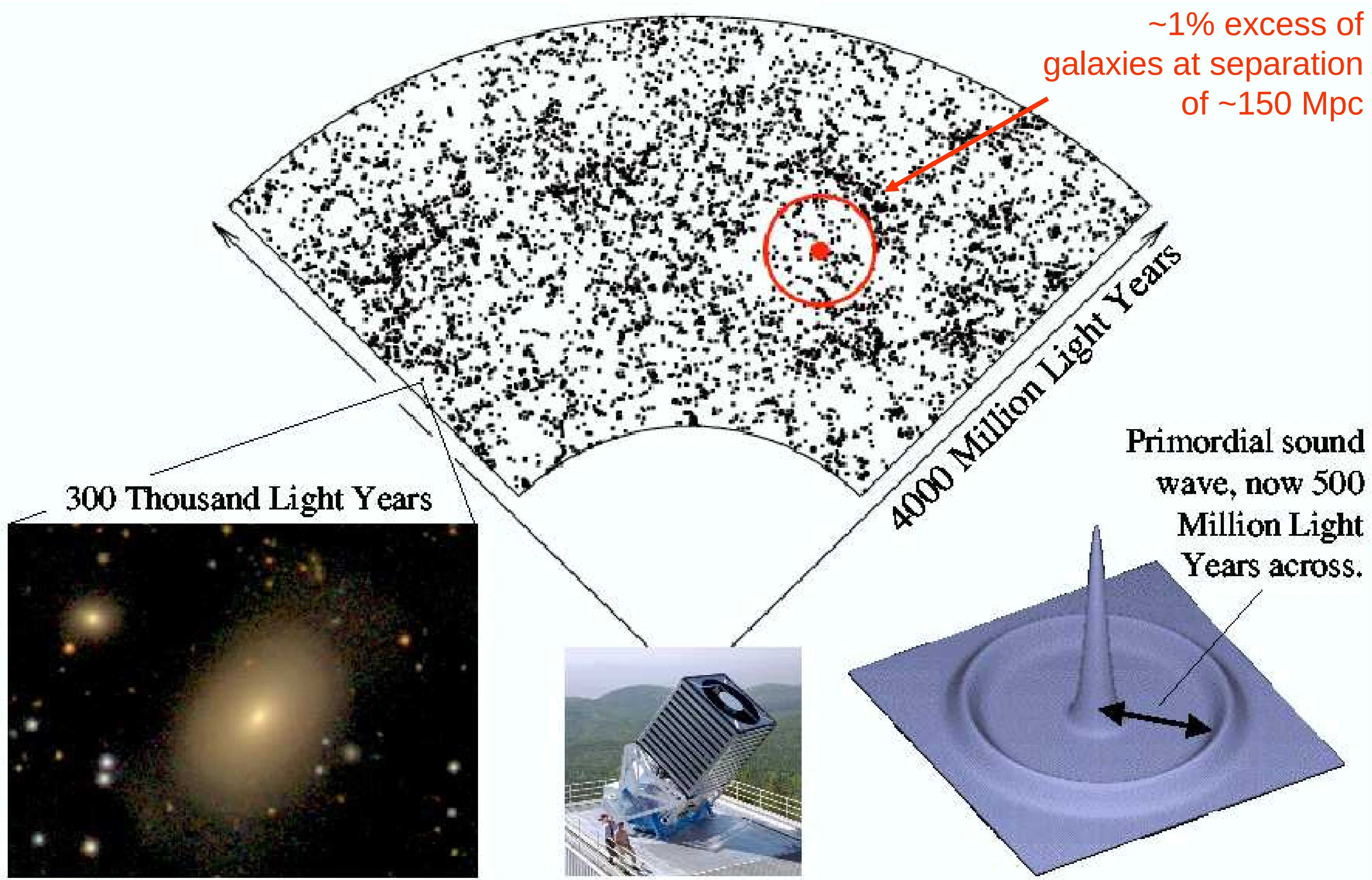
Fits to the *WMAP* TE power spectrum



Fits to the *SDSS* galaxy power spectrum



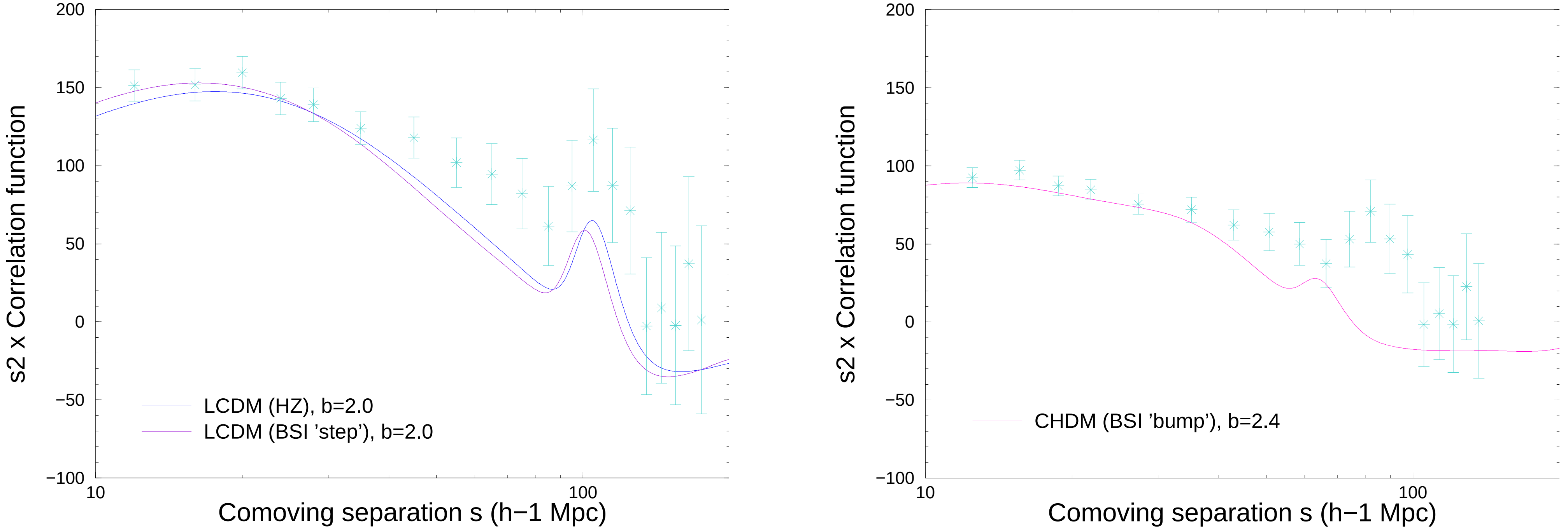
New Test: Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies



~1% excess of galaxies at separation of ~150 Mpc

In the model with no dark energy, the baryon bump is at the ~same *physical* scale but at a different location in observed (redshift) space

(NB: Ang. correlation fn. scales differently for *Λ*CDM and CHDM – Alcock-Paczynski effect)



We can match the angular size of the 1st acoustic peak at z ~ 1100 by taking h ~ 0.5, but we *cannot* then also match the angular size of the baryonic feature at z ~ 0.35

If confirmed (>5σ) this *can* rule out alternatives to the *Λ*CDM model

… as can (>5σ) detection of the late ISW effect-induced correlations with the CMB

Conclusions

WMAP has provided evidence that the CMB temperature fluctuations *were* generated by inflation (ie. on scales *super*-horizon at last scattering)

- However cannot simultaneously determine ***both*** the primordial spectrum ***and*** the cosmological parameters from CMB (and LSS) data

We do not know the physics of inflation hence not justified to ***assume*** that the generated density perturbation is scale-free (and then conclude that CMB & LSS data are consistent with the ΛCDM model!)

- Must resolve degeneracies ***experimentally***, using e.g. polarization data and ***independent*** measurements of cosmological parameters

The data provides intriguing hints for ***non-trivial inflationary dynamics*** … if the 'glitches' are confirmed, this may provide the first ***direct*** link between astronomical data and physics beyond the Standard Model