Physics Possibilities at INO

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Using Atmospheric Neutrinos as Source

- Study the (L/E) dependence of the Up/Down ratio, locate the first minimum of $P(\nu_\mu \rightarrow \nu_\mu)$ and determine $\Delta_{atm} = \Delta_{31}$ and $\sin^2 2\theta_{atm} = \sin^2 2\theta_{23}$ accurately.
- Search for the change induced in $P(\nu_\mu \rightarrow \nu_\mu)$ by matter effects and determine the sign of Δ_{31} (neutrino mass hierarchy).
- Search for the deviation of θ_{23} away from maximality.

- Search for possible CPT violation
- Study the flux of Ultra High Energy muons which will provide a check on Ultra High Energy Cosmic Rays.

(L/E) Dependence of Up/Down ratio

Atmospheric neutrinos are produced over a very wide range of energies and they enter detector after travelling a wide range of pathlengths.

In INO, we expect to have significant number of events in the energy range 2-10 GeV and pathlength range 2000 - 10000 Km (for upward going neutrinos).

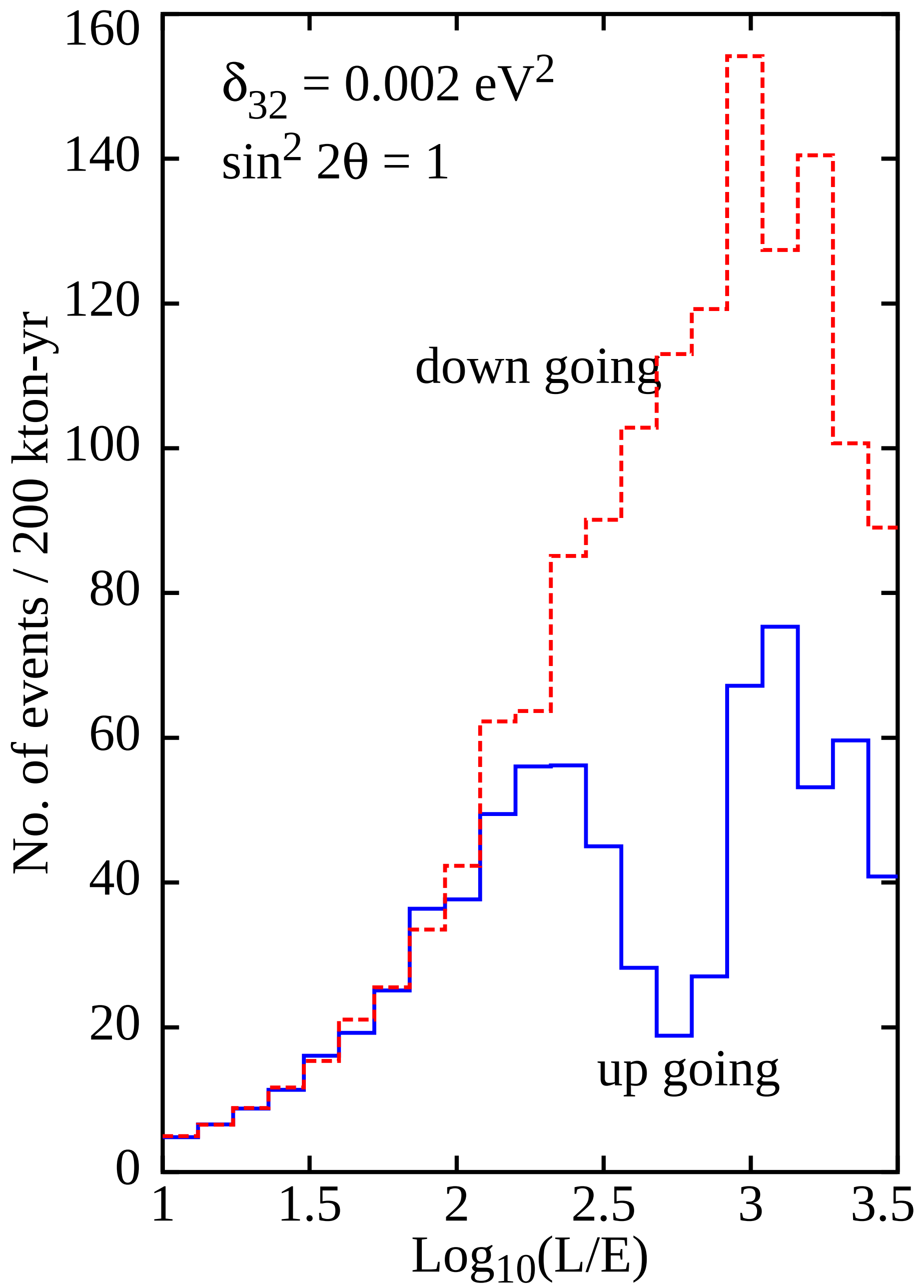
So the wide range in (L/E) available make atmospheric neutrinos a good source to study neutrino oscillations, provided the detector can detector can maeasure both L and E with reasonable accuracy.

An Iron calorimeter, which can have a good energy and directional resolution, is an ideal tool to study the (L/E) dependence of $P(\nu_\mu \to \nu_\mu)$ and locate the first minimum.

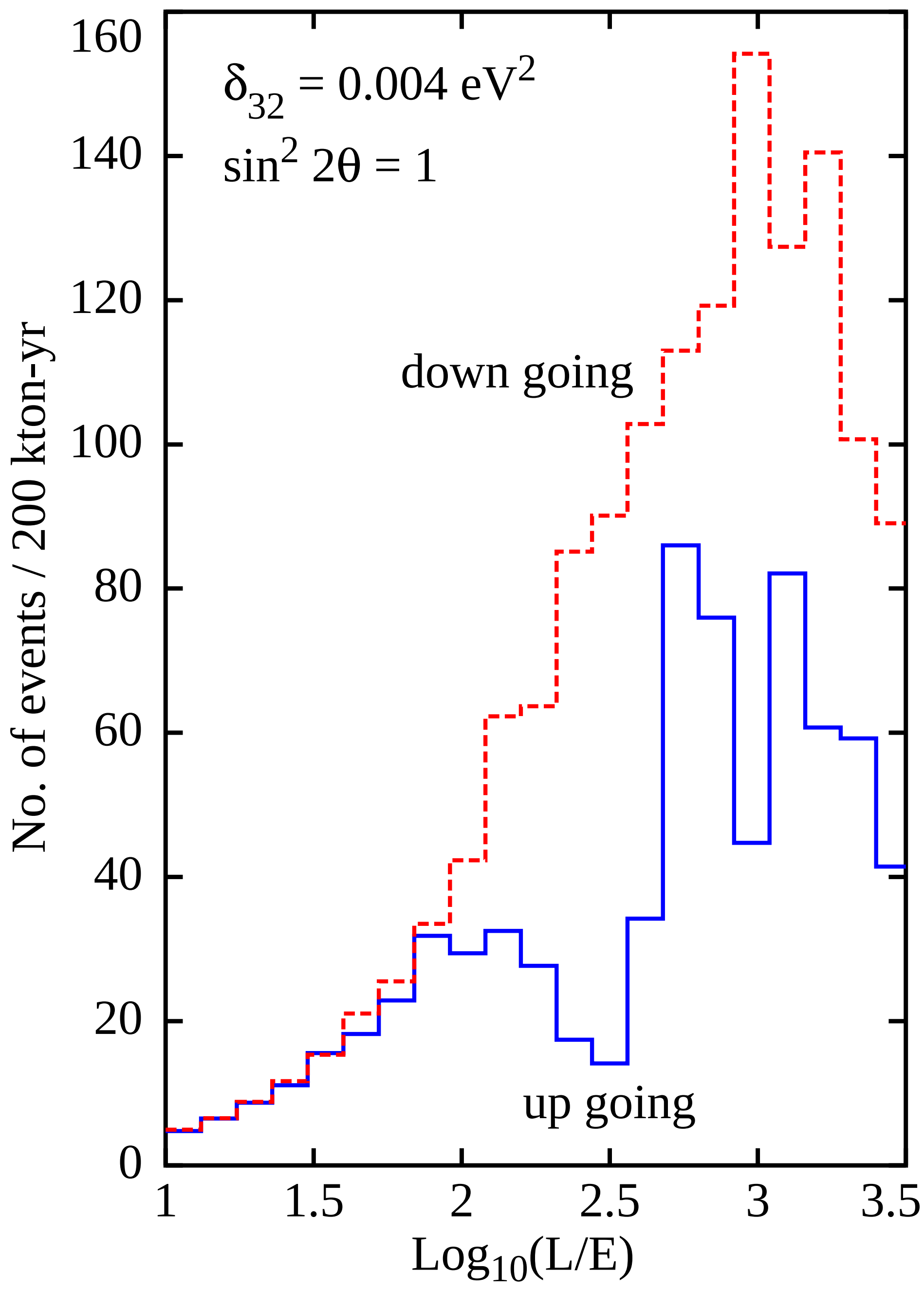
Position of the minimum determines Δ_{31} and the depth of the minimum determines $\sin^2 2\theta_{23}$.

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2(1.27\Delta_{31} L/E)$$

Muon charge identification, which requires magnetic field, is not necessary for the location of the minimum in $P(\nu_\mu \to \nu_\mu)$.



δ_{32} = 0.002 eV^2
$\sin^2 2\theta$ = 1
down going
up going
No. of events / 200 kton-yr
Log$_{10}$(L/E)
0
20
40
60
80
100
120
140
160
1
1.5
2
2.5
3
3.5



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No. of events / 200 kton-yr
Log$_{10}$(L/E)
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20
40
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100
120
140
160
1
1.5
2
2.5
3
3.5

In generating the above plots, it is assumed that (L/E) is measured with 50% accuracy.

For input values of $\Delta_{31} = 2\times 10^{-3}$ eV^2 and $\sin^2 2\theta_{23} = 1$, the returned values from the simulation, at 90% C.L., are

$$\begin{aligned} \Delta_{31} &= (2.0 \pm 0.3) \times 10^{-3}\ \mathrm{eV}^2 \\ \sin^2 2\theta_{23} &\geq 0.93 \end{aligned}$$

Importance of Matter Effects

Matter effects arise due to the passage of neutrinos through the dense matter and the consequent modification of the propagation of different flavours because of the different forward scattering amplitudes.

Matter effects provide the energy dependence which is needed to solve solar neutrino problem. And they establish that $\Delta_{sol} = \Delta_{21}$ is positive.

Establishing matter effects in atmospheric neutrino oscillations (or long baseline neutrino oscillations) is crucial to determining the sign of Δ_{31}.

Matter Effects in $P(\nu_\mu \to \nu_\mu)$

For algebraic simplicity, we set $\Delta_{21} = 0$. This leads to θ_{12} and δ_{CP} also dropping out of the expressions.

The figures and the final numbers presented here are all calculated using $\Delta_{21} = 8 \times 10^{-5}$ eV^2 and $\theta_{12} = 34°$. For simplicity, we set $\delta_{CP} = 0$. Non-zero values change the results by less than 5%.

In vacuum, we have, for non-zero θ_{13}

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23}(1 - \cos^2\theta_{13}\sin^2\theta_{23})\sin^2(1.27\Delta_{31}L/E)$$

Including the matter effects changes this to

$$P(\nu_\mu \to \nu_\mu)_{mat} = 1 - P^1_{\mu\mu} - P^2_{\mu\mu} - P^3_{\mu\mu},$$

where

$$\begin{aligned}
P^1_{\mu\mu} &= \sin^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2\left(0.64\frac{\Delta_{31}(1+A)-\Delta^m_{31}}{E}L\right) \\
P^1_{\mu\mu} &= \cos^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2\left(0.64\frac{\Delta_{31}(1+A)+\Delta^m_{31}}{E}L\right) \\
P^3_{\mu\mu} &= \sin^2 2\theta^m_{13}\sin^4\theta_{23}\sin^2(1.27\Delta^m_{31}L/E)
\end{aligned}$$

We search for values of L and E for which the change in in $P(\nu_\mu \to \nu_\mu)$ due to matter effects, is largest.

If (L/E) is such that one is in the vicinity of vacuum peak of $P(\nu_\mu \rightarrow \nu_\mu)$, i.e. $\Delta_{31}L/4E = \pi$ and $P(\nu_\mu \rightarrow \nu_\mu) = 1$.

In such a situation, $P^1_{\mu\mu}, P^2_{\mu\mu}$ and $P^3_{\mu\mu}$ are all non-zero because $\Delta^m_{31} \neq \Delta^{mat}_{31}$. Thus there is a possibility of large change induced by matter effects.

This change is further maximised if the energy also satisfies the resonance condition

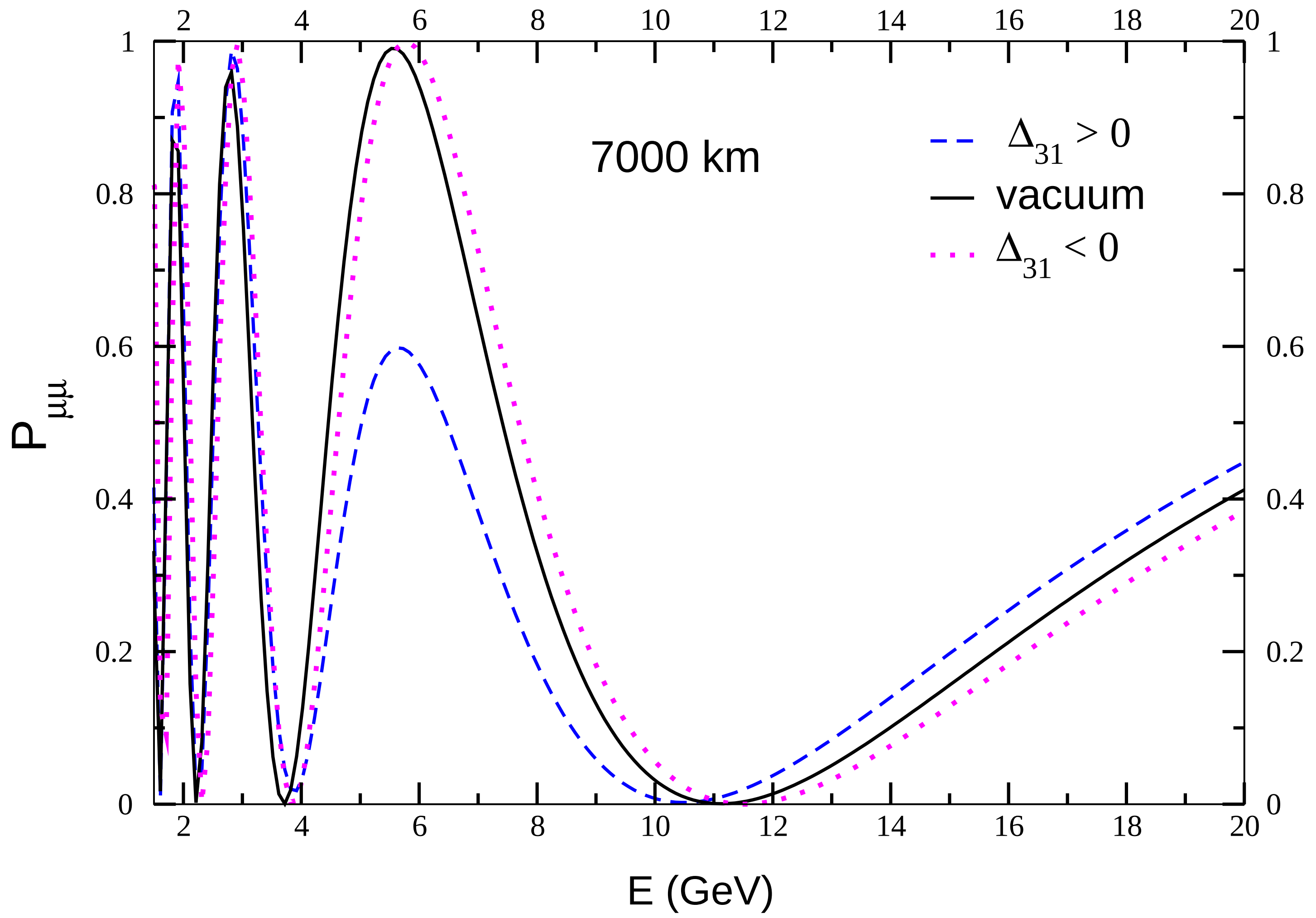
$$\begin{aligned} E = E_{res} &= \frac{\Delta_{31} \cos 2\theta_{13}}{2\sqrt{2} G_F n_e} \\ &= \frac{\Delta_{31}}{0.76 \rho 10^{-4}} \simeq 5 - 6 GeV \end{aligned}$$

So we conclude that the matter effects induce the largest change in $P(\nu_\mu \rightarrow \nu_\mu)$ if the energy is close to the resonance energy and pathlength is such that $\Delta_{31}L/4E_{res} \simeq \pi$.

Combining the expression for E_{res} with the above condition for phase, we get the condition for pathlength $L_{\mu\mu}^{max}$ w here the change induced by matter effects is maximum.

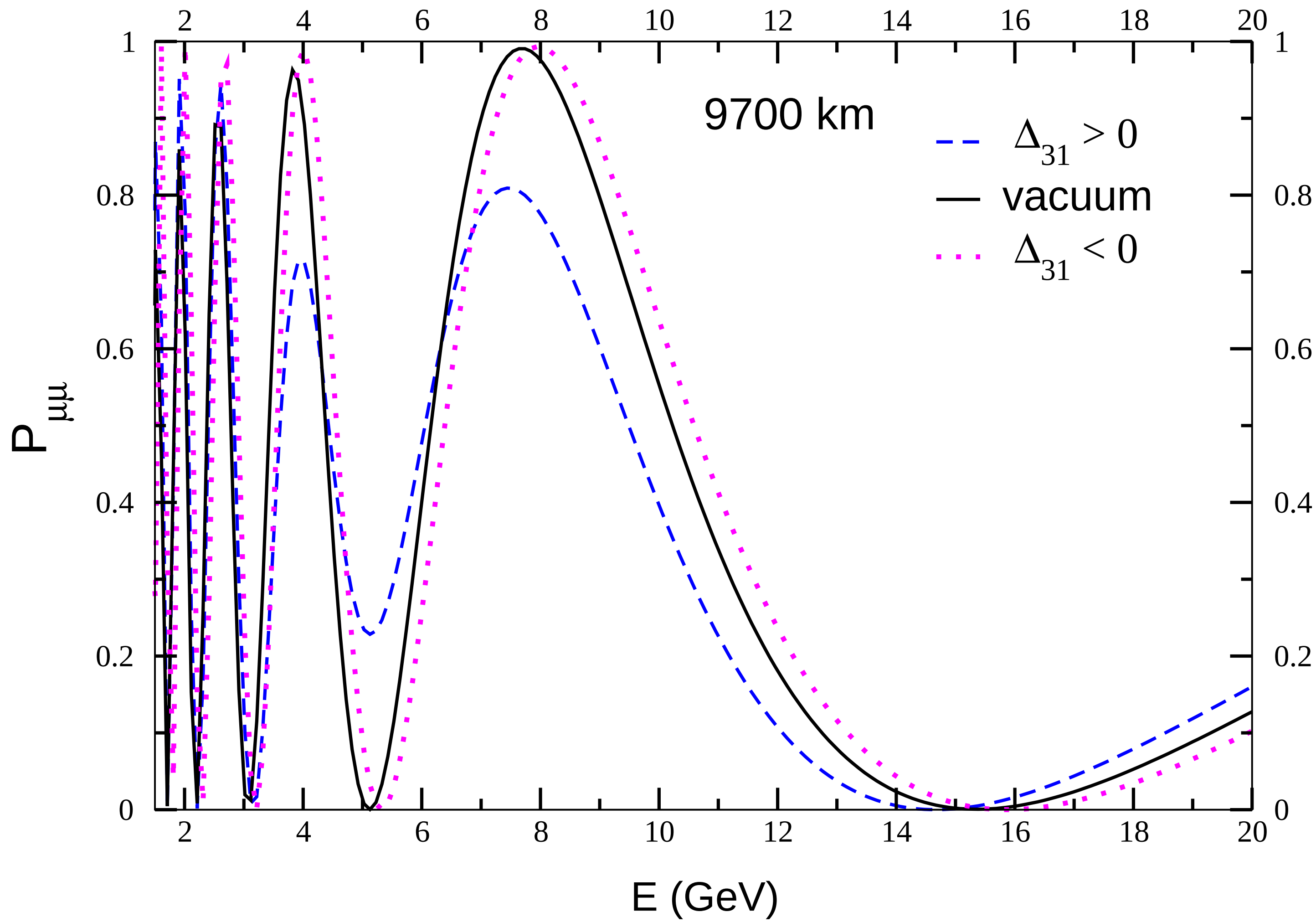
$$L_{\mu\mu}^{max} = (\pi/\rho) \times 10^4 (\cos 2\theta_{13})\ Km \simeq 7000\ Km.$$

This implies that maximum change in $P(\nu_\mu \rightarrow \nu_\mu)$ occurs at the same pathlength for all values of θ_{13}.



Matter effects lead to a large change in $P(\nu_\mu \to \nu_\mu)$

(from 1 to 0.6) for L = 7000 Km and in the energy range 5 - 10 GeV. In our event rate calculations, we consider the above energy range and the L range of 6000 - 9700 Km. We have checked that the large suppression in the energy range 5 - 10 GeV, persists for all these pathlengths.



If Δ_{31} is positive (normal hierarchy), matter effects suppress $P(\nu_\mu \rightarrow \nu_\mu)$ but the survival probability of anti-neutrinos is unaffected.

If Δ_{31} is negative (inverted hierarchy), matter effects suppress anti-neutrino survival probability but leave $P(\nu_\mu \rightarrow \nu_\mu)$ unaffected.

A magnetized calorimeter can measure muon and anti-muon event rates separately. Independent of the hierarchy, muons of one sign should be commensurate with vacuum oscillation expectations and muons of the other sign should have suppression relative to vacuum oscillation expectations.

Muon Event Rates Including Matter Effects

Below we present two tables with the μ^+ and μ^- event rates binned in L and in E. These event rates have been calculated for an exposure of **1000 Kt-yr**. The input values of neutrino paramaters are $\Delta_{31} = +0.002$ eV^2, $\sin^2\theta_{23} = 0.5$, $\sin^2 2\theta_{13} = 0.1$, $\Delta_{21} = 8 \times 10^{-5}$ eV^2 and $\theta_{12} = 34°$.

In calculating these numbers, a resolution in energy of 15% and a resolution in pathlength of 15% was assumed.

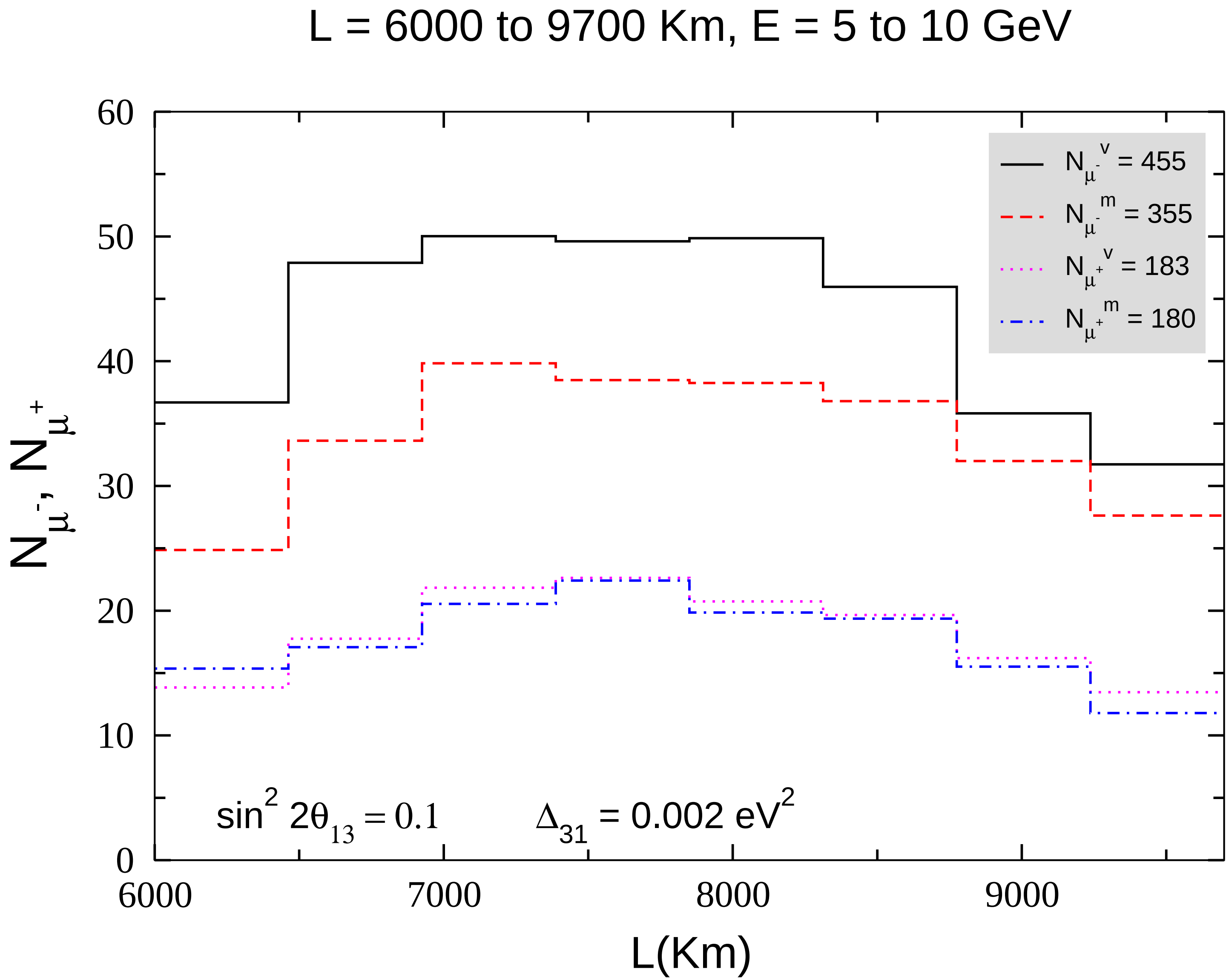
Tables are on pages 33 and 34.

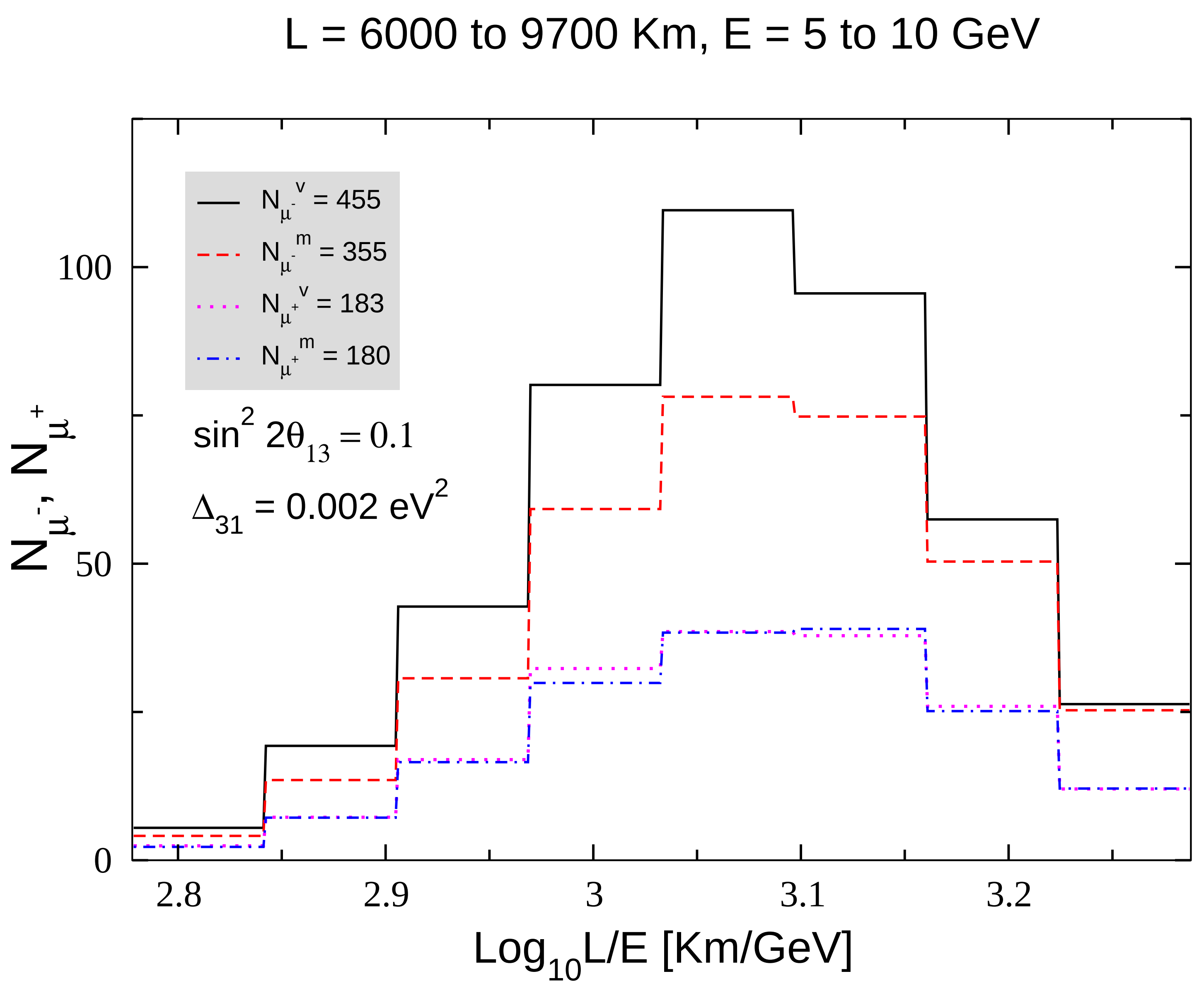
Some Notable points are

- The rates of μ^+, for both vacuum and matter oscillations are essentially the same, reflecting the fact that anti-neutrino survival probability is unaffected by matter effects if Δ_{31} is positive.
- The rates of μ^- show significant difference in energy range 5-10 GeV and pathlength range 6000 - 9700 Km (shown in bold).
- For the above selected range, integrated number of events in the case of vacuum oscillations is **455**

and in the case of matter oscillations is **355**, which is 5σ signal for matter effects.

- Integrating over the whole allowed range of E and L, dilutes the signal considerably (3690 vs 3540, 2.5 σ).


L = 6000 to 9700 Km, E = 5 to 10 GeV
$N_{\mu^-}^{v}$ = 455
$N_{\mu^-}^{m}$ = 355
$N_{\mu^+}^{v}$ = 183
$N_{\mu^+}^{m}$ = 180
$\sin^2 2\theta_{13} = 0.1$
Δ_{31} = 0.002 eV^2
N_{μ^-}, N_{μ^+}
L(Km)
0
10
20
30
40
50
60
6000
7000
8000
9000



L = 6000 to 9700 Km, E = 5 to 10 GeV
$N_{\mu^-}^{v}$ = 455
$N_{\mu^-}^{m}$ = 355
$N_{\mu^+}^{v}$ = 183
$N_{\mu^+}^{m}$ = 180
$\sin^2 2\theta_{13} = 0.1$
Δ_{31} = 0.002 eV^2
N_{μ^-}, N_{μ^+}
0
50
100
2.8
2.9
3
3.1
3.2
Log_{10}L/E [Km/GeV]

In calculating the sensitivity of the detector to determine the hierarchy, one has to vary the neutrino parameters over their allowed ranges.

To do this, we have used the following procedure. The number of μ^- events in the energy range 5-10 GeV and L range 6000-9700 Km is obtained and divided into eight bins in $\mathrm{Log_{10}(L/E)}$. This is our "experimental data" – $\mathrm{N^i_{ex} = N^i_{NH}}$.

We define

$$\chi^2 = \sum_{\mathrm{i}} \left(\frac{\mathrm{N^i_{ex} - N^i_{th}}}{\sigma^{\mathrm{i}}} \right)^2 ,$$

where σ is the uncertainty in $\mathrm{N_{ex}}$. In the calculation of the σ a systematic uncertainty of 10% is also added, which includes the cross-section error.

In order to test at what statistical significance the ”wrong hierarchy” can be disfavoured we calculate the theoretical expectation in each bin – $\mathrm{N^i_{th}} = \mathrm{N^i_{IH}}$ assuming ”wrong hierarchy”, in this case IH.

We allow the parameters θ_{13}, θ_{23} and Δ_{31} to vary within the following range:

(i) Δ_{31} is varied in the range $1.8 \times 10^{-3} - 2.34 \times 10^{-3}$

eV^2, which is the is the 3σ range which wil expectedly be allowed after MINOS, OPERA and ICARUS

(ii)$\sin^2\theta_{23}$ is varied in the range 0.4 - 0.62. This is the predicted range including the long baseline experiments MINOS+OPERA+ICARUS as well as JPARC-SK

(iii)$\sin^2\theta_{13}$ is varied from 0.0 to 0.05 (corresponding to the $\sin^2 2\theta_{13}$ range 0.0 to 0.19). The current 3σ bound on $\sin^2\theta_{13}$ is < 0.044 (or $\sin^2 2\theta_{13} < 0.17$), from

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Thus we marginalise our χ^2 over the three parameters θ_{13}, θ_{23} and Δ_{31} and determine the $\chi^2_{\rm min}$ for the "wrong hierarchy".

We rule out the "wrong hierarchy" at $p\sigma$ if this minimum χ^2 per degree of freedom is greater than p^2 for all allowed values of θ_{13}, θ_{23} and $|\Delta_{31}|$. Since the results depend on the choice of true parameter values, we present our results in Table for various choices of the input values of θ_{13}, i.e. $\theta_{13}^{\rm true}$, since this is the parameter which most significantly affects the sensitivity to matter effects and the mass hierarchy. The input values of θ_{23} and Δ_{31} are $\sin^2\theta_{23}^{\rm true} = 0.5$, $\Delta_{31}^{\rm true} = 0.002$ eV^2.

The above assumes that the cosmic ray fluxes will be well measured in ten years' time and the atmospheric neutrino fluxes can be predicted with much smaller errors than currently available. If the uncertainty in atmospheric neutrino flux prediction remains high, then one can use Up/Down event ratios to cancel the normalization uncertainty of the flux predictions.

Since the atmospheric neutrino fluxes depend only on the modulus of the cosine of the zenith angle, the muon event rates expected in case of no oscillations can be directly obtained from the experiment, by measuring the rates of downward going muons, binned according to the same energy and the same value of $|\cos\theta|$.

Thus the downward going neutrinos provide the necessary information on unoscillated fluxes. Figure shows the Up/Down muon event ratio versus L for L $=$ 6000-9700 Km, E $=$ 5-10 GeV in vacuum and in matter for both signs of Δ_{31}. For this range of energies and baselines, the downward event rates are calculated to be 713 for muons and 285 for anti-muons (same in matter and in vacuum). As mentioned earlier, the upward event rate for muons is 355 in matter and 455 in vacuum. Performing a similar χ^2 minimization for the Up/Down ratio as described earlier for the event rates gives the sensitivity to the hierarchy to be $\sim 3\sigma$ for $\sin^2 2\theta_{13}^{\rm true} = 0.1$ and 1000 kT yr exposure. Table lists the values of $\chi^2_{\rm min}$ and sensitivity using the ratio of Up/Down events for the E and L ranges discussed earlier, for three different values of $\theta_{13}^{\rm true}$.

References for Determination of Hierarchy at INO:

- R. Gandhi *et al*, Phys. Rev. Lett. **94** (2005) 051801.
- D. Indumathi and M. V. N. Murthy, Phys. Rev. **D71** (2005) 013001.
- R. Gandhi *et al*, hep-ph/0411252.

Other possibilities

- Measuring CPT violation due to difference in $P(\nu_\mu \to \nu_\mu)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_\mu)$. (Datta *et al*, Phys. Lett. B597 (2004) 356).
- Deviation of θ_{23} from maximality. Present measurements limit $|D = \sin^2\theta_{23} - 0.5| \leq 0.16$. INO can improve this to 0.08 (Choubey and Roy, hep-ph/0509197)
- Measuring the flux of ultra high energy muons (Gandhi and Panda, hep-ph/0512179).

With Neutrino Factory Beam as Source

- Very precise measurement of Δ_{31}, θ_{23} and θ_{13}. Or improving the limit on $\sin^2 2\theta_{13}$ to about 10^{-3}.
- Determination of sign of Δ_{31}
- Search for CP and CPT violation
- τ appearance and study of ν_τ interactions
- Search for Non-Standard Neutrino Interactions

E (GeV) ⇒ L (Km) ⇓	2 - 3 mat vac	3 - 5 mat vac	5 - 7 mat vac	7 - 10 mat vac	2 - 10 mat vac
2000 - 4000	172, 177	64, 71	19, 21	40, 40	295, 309
4000 - 6000	96, 103	160, 162	30, 33	5, 5	291, 303
6000 - 8000	124, 124	82, 84	78, 78	17, 21	301, 307
8000 - 9700	85, 85	82, 84	47, 44	38, 40	252, 253
9700 - 10500	44, 45	35, 37	10, 9	21, 23	110, 114
10500 - 12500	104, 101	94, 96	17, 19	47, 45	262, 261
2000 - 12500	625, 635	517, 534	201, 204	168, 174	1511, 1547

Table 1: Number of μ^+ events (with 15% smearing in E and L) in matter and in vacuum in restricted bins of E and L for $\Delta_{31} = 0.002$ eV^2, $\sin^2 2\theta_{13} = 0.1$, $\sin^2 \theta_{23} = 0.5$ and 1000 kT

E (GeV) ⇒ L (Km) ⇓	2 - 3 mat vac	3 - 5 mat vac	5 - 7 mat vac	7 - 10 mat vac	2 - 10 mat vac
2000 - 4000	398, 405	172, 162	54, 49	96, 92	720, 708
4000 - 6000	233, 242	333, 374	73, 78	17, 14	656, 708
6000 - 8000	292, 294	200, 197	**141, 190**	**37, 52**	670, 733
8000 - 9700	209, 205	224, 200	**103, 110**	**75, 103**	610, 618
9700 - 10500	110, 108	68, 89	42, 23	50, 56	270, 276
10500 - 12500	237, 240	205, 240	51, 49	122, 120	615, 649
2000 - 12500	1479, 1494	1202, 1262	463, 499	397, 437	3541, 3692

Table 2: Number of μ^- events (with 15% smearing in E and L) in matter and in vacuum in restricted bins of E and L for $\Delta_{31} = 0.002$ eV^2, $\sin^2 2\theta_{13} = 0.1$, $\sin^2\theta_{23} = 0.5$ and 1000 kT

$\sin^2 2\theta_{13}^{true}$	$\chi^2_{\rm min}$, $\sigma_{\rm NH-vac}$ (500 kT yr)	$\chi^2_{\rm min}$, $\sigma_{\rm NH-vac}$ (1000 kT yr)
0.02	1.7, 1.3σ	5.9, 2.4σ
0.05	4.2, 2.0σ	8.6, 2.9σ
0.1	8.9, 2.9σ	14.0, 3.7σ

Table 3: Values of $\chi^2_{\rm min}$ and corresponding values of σ sensitivity for event rates computed by taking $\rm N^i_{ex} = N^i_{NH}$, $\rm N^i_{th} = N^i_{vac}$ (or equivalently $\rm N^i_{th} = N^i_{IH}$) for 3 different values of $\sin^2 2\theta_{13}^{\rm true}$ for the E and L range E $=$ 5 - 10 GeV, L $=$ 6000 - 9700 Km (see text for details). $\Delta_{31}^{\rm true} = 0.002$ eV^2 and $\sin^2\theta_{23}^{\rm true} = 0.5$.

$\sin^2 2\theta_{13}^{true}$	$\chi^2_{\rm min}$, $\sigma_{\rm NH-vac}$ (500 kT yr)	$\chi^2_{\rm min}$, $\sigma_{\rm NH-vac}$ (1000 kT yr)
0.02	0.9, 0.9σ	4.0, 2.0σ
0.05	2.3, 1.5σ	5.0, 2.2σ
0.1	5.4, 2.3σ	8.5, 2.9σ

Table 4: Values of $\chi^2_{\rm min}$ and corresponding values of σ sensitivity for U/D ratios computed by taking $\rm N^i_{ex} = N^i_{NH}$, $\rm N^i_{th} = N^i_{vac}$ (or equivalently $\rm N^i_{th} = N^i_{IH}$) for 3 different values of $\sin^2 2\theta_{13}^{\rm true}$ for the E and L range E $=$ 5 - 10 GeV, L $=$ 6000 - 9700 Km (see text for details). $\Delta_{31}^{\rm true} = 0.002$ eV^2 and $\sin^2\theta_{23}^{\rm true} = 0.5$.